No assumption on the DGP of ξ_{jt}

After recovering δ_{jt} from the observed market share s_{jt}. We regress it on the product characteristics.

The **estimation equation** is

$$\delta_{jt} = x_{jt}\beta + p_{jt}\alpha + \xi_{jt}$$

where x_j is the product characteristics, p_j is the price, and ξ_{jt} unobserved product characteristics.

We can use the following moment conditions to estimate the parameters:

$$E(Z'_{jt}\xi_{jt}) = 0$$

where Z_{jt} is the exogenous variable + instruments.

The empirical moments condition is

$$\frac{1}{N}Z'\xi = 0$$

where Z is $N \times J$ and ξ is $N \times 1$.

Impose AR(1) structure on ξ_{jt}

Question: why do we want to impose an AR(1) structure on the unobserved product characteristics ξ_{jt}?

Now, we assume that ξ_{jt} follows an AR(1) process:

$$\xi_{jt} = \rho\xi_{j,t-1} + \omega_{jt}$$

where ω_{jt} is the innovation term (error term) that is uncorrelated with x_{jt} and p_{jt}.

For δ_{jt} and δ_{jt-1}, we have

$$\delta_{jt} = x_{jt}\beta + p_{jt}\alpha + \xi_{jt} = x_{jt}\beta + p_{jt}\alpha + \rho\xi_{j,t-1} + \omega_{jt} \ (1)$$

$$\rho\delta_{j,t-1} = \rho x_{j,t-1}\beta + \rho p_{j,t-1}\alpha + \rho\xi_{j,t-1} \ (2)$$

The new **estimation equation** (1)-(2) is

$$\delta_{j,t} - \rho\delta_{j,t-1} = \beta x_{jt} - \rho\beta x_{j,t-1} + \alpha p_{jt} - \rho\alpha p_{j,t-1} + \omega_{jt}$$

Thus the parameters we want to estimate is β, α, and ρ. This corresponds exactly to the model in blundellbond2000.

We rewrite as

$$\delta_{j,t} = \pi_1 x_{jt} + \pi_2 x_{j,t-1} + \pi_3 p_{jt} + \pi_4 p_{j,t-1} + \pi_5\delta_{j,t-1} + \omega_{jt}$$

where $\omega_{jt} \sim$ MA(0). We have the restriction that

$$\pi_4/\pi_3 = \pi_2/\pi_1 = -\pi_5$$

which can be tested after the estimation or imposed in the estimation.

Now we can perform standard GMM without any additional instruments because ω_{jt} is orthogonal to

$$z_{jt} = [x_{jt}, x_{j,t-1}, p_{jt}, p_{j,t-1}, \delta_{j,t-1}]$$

If we assume that only the price coefficient $\alpha \sim N(\alpha_0, \sigma^2)$\$ is the random coefficient, for each σ, we get the gmm objective function value by minimizing the empircal counterpart of

$$E[z'_{jt}\omega_{jt}] = E[z'_{jt}(\delta_{jt} - z_{jt}\pi)] = 0$$

We can then find the σ that minimizes the objective function value.

Question: Do we need extra instruments? If yes, what's the point of imposing the AR(1) structure?

Recall

$$\xi_{jt} = \rho \xi_{j,t-1} + \omega_{jt}$$

If ω_{jt} is not orthogonal to x_{jt} and p_{jt}, but orthogonal to some other instruments Z_{jt},

$$E[Z'_{jt}(\xi_{jt} - \rho \xi_{j,t-1})] = 0$$
$$E\{Z'_{jt}[\delta_{jt} - x_{jt}\beta - p_{jt}\alpha - \rho(\delta_{j,t-1} - x_{j,t-1}\beta - p_{j,t-1}\alpha)]\} = 0$$

Question: Previously we assume each year constitutes independent market. Mow we look at them from a panel perspective, which means that we impose some (dependence) structure on ξ_{jt} from year to year. Do we need panel where all products j are observed consecutively. What if some j are not observed in the two consecutive years?

Simulation and Estimation

Simulation

1. We **set** the true value of β, α, ρ, and σ.
2. Then we generate ξ_{jt} which is correlated to p_{jt} for example. (The correlation needs to be specified.)
3. We generate ξ_{jt+1} based on ξ_{jt} and some innovation term ω_{jt+1}. ξ_{jt+1} is still correlated with p_{jt+1}.
4. We generate some instruments Z_1 for ξ, and some instruments Z_2 for ω. **Question**: is it that by default, Z_1 should also be valid for ω, but not the other way around?
5. To get the market share, we use

$$s_j = \int \frac{\exp(\delta_j + \sigma p_j)}{1 + \sum_{j'} \exp(\delta_{j'} + \sigma p_{j'})} dF(\sigma)$$

   where $\delta_j = x_j \beta + p_j \alpha + \xi_j$.

6. Then we have the observed market share s_j, the product characteristics x_j, and the price p_j for each year. And we can pretend that we don't know the true value and estimate the paremeters β, α, ρ, and σ.

Estimation

For estimation, we can either 1. use Z_1 such that $E[Z'_{jt+1}\xi_{jt+1}] = 0$ 2. use Z_1 such that $E[Z'_{jt+1}\omega_{jt+1}] = 0$ 3. use Z_2 such that $E[Z'_{jt+1}\omega_{jt+1}] = 0$

Compare the results from the three methods.

Question: But why? Is it easier to find instruments for ω in reality? If we can find Z_1 why bother using $E[Z'_{jt+1}\omega_{jt+1}] = 0$ as the moment condition, rather than just $E[Z'_{jt+1}\xi_{jt+1}] = 0$? It's only useful when we can't find Z_1 but can find Z_2?