## Unsupervised-HW

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2021/3/19

## 10.3

```
##(a)
x1 \leftarrow c(1,1,0,5,6,4)
x2 \leftarrow c(4,3,4,1,2,0)
df <- data.frame(x1,x2)</pre>
plot(df)
                               0
                               0
       \mathfrak{S}
\chi
       α .
                                                                                                        0
                                                                                         0
                                                                           0
                0
                               1
                                             2
                                                            3
                                                                           4
                                                                                         5
                                                                                                        6
```

```
##(b)
```

```
set.seed(99)
in_clusters <- sample(2,nrow(df),replace=T)</pre>
in_clusters
```

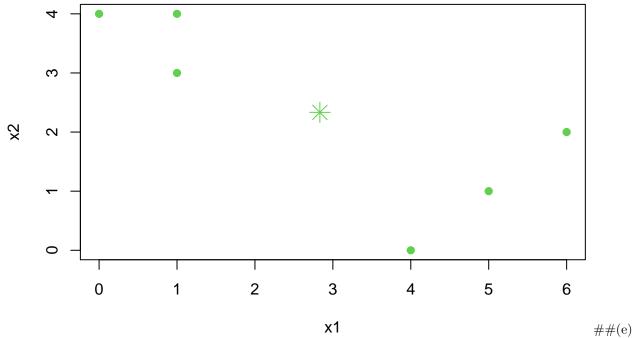
**x**1

## [1] 2 1 2 2 2 1

```
plot(x1,x2,col=in_clusters,pch=20,cex=1.5)
      \mathcal{C}
\overset{\mathsf{X}}{\sim}
              0
                          1
                                       2
                                                   3
                                                                            5
                                                               4
                                                                                        6
                                                  x1
df1 <- data.frame(df,in_clusters)</pre>
##(c)
cent_1 <- c(mean(df1[df1$in_clusters==1,1]),mean(df1[df1$in_clusters==1,2]))</pre>
cent_2 <- c(mean(df1[df1$in_clusters==2,1]),mean(df1[df1$in_clusters==2,2]))</pre>
print(paste0("Centroid for cluster 1 is: ", "(", cent_1[1], ",", cent_1[2], ")"))
## [1] "Centroid for cluster 1 is: (2.5,1.5)"
print(paste0("Centroid for cluster 2 is: ", "(", cent_2[1], ",", cent_2[2], ")"))
## [1] "Centroid for cluster 2 is: (3,2.75)"
```

plot(x1, x2, col = in\_clusters, pch = 20, cex = 1.5)
points(cent\_1[1], cent\_1[2], pch = 8, cex = 2, col = 2)
points(cent\_2[1], cent\_2[2], pch = 8, cex = 2, col = 3)

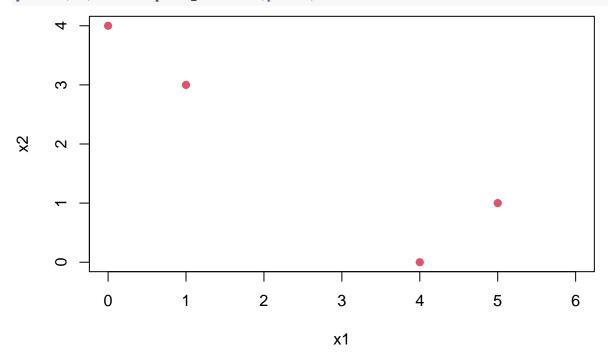
```
euc_dist <- function(v,z){</pre>
  sqrt(sum(v-z)^2)
df1$updated_cluster <- c()</pre>
for (i in 1:nrow(df1)) {
  d1 <- euc_dist(c(df1[i,1],df1[i,2]),c(cent_1[1],cent_1[2]))</pre>
  d2 <- euc_dist(c(df1[i,1],df1[i,2]),c(cent_2[1],cent_2[2]))</pre>
  if (d1<=d2){
    df1$update_cluster[i] <- 1</pre>
  }else{
    df1$updated_cluster[i] <- 2</pre>
  }
}
updated_centroid1 <- c(mean(df1[df1$updated_cluster == 1,1]), mean(df1[df1$updated_cluster == 1,2]))</pre>
updated_centroid2 <- c(mean(df1[df1$updated_cluster == 2,1]), mean(df1[df1$updated_cluster == 2,2]))
plot(x1, x2, col = df1$updated_cluster+1, pch = 20, cex = 1.5)
points(updated_centroid1[1], updated_centroid1[2], pch = 8, cex = 2, col = 2)
points(updated_centroid2[1], updated_centroid2[2], pch = 8, cex = 2, col = 3)
```



we don't have any changes.

