HACETTEPE UNIVERSITY

Department of Electrical and Electronics Engineering

ELE 409: Digital Signal Processing Laboratory

EXPERIMENT 2 - ANALYSIS OF DISCRETE-TIME SYSTEMS

I. PURPOSE

The main purpose of this experiment is to study the interrelationship between the transfer function, difference equation and the impulse response of a discrete-time system. One other purpose of this experiment is to study the maximum/minimum phase property.

- Learn how to use the following built-in functions and the reason why do we use:
  audioread, fft, linspace, fftshift, freqz, impz, stepz, roots, randn, zplane, toeplitz, fliplr, subplot.
- Pay particular attention on:
  - the relationship between transfer function and difference equation,
  - locations of poles and zeros and how these locations effect the type of the system,
  - properties of uniformly and Gaussian distributed signals,
  - calculation of impulse and step responses and relation to magnitude and phase responses.
  - matrix operations and recursive functions
- Frequency range should be $[-\frac{f_s}{2}, \frac{f_s}{2}]$ in your frequency domain figures.

II. PRELIMINARY WORK

1. Consider the transfer function below:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

   By taking inverse z transform of both sides obtain the parametric difference equation showing the input-output relationship.

2. Using the difference equation obtained above, write a Matlab function $\mathbf{y} = \text{inout}(\mathbf{b},\mathbf{a},\mathbf{x},L)$ that calculates the first L samples of the output of a parametric difference equation where $\mathbf{x}$ and $\mathbf{y}$ are input and output vectors of the system, respectively. $\mathbf{b}$ and $\mathbf{a}$ are the coefficient vectors which define the transfer function of the system and are defined as;

$$\mathbf{b} = [b_0 \quad b_1 \quad b_2 \quad \ldots \quad b_M] \quad \text{and} \quad \mathbf{a} = [a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_N]$$

   If length of x vector is L_x, ($x[n]$ is defined for $n = 0, 1, ..., L_x - 1$) then assume than $x[n]$ is zero for $n < 0$ and $n > L_x - 1$, besides assume that $y[n]$ is zero for $n < 0$. Although $y[n]$ may be nonzero for $n > L - 1$ (Why ?), your function should compute only the first L samples of y. Note that length of $\mathbf{y}$, L, may be different than the length of $\mathbf{x}$.

   Test your *inout* function using ARMA coefficients given by $\mathbf{b} = \begin{bmatrix}1 & -0.4944 & 0.64\end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix}1 & -1.3335 & 0.49\end{bmatrix}$.

3. Write a recursive MATLAB function $\mathbf{y} = myAR(\mathbf{x},\mathbf{a})$ using the following

$$y[n] = x[n] - \sum_{j=1}^{N} a_j y[n-j]$$

   where $\mathbf{x}$, $\mathbf{y}$ are of length K, whereas $\mathbf{a}$ is $(N+1)$-by-1 vector. Note that recursive function is such functions that call themselves until some condition is satisfied.

   Test your function with input $\mathbf{x}$ as an impulse signal and an appropriate vector $\mathbf{a}$ to generate output $\mathbf{y}$ as Fibonacci series, i.e., $\mathbf{y} = \begin{bmatrix}1 \ 1 \ 2 \ 3 \ 5 \ \ldots\end{bmatrix}^T$.

4. Write a MATLAB function $\mathbf{y} = myMA(\mathbf{x},\mathbf{b})$ implementing the following equation

$$y[n] = \sum_{i=0}^{M} b_i x[n-i]$$

   without using any loops. *Hint*: you can use built-in function *toeplitz*.

   Test your function for calculating first- and second- order derivatives of ramp input, $\mathbf{x}$, with $\mathbf{b} = \begin{bmatrix} 1 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$, respectively.

5. Generate zero mean, unit variance Gaussian distributed signal of length 1024. You may use MATLABs *randn()* function. Name the signal $\mathbf{x}$, plot $\mathbf{x}$ and its magnitude spectrum.

6. (a) Pass the signal $\mathbf{x}$ generated in 5, through each system given below. Plot input and output spectra. Comment on the plots.

$$\mathbf{b1} = \begin{bmatrix} 1 & -0.4944 & 0.64 \end{bmatrix} \quad \mathbf{a1} = \begin{bmatrix} 1 & -1.3335 & 0.49 \end{bmatrix}$$
$$\mathbf{b2} = \begin{bmatrix} 1 & 0.4944 & 0.64 \end{bmatrix} \quad \mathbf{a2} = \begin{bmatrix} 1 & 1.3335 & 0.49 \end{bmatrix}$$

   (b) Combine the two systems in parallel and repeat part (a).

   (c) Combine the two systems in a cascade manner and repeat part (a).

   (d) Comment on the differences of combined systems in parts (b) and (c).

7. For the transfer function in first question, let the numerator be equal to one. Determine the transfer function when you place the poles of the system at:

   (a) $p_{1,2} = 0.8\ e^{\pm j0.1\pi}$

   (b) $p_{1,2} = 0.8\ e^{\pm j0.5\pi}$

   (c) $p_{1,2} = 0.8\ e^{\pm j0.9\pi}$

   (d) $p_{1,2} = 0.1\ e^{\pm j0.5\pi}$

   (e) $p_{1,2} = 0.9\ e^{\pm j0.5\pi}$

   For each of the cases, compute and plot the magnitude response of the system by making use of the MATLAB function *freqz*. Comment on the relation between the location of the poles and the magnitude response. Compute the frequency response at 100 points.

8. For each of the transfer functions below; determine the pole-zero locations, compute and plot the impulse and unit step response of the system, and state whether the system is stable and/or max (min) phase. Compute the system responses for the first 30 samples.

   (a) $H(z) = \frac{1+0.7264\ z^{-1}+0.64\ z^{-2}}{1-0.6356\ z^{-1}+0.49\ z^{-2}}$

   (b) $H(z) = \frac{1+1.1350\ z^{-1}+1.5625\ z^{-2}}{1-0.6356\ z^{-1}+0.49\ z^{-2}}$

   (c) $H(z) = \frac{1+0.7264\ z^{-1}+0.64\ z^{-2}}{1-1.3620\ z^{-1}+2.25\ z^{-2}}$

9. Compare the magnitude response of systems (a) and (b) in the question 8 and comment on it. Support your answer with theoretical reasoning.