When to book a flight?

A response to dynamic pricing and uncertainty

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1 Introduction

It is a well-known fact that airline companies adopt dynamic pricing to optimize revenue. The same strategy applies to other markets with perishable inventories as well, such as tickets for entertainment events. It has been estimated that dynamic pricing increases airline's revenue by 7.6% compared to uniform pricing (Williams, 2022). In addition, many economic models have studied the optimal pricing strategy for firms, assuming that consumer arrivals follow a Poisson process with time-varying willingness to pay and price elasticity.

Though it is not without good reason to model the consumer side in this way and ignore forward-looking behaviors, it is in my own interest to shed light on the traveler's intertemporal decision—what and when to buy? Anecdotal experience suggests that travelers may start checking prices a few months ahead but wait until a certain time to purchase. While prices may be low early on, they delay purchase until they are more certain about their travel plans. While the entire price change is unknown to travelers at the time of decision, they form some expectations about the price dynamics to make this kind of trade-off between cost and uncertainty. In fact, by setting a Google flight tracker, one can observe the ups and downs of the prices of desired flights, which fluctuate more or less around a low level at first and then increase as the departure date approaches.

In this paper, I aim to model the consumer's dynamic decision-making process as each individual solving an optimal stopping problem. By doing so, I seek to answer the question of how consumers trade off between minimizing costs and reducing uncertainty. Through estimating the parameters that govern their decision-making, I plan to conduct counterfactual analyses on consumer welfare where firms adjust their pricing strategies (e.g., removing uncertainty and implementing only a monotonically increasing price structure) or modify their flexibility options.

The paper is structured as follows: Section 2 reviews the related literature and highlights the unique features of the problem addressed in this paper. In Section 3, I present the model, starting with a simplified case. Section 4 discusses the additional complexities that can be incorporated into the model.

2 Related Literature

This section revisits several classical dynamic discrete choice problems. By classifying and summarizing these examples, I aim to highlight the properties of the problem addressed in this paper and the ways in which it differs from existing literature.

Base Case: One-Time Purchase of Non-Storable Goods In the base case, a consumer enters the market, makes a purchase decision a based on product characteristics x and personal attributes z, and then exits the market. The purchased goods are intended for immediate consumption and cannot be stored. Consequently, there is no inventory, and the decision-making process is static, with no intertemporal considerations. For example, studies that make this assumption include those on cereal purchases (Nevo, 2001), or car purchases (Berry et al., 1995).

Repeated Decision with Consumption Utility for Storable Goods This scenario introduces **stockpiling** behavior, as seen in the purchase of goods like detergent (Hendel and Nevo, 2006) or soft drinks. Here, the consumer enters the market with an existing inventory level i_t and faces individual inventory costs or limits. Their purchase decision depends on both the product characteristics x_t and the current inventory i_t. For instance, the product price fluctuates between a regular price p_r and a sale price p_s. At time t, if the consumer chooses to buy, they pay the current price and accumulate inventory to consume over time, incurring holding costs or facing storage constraints. If they choose not to buy, they consume from their existing inventory.

The dynamic aspect arises from the trade-off between purchasing at a lower price (e.g., during sales) and incurring the cost of holding additional inventory. The consumer must weigh the immediate cost savings against the future costs and benefits associated with stockpiling.

One-Shot Decision with Flow Utility for Durable Goods In this case, the consumer makes a one-time purchase decision, after which they accrue holding utility from the durable good over time. For example, consider purchasing a car (Grigolon et al., 2018) or installing solar panels (De Groote and Verboven, 2019).[1] The decision involves a trade-off between current costs and future benefits. The consumer evaluates whether the present price justifies the expected long-term utility of owning the product, considering potential changes in future costs or benefits.

Repeated Decision with Flow Utility for Durable Goods Here, consumers face a recurring decision to purchase or replace a durable good, such as in the bus engine replacement problem (Rust, 1987) or the camcorder industry (Gowrisankaran and Rysman, 2012). At any given time, they hold and use only one product. If they decide to purchase, the

[1] Patent renewal (Pakes, 1984) is similar, though distinct, as the decision-maker exits by not paying, whereas in the purchase example, the agent exits by buying.

holding utility derives from the newly acquired product f_j; otherwise, it comes from the existing product f_0.

In this scenario, the characteristics of the owned product evolve over time (e.g., depreciation, as in the bus engine case), while market options may improve (e.g., technological innovation, as in the camcorder industry). Each period, the consumer evaluates whether to purchase a new product now (incurring an upfront cost but benefiting from higher quality) or delay the purchase (accepting lower holding utility in the short term but potentially gaining from lower future prices or better-quality options). This dynamic decision-making process involves balancing current utility against expected future gains.