##(f)

plot(x1,x2,col=df1\$update\_cluster+1,pch=20,cex=1.5)



10.5

Left: The left side plot shows unscaled variables. In this case, the number of socks becomes more important than the number of computers. With K=2, the K-Means clustering will result in two clusters separately for socks and computer purchases. Center: Since the variables are scaled, in this case, the purchase of computers becomes as important as socks. Here, the K-Means clustering will produce two clusters - one of people who have purchased a computer and

the other of people who haven't. Right: In this case, K-Means clustering will produce clusters of socks purchases and computer purchases separately because there is a huge difference in the price of socks and computers.

#### 10.6

##(a) 90% of the information in the original data is lost in projecting the tissue sample observations onto the first principle component. Or, 90% of the original data is not contain the first priciple component.

##(b) Since each patient sample was rin on either of the machine A and B, the machine used could be used as a feature in the PCA. We check if there is an improvement in the PVE after adding the machine used as a feature.

```
##(c)
set.seed(9)
control <- matrix(rnorm(50*1000),ncol=50)</pre>
treatment <- matrix(rnorm(50*1000),ncol=50)</pre>
x <- cbind(control, treatment)</pre>
x[1,] \leftarrow seq(-18,18 -.36,.36)
pca <- prcomp(scale(x))</pre>
summary(pca)$importance[,1]
##
       Standard deviation Proportion of Variance Cumulative Proportion
##
                  3.159783
                                            0.099840
                                                                      0.099840
X \leftarrow rbind(x,c(rep(0,50),rep(10,50)))
pca_out <- prcomp(scale(X))</pre>
summary(pca_out)$importance[,1]
##
       Standard deviation Proportion of Variance Cumulative Proportion
##
                   3.391937
                                             0.115050
                                                                      0.115050
```

The proportion of variance explained by the first principle component is 9.98%. Including the machine used as a feature, coding 0 for A and 10 for B, the PVE increased to 11.5%.

### 10.8

```
##(a)
# the sdev approach to PVE
data("USArrests")
pca_usa <- prcomp(USArrests,scale. = T)
pca_usa$sdev

## [1] 1.5748783 0.9948694 0.5971291 0.4164494
# variance
pca_var <- pca_usa$sdev^2
pca_var

## [1] 2.4802416 0.9897652 0.3565632 0.1734301
#PVE
pve <- pca_var/sum(pca_var)
pve

## [1] 0.62006039 0.24744129 0.08914080 0.04335752</pre>
```

```
##(b)
# the prcomp PVE approach
usa_scaled <- scale(USArrests)
loadings <- pca_usa$rotation
sum_var <- sum(apply(as.matrix(usa_scaled)^2,2,sum))
apply((as.matrix(usa_scaled)%*%loadings)^2,2,sum)/sum_var

## PC1 PC2 PC3 PC4</pre>
```

**##** 0.62006039 0.24744129 0.08914080 0.04335752

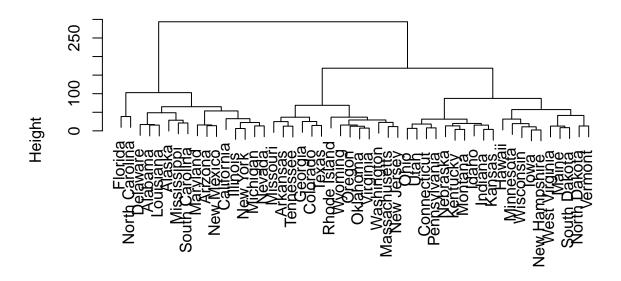
The PVE for each principal component form both the approaches is the same.

