Estimate Dynamic Models with CCP

Chuqing Jin*

Due date: Feb. 7th 2025.

Download the data on engine replacements in city buses in Madison, WI from Moodle/Dropbox. The data are a subset of the data used in Rust (1987). The data are organized in 2 columns. The first column is the number of miles the bus has traveled since its last engine overhaul. The second column takes on 3 values: 0 for no change, 1 for an engine overhaul and 2 for the beginning of observations for a new bus. The data set covers 58 buses. These data are essentially the complement of the data he labels "Groups 1, 2, 3 and 4."

Assume the profit to Harold Zurcher of running a bus is:

$$\pi(s) = (-RC + \nu_1)\, Y + (-\mu s + \nu_0)\,(1 - Y),$$

where $Y = 1$ indicates overhaul and $Y = 0$ indicates no overhaul. Cumulative mileage is denoted by s. RC and μ are parameters to estimate. Let s' denote mileage next period. Assume $s' = 0$ if $Y = 1$, and $s' = s + \varepsilon$ otherwise, where $\varepsilon \sim \mathcal{N}(\rho, \sigma^\rho)$. Assume that $\nu_Y \sim$ Type I EV.

For this problem set, divide mileage by 10,000 .

1. Estimate a probit model of the choice to invest as a function of a constant term and mileage. Graph the probability of investment as a function of mileage, up to 400,000 miles (that is up to 40 units). Experiment with including mileage-squared and mileage-cubed. How about the log of mileage? Which is your preferred specification?

   Note that the data is organized so that all engine overhaul is marked in the first period of the new engine. Therefore, you need to adjust the timing of one of the variables (lag/lead) in order for this regression to make sense.

2. What is the average increase in miles for buses that do not invest? What is the standard deviation? That is, what is ρ and σ^ρ ?

3. Based on the ideas of Bajari, Benkard & Levin (2007), construct the elements of the value function via simulation. Let K be the number of simulations. In this case, the value function can written as a function of three elements. The first element captures the present discounted stream of replacement costs:

   $$x_i^1 = \frac{1}{K} \sum_{k=1}^{K} \sum_{\tau=1}^{T} \beta^\tau \left(-Y_{i\tau}^k\right)$$

   The second element captures the stream of mileage costs:

   $$x_i^2 = \frac{1}{K} \sum_{k=1}^{K} \sum_{\tau=1}^{T} \beta^\tau \left(-s_{i\tau}^k (1 - Y_{i\tau}^k)\right).$$

*This problem set is built on Marc Rysman's problem set.

The last term is:

$$x_i^3 = \frac{1}{K} \sum_{k=1}^{K} \sum_{\tau=1}^{T} \beta^\tau E\left[\nu_{Y_{i\tau}^k i\tau} | s_{i\tau}^k, Y_{i\tau}^k\right].$$

Note that the distributional assumption for ν implies that $E\left[\nu_{Y_{it}it} \mid s_{it}, Y_{it}\right] = \gamma - \ln\left(Pr\left(Y_{it} \mid s_{it}\right)\right)$ where γ is Euler's constant.[1] So you can compute this from your result in 1 .

For each observation in the data, construct each of these three variables **twice**. The first is for the case where the Zurcher invests in the first period and the second is for the case in which Zurcher does not invest in the first period. That is, $x_{i1}^1 = -1 + x_i^1(0)$ and $x_{i0}^1 = x_i^1(s')$. The term x_{i1}^1 assumes Zurcher replaces in the first period, so x_i^1 is computed starting from a state of $s = 0$, whereas x_{i0}^1 assumes Zurcher does not replace in the first period, so x_i^1 is computed starting from s'. Similarly, compute x_{iY}^2 and x_{iY}^3. For x_{iY}^3, the current period value is zero for both $Y = 1$ and $Y = 0$.

Set the discount rate to 0.99 . Simulate forward 100 periods. Use 10 paths. Simulate from the results you find in 1 and 2 .

Report the average over observations i of the 6 variables, x_{iY}^j for $j \in \{1, 2, 3\}$ and $Y \in \{0, 1\}$.

4. Let V_i be the vector of differences. Thus:

$$V_i = \begin{bmatrix} x_{i1}^1 - x_{i0}^1 & x_{i1}^2 - x_{i0}^2 & x_{i1}^3 - x_{i0}^3 \end{bmatrix}$$

Then, the difference in value for choosing to invest is: $V_i\theta + \varepsilon_{i1} - \varepsilon_{i0}$, where $\theta = [RC \quad \mu \quad 1]$. The probability of investment is:

$$P\left(a_i = 1 \mid s_i, \theta\right) = \frac{\exp\left(V_i\theta\right)}{1 + \exp\left(V_i\theta\right)} \quad P\left(a_i = 0 \mid s_i, \theta\right) = \frac{1}{1 + \exp\left(V_i\theta\right)}$$

Find

$$\theta^* = \arg\max_\theta \sum_i^n \ln\left(P\left(a_i \mid s_i, \theta\right)\right)$$

Report θ^*.

Note that we argued in class that a linear model is more attractive for this step, but this objective function is not linear. However, an advantage of this objective function is that this problem can be completed in Stata using the `logit` command. Check out the `offset` and `noconstant` options. Of course, you can use the logit model in other programming languages (e.g., `glm` in R).

5. Suppose you solved the Bellman equation in Rust (1987) at these parameters, and then graphed the probability in 1 . Describe in words why that might find a different result than 1. Speculate as to how the graph might look different.

[1] This holds if ν follows standard Type I EV with location parameter = 0 and scale parameter = 1.

References

Bajari, P., Benkard, L., & Levin, J. (2007). Estimating dynamic models of imperfect competition. *Econometrica*, 75, 1331-1370.
Rust, J. (1987). Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. *Econometrica*, 55, 999-1033.