One-Shot Decision with Final Consumption Utility for Perishable Goods This case primarily applies to goods like event tickets (Sweeting, 2012) or transport tickets (Board and Skrzypacz, 2016; Gershkov et al., 2018). Such goods have no residual value after their designated usage period, making them "perishable." Since ticket sales typically begin well in advance of the final usage date, consumers often start considering purchases several periods before the deadline.

The consumer purchases only one ticket and consumes it at the final usage date, meaning there is no ongoing utility from holding the ticket. The decision is not about balancing current versus future flow utility but rather about minimizing costs under price fluctuations. Consumers anticipate price trends (e.g., initial variability followed by an eventual increase) and may delay their purchase to secure a lower price.

Additionally, the time gap between purchase and consumption introduces uncertainty. As the final usage date approaches, the consumer gains better information about their plans or needs, allowing them to make a more informed decision. Thus, the trade-off involves evaluating price dynamics alongside the uncertainty of future utility at the time of purchase.

The next section will formalize the dynamic problem the consumer faces when balancing cost and uncertainty in each period after they enter the market (begin checking prices). First, I will consider a simplified case and then gradually introduce more complexities.

3 Model

I focus on airline ticket purchase in this section. To illustrate the idea, figure 1 depicts the simplest case. In this toy model with discrete time, I denote the last period as $t = 0$, which corresponds to the departure date. The value of t represents the number of periods before departure.

Background Consider a traveler who wants to fly in February 2025 ($t = 0$). Suppose that in December 2024 ($t = 2$), the traveler enters the market by searching for flights online. She then sets a price alert on Google Flights tracker [2] for all the flights around that time. In the following two months ($t = 2, 1, 0$), she decides whether to buy based on the current price p_t

[2] Add citation or URL for Google Flights tracker here.

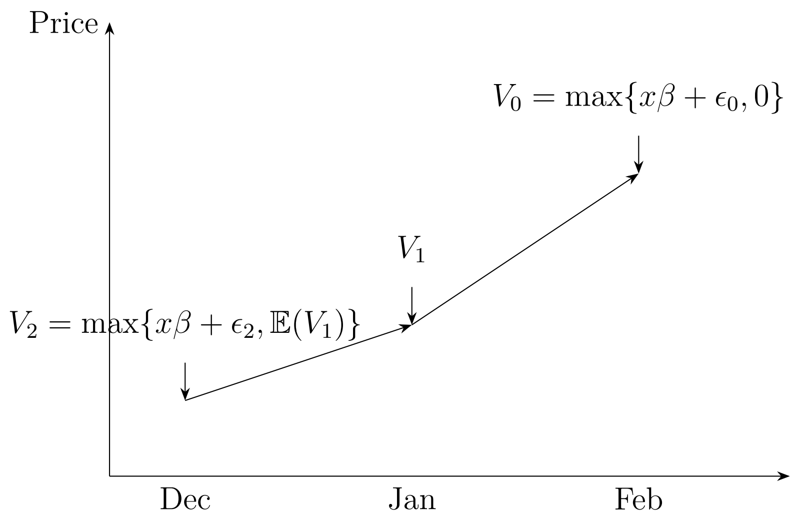

Figure 1: Timeline of airline ticket purchase

and her travel plans. She has a binary choice a, fly or not to fly. Therefore, her choice set is defined by $\{0, 1\}$. If she purchases a ticket, she will only realize her consumption utility at the last period ($t = 0$), which corresponds to $x\beta$.

For example, it might be the case that in December, she buys the ticket immediately at price p_{t2}. As time progresses to January $t = 1$, the situation evolves as she realizes that she would be occupied at that time. If she were to make a decision now $t = 1$, she would not fly. However, the ticket she bought at $t = 2$ is non-modifiable and non-refundable (unfortunately). When she reaches $t = 0$, she realized utility $x\beta + \epsilon_{t0}$.

State At time t, the known state of the individual includes:

- p_t: The current price of the ticket.
- ϵ_t: The random shock observed in period t.
- Bought: Whether the ticket has already been purchased.
- x: The product characteristics that determine the utility of flying.

The uncertainty arises from the next period $t - 1$ price and shock.

- The future price p_{t-1}. I assume that the price goes up in, following AR(1) process with $p_{t-1} = \rho p_t + \nu_{t-1}$ and $\rho > 1$.
- The future shock ϵ_{t-1}. I assume that they are T1EV with variance σ_t^2 and $\sigma_t^2 > \sigma_{t-1}^2$. The variance goes down as the departure date approaches.

Action

- $a_t = 1$: Buy the ticket in period t.
- $a_t = 0$: Wait until the next period to decide.

