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Evolution of Cooperation in Multi-level Public Goods Game with Mobility

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Contents

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Abstract

The provision of public goods at different scales is a realistic but partially developed topic. Using Agent-based Modelling, this paper examines whether the inclusion of mobility of individuals in Multi-level Public Goods Game allow for the survival of altruistic strategy. Our simulation results provide evidence that mobility is capable of sustaining contribution even when the probability is low. Although our results do not hold under mutation, we have introduced a possible candidate for an evolutionary stable strategy which sustains contribution.

1 Introduction

Public goods are a set of goods that can be consumed by a collection of people (Samuelson, 1954). Classic Public Goods Game has an unique Nash Equilibrium, where everyone free-rides. Dawes (1980) points out that collaboration and defection are usually seen as the core problem of every social dilemma. On this basis, Public Goods Game has the capacity to describe the collective cooperation behavior (Santos et al., 2006; Nowak et al., 2004). Classic Public Goods Game has problems of free-riding, exclusive cost, common tragedy, finance and distribution, etc. (Latham, 1967; Hardin, 1968; Roberts, 1987). However, the demand of public infrastructure exists, for example, constructing a local public library (De Witte and Geys, 2011). One solution to this problem is Pigou taxation, which corrects externalities and private costs through taxing or subsidizing economic agents (Pigou, 1948). But as with all governmental intervention, Pigou taxation induces inefficiency (Carlton and Loury, 1980). Besides, local governments are faced with limited financial budget, unable to sustain provision of public goods entirely.

From the three dimensions of private supply (Demsetz, 1970; Staaf, 1983), voluntary supply (Young, 1982; Falkinger et al., 2000) and joint supply (Buchanan, 1967, 1966; Demsetz, 1970), researchers explained that, under some specific circumstances (e.g., giving contributors the ability of excluding non-contributors without exclusive cost, involving silent agreements, united actions of contributors and voters, etc.), public goods do not have to be provided by the government. Therefore, the collective contribution to public goods may promote people's accessibility to public goods, which thereby improve the collective welfare (Desai and Olofsgård, 2019). In this paper, we explore the possibility of sustaining collective contribution to public goods, and thereby alleviate the financial pressure on local governments. As such, we avoid mechanisms which include monetary rewards to contributors and minimized the government's role in facilitation, such as identifying free-riders and enforcing punishment.

As a powerful tool, Evolutionary Game Theory expands the basic framework of Game Theory and study the evolution of cooperation behavior of selfish individuals (Hofbauer and Sigmund, 1998). It provides possible explanations to the phenomenon appeared in experiments (Perc and Szolnoki, 2010), where not all individuals contribute nothing. In addition to the standard evolution strategy, by introducing more reasonable evolution rules, for instance, expanding pair interaction to inter-group interaction (Dugatkin and Mesterton-Gibbons, 1996; Traulsen et al., 2008), or involving the concept of spatial interaction (Nowak and May, 1992), models are more consistent with real scenarios, and the explanatory potency of models has been dramatically improved. Moreover, individual's heterogeneous characteristics, for example, willingness to volunteer (Szabó and Hauert, 2002; Hauert et al., 2002), mobility (Yan, 2017), reputation (Yang and Yang, 2019), wealth (Tu, 2018; Wang et al., 2018), learning & teaching abilities (Szolnoki and Szabó, 2007; Wang and Chen, 2015), etc., can also affect interaction behaviors between agents. To sum up, Evolutionary Game Theory gives us the capacity to involve factors that are necessary for a more realistic analysis.

In reality, people are offered public goods from different levels, such as community and municipal public goods (Ge et al., 2018) . Therefore, games are played on different hierarchical levels (e.g., Blackwell and McKee (2003) find that people's contribution to a global public good and to a group public good changes conditional on the corresponding average per capita return). These types of Public Goods Games are known as Multi-level Public Goods Games (Wang et al., 2011).

In a classic Public Goods Game, individuals are confined to the same group of players and do not possess mobility. This is highly unlikely as individuals may move to a better place when the habitat becomes unsuitable for living. This behavior can also be found in nature. For example, insects often migrate as a response to changes in habitat availability, quality, and level of crowding (Shaw, 1970; Drake et al., 1995). Nowak and May (1992) find that, within a spatial structure, individuals can move and form clusters to amplify the effect of collective contribution, so that the negative effect of defections can be outweighed. Yet when the spatial structure is moved away, the collective behavior will return to non-contribution. Although there is a number of studies on multi-level (Wang et al., 2011; Caplan et al., 2000; Güth and Sääksvuori, 2012) and spatial (Szolnoki et al., 2011b; Szabó and Hauert, 2002) Public Goods Game, research with a simultaneous introduction of both aspects are relatively scarce. Therefore, we want to examine from the dynamic evolution perspective whether the presence of multi-level public goods provides a desirable environment for the survival of altruistic strategy after the introduction of a movement mechanism.

