

Standard

I always need to remind myself of the standard BLP estimation procedure.

The subscripts are i for individual, j for product, t for time (market).

1. Given

$$u_{ijt} = x_{jt}\beta + \xi_{jt} + \mu_{ijt} + \epsilon_{jt} = \delta_{jt} + \mu_{ijt} + \epsilon_{jt}$$

2. The random coefficient is contained in μ_{ijt} . Here I assume the only random coefficient is that of the constant term $\mu_{ijt} \sim N(0, \sigma^2)$.
3. So we guess it first. Compute δ_{jt} from the observed market share s_{jt} .
4. Regress δ_{jt} on x_{jt} to get β (with instruments).
5. Having estimated the linear parameters β , we now move on to estimate the nonlinear one σ . We need to find another instrument for σ . We can construct GMM objective function written as $G(\sigma)WG(\sigma)'$.
6. We minimize the GMM objective function to get an estimate of σ .

One can notice that the estimation of the linear parameters β is done separately from the estimation of the nonlinear parameter σ . I would apply the same idea in the next case.

AR(1) Structure

1. Given

$$u_{ijt} = x_{jt}\beta + \xi_{jt} + \mu_{ijt} + \epsilon_{jt} = \delta_{jt} + \mu_{ijt} + \epsilon_{jt}$$

The unobserved product characteristics ξ_{jt} follows an AR(1) process:

$$\xi_{jt} = \rho\xi_{j,t-1} + \omega_{jt}$$

2. Therefore, the non linear parameters are ρ and σ . One from the AR(1) process and one from the random coefficient.
3. As before, we guess σ first. Compute δ_{jt} from the observed market share s_{jt} .
4. Previously we have this moment condition

$$E(x_{jt}(\delta_{jt} - x_{jt}\beta)) = 0$$

to estimate β . Now we have a new equation since we have further imposed the AR(1) structure on the residual ξ_{jt} , which is

$$\delta_{jt} - \rho\delta_{j,t-1} = \beta x_{jt} - \rho\beta x_{j,t-1} + \omega_{jt}$$

5. We first guess a value for ρ . We make use of this new equation which gives the moment condition to estimate the linear parameters β .

$$E[x_{jt}(\delta_{jt} - \rho\delta_{j,t-1} - (\beta x_{jt} - \rho\beta x_{j,t-1}))] = 0$$

6. Having estimated the linear parameters β , we move on to estimating nonlinear ones, σ and ρ . We need to find two additional instruments for them to construct the GMM objective function.
7. We minimize the GMM objective function $G(\sigma, \rho)WG(\sigma, \rho)'$ to get estimates of σ and ρ .
8. Notice that the linear and nonlinear parameters are estimated separately. The last step is to estimate them jointly to get the correct standard errors. We can construct $G(\beta, \sigma, \rho)WG(\beta, \sigma, \rho)'$ with the previous estimates (from previous separate estimation) as the initial values.