State Transition

- If $a_t = 1$: The decision process ends, and no further actions are possible.
- If $a_t = 0$: The state transitions to the next period $t - 1$, and a new shock ϵ_{t-1} is observed.

Rewards

- If $a_t = 1$: Immediate reward is:

$$R_t(a_t = 1) = V_0 - \alpha p_t - \epsilon_1,$$

  where V_0 is the final utility of flying, adjusted by the ticket price and the shock at t.
- If $a_t = 0$: No reward is received in the current period.

Value Function The value function V_t represents the maximum expected utility at time t, given the state:

$$V_t = \max_{a_t} \mathbb{E}[R_t(a_t) + V_{t-1}].$$

- If $a_t = 1$: The process terminates with reward $R_t(a_t = 1)$.
- If $a_t = 0$: The individual moves to $t - 1$, and the expectation incorporates uncertainty about the future.

Recursive Formulation The recursive relationship for the value function is:

$$V_t = \max \left\{ R_t(a_t = 1), \mathbb{E}[V_{t-1} | \text{no purchase}] \right\}.$$

- Terminal period $t = 0$:
  - If the ticket is not purchased, utility is zero. If purchased:

$$V_0 = x\beta - \alpha p_0 - \epsilon_0.$$

  - Expected value at $t = 0$:

$$\mathbb{E}[V_1] = \int \int \max \left\{ x\beta - \alpha p_0 - \epsilon_0, 0 \right\} f(\epsilon_0) f(p_0) d\epsilon_0 dp_0,$$

    where $f(\epsilon_0), f(p_0)$ is the probability density function of the shock and the price.

- Intermediate period $t > 0$:
  - At $t = 1$, the value function is:

$$V_1 = \max \{x\beta - \alpha p_1 - \epsilon_1, \mathbb{E}[V_0]\}.$$

  - At $t = 2$, the value function is:

$$V_2 = \max \{x\beta - \alpha p_2 - \epsilon_2, \mathbb{E}[V_1]\}.$$

Choice Probability Each individual i's decision is based on the dynamic program, where they maximize their expected utility given the current information. The probability of individual i taking action $a_t = 1$ is given by:

$$P(a_t = 1|p_t) = \frac{\exp((x\beta - \alpha p_t)/\sigma_t)}{\exp((x\beta - \alpha p_t)/\sigma_t) + \exp(\mathbb{E}[V_{t-1}]/\sigma_t)}.$$

Estimation We observe in the data the flight characteristics (in this case, only a flight dummy), the price of the ticket, and the purchase decision at each period. The parameters to be estimated include:

- β: the coefficient on the flight characteristics.
- α: the price coefficient.
- ρ, σ_ν: the price process parameters.
- σ_t: the standard deviation of the shock at each period.

The most straightforward approach is to estimate by maximum likelihood with individual purchase data. Other estimation techniques are to be investigated.

4 Discussion

The previous section presents the simplified scenario where the traveler makes only a binary choice. To construct the full choice set for an individual who has purchased a ticket, I include all the flights available with the same origin and destination, with departure dates in proximity.

The market/choice set considered consists of flights by different companies, with various characteristics (the number of layovers, duration, the departure (arrival) time of the day, airline brand, etc.). With respect to layovers, I would like to include both direct and connecting flights because many international flights involve layovers. Since prices for international flights are usually high and price changes can be up to 20% [3], it gives travelers

[3] This is according to personal experience. The statistics need to be verified.

higher incentives to think about the timing of purchase. In addition, for the same flight, there are usually different flexibility options, which can be included in the x_j. Meanwhile, the flexibility property can affect how the shock impacts consumers. Therefore, the size of the shock can be modeled as a function of the flexibility of the ticket with $\epsilon_t(a_t) = \epsilon_t(x_j(a_t))$, where $x_j(a_t)$ is the flexibility of the ticket choice a_t.

In the toy model, (I assume that) the consumer assumes that the price follows an AR(1) process. The price dynamics can be made more realistic, for example, by assuming the price fluctuates around a certain level before a certain period ($t > t_c$) and then increases more or less monotonically.

In papers that focus on the firm side pricing strategy (Williams, 2022; Betancourt et al., 2022), it is a common assumption that willingness to pay (WTP) consumers arrive early (leisure traveler) and high WTP consumers arrive late (business traveler).[4] This variation in WTP over time can be incorporated into consumer utility models by introducing a heterogeneous price coefficient that depends on both the time of arrival and individual characteristics.

[4]This pattern may be reversed in the case of event or performance ticket sales. Early buyers are often avid fans who closely follow the performers and assign greater value to the event, leading to a higher willingness to pay. In contrast, late buyers may discover the event later and have a lower willingness to pay.

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