#### 10.9

plot(hc.complete)

```
##(a)
set.seed(9)
hc.complete <- hclust(dist(USArrests), method = "complete")</pre>
```

## **Cluster Dendrogram**



```
dist(USArrests)
hclust (*, "complete")
```

##(b)

```
hc_cut <- cutree(hc.complete,3)
clusters <- split(data.frame(names(hc_cut),hc_cut),as.factor(hc_cut))
clusters</pre>
```

```
## California
                       California
## Delaware
                         Delaware
## Florida
                          Florida
## Illinois
                         Illinois
## Louisiana
                        Louisiana
## Maryland
                         Maryland
## Michigan
                         Michigan
## Mississippi
                      Mississippi
## Nevada
                           Nevada
## New Mexico
                       New Mexico
## New York
                         New York
## North Carolina North Carolina
## South Carolina South Carolina
##
## $^2`
##
                  names.hc_cut. hc_cut
## Arkansas
                       Arkansas
                                      2
                                      2
## Colorado
                       Colorado
## Georgia
                        Georgia
                                      2
                                      2
## Massachusetts Massachusetts
## Missouri
                       Missouri
                                      2
## New Jersey
                     New Jersey
                                      2
## Oklahoma
                                      2
                       Oklahoma
## Oregon
                         Oregon
                                      2
                                      2
## Rhode Island
                  Rhode Island
## Tennessee
                      Tennessee
                                      2
## Texas
                          Texas
                                      2
## Virginia
                       Virginia
                                      2
                                      2
## Washington
                     Washington
                                      2
## Wyoming
                        Wyoming
##
## $`3`
##
                 names.hc_cut. hc_cut
## Connecticut
                    Connecticut
                                      3
## Hawaii
                         Hawaii
                                      3
## Idaho
                          Idaho
## Indiana
                        Indiana
                                      3
## Iowa
                           Iowa
                                      3
## Kansas
                         Kansas
                                      3
## Kentucky
                                      3
                       Kentucky
## Maine
                          Maine
                                      3
                                      3
## Minnesota
                      Minnesota
## Montana
                        Montana
                                      3
## Nebraska
                       Nebraska
                                      3
## New Hampshire New Hampshire
                                      3
## North Dakota
                  North Dakota
                                      3
## Ohio
                                      3
                           Ohio
## Pennsylvania
                   Pennsylvania
                                      3
## South Dakota
                                      3
                   South Dakota
## Utah
                                      3
                           Utah
                                      3
## Vermont
                        Vermont
                                      3
## West Virginia West Virginia
## Wisconsin
                      Wisconsin
                                      3
```

1

1

1

1

1

1

1

1

1

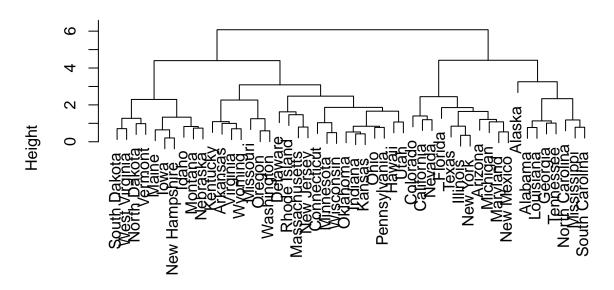
1

1

```
##(c)
```

```
hc_scaled <- hclust(dist(scale(USArrests)),method = "complete")
plot(hc_scaled)</pre>
```

## **Cluster Dendrogram**



# dist(scale(USArrests)) hclust (\*, "complete")

##(d)

