STAT 302: Bonus for HW
Section 201 - Ed Kroc

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A federal committe of three people is to be randomly selected from a group consisting of four Liberals, three Conservatives, and two NDPers (sorry Greens and Bloc). Let L denote the number of Liberals on the committee, and let C denote the number of Conservatives on the committee.

(c) [BONUS for HW:] Let N denote the number of NDPers on the committee. Find the joint probability mass function of L, C, and N by directly generalizing our definitions. Compute the marginals for each party.

Solution

The joint probability function is:

Table when N = 0

Pr(L=l, C= c)	L=0	L=1	L=2	L = 3
C=0	0	0	0	4/84
C=1	0	0	18/84	0
C=2	0	12/84	0	0
C=3	1/84	0	0	0

Table when N = 1

Pr(L=l, C= c)	L=0	L=1	L=2	L = 3
C=0	0	0	12/84	0
C=1	0	24/84	0	0
C=2	6/84	0	0	0
C=3	0	0	0	0

Table when N = 2

Pr(L=l, C= c)	L=0	L=1	L=2	L = 3
C=0	0	4/84	0	0
C=1	3/84	0	0	0
C=2	0	0	0	0
C=3	0	0	0	0

The marginal probability functions are:

For Conservatives:

$$\begin{aligned} p_C(c) &= \sum_{all\ \ l,n} p(c,l,n) \\ &= p(c,0,0) + p(c,0,1) + p(c,0,2) \\ &\quad + p(c,1,0) + p(c,1,1) + p(c,1,2) \\ &\quad + p(c,2,0) + p(c,2,1) + p(c,2,2) \end{aligned} \tag{1}$$

C=c	C=0	C=1	C=2	C = 3
$p_C(c)$	20/84	45/84	18/84	1/84

For Liberals:

$$\begin{aligned} p_L(l) &= \sum_{all\ \ c,n} p(c,l,n) \\ &= p(0,l,0) + p(0,l,1) + p(0,l,2) \\ &\quad + p(1,l,0) + p(1,l,1) + p(1,l,2) \\ &\quad + p(2,l,0) + p(2,l,1) + p(2,l,2) \\ &\quad + p(3,l,0) + p(3,l,1) + p(3,l,2) \end{aligned} \tag{2}$$

L=l	L=0	L=1	L=2	L = 3
$p_L(l)$	10/84	40/84	30/84	4/84

For NDPers:

$$\begin{aligned} p_N(n) &= \sum_{all\ \ c,l} p(c,l,n) \\ &= p(0,0,n) + p(0,1,n) + p(0,2,n) + p(0,3,n) \\ &\quad + p(1,0,n) + p(1,1,n) + p(1,2,n) + p(1,3,n) \\ &\quad + p(2,0,n) + p(2,1,n) + p(2,2,n) + p(2,3,n) \\ &\quad + p(3,0,n) + p(3,1,n) + p(3,2,n) + p(3,3,n) \end{aligned} \tag{3}$$

N=n	N=0	N=1	N=2
$p_N(n)$	35/84	42/84	7/84