L'Hôpital's (Selection) Rule

An Empirical Bayes Application to French Hospitals

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Introduction

Motivation

If all you have is a hammer, everything looks like a nail.

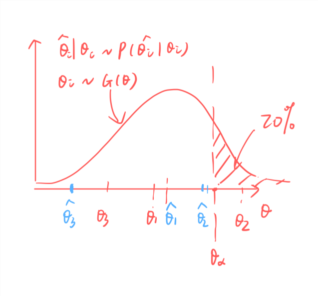
Nail: Measuring efficiency of individual units

- *Productivity/Efficiency*: Factories, Schools, Hospitals etc.
- *Ownership*: Public (Teaching, Ordinary) vs. Private (For profit, Non-profit).
- *Methodology*: Following Croiset and Gary-Bobo (2024), we use the *conditional input demand function*. The less input is needed to produce the same amount of output, the more efficient the hospital is. ▸ Reasons

Hammer: Invidious decision

- *League table mentality*: Ranking & Selection.(Gu and Koenker, 2023)
- *Noisy estimates*: e.g. estimated fixed effect $\hat{\theta}_i$. (Chetty et al., 2014; Kline et al., 2022)
- *Compound Decision/ Empirical Bayesian*: Compound decision framework (Robbins, 1956), (Non-parametric) Estimation of the prior distribution of θ_i. (Koenker and Mizera, 2014; Gu and Koenker, 2017)

Hammer on the nail: Questions



- Out of the top 20% hospitals in France in terms of labor employment efficiency, how many of them are public/private hospitals?
- What would be the selection outcome if I want to control the (expected) percentage of mistakes?
- Does different selection rule produce different results? And to what degree?

Recap

1. Estimate the efficiency with *factor demand function*.
   - LHS X: Labor input (number of full time equivalent nurses).
   - RHS Y: Hospital output (e.g., inpatient/outpatient stays, medical sessions).

$$\log(x)_{it,\text{nurses}} = \log(y)_{it,\text{output}}\beta + \theta_i + \varepsilon_{it} \quad \text{where} \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2)$$

2. Estimate the prior distribution G of θ_i with nonparametric maximum likelihood estimator (NPMLE).

3. Select the 20% most efficient hospitals utilizing the estimated G.

Data

Hospital Types

The Annual Statistics of Health Establishments (SAE)[1], 2013-2022 [2].

Year	Teaching	Normal Public	Private For Profit	Private Non Profit	Total
2013	198	1312	1305	1382	4197
2014	201	1274	1293	1349	4117
2015	211	1275	1297	1349	4132
2016	212	1266	1297	1313	4088
2017	211	1249	1297	1306	4063
2018	214	1247	1296	1288	4045
2019	214	1236	1287	1281	4018
2021	219	1222	1293	1264	3998
2022	220	1220	1296	1259	3995

- Teaching hospitals may be innately very different from others (training, research).

[1] La Statistique annuelle des établissements (SAE)
[2] 2020 missing due to Covid-19

Output

The list of 8 medical output.

Output	Teaching	Normal Public	Private For Profit	Private Non Profit	Total
STAC inpatient	25.17%	43.09%	23.64%	8.1%	100%
STAC outpatient	18.4%	19.46%	52.95%	9.18%	100%
Sessions	14.49%	21.96%	34.4%	29.16%	100%
Outpatient Consultations	36.8%	52.45%	0.23%	10.52%	100%
Emergency	21.4%	60.06%	13.37%	5.17%	100%
Follow-up care and Long-term care	7.6%	19.47%	37.95%	34.98%	100%
Home hospitalization	13%	17.38%	12.4%	57.22%	100%
Psychiatry stays	6.53%	62.26%	12.93%	18.28%	100%

- Hospitals differ not only in efficiency but also in the mix of services they provide.

Panel Data Estimation

Literature [3]

1. Within group estimator: Feedback effect
2. First difference GMM (Anderson and Hsiao, 1982; Arellano and Bond, 1991): Weak instruments
3. System GMM (Arellano and Bover, 1995; Blundell and Bond, 1998): Increased power of Sargan overidentification test to reject the null Blundell and Bond (2000)

[3] or M2ETE course review...?

Panel data estimation

Assume that $\mathbb{E}[\varepsilon_{it}|\theta_i, x_{i1}, \dots, x_{i,t-1}] = 0$.

