

# Optimal Urban Transportation Policy

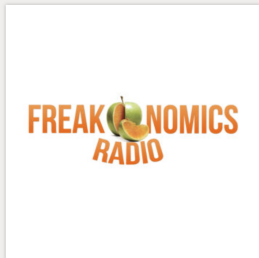
Evidence from Chicago

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Presented by Zixuan

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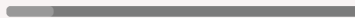
EPISODE 513

## Should Public Transit Be Free? (Update)

It boosts economic opportunity and social mobility. It's good for the environment. So why do we charge people to use it? The short answer: it's complicated. Also: We talk to the man who gets half the nation's mass-transit riders where they want to go (most of the time).



00:00



58:10



When it comes to transportation policy,

- Free public transit?
- High quality service?
- Road pricing (cordon tax)?

# Introduction

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We consider three modes of transportation:

- **Public transit:** fare  $p$ , frequency  $k$ 
  - Train/Metro
  - Bus
- Driving: **road pricing**  $p$
- Ride hailing (taxi, uber): fare  $p$

## Question

Assuming that the government has control over the **public transit** and **road pricing**, what is the optimal price  $p$  and quality  $k$  that maximizes **social welfare**, while taking into account **environmental costs** and **budget constraint**?

For each transportation mode, define

- $p_j$ : the price (fare, road tax etc.)
- $k_j$ : the frequency (quality measure)
- $q_j$ : the total demand for mode  $j$
- $t_j$ : the time spent on mode  $j$

The social welfare function  $U(p, t) - C(q, k) - E(q, k)$  consists of three components:

- Gross utility:  $U(p, t)$
- Cost of providing the service:  $C(q, k)$
- Environmental cost:  $E(q, k)$

The budget constraint is given by

$$B + \sum_{j \in G} (p_j q_j - C_j) \geq 0$$

## Problem

The optimal transportation policy is essentially solving for the set of **prices  $p$**  and **frequencies  $k$**  of the following the constrained problem

$$\begin{aligned} \max_{p_j, k_j} \quad & U(p, t) - C(q, k) - E(q, k) \\ \text{s.t.} \quad & B + \sum_{j \in G} (p_j q_j - C_j) \geq 0 \end{aligned}$$

# Model

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## Demand side $q(p, t)$

A traveler with type  $\theta$  chooses mode  $j^*$  such that

$$j^*(\theta) = \arg \max_{j \in \mathcal{J}(\theta) \cup \{0\}} u_j(t_j, \theta) - p_j$$

Explicitly,

$$\max_{j \in \mathcal{J}_m^i \cup \{0\}} \xi_{mj} + \alpha_T \cdot T_{mj} + \alpha_p^i \cdot p_{mj} + \epsilon_{mj}^i$$

The choice probability is given by

$$\mathbb{P}_{mj}^i = \frac{\exp\left(\frac{\delta_{mj}^i}{1-\rho}\right)}{\left[\sum_{j \in g} \exp\left(\frac{\delta_{mj}^i}{1-\rho}\right)\right]^\rho \cdot \left(\sum_g \left[\sum_{j \in g} \exp\left(\frac{\delta_{mj}^i}{1-\rho}\right)\right]^{(1-\rho)}\right)}$$

Given vectors of  $p$  and  $t$ , demand for mode  $j$  is given by

$$q_j = q_j(p, t) = \int_{\Theta_j(p, t)} f(\theta) d\theta$$

Explicitly,

$$\mathbb{P}_{mj} = \int \mathbb{P}_{mj}^i dF_m(\alpha_p^i), \quad q_{mj} = N_m \cdot \mathbb{P}_{mj}$$

## Demand side $q(p, t)$

Gross utility is given by the sum of each mode's utility

$$U(p, t) = \sum_{j \in J} \int_{\Theta_j(p, t)} u_j(t_j, \theta) f(\theta) d\theta$$

### Demand & Supply

Demand can be written as a function of price  $p$  and travel time  $t$ . That is,

$$q = q(p, t)$$

However note that travel time  $t$  is not exogenously given. It is affected by the demand  $q$  as well as the frequency  $k$ . That is,

$$t = t(p, k)$$

To make the travel time function explicit,

$$T_{mj} = \gamma \cdot (T_{mj}^{\text{walk}} + T_{mj}^{\text{wait}}) + T_{mj}^{\text{vehicle}}$$

For waiting time,

- public transit: taking into account the reliability
- ride hailing: taking into account idle drivers in the vicinity

For in vehicle time, it takes into account **congestion**. When the total flow of vehicle on an edge is

- below a certain threshold: the in vehicle time is the *free flow* time suggested by Google Maps.
- above: a function of the total flow on the edge.