Analyzing the interaction between heterogeneous individuals from a dynamic perspective requires enormous calculations especially when we introduce external interventions into the model. To study individuals' behavior in this intricate structure, we need to approach it differently and utilize more intuitive and adaptive tools. Agent-based Modeling models agents' automatic decision-making process. Repeated competitive interactions between agents is a main feature of ABM. Bonabeau (2002) divided the application of ABM in four areas: (1) flows simulation (e.g., Fu and Hao (2018); Gabardo et al. (2019)); (2) market simulation (e.g., Leal and Napoletano (2017); Teglio et al. (2017)); (3) organizational simulation (e.g., Kangur et al. (2017)) and (4) diffusion simulation (e.g., Kandiah et al. (2019); Sun et al. (2019)). Hence, ABM has the following advantages compared to traditional models: (1) ABM can simulate "emergence", which means, the results of a large number of simple reactions superimposed on each other; (2) ABM can simulate the dynamic evolution process with conditions closer to reality; (3) the setting of ABM is more flexible (Bonabeau, 2002). Therefore, ABM is an effective method to simulate the dynamic evolution process in our model.

To create an environment and mechanism which can sustain continuous contribution, we propose a Multi-level Public Goods Game where contributors have the probability to move with a movement cost. As predicted by theoretical models, our simulation results show that contributors will not survive under the classic settings. With this multi-level structure, contributors will be more likely to survive, regardless of his mobility probability. To examine if our strategy is an evolutionary stable strategy, we introduce the mutation of offspring. The simulation results show that the survival rate of contributors decrease with a probabilistic mutation of offspring. However, in those simulations whereby contributors survive, their existence persist for more than 10000 time periods.

The remainder of this paper is organized as follows. In the next section, we describe our Multi-level Public Goods Game model and the evolutionary replicator dynamics. In section 3 we introduce the setting of the ABM and model calibration before presenting the simulation results. Lastly, we summarize our findings in section 4.

2 Model

Our model of cooperation bears similarity to the study of Caplan et al. (2000), which shows, in a theoretical way, that if regional governments in EU positively contribute to the federal public goods, considering the subsequent labor mobility, there will be an efficient sub-game perfect equilibria. In our study, we adopt the Evolutionary Game Theory approach which includes two parts. First being competition, which is the game played among individuals. Second being the natural selection process, also known as the replicator dynamics.

2.1 Competition: The Public Good Games

Building on top of the classic Public Goods Game, we divide the population into smaller divisions which we refer as a "community". Each community offers a community-level public good which only its member can contribute to and benefit from. Individuals in our model are, therefore, both members of the nation (which consists of the entire population) and members of the community which they live in. There are in total three different strategies which individuals can adopt, namely, contribute to the national-level public good (N), contribute to the community-level public good (C), and lastly to free-ride (F). The payoff function of this game, at discrete time interval, is then defined as the following:

$$\text{Payoff} = w - s^j - e + N * \frac{x^N}{n^N} + C_i * \frac{x^C}{n_i^C}$$

$$\text{where } j \in \{N, C\}$$

$$i \text{ denotes the code of a community, } i \in [1, ..., m]$$

w denotes the wealth of an individual, which is also the initial endowment at the first time period. s^j denotes the number of units which an individual contributes to either the national-level public good or the community-level public good. As such, $s^j = 0$ for a free-rider. e is the consumption at every time period. N and C_i represents the total contribution to the national or the community-level public good of its residence. x^j denotes the multiplier of the respective public good and n_i^j is the population in the nation or in each community. The multiplier takes values of $1 < x^C < x^N < n^C < n^N$. It is required that the public good multiplier is smaller than the total number of players contributing. When this condition is not satisfied, it is profitable to contribute without any other individuals contributing. An realistic conditional intuition into why $x^N > x^C$, is that we presume that there exists scale effect on public goods. National public goods to may be national defence or transportation network across the country which has impact on all members of the nation. While on the other hand, community-level public goods impact a much smaller scale.

Within a community, individuals are more likely to cooperate due to the lower in-group variation as compared to the huge variation across groups. Commonly, there is more frequent interactions among members of a community, thus individuals gain more information about the community members' behaviours. Trust may also be built and this paves for cooperation. Furthermore, individuals may value those from the same community more, and are more willing to contribute.