hc\_scaled\_cut <- cutree(hc\_scaled,3)
clusters\_scaled <- split(data.frame(names(hc\_scaled\_cut),hc\_scaled\_cut),as.factor(hc\_scaled\_cut))
clusters\_scaled</pre>

```
## $`1`
##
                   names.hc_scaled_cut. hc_scaled_cut
## Alabama
                                Alabama
                                                      1
## Alaska
                                 Alaska
                                                      1
## Georgia
                                Georgia
                                                      1
## Louisiana
                              Louisiana
                                                      1
## Mississippi
                            Mississippi
                                                      1
## North Carolina
                         North Carolina
                                                      1
## South Carolina
                         South Carolina
                                                      1
## Tennessee
                              Tennessee
##
## $\2\
##
              names.hc_scaled_cut. hc_scaled_cut
## Arizona
                            Arizona
## California
                         California
                                                 2
## Colorado
                           Colorado
                                                 2
                                                 2
## Florida
                            Florida
## Illinois
                           Illinois
                                                 2
                                                 2
## Maryland
                           Maryland
## Michigan
                           Michigan
                                                 2
```

```
## Nevada
                                                  2
                             Nevada
## New Mexico
                                                  2
                         New Mexico
## New York
                           New York
                                                  2
## Texas
                              Texas
                                                  2
##
## $`3`
##
                  names.hc_scaled_cut. hc_scaled_cut
                              Arkansas
## Arkansas
## Connecticut
                           Connecticut
                                                     3
## Delaware
                              Delaware
                                                     3
## Hawaii
                                 Hawaii
                                                     3
## Idaho
                                                     3
                                  Idaho
                               Indiana
## Indiana
                                                     3
                                                     3
## Iowa
                                   Iowa
## Kansas
                                 Kansas
                                                     3
                                                     3
## Kentucky
                              Kentucky
## Maine
                                 Maine
                                                     3
                                                     3
## Massachusetts
                         Massachusetts
                                                     3
## Minnesota
                             Minnesota
                              Missouri
                                                     3
## Missouri
                                                     3
## Montana
                               Montana
## Nebraska
                              Nebraska
                                                     3
                         New Hampshire
                                                     3
## New Hampshire
## New Jersey
                            New Jersey
                                                     3
                                                     3
## North Dakota
                          North Dakota
## Ohio
                                   Ohio
                                                     3
## Oklahoma
                              Oklahoma
                                                     3
## Oregon
                                 Oregon
                                                     3
                                                     3
## Pennsylvania
                          Pennsylvania
## Rhode Island
                          Rhode Island
                                                     3
## South Dakota
                                                     3
                          South Dakota
## Utah
                                   Utah
                                                     3
                                                     3
## Vermont
                               Vermont
                                                     3
## Virginia
                              Virginia
                                                     3
## Washington
                            Washington
                                                     3
## West Virginia
                         West Virginia
## Wisconsin
                             Wisconsin
                                                     3
## Wyoming
                               Wyoming
                                                     3
```

### table(hc\_cut,hc\_scaled\_cut)

```
## hc_scaled_cut
## hc_cut 1 2 3
## 1 6 9 1
## 2 2 2 10
## 3 0 0 20
```

Scaling the variable affect the clusters obtained. It is better to scale the variable because they are measured on different units.

## 10.10

```
##(a)
```

```
set.seed(9)
```

```
sim_data <- matrix(sapply(1:3, function(x) rnorm(20*50,mean = 0,sd=0.001)),ncol=50)</pre>
class <- unlist(lapply(1:3,FUN=function(x) rep(x,20)))</pre>
sim_data <- data.frame(sim_data)</pre>
sim_datatrue_lables <- c(rep(1,20),rep(2,20),rep(3,20))
##(b)
sim_pca <- prcomp(sim_data,scale. = T,center = T)</pre>
plot(sim_pca$x[,1:2],col=class,xlab = "Z1",ylab="Z2",pch=20)
      ^{\circ}
22
      0
     7
                              -2
                                                   0
                                                                       2
           -4
                                                 Z1
                                                                                           ##(c)
set.seed(9)
sim_kmeans <- kmeans(sim_data,3)</pre>
table(sim_data$true_lables,sim_kmeans$cluster)
##
##
##
        0
          0 20
##
           0 20
     3 10 10 0
Cluster the observations correctly.
\#\#(d)
set.seed(9)
sim_kmeans2 <- kmeans(sim_data,2)</pre>
table(sim_data$true_lables,sim_kmeans2$cluster)
##
##
        1 2
        0 20
##
     2 0 20
##
```

```
3 20 0
##
\#\#(e)
set.seed(9)
sims_kmeans4 <- kmeans(sim_data,4)</pre>
table(sim_data$true_lables,sims_kmeans4$cluster)
##
##
        1 2 3 4
##
     1 0 0 20 0
##
     2 0 0 0 20
##
     3 10 10 0 0
This one dones't perform as well as the above two.
\#\#(f)
set.seed(9)
km_out <- kmeans(sim_pca$x[,1:2],3)</pre>
table(sim_data$true_lables,km_out$cluster)
##
##
        1 2 3
##
     1 4 4 12
     2 7 4 9
##
     3 7 8 5
many miss classified observations.
##(g)
set.seed(9)
km_out_1 <- kmeans(scale(sim_data),3)</pre>
table(sim_data$true_lables,km_out_1$cluster)
##
##
        1
           2 3
##
     1 8 4 8
##
     2 10 3 7
     3 9 8 3
##
```