### Equilibrium

Given price  $p$  and frequency  $k$ , an equilibrium is a vector of  $q$  and  $t$  such that

$$q = q(p, t), \quad t = t(p, k)$$

Solving the equilibrium is essentially finding the fixed point of the function

$$f_{p,k}(q) = q(p, t(p, k))$$

The author uses a limited-memory version of Broyden's method to find the root of  $f_{p,k}(q) - q = 0$ .

1. The optimal policy  $p^*, k^*$  is derived from iteratively maximizing problems that approximate the Lagrangian of the main problem.
2. Every evaluation of the Lagrangian requires solving for the equilibrium  $q^*$  and  $t^*$ .

# Estimation

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Recall

$$U_{mj}^i = \xi_{mj} + \alpha_T \cdot T_{mj} + \alpha_p^i \cdot p_{mj} + \epsilon_{mj}^i$$

where  $\alpha_p^i = \frac{\alpha_p}{y_i^{1-\alpha_{py}}}$  and  $\xi_{mj}$  is the unobserved demand shock.

- Endogeneity of price: the price for ride hailing is endogenous, while assuming that the price of public transit and driving is exogenous to  $\xi_{mj}$ .
- Endogeneity of travel time: higher  $\xi_{mj}$  is associated with higher travel time. If this source of endogeneity is not addressed, the time coefficient is biased towards 0.



We have two types of moment conditions.

The first one targets the price coefficient  $\alpha_p$ .

$$\mathbb{E} [(\hat{\eta}_{mj} - \tilde{\eta}_{mj}) \mathbb{1}\{j = \text{ride-hail}, m \in \mathcal{M}_\tau\}] = 0$$

where  $\hat{\eta}_{mj}$  is the ride-hailing elasticity from our differences-in-differences estimate (Appendix B),  $\tilde{\eta}_{mj}$  is the model-implied elasticity, and  $\mathcal{M}_\tau$  are the markets affected by the surcharge policy.

The second one targets the travel time coefficient  $\alpha_T$  and substitution parameter  $\rho$ .

1. instrument for travel time: free-flow time  $T_{mj}^0$  which do not depend on the vehicle flow and therefore not affected by within day demand shock.
2. instrument for the substitution parameter  $\rho$ : Gandhi and Houde (2019)

$$\mathbb{E} [Z_{mj}\xi_{mj}] = 0$$

**Table 1:** Demand estimation results

	Pooled					Peak/Off-Peak	
	(1)	(2)	(3)	(4)	(5)	Peak	Off-peak
$\alpha_T$	-1.068 (0.011)	-1.692 (0.022)	-2.345 (0.023)	-2.415 (0.023)	-1.928 (0.018)	-1.824 (0.022)	-1.872 (0.027)
$\alpha_p$	-0.058 (0.001)	-0.155 (0.002)	-8.461 (0.492)	-3.416 (0.111)	-2.078 (0.09)	-2.388 (0.225)	-1.657 (0.068)
$\alpha_{py}$	.	.	-1.262 (0.039)	-0.588 (0.02)	-0.414 (0.022)	-0.696 (0.048)	-0.152 (0.022)
$\rho$	.	.	.	.	0.262 (0.012)	0.376 (0.017)	0.162 (0.017)
Estimator	OLS	IV	GMM	GMM	GMM	GMM	GMM
Policy Moment			✓	✓	✓	✓	✓
Car Ownership				✓	✓	✓	✓
Nest					✓	✓	✓
Avg. VOT	18.41	10.89	23.88	14.65	13.47	19.47	9.81
VOT (Bot. Quintile)	.	.	2.44	3.26	3.62	3.9	3.42
VOT (Top Quintile)	.	.	64.24	32.36	27.94	45.32	18.09
Avg. Price Elast.	-0.2	-0.53	-0.5	-0.61	-0.65	-0.55	-0.72
Avg. Time Elast.	-0.58	-0.91	-1.26	-1.27	-1.29	-1.44	-1.07
M	92,284	92,284	91,908	91,561	91,561	42,989	48,572
N	281,755	281,755	281,042	280,185	280,185	136,337	143,848