After the inclusion of this new community-level public goods, the payoff of the game should still resemble the original Public Goods Game. There must exist a dominant strategy, which is to free-ride. Following which, the Nash Equilibrium will be at zero contribution. This dominant strategy creates a social dilemma which resembles that of prisoners' dilemma whereby the social optimal outcome is not a Nash equilibrium. However, observations in real life and experiments in lab provide evidence that individuals are willing to contribute (Andreoni et al., 1993). This observation of altruistic behaviour is even present in one-shot games, as shown by Fischbacher et al. (2001). In experiments with repeated reactions, Neugebauer et al. (2009); Fehr and Gachter (2000) show that individuals contribute but their contribution declines overtime. Although contributing may be viewed as irrational for payoff-maximizing individuals in classic game theory, we approach this problem using Evolutionary Game Theory with the aim of providing evidence that contributing to the community-level public good is an Evolutionary Stable Strategy.

When solving for Nash Equilibrium, we have to assume rationality of individuals and common knowledge. These assumptions are relaxed in Evolutionary Game Theory. Instead of having individuals who consciously make decisions by weighing expected utility, we assume that individuals adopt a fixed strategy which is an inheritable phenotype (Vincent and Brown, 2005). This strategy may include responses and assessment conditioned on the strategy and payoff of the other individuals, but this reaction is predetermined and fixed. Take the "tit-for-tat" strategy for an example, an individual's action is determined by the previous action of his opponents, which may vary from cooperate to defect in between rounds. However, the mechanism which decides the individual's action at every interaction with his opponents remain the same. Additionally, we provide a brief intuition into why well-known strategies like the "tit-for-tat" and Grimm Trigger weren't used in our paper in solving repeated reactions. Firstly, when there are more than 2 players, "tit-for-tat" is not well-defined to cater to situations where by one opponent contributes while the other does not. Secondly, the Grimm Trigger defines a range of discount factor where contribution is a rational strategy. In reality, the heterogeneous human population consists of individuals with different discount factor and thus, they behave differently under the same circumstances.

Nevertheless, being altruistic alone does not allow individuals to survive in the long term. Axelrod and Hamilton (1981) point out that strategies need to include reciprocity whereby those who free-ride are punished and those who contributed are rewarded. Existing literature focused at reputation building, reward and punishment targeted at free-riders (Auerswald et al., 2018; Szolnoki et al., 2011a; Quan et al., 2018; Fu et al., 2019; Hauert, 2010). These mechanisms have significant effect on promoting contribution behaviors. However, in a society whereby individuals only differ in their strategies, anonymity makes it unfeasible to identify free-riders. Thus, we are incapable of punishing those free-riders directly. Furthermore, rewarding the contributors require extra monetary inputs from the local government.

A movement strategy which we would like to introduce would be that community-level public goods contributors receive private information that their accumulated payoff is decreasing. This information can be understood as a form of reward in which only contributors are entitled to. However, no contributors are awarded additional monetary payoff. As such, a contributor indirectly punishes the free-riders by leaving. In the following section, we would present statistical data of how this movement strategy will help to establish an Evolutionary Stable Equilibrium without an institution issuing rewards or punishments.

2.2 Replicator Dynamics

Evolutionary Stable Strategy is defined as a strategy which cannot be invaded by a small number of individuals playing an alternative strategy (Smith and Price, 1973). Furthermore, it corresponds to a strategy that will be adopted by fully-informed rational players (Binmore and Samuelson, 1992). Thus, Evolutionary Stable Equilibrium is a refinement of Nash Equilibrium.

In Evolutionary Game Theory, utility is perceived as survival fitness or reproductive value (Weibull, 1997). Consumption, e, can be interpreted as the energy needed for survival. Individuals with high survival fitness will reproduce and pass down their strategy to ensure the survival of their kind. We have set a fixed threshold whereby individuals with fitness level above it, give birth to an offspring in its neighbouring grid. If there is no empty spot, no offspring will be born. Thus, for each community, there is a maximum number of individuals allowed. At zero mutation, offspring is identical to its parent and adopt the parent's strategy. There are two ways to understand this setup. One explanation would be that our actions are determined by genes and thus decided by our parents. Arseneault et al. (2003) provided evidence that anti-social behaviour is highly heritable for those below 5 years old. Another more acceptable explanation would be that individuals and their surrounding environment are interdependent. Their experience of interacting with others and how they fair in comparison to others is crucial (Markus and Kitayama, 1991). When fitness level falls to zero, or when the maximal age is reached, individuals die. This setup is constructed to resemble human society and interaction of humans.

