N-Gon Problem

Triangle Problem (Exercise 7.14 - Stat110):
A stick is broken into n pieces by picking $n-1$ points independently and uniformly along the stick, and breaking the stick at those $n-1$ points. What is the probability that the n pieces can be assembled into a closed polygon?

Solution:
Assume without loss of generality that the stick has a length of 1 meter. Imagine instead that we have a rope of length 1 meter and each break of the stick is represented by marking the rope with a dash. Then, we connect the beginning and end of the rope to form a circle. This joining point will also be marked with a dash. The n pieces the stick would have broken into are represented by the lines between the n dashes on the rope. In order for the n pieces to form a polygon, the longest piece has length less than $\frac{1}{2}$. This implies that the n pieces cannot form a polygon if the longest piece has length equal to or more than $\frac{1}{2}$. This equates to all the marks on the rope belonging to the same semicircle. We can compute this probability and then take the complement.

Let us denote the marks as $M_1, M_2, \cdots, M_n$. We will view these marks in a clockwise fashion. If we take any mark M_i, we can calculate the probability that the n marks are contained in the semicircle which has M_i at its leftmost tip to be $\frac{1}{2^{n-1}}$. Since only one mark can be "leftmost", these events are disjoint and thus the total probability that these n marks lie within the same semicircle is simply $\frac{n}{2^{n-1}}$ by linearity. If we take the complement, we find that the probability that these n marks do not lie within the same semicircle, which equals the probability that longest piece of rope has length less than $\frac{1}{2}$, is $\boxed{1 - \frac{1}{2^{n-1}}}$