First Difference GMM: use lagged level as instruments for current difference

$$\mathbb{E}[x_{i,t-2}(\Delta y_{it} - \beta \Delta x_{it})]$$

System GMM: use lagged difference as instruments for current levels

$$\mathbb{E}[\Delta x_{i,t-1}(y_{it} - \beta x_{it})] \quad \text{if} \quad \mathbb{E}\left[\Delta x_{i,t-1}(\theta_i + \varepsilon_{i,t})\right] = 0$$

Results

Dependent Variable:	Nurses			
	Within Group	First Difference	FD GMM	SYS GMM
Model:	(1)	(2)	(3)	(4)
Variables				
STAC inpatient	0.10***	0.07***	0.13***	0.48***
	(0.00)	(0.01)	(0.03)	(0.02)
STAC outpatient	0.02***	0.01***	0.02	0.05*
	(0.00)	(0.00)	(0.01)	(0.02)
Medical sessions	0.02***	0.02***	0.02***	0.06***
	(0.00)	(0.00)	(0.00)	(0.01)
External consultations	0.00	0.00	0.01	0.07***
	(0.00)	(0.00)	(0.01)	(0.01)
Emergency	0.01***	0.01	0.05	−0.04**
	(0.00)	(0.00)	(0.04)	(0.01)
Long-term & follow-up	0.01***	0.01***	−0.06**	0.07***
	(0.00)	(0.00)	(0.02)	(0.01)
Home care	0.01***	0.02**	−0.00	0.01
	(0.00)	(0.01)	(0.02)	(0.02)
Psychiatric care	0.02***	0.01	0.01	0.07**
	(0.00)	(0.01)	(0.02)	(0.02)
Fit statistics				
n	1690	1690	1690	1690
T	9	9	9	9
Sargan Test: chisq			128.32	360.97
Sargan Test: df			48.00	104.00

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Compound decision and Empirical Bayes

Compound Decision Framework

Observe:

$$\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \ldots, \hat{\theta}_n)$$
$$\text{where} \quad \hat{\theta}_i | \theta_i \sim P_{\theta_i}$$

Decision:

$$\delta(\hat{\boldsymbol{\theta}}) = (\delta_1(\hat{\boldsymbol{\theta}}), \ldots, \delta_n(\hat{\boldsymbol{\theta}}))$$

Loss:

$$L_n(\theta, \delta(\hat{\boldsymbol{\theta}})) = \sum_{i=1}^{n} L(\theta_i, \delta_i(\hat{\theta})).$$

Compound risk

Risk (Expectation of loss):

$$\begin{aligned} R_n(\theta, \delta(\hat{\boldsymbol{\theta}})) &= \mathbb{E}[L_n(\theta, \delta(\hat{\boldsymbol{\theta}}))] \\ &= \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_{\theta_i|\hat{\theta}_i}[L(\theta_i, \delta_i(\hat{\boldsymbol{\theta}}))] \quad \text{Separable decision rule } \delta \\ &= \frac{1}{n}\sum_{i=1}^{n} \int L(\theta_i, \delta_i(\hat{\theta}_i))dP(\hat{\theta}_i|\theta_i) \\ &= \int\int L(\theta_i, \delta(\hat{\theta}_i))dP(\hat{\theta}_i|\theta_i)dG_n(\theta) \end{aligned}$$

where $G_n(\theta)$ is the empirical distribution[4] of a set of θ_i drawn from the true distribution $\theta \sim G$.

[4] $E_{G_n}(f(x)) = 1/n\sum_i f(x_i)$

From compound decision to empirical Bayes

- Bayes risk: Replace G_n by the distribution G (though both of which are unknown...)

$$\int \int L(\theta_i, \delta(\hat{\theta}_i)) dP(\hat{\theta}_i | \theta_i) dG(\theta)$$

- Empirical Bayes:Empirically estimate from the data $\hat{\boldsymbol{\theta}}$ the G (or G_n) to construct the optimal Bayes decision rule $\delta_G(\hat{\theta})$ (or $\delta_{G_n}(\hat{\theta})$).

Literature

The parameter θ_i follows a certain distribution G. We only observe the noisy estimate of it $\hat{\theta}_i \sim P(\hat{\theta}_i|\theta_i)$ whose type is known. In order to have an estimate of $\hat{G}$

- Naive plug in estimator based on $\hat{\theta}$: larger tail
- Bias corrected naive plug in estimator based on $\hat{\theta}$ (Jochmans and Weidner, 2024).
- Nonparametrically estimate the marginal distribution $P(\hat{\theta}_i)$
  - Deconvolution: From $P(\hat{\theta}_i|\theta_i)$ and $P(\hat{\theta}_i)$, recover $G(\theta_i)$.
  - Shrink the noisy $\hat{\theta}$ following Robbins (1956). Naive plug in estimator based on the shrinked $\hat{\theta}$.
- Nonparametric estimation of G as a convex problem (Kiefer and Wolfowitz, 1956; Koenker and Mizera, 2014).

Estimate G

Kiefer and Wolfowitz (1956) established the nonparametric maximum likelihood estimator (NPMLE)

$$\hat{G} = \operatorname*{arg\,min}_{G \in \mathcal{G}} \left\{ -\sum_{i=1}^{n} \log g(\hat{\theta}_i) \middle| g(\hat{\theta}_i) = \int P(\hat{\theta}_i|\theta) dG(\theta) \right\}$$

where

1. $P(\hat{\theta}_i|\theta_i)$ is the probability density function of $\hat{\theta}_i$ conditional on the *true* parameter θ_i
2. $g(\hat{\theta}_i)$ is the marginal PDF of $\hat{\theta}_i$.

