Dynamic Discrete Choice

Problem Set 2
Empirical Industrial Organization

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The value function is given by

$$\begin{aligned} V(i,c,p,\epsilon_t) &= \max_{x\in\{0,1\}} \{u(i,c,p,x)+\epsilon(x)+\beta\mathbb{E}[V(i',c',p',\epsilon_{t+1})|i,c,p,x]\} \\ &= \max_{x\in\{0,1\}} \left\{u(i,c,p,x)+\epsilon(x)+\beta\sum_{i',c',p'}\mathbb{E}_{\epsilon_{t+1}}[V(i',c',p',\epsilon_{t+1})|i,c,p,x]\Pr(i',c',p'|i,c,p,x)\right\} \end{aligned}$$

The utility function $u(i,c,p,x)$ is given by

$$u(i,c,p,x) = -\lambda\mathbb{1}(c>0)\mathbb{1}(i=0)+\alpha c - xp$$

In terms of the variables (data that we have),

- i is the inventory level.
- c is the consumer's purchase decision (firm's sales).
- p is the price.
- x is the firm's purchase decision.
- $\epsilon(x)$ is choice specific random utility shock

In terms of the parameters (to be estimated, but actually given in this problem),

- $\lambda = -3$ is the penalty of stocking out (the consumer wants to buy, but the firm does not have the product).
- $\alpha = 2$ is the marginal utility of selling the product.
- $\beta = 0.99$ is the discount factor.

The variables follow a certain process. Here, we assume that the variables follow discrete Markov process. The variables in the next period:

- Inventory i will be the current level + firm's purchase - sales:

$$i' = \min\{\bar{i} = 4, i + x - c\}$$

- Consumer's purchase decision c (firm's sales):

$$c' = \begin{cases} 0 & \text{with probability } \gamma = 0.5 \\ 1 & \text{with probability } 1 - \gamma = 0.5 \end{cases}$$

- Price p with two discrete states $p_s = 0.5$ and $p_r = 2$:

$$\Pi = \begin{pmatrix} 0.75 & 0.25 \\ 0.95 & 0.05 \end{pmatrix}$$

1 Question 1: Transition Probability

Since we have discrete state variables (i, c, p), the transition probability can be expressed in matrix form. Moreover, the transition of c and p are independent of each other, i and x. We only specify the transition of i here which takes values from 0 to 4.
When $x = 0$,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

When $x = 1$,

$$\begin{pmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Then the transition probability matrix for state s is given by the Kronecker product of the transition matrices of i, c, and p, which is $P_s(x) = P_i(x) \otimes P_c \otimes P_p$.

2 Question 2: Expected Value Function

2.1 Expected/Intermediate Value Function $\bar{V}(i, c, p)$

We denote $\bar{V}(i, c, p) = \mathbb{E}_\epsilon[V(i, c, p, \epsilon)]$ as the expected value function (I used to call it intermediate value function).

$$\bar{V}(i,c,p) = \sum_{x\in\{0,1\}} P(x|i,c,p)\left\{u(i,c,p,x) + \mathbb{E}[\epsilon(x)|i,c,p,x] + \beta \sum \bar{V}(i',c',p')\,\Pr(i',c',p'|i,c,p,x)\right\}$$

2.2 Numerical solution of $\bar{V}(i,c,p)$

3 Question 3: Simulation

4 Question 4: Estimate $\bar{V}(i,c,p)$ using CCP method