Problem Set 2: Dynamic Discrete Choice
Empirical Industrial Organization

In this assignment you have to simulate a simple version of an inventory control problem. The objective is to familiarize with the computation of the dynamic programming solution and compare it with a CCP method.

You can work in groups of 3 people, each group has to send a pdf report as well as the code files used to generate the results. When reporting results, present them in tables that are easy to read and discuss your findings. Do not simply attach a printout of the computer program you used.

Deadline: Please send your problem set before **January 10th**.

Consider the following dynamic model of inventory control:

$$V(i, c, p, \varepsilon_t) = \max_{x \in \{0,1\}} \alpha c - \lambda \mathbf{1}(c > 0)\mathbf{1}(i = 0) - xp + \varepsilon(x) + \beta \sum_{c',p'} E_{\varepsilon'}[V(i', c', p', \varepsilon')] \Pr(c', p'|c, p, x)$$

subject to

$$i' = \min\{\bar{i}, i + x - c\}$$

$$c' = \begin{cases} 0 & \text{with probability } \gamma \\ 1 & \text{with probability } 1 - \gamma \end{cases}$$

$$p' = \begin{cases} p_s = 0.5 & \text{with probability } \pi(p) \\ p_r = 2 & \text{with probability } 1 - \pi(p) \end{cases}$$

where λ is a penalty for stocking out and $\varepsilon(x)$ is an extreme-value distributed random utility shock associated with choice x. Consumption and price state variables follow discrete Markov processes. Assume that consumers need the good with probability $\gamma = \frac{1}{2}$. The Markov process for the price is given by the following matrix:

$$\Pi = \begin{pmatrix} 1 - \pi(p_r) & \pi(p_r) \\ 1 - \pi(p_s) & \pi(p_s) \end{pmatrix} = \begin{pmatrix} 0.75 & 0.25 \\ 0.95 & 0.05 \end{pmatrix}$$

where the first column corresponds to transitions to the regular price p_r and the second column corresponds to transitions to the sales price p_s. Consider $p_r = 2$ and $p_s = \frac{1}{2}$.

Assume $\beta = 0.99$, $\alpha = 2$, $\lambda = -3$, $\bar{i} = 4$, and $\gamma = \frac{1}{2}$.

1. Define the discrete state vector as $s = (i, c, p)$. What are the choice-specific state-to-state transition probability matrices (i.e. $F(s'|s, x = 0)$ and $F(s'|s, x = 1)$)?

2. Write down the contraction mapping that implicitly defines the expected value function $v(s) = E_\varepsilon[V(s,\varepsilon)]$. Numerically solve for the value function, and tabulate its value against the discrete state variable s.

3. Simulate a sequence of $T = 10,000$ periods from the model. Report the following summary statistics: frequency of positive purchases, probability of purchasing on sale, average duration between sales, and average duration between purchases.

4. Use the simulated sequence of choices and states to calculate the conditional-choice probability vector $\hat{P}(s)$. At the true parameters, calculate the implied expected value function $\hat{v}(s|\hat{P})$ using the CCP mapping. Compare and tabulate this estimate with the true value function calculated in question 2. Discuss the differences.