Estimate G

Convex optimization

This is an infinite-dimensional convex optimization problem with a strictly convex objective subject to linear constraints.

$$\min_{f=dG} \left\{ -\sum_i \log g(y_i) \middle| g(y_i) = T(f),\ K(f) = 1,\ \forall i \right\}$$

where $T(f) = \int \mathbb{P}(y_i|\theta) f d\theta$ and $K(f) = \int f d\theta$.

Consistency is proven by Kiefer and Wolfowitz (1956). Efficient computation method introduced by Koenker and Mizera (2014). Implemented with ***MOSEK*** created by Andersen and Andersen (2010).

The Selection Task

- Capacity: Select the bottom 20% (the smaller the θ_i, the more efficient) of the *true* θ_i. Since we assume that $\theta_i \sim G$, those i whose $\theta_i < G^{-1}(0.2)$

- False discovery rate: Control the overall false discovery rate at 20%,

$$\frac{\mathbb{E}_G\left[1\left\{\theta_i > \theta_\alpha, \delta_i = 1\right\}\right]}{\mathbb{E}_G\left[\delta_i\right]} \leq \gamma$$

1. Nominator: Selected but whose *true* value $> G^{-1}(0.2)$.
2. Denominator: Selected.

Problem Formulation

The loss function is

$$L(\delta, \theta) = \sum h_i(1 - \delta_i) + \tau_1 \left(\sum (1 - h_i)\delta_i - \gamma\delta_i \right) + \tau_2 \left(\sum \delta_i - \alpha n \right)$$

where $h_i = 1\{\theta_i < \theta_\alpha = G^-1(\alpha)\}$.

1. h_i is an indicator of whether the *true* value belong to the set.
2. δ_i is an indicator of whether i is being selected.

Problem Formulation

Therefore, the problem is to find δ such that

$$\begin{aligned}
\min_{\delta} \quad & \mathbb{E}_G \mathbb{E}_{\theta|\hat{\theta}} [L(\delta, \theta)] \\
=& \mathbb{E}_G \sum \mathbb{E}(h_i)(1-\delta_i) + \tau_1 \left(\sum (1 - \mathbb{E}(h_i))\delta_i - \gamma\delta_i \right) \\
& + \tau_2 \left(\sum \delta_i - \alpha n \right) \\
=& \mathbb{E}_G \sum v_\alpha(\hat{\theta})(1-\delta_i) + \tau_1 \left(\sum (1 - v_\alpha(\hat{\theta}))\delta_i - \gamma\delta_i \right) + \tau_2 \left(\sum \delta_i - \alpha n \right)
\end{aligned}$$

where $v_\alpha(\hat{\theta}) = \mathbb{P}(\theta < \theta_\alpha | \hat{\theta})$ is the posterior tail probability.

Derive tail probability v_α

Pick hospital i whose *true* inefficiency value is θ_i, which we don't observe. We only observe a sequence of Y_{it} where

$$Y_{it} = \theta_i + \varepsilon_{it} \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2) \quad (\theta_i, \sigma_i^2) \sim G$$

Neither θ_i nor σ_i^2 is known. But the sufficient statistics are

$$Y_i = \frac{1}{T_i}\sum_{t=1}^{T_i} Y_{it} \quad \text{where} \quad Y_i|\theta_i, \sigma_i^2 \sim \mathcal{N}(\theta_i, \sigma_i^2/T_i)$$

$$S_i = \frac{1}{T_i - 1}\sum_{t=1}^{T_i}(Y_{it} - Y_i)^2 \quad \text{where} \quad S_i|\sigma_i^2 \sim \Gamma(r_i = (T_i - 1)/2, 2\sigma_i^2/(T_i - 1))$$

In our factor demand function specification, we have $Y_{it} = \log(x_{it}) - \beta \log(y_{it})$. ▸ Appendix

TP and Constraints

Given the two sufficient statistics, the posterior tail probability is

$$\begin{aligned} v_\alpha(\hat{\theta}_i) &= v_\alpha(Y_i, S_i) \\ &= P(\theta_i < \theta_\alpha | Y_i, S_i) \\ &= \frac{\int_{-\infty}^{\theta_\alpha} \Gamma(s_i|r_i, \sigma_i^2) f(y_i|\theta_i, \sigma_i^2) dG(\theta_i, \sigma_i^2)}{\int_{-\infty}^{\infty} \Gamma(s_i|r_i, \sigma_i^2) f(y_i|\theta_i, \sigma_i^2) dG(\theta_i, \sigma_i^2)} \end{aligned}$$

We want to find a cutoff λ such that both constraints are satisfied [5]:

- Capacity: $\int \int P(v_\alpha(Y_i, S_i) > \lambda) dG(\theta_i, \sigma_i^2) \leq \alpha$
- FDR: $\int \int \frac{E[1\{v_\alpha(Y_i,S_i)>\lambda\}(1-v_\alpha(Y_i,S_i))]}{E[1\{v_\alpha(Y_i,S_i)>\lambda\}]} dG(\theta_i, \sigma_i^2) \leq \gamma$

[5] Relaxed discrete optimization problem, following (Basu et al., 2018)