3 Agent-based Simulation

We approach our model of cooperation by using Agent-based Modelling, which is a commonly used simulation method to solve Evolutionary Game Theory problems. In our simulation, we induce mobility to test whether cooperation can be sustained in Multi-level Public Goods Game. First, We test the classic Public Goods Game and multi-level model without mobility as baseline results, then we adjust the probability of movement to test for robustness. Finally, we include mutation of offspring, to test if the strategy is evolutionary stable. We have in total fourteen rounds of games. In each round, we run the model 300 times to check if the results are statistically significant, and set time limit at 10000 period to ensure the results' reliability.

To implement and run the model, we use the $NetLogo6.0.4$ Platform (Wilensky et al., 1999), which is a multi-agent programmable modelling environment. The sequence of the model execution is outlined in Figure 1 and described as following: at time t, agents play individual strategy 1 according to their identities, that is, free-riders would not contribute; national-level contributors always contribute to national-level goods; and community-level contributors always contribute to community-level goods. After this stage, agents die if $fitness \leq 0$. The decrease of fitness is a result of having too many free-riders in a community and the dividend from public goods is lower than contribution. Then, community-level contributors play individual strategy 2 – upon noticing their fitness levels are decreasing, they can choose to migrate to another community to obtain a potential higher payoff. Then, the living agents will give birth to an offspring if their fitness level exceed a certain threshold. After which, agents age one unit. By then, agents whose age exceeds a certain threshold die. The model forwards to time $t+1$ and a new cycle of dynamics begin. The model calibration and simulation results are discussed in the following subsections.

3.1 Model Calibration

The identification of our model's parameters is mostly based on the theoretical and experimental papers on Public Goods Game. In our simulation, we construct the refer to the population as a "nation", which is divided into sixteen different communities. The communities are identified by distinct colors during the simulation process. The maximal population of a nation is 784 agents, and the maximal population of each community is 49 agents. The initial setting can be seen in