The estimation equation is

$$\log T_{ehj}^{\text{vehicle}} = a_e + \beta_j \log F_{eh} + \epsilon_{ehj}$$

**Table 2:** Traffic congestion estimation results

	Bus			Car		
	(1)	(2)	(3)	(4)	(5)	(6)
Log Flow	0.092*** (0.006)	0.059*** (0.006)	0.101*** (0.008)	0.128*** (0.005)	0.100*** (0.005)	0.168*** (0.004)
Edge FE	✓	✓	✓	✓	✓	✓
Weather controls		✓	✓		✓	✓
IV			✓			✓
within $R^2$	0.093	0.129	0.116	0.411	0.529	0.443
First-stage F		2767.096			4487.258	
Observations	7962	7962	7962	11739	11739	11739

1. Estimate demand side parameters  $\theta$  to get  $q(p, t)$
2. Estimate supply side/congestion parameters  $\beta$  to get  $t(p, k)$
3. Solve the optimal policy  $p^*$  and  $k^*$ .
4. Compare the welfare under the status quo and the optimal policy.

# Results

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The social planner's problem can be rewritten as the Lagrangian

$$\mathcal{L}(p, k, \lambda) = U(p, t(q, k)) - C(q, k) - E(q, k) + \lambda \left( B + \sum_{j \in G} (p_j q_j - C_j(q_j, k_j)) \right)$$

where  $q^*$  is determined by the equilibrium condition  $f_{p,k}(q) - q = 0$  (implicit function of  $p$  and  $k$ ).



**Proposition 1.** Prices under the solution of the social planner's problem (4) are given by:

$$p_j = \overbrace{C_j^q + E_j^q}^{\text{Mg. cost and env. externality}} - \overbrace{\sum_l u_l^T \cdot T_{lj}^q}^{\text{Network effects}} + \overbrace{\tilde{M}_j^q}^{\text{Diversion}} + \frac{\lambda}{1+\lambda} \cdot \left( \underbrace{\sum_{k \in \mathcal{J}_G} q_k \cdot \Omega_{kj} - E_j^q}_{\text{Market power markup}} - \underbrace{\sum_l (\tilde{u}_l^T - u_l^T) \cdot T_{lj}^q}_{\text{Spence distortion}} + \underbrace{\tilde{M}_j^q - M_j^q}_{\text{Diversion distortion}} \right) \quad (5)$$

where  $\lambda$  is the Lagrange multiplier for the budget constraint,  $\tilde{u}_j^T$  is a weighted sum of the derivative of gross utility among marginal travelers with respect to mode- $j$  travel time, and  $M_j^q$  and  $\tilde{M}_j^q$  are defined as:

$$M_j^q \equiv \sum_{k \neq j} D_{kj} \left( C_k^q + E_k^q - \sum_l u_l^T \cdot T_{lk}^q - p_k \right) \quad (6)$$

$$\tilde{M}_j^q \equiv \sum_{l \in \mathcal{J}_G \setminus j} D_{lj} \left( C_l^q + \sum_{k \in \mathcal{J}_G} q_k \cdot \Omega_{kj} - \sum_m \tilde{u}_m^T \cdot T_{ml}^q - p_l \right). \quad (7)$$

## C.2 Proof of Propositions 1 and 2

*Proof.* The Lagrangian for the social planner's problem is:

$$U(\mathbf{q}, T(\mathbf{q}, \mathbf{k})) - C(\mathbf{q}, \mathbf{k}) - E(\mathbf{q}, \mathbf{k}) - \lambda \left( \sum_{j \in \mathcal{J}_G} [C_j(q_j, k_j) - p_j(\mathbf{q}, T(\mathbf{q}, \mathbf{k}))q_j] - B \right).$$

In this expressions,  $\mathbf{q}$  is a function of  $(\mathbf{p}, \mathbf{k})$  given by market equilibria.