Recap

1. We have a $N \times T$ panel. Y_{it} is an observation of hospital i's inefficiency term θ_i. Say $Y_i | \theta_i, \sigma_i \sim \mathcal{N}(\theta_i, \sigma_i^2)$.
2. Given a set of Y_i, perform NPMLE to get an estimate of $G(\theta, \sigma^2)$.
3. Given the estimated prior G, derive the explicit form of posterior tail probability $v_\alpha(Y_i, S_i)$ and the two constraints.
4. Solve the selection problem and find the optimal δ^*

$$\min_{\delta} \mathbb{E}_G \sum v_\alpha(\hat{\theta})(1 - \delta_i) + \tau_1 \left(\sum (1 - v_\alpha(\hat{\theta}))\delta_i - \gamma \delta_i \right) + \tau_2 \left(\sum \delta_i - \alpha n \right)$$

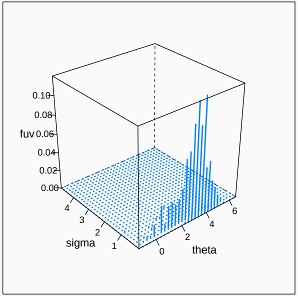
5. The decision rule is defined by the cutoff λ^*

$$\delta^*(y_i, s_i) = 1\{v_\alpha(y_i, s_i) > \lambda^*\}$$

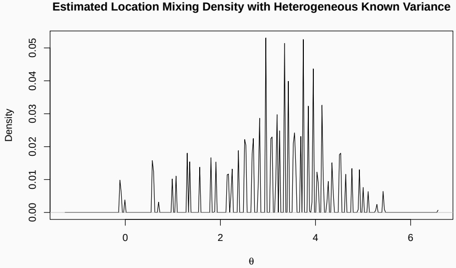
Results

The estimated $\hat{G}$

Case 1: $G(\theta, \sigma^2)$ for $v_\alpha(Y_i, S_i)$

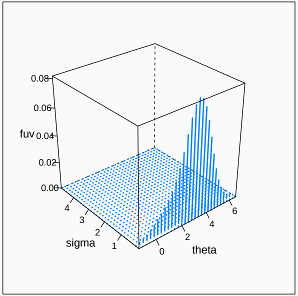


Case 2: $G(\theta)$ for $v_\alpha(Y_i)$

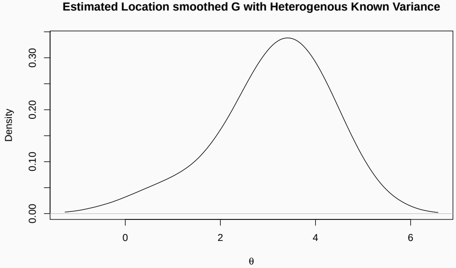


The estimated $\hat{G}$

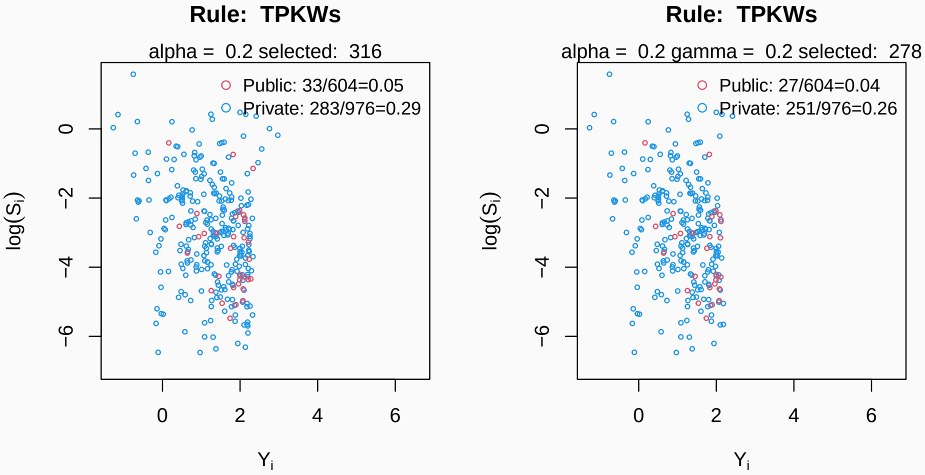
Case 1: $G(\theta, \sigma^2)$ for $v_\alpha(Y_i, S_i)$



