MRes Advanced Environmental Economics
Problem Set

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1. Working with spatial data

   (a) Answered in code.

   (b) Answered in code.

   (c) We plot the spatial distribution of cropland by irrigation status in 2021 in the province of Almeria in Figure 1 below.

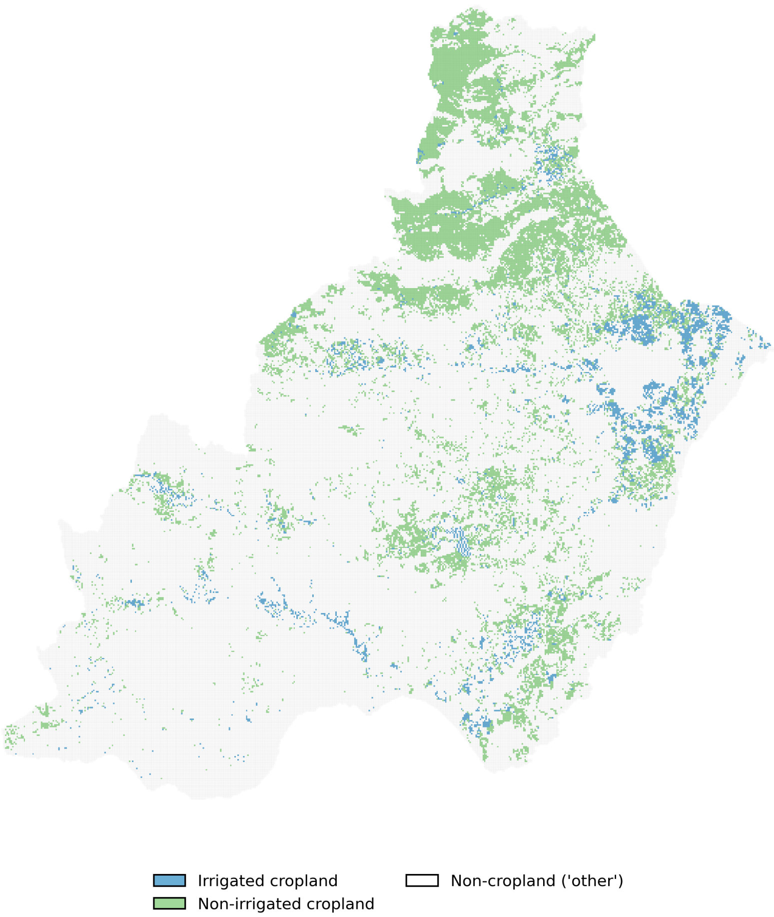

Figure 1: Cropland by irrigation status in Almeria, 2021

(d) We report the total cropland area devoted to each crop (averaging across all years) in Table 1 below and plot changes in irrigated area over time by crop and across all crops in Figures 2 and 3. Since we observed in the data that some land classified as 'Other' in land use was also irrigated, we additionally include land use category 'Other' in the plots as well.

Table 1: Area by crop

Crop	Mean annual area (in hectares)
Arable	149535.156250
Citric	9535.937500
Nuts	59607.031250
Olive	9780.468750