The first order condition for  $p_j$  is:

$$\sum_l \frac{\partial q_l}{\partial p_j} \left[ \frac{\partial U}{\partial q_l} + \sum_m u_m^T T_{ml}^q - C_l^q - E_l^q + \lambda \left( p_l + \sum_m q_m \frac{dp_m}{dq_l} - C_l^q \right) \right] = 0. \quad (16)$$





Table 4: Counterfactual results

		Status Quo (1)	Transit (2)	Transit, Budget (3)	Road Pricing (4)	Transit + Road Pricing (5)
<b>Panel A: Prices</b>						
Avg. Price (\$)	Bus	1.09	-0.33	0.65	1.09	0.16
	Train	1.33	-0.37	1.02	1.33	0.26
Road Tax (\$/km)		0	0	0	0.35	0.32
<b>Panel B: Wait Times and Frequencies</b>						
Avg. Wait (min)	Bus	7.06	7.15	8.19	7.06	7.15
	Train	4.37	4.05	4.55	4.37	4.06
Δ Frequency	Bus	0%	-1.33%	-13.88%	0%	-1.24%
	Train	0%	9.21%	-2.58%	0%	8.91%

Panel C: Trips

	Bus	3.7	5.0	3.8	4.2	5.0
Number	Train	2.7	3.5	2.8	2.9	3.5
Of Trips	Ride-hailing	3.0	2.9	3.0	3.1	3.1
(M/week)	Car	21.3	20.5	21.2	17.6	17.4
	Total	30.6	31.9	30.7	27.8	28.9

Panel D: Welfare

Δ Welfare (\$M/week)	0	1.54	0.39	4.57	5.27
Δ CS (\$M/week)	0	12.65	0.03	-29.11	-18.54
Δ City Surplus (\$M/week)	0	-10.99	0	28.31	18.96
Δ Transit Surplus (\$M/week)	0	-10.99	0	0.82	-6.32
Road Taxes (\$M/week)	0	0	0	27.49	25.28
Δ Externalities (\$M/week)	0	-0.62	-0.35	-3.59	-3.69

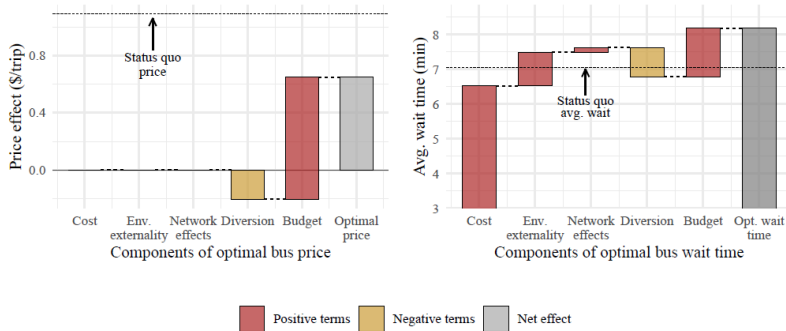


Figure 9: Optimal bus price and wait time decomposition for *Transit + Budget*

*Notes:* This graph shows a decomposition of the optimal prices and travel times for buses corresponding to our theoretical decomposition in Section 3.2. Red bars indicate terms that lead prices and travel times to be higher and yellow bars indicate terms that lead them to be lower.

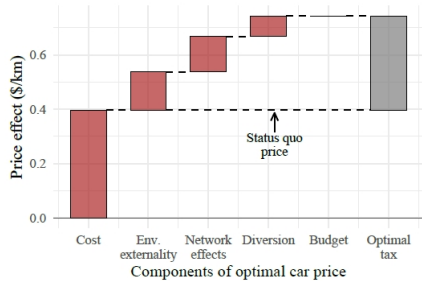


Figure 10: Optimal car price decomposition in the *Road Pricing* scenario

*Notes:* This figure shows the price decomposition for cars, following our theoretical derivations in Section 3.2. Red bars indicate terms that lead optimal car prices to be higher.

# Conclusion

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- Spill over effect across different hours (market)
- Joint decision of outbound and inbound trips
- Relocation effect
- Route reoptimization

Government can undo the “monopoly” distortions that arise due to **budget constraint** by using road pricing revenues to **cross-subsidize** public transit.

Indeed, recent transit policies in London and New York explicitly designate the revenues from road pricing to fund public transit. **Our results highlight that such combined policy approaches can eliminate inefficiencies.**

Thanks

### References

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Gandhi, A. and Houde, J.-F. (2019). Measuring substitution patterns in differentiated-products industries. *NBER Working paper*, (w26375).