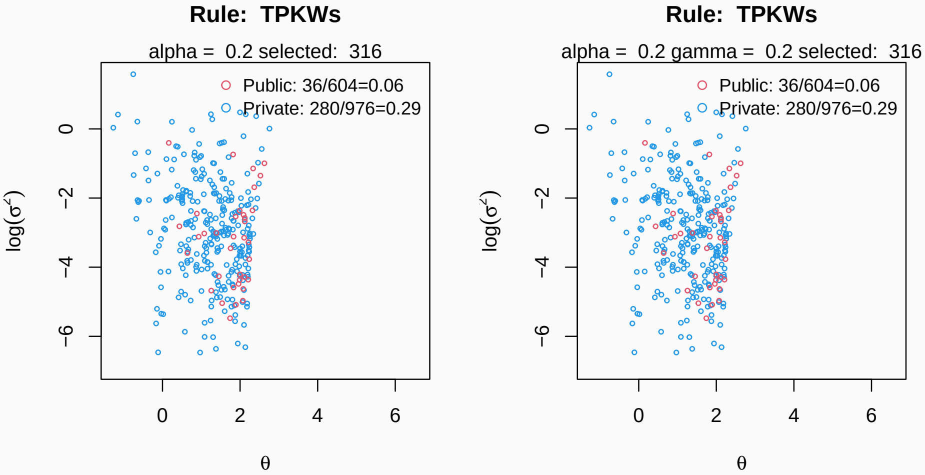
Case 2: $G(\theta)$ for $v_\alpha(Y_i)$



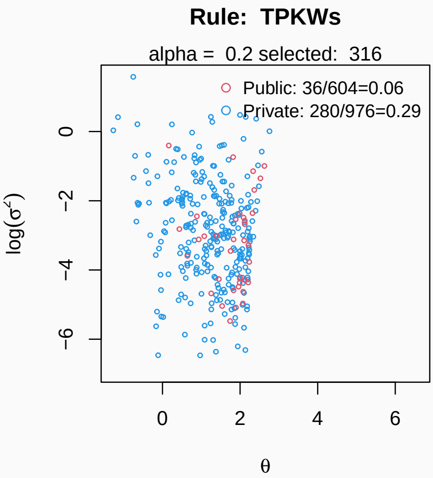
$G(\theta, \sigma^2)$: Posterior Tail probability (0.2,0.2)

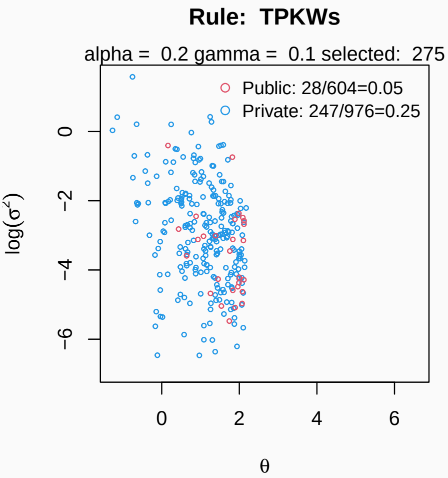


$G(\theta)$: Posterior Tail probability (0.2,0.2)

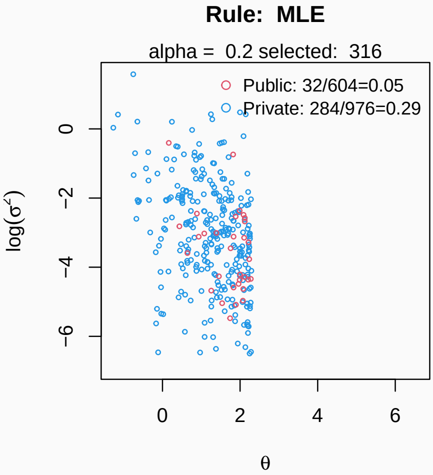


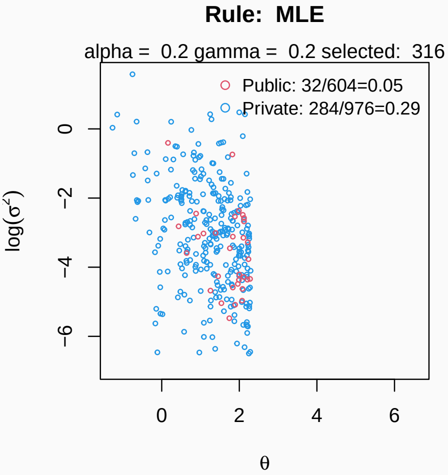
$G(\theta)$: Posterior Tail probability (0.2,0.1)





"Face value"


Rule: MLE
alpha = 0.2 selected: 316
Public: 32/604=0.05
Private: 284/976=0.29
0
−2
−4
−6
log(σ²)
0
2
4
6
θ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θ

Conclusion

Conclusion

- Whether to control for **False discovery rate**$\rightarrow$Control for FDR shrinks the selection set.
- Whether to assume known σ_i makes a difference$\rightarrow$ Assume unknown σ_i makes the FDR constraints bind, thus less selected than assuming σ_i known.
- Private (FP and NP) hospitals are indeed more ”efficient”$\rightarrow$ Caution.

Limitation

- Interpretation of the θ_i: The Schmidt and Sickles/Pitt and Lee models treat all time invariant effects as inefficiency. Greene (2005) treats time invariant components as only unobserved heterogeneity.
- Specification, endogeneity, normality assumption on ε_{it}, inference on $\hat{G}$ *etc.* ▸ Next

Thanks

Conditional Input Demand Function

In standard microeconomics, the profit maximization problem is

$$\max_{\vec{y}} \sum k_i y_i - \sum p_i x_i \quad \text{subject to} \quad f_i(x_1, x_2, \ldots, x_n) = y_i$$

where p_i is the price of input i and f is the cost function.

