

STAT 302: Bonus for HW

Section 201 - Ed Kroc

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A federal committee of three people is to be randomly selected from a group consisting of four Liberals, three Conservatives, and two NDPers (sorry Greens and Bloc). Let L denote the number of Liberals on the committee, and let C denote the number of Conservatives on the committee.

(c) [BONUS for HW:] Let N denote the number of NDPers on the committee. Find the joint probability mass function of L , C , and N by directly generalizing our definitions. Compute the marginals for each party.

Solution

The joint probability function is:

Table when $N = 0$

| $\Pr(L=l, C=c)$ | $L=0$ | $L=1$ | $L=2$ | $L=3$ |
|-----------------|--------|---------|---------|--------|
| $C=0$ | 0 | 0 | 0 | $4/84$ |
| $C=1$ | 0 | 0 | $18/84$ | 0 |
| $C=2$ | 0 | $12/84$ | 0 | 0 |
| $C=3$ | $1/84$ | 0 | 0 | 0 |

Table when $N = 1$

| $\Pr(L=l, C=c)$ | $L=0$ | $L=1$ | $L=2$ | $L=3$ |
|-----------------|--------|---------|---------|-------|
| $C=0$ | 0 | 0 | $12/84$ | 0 |
| $C=1$ | 0 | $24/84$ | 0 | 0 |
| $C=2$ | $6/84$ | 0 | 0 | 0 |
| $C=3$ | 0 | 0 | 0 | 0 |

Table when $N = 2$

| Pr(L=1, C= c) | L=0 | L=1 | L=2 | L = 3 |
|---------------|------|------|-----|-------|
| C=0 | 0 | 4/84 | 0 | 0 |
| C=1 | 3/84 | 0 | 0 | 0 |
| C=2 | 0 | 0 | 0 | 0 |
| C=3 | 0 | 0 | 0 | 0 |

The marginal probability functions are:

For Conservatives:

$$\begin{aligned}
p_C(c) &= \sum_{all\ l,n} p(c, l, n) \\
&= p(c, 0, 0) + p(c, 0, 1) + p(c, 0, 2) \\
&\quad + p(c, 1, 0) + p(c, 1, 1) + p(c, 1, 2) \\
&\quad + p(c, 2, 0) + p(c, 2, 1) + p(c, 2, 2)
\end{aligned} \tag{1}$$

| C=c | C=0 | C=1 | C=2 | C = 3 |
|----------|-------|-------|-------|-------|
| $p_C(c)$ | 20/84 | 45/84 | 18/84 | 1/84 |

For Liberals:

$$\begin{aligned}
p_L(l) &= \sum_{all\ c,n} p(c, l, n) \\
&= p(0, l, 0) + p(0, l, 1) + p(0, l, 2) \\
&\quad + p(1, l, 0) + p(1, l, 1) + p(1, l, 2) \\
&\quad + p(2, l, 0) + p(2, l, 1) + p(2, l, 2) \\
&\quad + p(3, l, 0) + p(3, l, 1) + p(3, l, 2)
\end{aligned} \tag{2}$$

| L=1 | L=0 | L=1 | L=2 | L = 3 |
|----------|-------|-------|-------|-------|
| $p_L(l)$ | 10/84 | 40/84 | 30/84 | 4/84 |

For NDPers:

$$\begin{aligned}
p_N(n) &= \sum_{all\ c,l} p(c, l, n) \\
&= p(0, 0, n) + p(0, 1, n) + p(0, 2, n) + p(0, 3, n) \\
&\quad + p(1, 0, n) + p(1, 1, n) + p(1, 2, n) + p(1, 3, n) \\
&\quad + p(2, 0, n) + p(2, 1, n) + p(2, 2, n) + p(2, 3, n) \\
&\quad + p(3, 0, n) + p(3, 1, n) + p(3, 2, n) + p(3, 3, n)
\end{aligned} \tag{3}$$

| N=n | N=0 | N=1 | N=2 |
|----------|-------|-------|------|
| $p_N(n)$ | 35/84 | 42/84 | 7/84 |