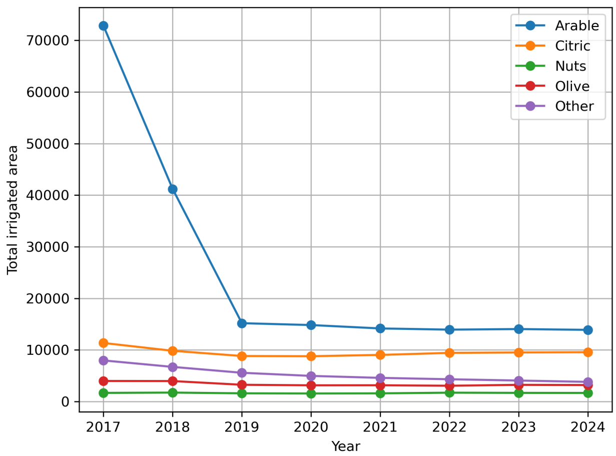

Figure 2: Total irrigated area by crop over time

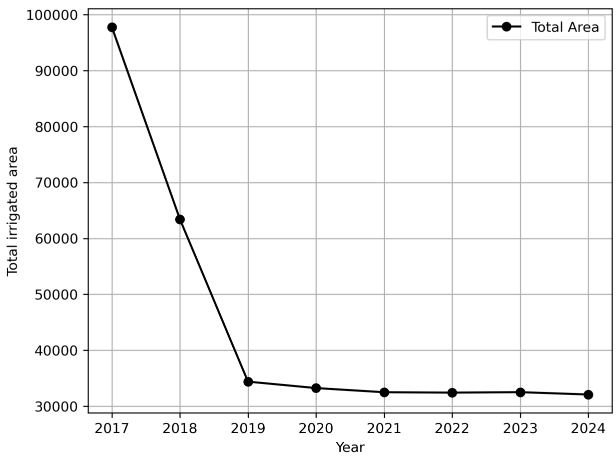

Figure 3: Total irrigated area over time

We find that the total irrigated area has decreased from 2017 to 2019 and remained stable afterwards. This decrease was largely driven by a decrease in irrigation of arable crops. This may indicate especially harsh weather in 2017-2018 necessitating an increase in irrigation.

3. Identification

(a) Taking the difference between two crops j and a, we have

$$\begin{aligned}\psi_j(k,\omega) - \psi_a(k,\omega) =& [V(k,w) - \pi_j(k,\omega) - \beta V(j,\omega') - \beta e^v(j,\omega,\omega')] \\ &- [V(k,w) - \pi_a(k,\omega) - \beta V(a,\omega') - \beta e^v(a,\omega,\omega')] \\ =& \pi_j(k,\omega) - \pi_a(k,\omega) - \beta[V(j,\omega') - \beta V(a,\omega')] \\ &- \beta[e^v(j,\omega,\omega') - \beta e^v(a,\omega,\omega')]\end{aligned} \tag{1}$$

(b) The difference between conditional value functions of j_{re} from different field states k and k' may be expressed as

$$\begin{aligned}v_{j_{re}}(k_{imt},\omega_{mt}) - v_{j_{re}}(k'_{imt},\omega_{mt}) =& (\pi_{j_{re}}(k_{imt},\omega_{mt}) + \beta\mathbb{E}[V(j_{re},\omega_{m,t+1}|j_{re},k_{imt},\omega_{mt}]) \\ &- (\pi_{j_{re}}(k'_{imt},\omega_{m,t+1}) + \beta\mathbb{E}[V(j_{re},\omega_{m,t+1}|j_{re},k'_{imt},\omega_{mt}]) \\ =& \pi_{j_{re}}(k_{imt},\omega_{mt}) - \pi_{j_{re}}(k'_{imt},\omega_{mt})\end{aligned} \tag{2}$$

(c) Substituting Equation 2 into Equation 1, we derive

$$\begin{aligned}\psi_j(k,\omega)-\psi_a(k,\omega)=&\pi_j(k,\omega)-\pi_a(k,\omega)-\beta[\pi_J(j,\omega')-\pi_J(a,\omega')+\psi_J(j,\omega')-\psi_J(a,\omega')]\\&-\beta[e^v(j,\omega,\omega')-e^v(a,\omega,\omega')]\end{aligned} \tag{3}$$

Grouping ψ terms on the LHS,

$$\begin{aligned}\psi_j(k,\omega)-\psi_a(k,\omega)+\beta\psi_J(j,\omega')-\beta\psi_J(a,\omega')=&\pi_j(k,\omega)-\pi_a(k,\omega)-\beta[\pi_J(j,\omega')-\pi_J(a,\omega')]\\&-\beta[e^v(j,\omega,\omega')-e^v(a,\omega,\omega')]\end{aligned} \tag{4}$$