The cost minimization problem is thus

$$\min_{\vec{x}} \sum p_i x_i \quad \text{subject to} \quad f_i(x_1, x_2, \ldots, x_n) = y_i$$

Thus, the factor demand function/correspondence is

$$x_i = x_i(p_1, p_2, \ldots, p_n, y_1, y_2, \ldots, y_m)$$

▸ Back

Input demand function vs Production function

- We can remain agnostic as to the nature of the appropriate formula for the aggregation of outputs and use as many different products as desired.
- When input prices have low variability. Conditional factor demand can be estimated without information on input prices. Even if we add prices, due a lack of variability, the price parameters will be poorly estimated.
- From $x_i = x_i(p_1, p_2, \ldots, p_n, y_1, y_2, \ldots, y_m)$, we do not need to observe a complete list of inputs. But we do need to observe all input prices (can be ignored if almost no variability) and all outputs. While in the production function, it is the other way around (need to observe all inputs). Since, in our case, output is more *observable* than input (because capital is not easily observed), this approach is preferred.

First glance

Dependent Variable:	Nurses	
	Pool	Pool exclude
Model:	(1)	(2)
Variables		
Constant	1.39***	1.36***
	(0.025)	(0.024)
STAC inpatient	0.279***	0.293***
	(0.005)	(0.005)
STAC outpatient	0.050***	0.034***
	(0.003)	(0.003)
Medical sessions	0.061***	0.063***
	(0.002)	(0.002)
External consultations	0.057***	0.040***
	(0.002)	(0.001)
Emergency	0.016***	0.023***
	(0.001)	(0.001)
Long-term & follow-up	0.076***	0.072***
	(0.002)	(0.002)
Home care	0.016***	0.028***
	(0.003)	(0.003)
Psychiatric care	0.073***	0.075***
	(0.003)	(0.004)
Fit statistics		
Observations	13,402	12,279
R^2	0.821	0.819

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Dependent Variable:	Nurses	
	Dummy	Dummy exclude
Model:	(1)	(2)
Variables		
Constant	1.58***	1.50***
	(0.027)	(0.028)
STAC inpatient	0.276***	0.290***
	(0.005)	(0.005)
STAC outpatient	0.057***	0.048***
	(0.004)	(0.004)
Medical sessions	0.063***	0.068***
	(0.002)	(0.002)
External consultations	0.027***	0.028***
	(0.002)	(0.002)
Emergency	0.021***	0.018***
	(0.001)	(0.001)
Long-term & follow-up	0.069***	0.067***
	(0.002)	(0.002)
Home care	0.026***	0.025***
	(0.003)	(0.003)
Psychiatric care	0.063***	0.071***
	(0.003)	(0.004)
Private Forprofit	-0.270***	-0.245***
	(0.027)	(0.027)
Private Nonprofit	-0.180***	-0.160***
	(0.021)	(0.022)
Teaching	0.707***	
	(0.021)	
Fit statistics		
Observations	13,402	12,279
R^2	0.838	0.821

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Second glance

Dependent Variable:	Nurses			
	Teaching	Public	Forprofit	Nonprofit
Model:	(1)	(2)	(3)	(4)
Variables				
Constant	3.28***	1.40***	1.41***	1.00***
	(0.123)	(0.099)	(0.041)	(0.059)
STAC inpatient	0.108***	0.328***	0.261***	0.343***
	(0.016)	(0.018)	(0.007)	(0.012)
STAC outpatient	0.131***	0.078***	0.048***	0.046***
	(0.013)	(0.005)	(0.005)	(0.010)
Medical sessions	0.058***	0.049***	0.075***	0.093***
	(0.008)	(0.003)	(0.002)	(0.006)
External consultations	0.018**	0.025***	-0.003	0.002
	(0.009)	(0.004)	(0.006)	(0.005)
Emergency	0.049***	-0.008**	0.034***	0.024***
	(0.004)	(0.003)	(0.002)	(0.004)
Long-term & follow-up	0.058***	0.051***	0.057***	0.117***
	(0.005)	(0.003)	(0.003)	(0.007)
Home care	0.020**	0.029***	0.049***	-0.012
	(0.010)	(0.003)	(0.007)	(0.009)
Psychiatric care	0.029***	0.071***	0.076***	0.049***
	(0.005)	(0.004)	(0.007)	(0.017)
Fit statistics				
Observations	1,123	5,260	4,415	2,604
R^2	0.780	0.863	0.742	0.754

Heteroskedasticity-robust standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Back

Panel data Estimator

- Strict exogeneity: Within Group/First Diffrence

$$E[\epsilon_{it}|x_{i1},\ldots,x_{iT},\theta_i] = 0$$

- Relaxed: First Difference GMM, System GMM.

$$E[\epsilon_{it}|x_{i1},\ldots,x_{it-p},\theta_i] = 0$$

Issues: Weak instruments (Blundell and Bond, 1998) and the proliferation of instruments (Roodman, 2007).

