sorry for reviewing discrete choice model again.

we have x_i y_j and z_{ij}

Micro data

Cross sectional (single)

$$u_{ij} = \alpha_i + \alpha x_i + \beta_j + \beta y_j + \zeta x_i y_j + \gamma z_{ij} + \epsilon_{ij}$$

We can estimate $\alpha, \beta_j, \beta, \zeta, \gamma$.

- application: school choice

Multiple markets (across location l)

If we have micro data, this is the same as the single cross sectional case. Just pool everything tgt.

Multiple markets (across time t)

$$u_{ijt} = \alpha_i + \alpha x_{it} + \beta_j + \beta_t + \beta y_j + \zeta x_{it} y_j + \gamma z_{ijt} + \epsilon_{ijt}$$

Same idea... we may be able to identify the α_i if x_{it} changes over time.

Market share data

There's no individual here :) but we can still write individual utility just for the sake of it.

$$u_{ij} = \alpha_i + \alpha x_i + \beta_j + \beta y_j + \zeta x_i y_j + \gamma z_{ij} + \epsilon_{ij}$$

Cross sectional (single)

The market share takes the same form as the choice probability but no individual characteristics x_i.

$$u_{ij} = \beta_j + \beta y_j + \epsilon_{ij}$$

Thus

$$s_j = \frac{\exp(\beta_j + \beta y_j)}{\sum_{j'} \exp(\beta_{j'} + \beta y_{j'})}$$

$$\log s_j - \log s_0 = \beta_j + \beta y_j$$

The moment condition is that

$$E[\beta_j y_j] = 0$$

Here each j is one observation that can be used to estimate the coeffecients β. Then

$$\hat{\beta}_j = \log s_j - \log s_0 - \hat{\beta} y_j$$

Multiple markets (across location l)

$$u_{ijl} = \beta_{jl} + \beta y_j + \epsilon_{ijl}$$

Thus

$$s_{jl} = \frac{\exp(\beta_{jl} + \beta y_j)}{\sum_{j'l} \exp(\beta_{j'l} + \beta y_{j'})}$$

$$\log s_{jl} - \log s_{0l} = \beta_{jl} + \beta y_j$$

Here each jl is one observation that can be used to estimate the coeffecient β.

The moment condition is that

$$E[\beta_{jl} y_j] = 0$$

Multiple markets (across time t)

Same as above but with t. However, we may want to assume some structure between β_{jt} and β_{jt-1} which may be of help in estimation.

Appendix

Multinomial logistic regression

According to the wikipedia page,there are **three** ways to look at this regression model: 1. As a set of independent binary regressions (the odds-ratio)

$$\ln \frac{\Pr(Y_i = k)}{\Pr(Y_i = K)} \,=\, \boldsymbol{\beta}_k \cdot \mathbf{X}_i, \qquad 1 \le k < K$$

1. As a log linear model

$$\ln \Pr(Y_i = k) = \boldsymbol{\beta}_k \cdot \mathbf{X}_i - \ln Z, \qquad 1 \le k \le K.$$

1. As a latent variable model

$$Y_{i,k}^{*} = \boldsymbol{\beta}_k \cdot \mathbf{X}_i + \varepsilon_k \quad , \quad k \le K$$

Check this chapter of the train textbook for the derivation of the choice probability. Then estimation is done by maximum likelihood.