Mathematics Applied In Our Life

Game Theory（博弈论）

Probability Theory（概率论）

Game

Your rival

You

	α	β
α	B- , B-	A , C
β	C , A	B+ , B+

Prisoner's Dilemma

prisoner 2

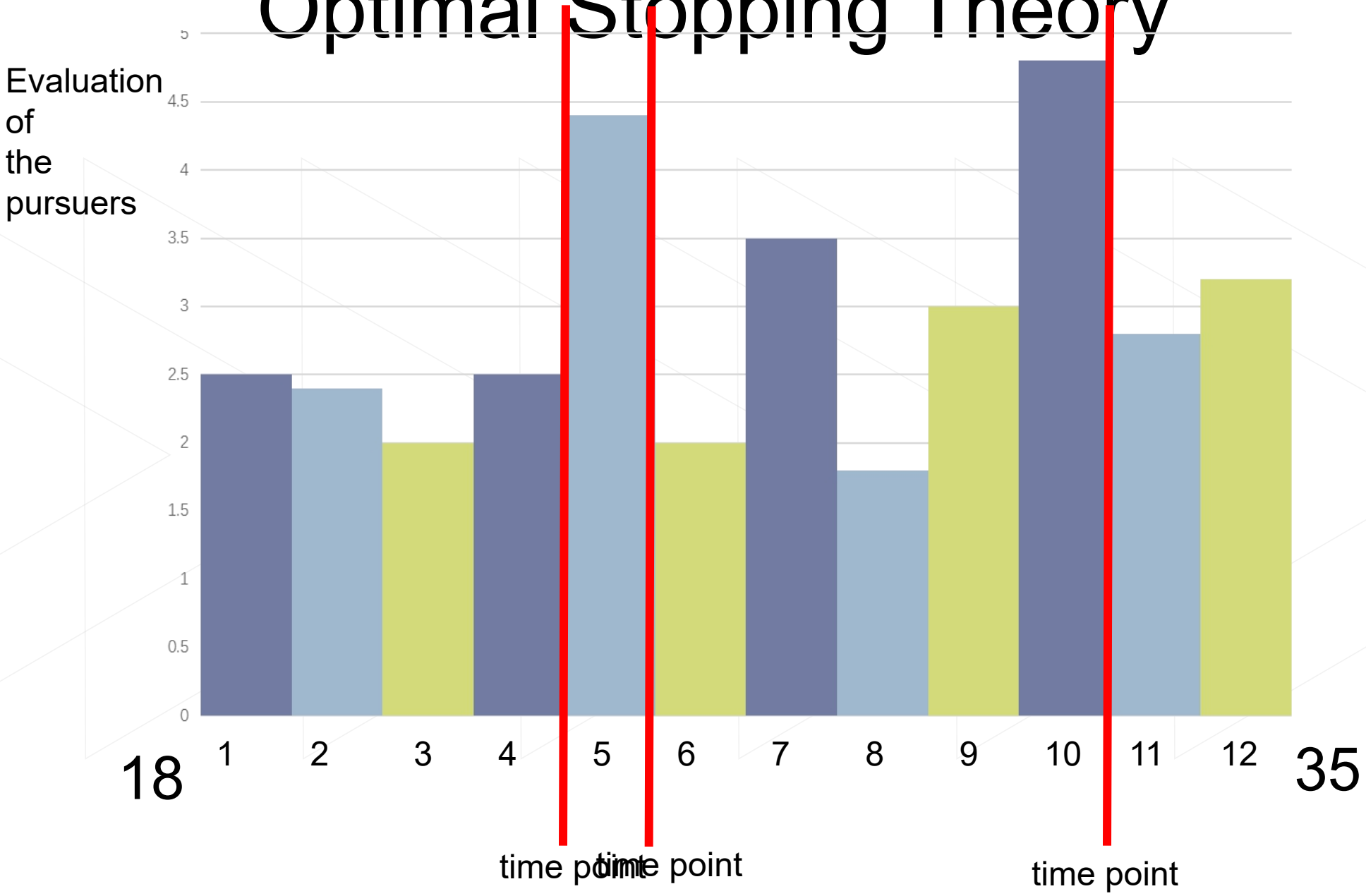
	stay silent	**betray**
stay silent	-2 , -2	-10 , 0
betray	0, -10	-5 , -5

prisoner 1

- rational choice can lead to bad outcomes

- punishment
- Tip For Tat

Optimal Stopping Theory



18

35

Assumptions of the model

① Only boys can tell girls he love her.

② Girls meet all her pursuers during this period.

③ The arrival time of the pursuers obey uniform distribution.

④ Girls can only accept or reject. Broken mirror couldn’t join together.

⑤ Girls aim to maximize the probability of accepting the best pursuer.

Define

n *Total number of pursuers*

k *The number of pursuers ahead certainly rejected in this strategy*

P_k *The probability of successfully selecting the best partner of all*

$$P_k = \sum_{i=k+1}^{n} P(i^{th} \text{ pursuer is best and is selected})$$

$$= \sum_{i=k+1}^{n} P(i^{th} \text{ pursuer is best}) P(i^{th} \text{ pursuer is selected} \mid \text{it is best})$$

$$= \sum_{i=k+1}^{n} \frac{1}{n} P(\text{best of first } i-1 \text{ appears before } k+1)$$

$$= \sum_{i=k+1}^{n} \frac{1}{n} \cdot \frac{k}{i-1} = \frac{k}{n} \sum_{i=k+1}^{n} \frac{1}{i-1}$$

$\Leftarrow$ *we want to find the ideal* $\frac{k}{n}$

Define

n *Total number of pursuers*

k *The number of pursuers ahead certainly rejected in this strategy*

P_k *The probability of successfully selecting the best partner of all*

$$P_k = \frac{k}{n}\sum_{i=k+1}^{n}\frac{1}{i-1}$$

$$set\ x = \frac{k}{n}, n \to \infty$$

$$p_k = x\int_1^x \frac{dt}{t} = -x\ln x$$

$$set\ \frac{dp_k}{dx} = -\ln x - 1 = 0 \Rightarrow x = \frac{1}{e}$$

$$\frac{1}{e} \approx 0.37$$

Optimal Stopping Theory

最优停止理论