Exercise 8.5.2

(a) plugin estimator: $\hat{p} = E(\hat{Y}_n) = n^{-1}\sum_{i=1}^{n} X_i = \bar{X}_n$

plugin estimator: $\sqrt{\hat{p}(1-\hat{p})} = \sqrt{\bar{X}_n(1-\bar{X}_n)}$

(b) $\hat{p} \pm z_{5\%}\sqrt{\bar{X}(1-\bar{X})}$

(c) $\theta = p - q$

$\hat{\theta} = \hat{p} - \hat{q} = \bar{X}_n - \bar{Y}_m$

$se = \sqrt{V(\hat{p}-\hat{q})} = \sqrt{V(\hat{p}) + V(\hat{q})} = \sqrt{\hat{p}(1-\hat{p}) + \hat{q}(1-\hat{q})}$

$= \sqrt{\bar{X}_n(1-\bar{X}_n) + \bar{Y}_m(1-\bar{Y}_m)}$

(d) $\hat{\theta} \pm z_{5\%}\,\hat{se}(\hat{\theta}) = (\bar{X}_n - \bar{Y}_m) \pm z_{5\%}\sqrt{\bar{X}_n(1-\bar{X}_n) + \bar{Y}_m(1-\bar{Y}_m)}$

Exercise 8.5.4

$$\hat{F}_n(x) = \frac{\sum_{i=1}^{n} I(X_i < x)}{n}$$

$$I(X_i < x) = \begin{cases} 1 & \text{if } X_i \le x \\ 0 & \text{if } X_i > x \end{cases}$$

$$\sqrt{n}(\bar{Y}_n - \mu_Y) \sim N(0, \sigma_Y^2)$$

$$\bar{Y}_n \sim N(\mu_Y, \sigma_Y^2/n)$$

$\bar{\mu}_Y$: $E(\hat{\mu}_Y) = E(\bar{Y}_n) = n^{-1}\sum_{i=1}^{n} E(I(X_i \le x)) = n^{-1}\sum_{i=1}^{n} F(x) = \hat{F}_n(x)$

σ_Y^2: $E(\sigma_Y^2) = E(V(\bar{Y}_n)) = n^{-1}\sum_{i=1}^{n} E((Y_i - \bar{Y}_n)^2) = n^{-1}\sum_{i=1}^{n}(E(Y_i^2) -$

$2E(Y_i\bar{Y}_n) + E(\bar{Y}_n^2)) \le n^{-1}\sum_{i=1}^{n}(E(Y_i) + E(\bar{Y}_n^2)) \le 2$

So, for large n, the limiting distribution has variance that goes to zero, so $\bar{Y}_n \to \mu_Y$, $I(X_i < x) \to F(x)$. Then $\hat{F}_n(x) \to n^{-1}\sum_{i=1}^{n} F(x) = F(x)$, so F is the limiting distribution of F_n.