STAT302: Introduction to Probability **Winter 2017-18**

Assignment 1

Due date: see course website (Under Schedule).

Instructions

Academic integrity policy: We encourage you to discuss verbally with other students about the assignment. However, you should write your answers by yourself (in particular, without reading someone else's answer). If you use online resources (including asking questions on math exchange or similar), you *have* to cite them. If you have not done se already, read:
`http://learningcommons.ubc.ca/guide-to-academic-integrity/`

Instructions:

1. Include a cover sheet with your name, student number, course code, and assignment number.
2. Due date: see the course website.
3. Justify all your answers.
4. Write your answers in order, and clearly label the question you are answering, for example 1.1 for part 1 of question 1.
5. Assignments are due to be handed-in in the mailbox labelled 302 on the main floor of the ESB building. The mailbox is in the seating area by the computer labs (also a few steps from the public washroom of the main floor). The mailbox should be easy to find, but make sure you find it before the day the assignment is due. Do not bring your assignment at the beginning of the lecture: we do not collect them, the TAs pick them up from the mailbox only. Thank you!
6. No late assignments are accepted.

1 Polymerase chain reaction

1. PCR (Polymerase chain reaction) is an important technique critical for modern DNA sequencing. The idea is to use a sequence of *rounds* (biological reactions controlled by temperature changes) where (roughly speaking) every strand of DNA is duplicated. Once in a while a round fails, and for simplicity assume the amount of DNA stays constant in such round. Say we start with one billion copies of a DNA strand. Assume a round fails with probability half (in practise the failure rate would be much lower!). Under this simplistic model, what is the probability that there are exactly 8 billion copies immediately after 9 rounds? Find a general formula for the probability that there are exactly x billion copies after y rounds.
2. **(Optional)** Suppose you posit a different model for a failed round, where in such round the amount of DNA decreases by a factor two. Again assume equal probability for the failure or the success of a round. What is the probability that there are exactly 2 billion copies immediately after nine rounds? Find a general formula for the probability that there are exactly x copies after y rounds.

2 Hypothesis testing

1. A famous, controversial study performed the following experiment: Brochet, the author of the study, picked a few white wines, and dyed them in red. Then the wines were given to wine experts and the experts were asked to described the wines. Surprisingly, the wine experts used qualifications typically used for red wines (original study linked here, popular coverage linked here).

   Inspired by this, you invite over your wine lover friend. You pour eight wines, 4 red, 4 white, and ask her to identify, blindfolded, the 4 red wines. She correctly identifies 3 of the red wines and incorrectly selects 1 of the white wines. Thereafter, she claims to have debunked Brochet's study. What is the probability that your friend would have correctly identified at least 3 of the red wines if she were, in fact, guessing?

3 Gift exchange problem

1. Tom plans on throwing a holiday party for his friends. At this party, he decides he wants to organize a secret Santa style gift exchange. His plan is all follows: the gifts will be put into identical boxes (same color, same size, completely indistinguishable from one another) so no one is able to tell whose gift is in a box. The boxes in turn will be randomly distributed to each guest. Ideally, Tom would like for no one to end up with his/her own gift. Tom wants to find out what the probability is that no one is assigned his/her own gift after the exchange.

   Suppose there are four people at his party (including himself). In how many ways can the boxes be distributed so that exactly one person is assigned his/her own gift?

2. Tom is fickle. He changes his mind about the number of people he wants to invite. Suppose there are five people total now (including himself). In how many ways can the boxes be distributed so that no one is assigned his/her own gift? Do not use explicit enumeration in this question. Instead, introduce the number of ways, N_i, of having i people getting their own gift. What is $\sum_{i=0}^{5} N_i$?

3. If the boxes are randomly distributed in a party of 5, what is the probability that no one is assigned his/her own gift?

4. Tom knows that his best friend Finn, one of the other 5 people attending the party, gives exceptionally good gifts. However, Tom does not feel as confident in his own gift. What is the probability that Tom ends up receiving Finn′s gift, but Finn does not receive Tom′s gift?

4 Guessing answers on a test

1. Suppose you are given a test composed of ten problems. Not all problems on the test carry the same weight. The first eight problems are worth as much as each other and in total sum to sixty points. The last two problems are also worth as much as each other and in total sum to forty points. Each problem is a TRUE/FALSE problem. In order to pass, you need to score at least sixty points on the test.

   You forgot to study for the test. You decide to guess the answer to each problem. Your strategy is to not answer false for any two consecutive questions. In how many ways can you answer all ten questions on the test?

2. Your friend Nathan also forgot to study and decides to just guess the answer to each problem.

   Given that Nathan is able to correctly guess the answer to the last two problems, what is the probability that he passes the test?

3. Find the probability that Nathan passes the test. [Hint: First condition on the number of correctly answered questions among the last two problems.]