(d) Substituting $\psi_j(k,w)=\gamma-\ln(p_j(k,\omega))$, we derive

$$\begin{aligned}\ln\frac{p_a(k,\omega_t)}{p_j(k,\omega_t)}+\beta\ln\frac{p_J(a,\omega_{t+1})}{p_J(j,\omega_{t+1})}=&\pi_j(k,\omega_t)-\pi_a(k,\omega_t)-\beta[\pi_J(j,\omega_{t+1})-\pi_J(a,\omega_{t+1})]-\\&\beta[e^v(j,\omega_t,\omega_{t+1})-e^v(a,\omega_t,\omega_{t+1})]\\=&[\theta_j+\theta_{kj}+\theta^R R_j^e(\omega_t)+\xi_j(k,\omega_t)]-\\&[\theta_a+\theta_{ka}+\theta^R R_a^e(\omega_t)+\xi_a(k,\omega_t)]-\\&\beta[(\theta_J+\theta_{jJ}+\theta^R R_J^e(\omega_{t+1})+\xi_J(j,\omega_{t+1})-\\&(\theta_J+\theta_{aJ}+\theta^R R_J^e(\omega_{t+1})+\xi_J(a,\omega_{t+1})]-\\&\beta[e^v(j,\omega_t,\omega_{t+1})-e^v(a,\omega_t,\omega_{t+1})]\\=&\theta_j+\theta_{kj}-\theta_a-\theta_{ka}-\beta\theta_{jJ}+\beta\theta_{aJ}+\theta^R(R_j^e(\omega_t)-R_a^e(\omega_t))+\\&\xi_j(k,\omega_t)-\xi_a(k,\omega_t)-\beta[e^v(j,\omega_t,\omega_{t+1})-e^v(a,\omega_t,\omega_{t+1})]\end{aligned} \tag{5}$$

Replacing $k=j$ and $J=a$ gives

$$\begin{aligned}\ln\frac{p_a(j,\omega_t)}{p_j(j,\omega_t)}+\beta\ln\frac{p_a(a,\omega_{t+1})}{p_a(j,\omega_{t+1})}=&\theta_j+\theta_{jj}-\theta_a-\theta_{ja}-\beta\theta_a+\beta\theta_{aa}+\theta^R[R_j^e(w_t)-R_a^e(w_t)]\\&+\xi_a(j,\omega_t)-\beta\xi_a(j,\omega_{t+1})-\beta e^v(j,\omega_t,\omega_{t+1})+\beta e^v(a,\omega_t,\omega_{t+1})\end{aligned} \tag{6}$$

The resulting regression equation may be written

$$Y_{j,t} = \Delta\theta_{j,t} + \theta_R \Delta R_{j,t} + \tilde{\Delta}\xi_{j,t} + \Delta e_{j,t}, \tag{7}$$

where $Y_{j,t} = \ln \frac{p_a(j,\omega_t)}{p_j(j,\omega_t)} + \beta \ln \frac{p_a(a,\omega_{t+1})}{p_a(j,\omega_{t+1})}$, $\Delta\theta_{j,t} = \theta_j + \theta_{jj} - \theta_a - \theta_{ja} - \beta\theta_{ja} + \beta\theta_{aa}$, $\Delta R_{j,t} = R^e_j(w_t) - R^e_a(w_t)$, $\Delta\xi_{j,t} = \xi_a(j,\omega_t) - \beta\xi_a(j,\omega_{t+1})$, and $\Delta e_{j,t} = -\beta e^v(j,\omega_t,\omega_{t+1}) + \beta e^v(a,\omega_t,\omega_{t+1})$.

(e) The identifying variation is variation in $\Delta R_{j,t}$. Variation in this variable is driven by (i) variation in the prices of crop j relative to crop a over time, and (ii) variation in the yields of crop j relative to crop a depending on the aggregate state ω_t.

(f) Since $\Delta R_{j,t}$ is the expected and not the realized return, it only depends on the choice of land use and ω_t, it is uncorrelated with $\tilde{\Delta}\xi_{j,t}$. Moreover, by construction, it is mean-uncorrelated with $\Delta e_{j,t}$. However, using fixed effects estimation raises an endogeneity issue. Fixed effects estimation requires that

$$\mathbb{E}[\Delta e_{j,t} \Delta R_{j,t+1}] = 0, \tag{8}$$

but since the expectational error in the composite error is the difference between the value function realization in $t+1$ and its expected value at time t, $\varepsilon_{j,t}$ is endogenous to $R_{j,t+1}$. We therefore take first differences, deriving

$$\begin{aligned} Y_{j,t+1} - Y_{j,t} =& \Delta\theta_{j,t+1} - \Delta\theta_{j,t} + \theta_R(\Delta R_{j,t+1} - \Delta R_{j,t}) + \tilde{\Delta}\xi_{j,t+1} - \tilde{\Delta}\xi_{j,t} + \Delta e_{j,t+1} - \Delta e_{j,t} \\ =& \theta_R(\Delta R_{j,t+1} - \Delta R_{j,t}) + \tilde{\Delta}\xi_{j,t+1} - \tilde{\Delta}\xi_{j,t} + \Delta e_{j,t+1} - \Delta e_{j,t} \end{aligned} \tag{9}$$