$$\mathbb{E}[\Delta x_{i,t-1}(y_{it} - \beta x_{it})] \quad \text{if} \quad \mathbb{E}\left[\Delta x_{i,t-1}(\theta_i + \varepsilon_{i,t})\right] = 0$$

NPMLE Computation Methods

The primal problem:

$$\min_{f=dG} \left\{ -\sum_i \log g(y_i) \middle| g(y_i) = T(f),\ K(f) = 1,\ \forall i \right\}$$

where $T(f) = \int p(y_i|\theta) f d\theta$ and $K(f) = \int f d\theta$.
Discretize the support:

$$\min_{f=dG} \left\{ -\sum_i \log g(y_i) \middle| g = Af,\ 1^T f = 1 \right\}$$

where $A_{ij} = p(y_i|\theta_j)$ and $f = (f(\theta_1), f(\theta_2), \dots, f(\theta_m))$.
The dual problem:

$$\max_{\lambda,\mu} \left\{ \sum_i \log \lambda_1(i) \middle| A^T \lambda_1 < \lambda_2 1,\ (\lambda_1 > 0) \right\}$$

Normality assumption on ε_{it}

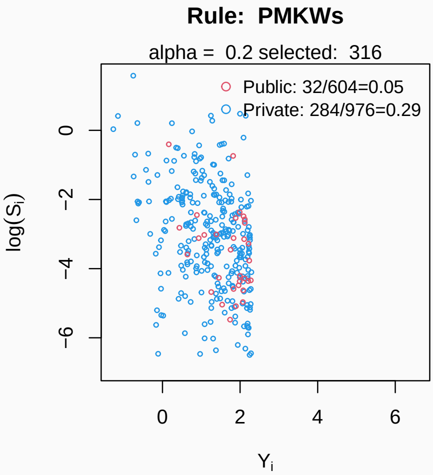
Estimate the fixed effect θ_i by

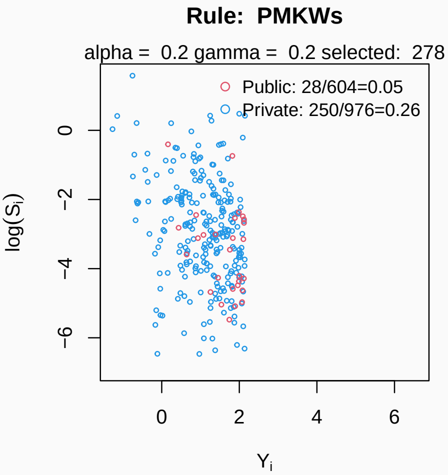
$$\begin{aligned}\hat{\theta}_i =& \frac{1}{T}\sum(\theta_i + \varepsilon_{it} + x_{it}(\beta - \hat{\beta})) \\ \xrightarrow{N\to\infty}& \theta_i + \frac{1}{T}\sum_t \varepsilon_{it}\end{aligned}$$

When T is relatively small (or even fixed), can't use central limit theorem to claim that $\hat{\theta}_i \xrightarrow{d} \mathcal{N}(\theta_i, \frac{\sigma_i^2}{T})$. $\longrightarrow$ Assume that $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_i^2)$.

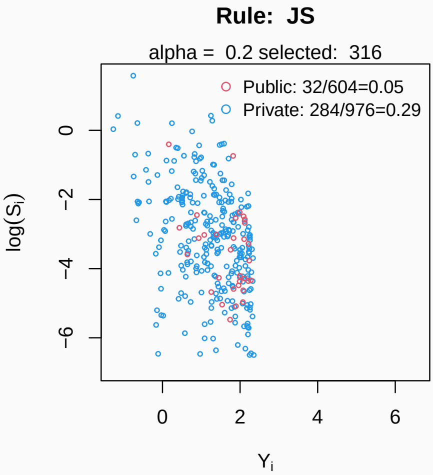
▸ Back (main) ▸ Back (end)

