Dynamic discrete choice solution methods

We want to express the intermediate value function in terms of the choice probabilities by making use of $E(A) = P(X)E(A|X) + P(X^c)E(A|X^c)$.

$$\bar{V}(x_t) = \sum_{a'} P(a'|x_t) \left\{ u(x_t, a') + E(\epsilon_{a'}|x, a') + \beta \sum_{x}^{'} \bar{V}(x') f(x'|x, a') \right\}$$

If we assume that $\epsilon_{a'}$ is i.i.d. and follows the type I extreme value distribution, then $E(\epsilon_{a'}|x, a') = \gamma - \ln(P(a'|x))$.

Conditional choice probability (CCP) method

> The goal is to express the intermediate value function $\bar{V}(x_t)$ in terms of the choice probabilities $P(a_t|x_t)$.

Intermediate value function

Recall the intermediate value function

$$\bar{V}(x_t) = \int V_t(x_t, \epsilon_t) g(\epsilon_t) d\epsilon_t$$

Let us define the dyanmic utility function as

$$\tilde{u}(x_t, a_t) = u(x_t, a_t) + \beta \int \bar{V}(x_{t+1}) f(x_{t+1}|x_t, a_t) dx_{t+1}$$

Then the value function can be written as

$$V(x_t, \epsilon_t) = \max_{a_t \in A} \tilde{u}(a_t, x_t) + \epsilon_t$$

Back to intermediate value function, it is

$$\bar{V}(x_t) = \int \left\{ \max_{a_t \in A} \tilde{u}(a_t, x_t) + \epsilon_t \right\} g(\epsilon_t) d\epsilon_t$$

Choice probability

Also the choice probability given state x_t is

$$P(a_t|x_t) = \frac{\exp\{\tilde{u}(x_t, a_t)\}}{\sum_{a_t \in A} \exp\{\tilde{u}(x_t, a_t)\}}$$

Finite period

1. Estimate $\hat{f}(x_{t+1}|x_t, a_t)$ and $\hat{P}(a_t|x_t)$ at every state point.
2. Compute the intermediate value function based on the $\hat{f}, \hat{P}$. Then estimate the structural parameters from the value function.

Infinite period

The full solution method is a **nested fixed-point algorithm**. We can swap the order of the two steps by making use of the CCP representation to obtain the **nested pseudo-likelihood** algorithm. From an initial guess of $P^0(a_t|x_t)$, 1. Obtain a new pseudo-likelihood estimate

$$\theta_K = \arg\max \sum_{i}^{n} \ln(\Phi(P^{K-1}(a_t|x_t)))$$

2. Update the conditional choice probabilities

$$P^k(a_t|x_t) = \Phi(P^{k-1}(a_t|x_t))$$

We give a more detailed explanation of the Φ function (policy iteration operator). 1. Given a set of choice probabilities P^k, we can obtain the value function $\bar{V}^k$ for the P^k. 2. Given the next period expected utility is $\bar{V}^k$, we can obtain the new choice probabilities P^{k+1}. 3. Then we have updated from P^k to P^{k+1}.