The moment condition then writes:

$$\mathbb{E}[Z_{j,t}(\Delta u_{j,t+1} - \Delta u_{j,t})] = 0, \tag{10}$$

where $\Delta u_{j,t}$ denotes the composite error term, $\tilde{\Delta}\xi_{j,t} + \Delta e_{j,t}$, and $Z_{j,t}$ is an instrument for $\Delta R_{j,t+1} - \Delta R_{j,t}$. A possible instrument may be $Z_{j,t} = \Delta R_{j,t}$, which is uncorrelated with the composite first-differenced error by the rational expectations assumption.

(g) We make these assumptions in the estimation of switching costs.

4. Estimation

(a) We deal with zero probabilities in the following way. First, following Scott (2013), we smooth choice probability estimates by taking an average of frequency estimates, weighted by distances between provinces (weight inversely proportional to distance). Second, since some zero probabilities remain in our data as some crop choices are never made across the two provinces we observe, we replace zero probabilities with ε, where $\varepsilon = 10^{-6}$.

(b) We separately regress realized subcrop yields on days over 10 °C, days over 30 °C, precipitation and interactions between the first two weather variables and precipitation, and year and province dummies. We then predict yearly yields for each subcrop in each province based on weather variables.

(c) We construct the dependent variable $Y_{j,t+1} - Y_{j,t}$ based on transition probabilities for each k, j, a, J combination in each province and year.

(d) Using the proposed instrument $\Delta R_{j,t}$, we estimate the elasticity to be 0.0003($s.e.$0.0000).

(e) Since the observable variables are at the province level, they do not allow for sufficient variation for yields to be estimated well. This results in large differences between predicted and actual returns.

(f) We recover the estimated switching costs following this procedure. First, we note for a $k - j - a - J$ combination, the constant in the RHS of the regression equation derived above evaluates to $\theta_j + \theta_{kj} - \theta_a - \theta_{ka} - \beta\theta_{jJ} + \beta\theta_{aJ}$. After obtaining these terms from a regression of $Y_{j,t} - \hat{\theta}_R \Delta R_{j,t}$ on a vector of ones for each $k - j - a - J$ combination.

Then, using the assumption that $\theta_{kj} = 0$ for $j = Other$ and for any state k, we are able to obtain $\theta_j + \theta_{kj} - \theta_a - \theta_{ka}$ for $k - j - a - J$ combinations where $J = other$. Taking the constant for a $k - j - a - J'$ combination, where $J' \neq Other$ and subtracting that from a $k - j - a - J$ combination, where $J = Other$, leaves us with $-\beta\theta_{jJ} + \beta\theta_{aJ}$ for each $k - j - a - J$ combination. Then, we simply compute $\beta\theta_{aJ} = -\beta\theta_{jJ} + \beta\theta_{aJ}$ for cases in which $j = J$ such that the switching cost is 0 to derive an estimate $\hat{\theta}_{aJ}$ for each $a - J$ combination possible.

(g) By considering irrigation switching as irrigated to non-irrigated or non-irrigated to irrigated transitions for the same crop, and crop switching as switching between crop types of the same irrigation status, we estimate a mean irrigation switching cost of -8.76 and a mean crop switching cost of -7.41.

(h) To estimate the elasticity for myopic agents, we simply set $\beta = 0$. We estimate an elasticity of $-0.0001(s.e.0.0000)$. To estimate the elasticity in a static setting, we re-compute choice probabilities not taking state transitions into account (i.e. we only compute the choice probability of j versus the choice probability of a based on their frequencies in the data, regardless of what choice was made in the past of future) for our LHS variable. We estimate an elasticity of $0.0000(s.e.0.0000)$. It is intuitive that the estimated elasticities under these different model assumptions are smaller as they do not account for the dynamic changes in returns that land owners consider when making decisions about which crop to plant.