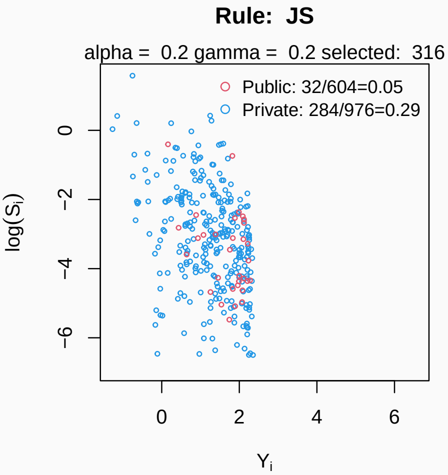
$G(\theta, \sigma)$: Posterior Mean



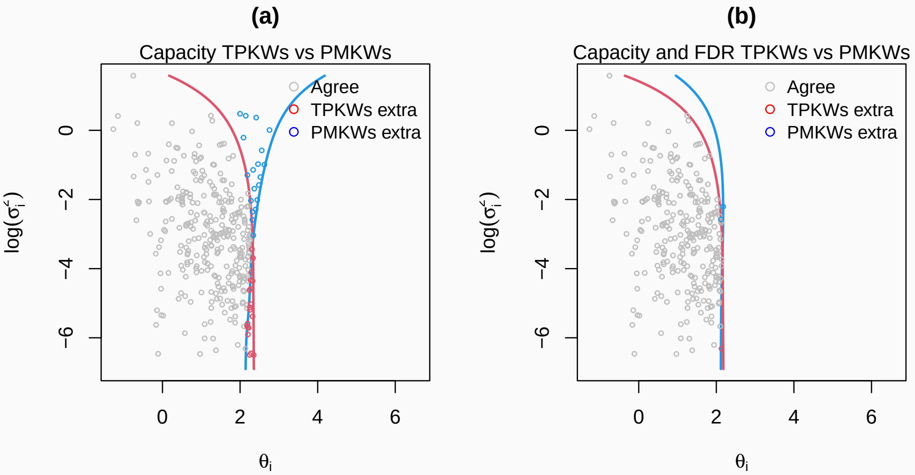


$G(\theta, \sigma)$: James-Stein Shrinkage



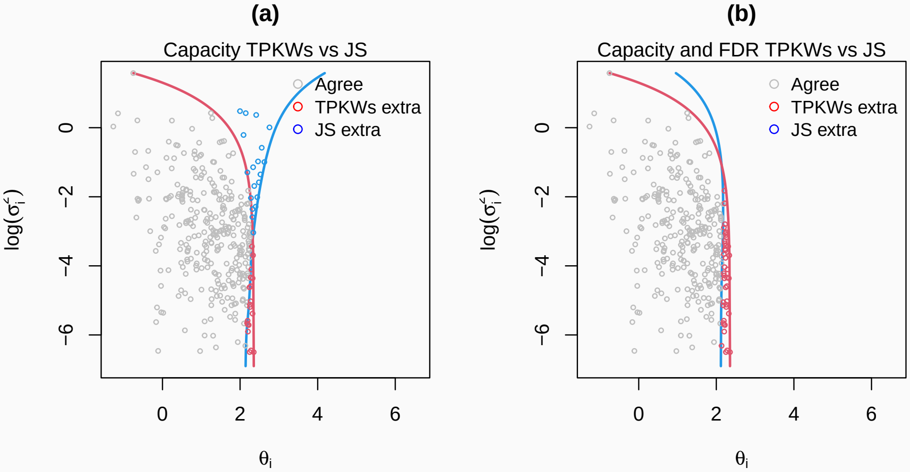


TP vs PM

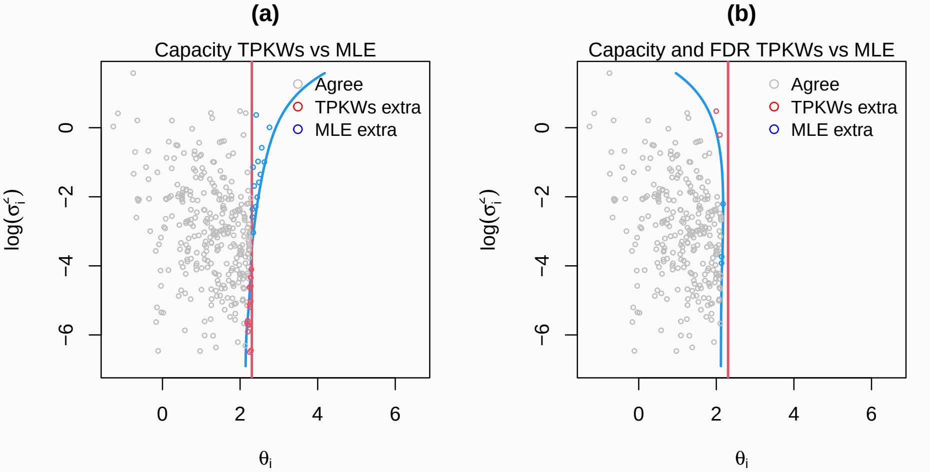

(a)
Capacity TPKWs vs PMKWs
Agree
TPKWs extra
PMKWs extra
$\log(\sigma_i^2)$
θ_i
(b)
Capacity and FDR TPKWs vs PMKWs
Agree
TPKWs extra
PMKWs extra
$\log(\sigma_i^2)$
θ_i

▸ Back

TP vs JS


(a)
Capacity TPKWs vs JS
Agree
TPKWs extra
JS extra
0
−2
−4
−6
0
2
4
6
$\log(\sigma_i^2)$
θ_i
(b)
Capacity and FDR TPKWs vs JS
Agree
TPKWs extra
JS extra
0
−2
−4
−6
0
2
4
6
$\log(\sigma_i^2)$
θ_i

TP vs MLE


(a)
Capacity TPKWs vs MLE
Agree
TPKWs extra
MLE extra
log(σ²ᵢ)
0
−2
−4
−6
0
2
4
6
θᵢ
(b)
Capacity and FDR TPKWs vs MLE
Agree
TPKWs extra
MLE extra
log(σ²ᵢ)
0
−2
−4
−6
0
2
4
6
θᵢ

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