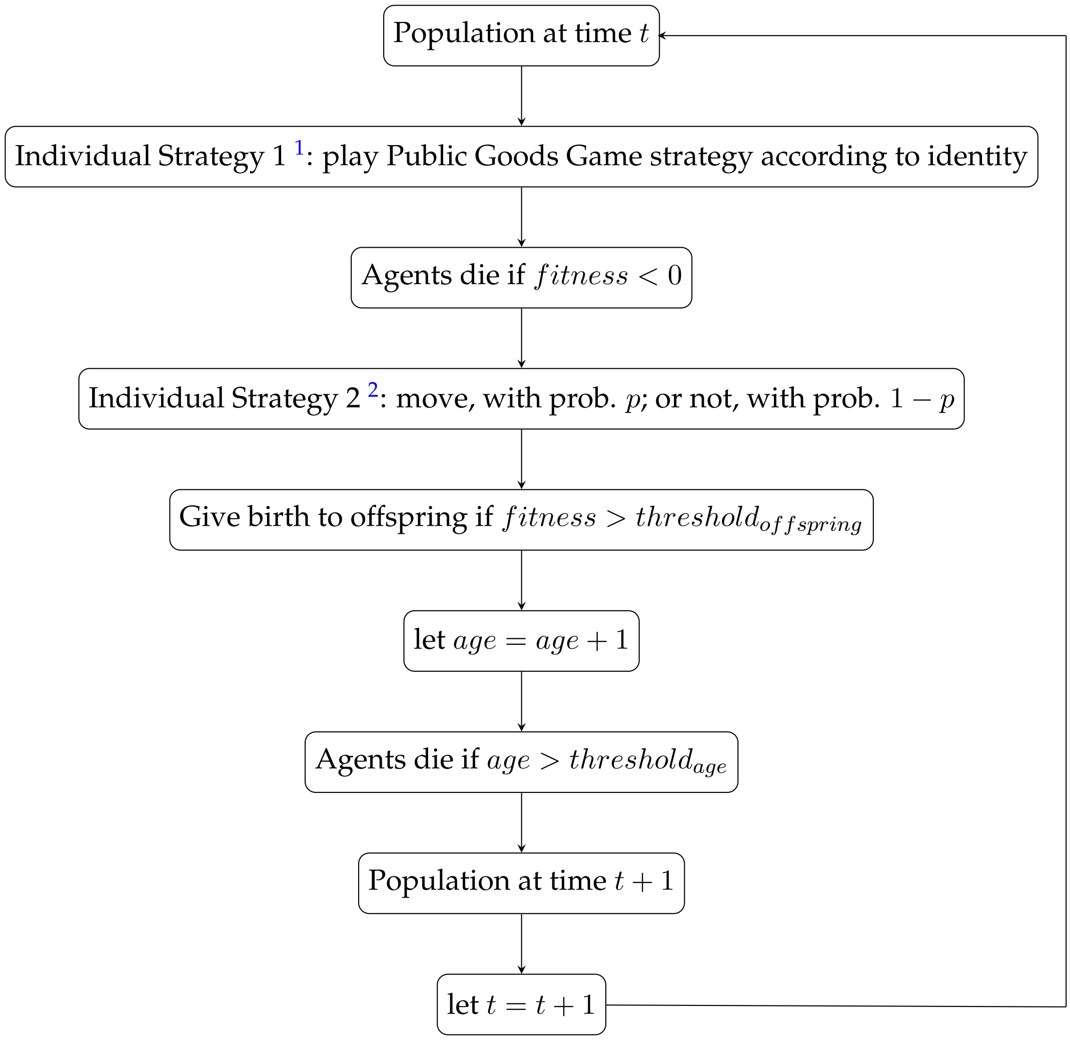


Figure 1: Model Working Flow
Notes: 1: For all kinds of agents; 2: Only for community-level contributors

figure 2, which presents the population structure in period, $t = 0$, of our game. In each simulation, we assume that the initial national population is 420, and agents are randomly distributed in each community. Although individuals should be indistinguishable among themselves, we have color-coded the agents to record their spacial behaviors. Red squares refer to agents who are national-level public goods contributors. Yellow squares refer to community-level public goods contributors and blue ones refer to free-riders. To simulate the Public Goods Game, we assume that the strategies of agents are fixed, which means that the individuals are assigned as "national-level contributor", "community-level contributors" and "free-riders". The agents' strategies remain unchanged throughout. Since our motivation is to determine if contributing to the community-level public goods is evolutionary stable, agents playing other strategies are kept low. We find that having around 60 free-riders is the minimal number which provides a meaningful population structure. Since the initial locations of agents are generated randomly, we would like to avoid the possibility of generating communities without any free-riders. Additionally, we keep the number of national-level contributors smaller than the community ones. The intuition is that there are in group favoritism among communities, so there will be more people willing to contribute to the community-level public goods. Hence, we keep the general population structure as 60 agents are national-level contributors, 300 community-level contributors, and 60 free-riders. The results are robust when we change the number of agents around the selected ones.

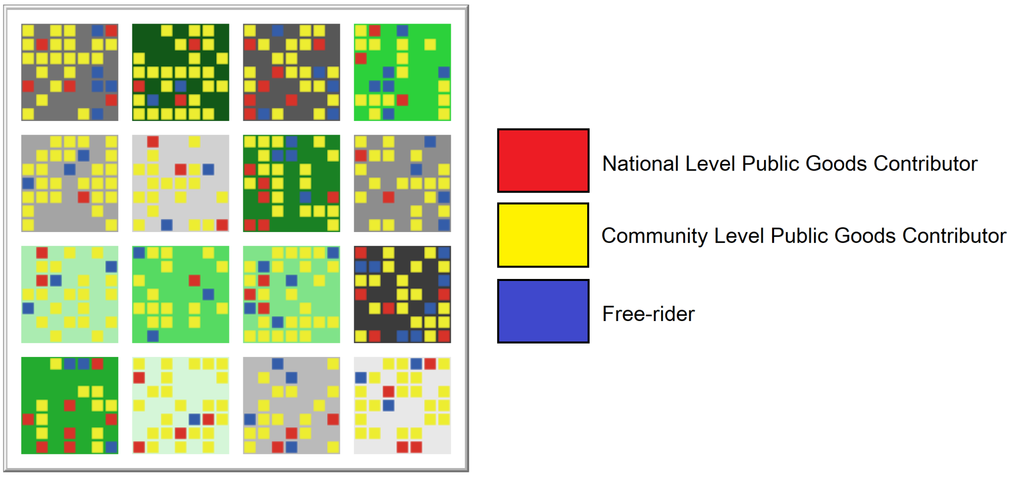


Figure 2: Initial Setting

The simulation is modelled as the evolutionary process of the human society. Hence, the payoff in classic Public Goods Game is framed as fitness of agents. The initial fitness w_0 of the agents is twenty units. Agents' contributions s^j are five units in each period if they are contributors. Agents receive payoff in the next period with a multiplier. The common multiplier used in public goods game is 2. In our setting, we have $x^C < x^N$, then we set $x^C = 2$ and $x^N = 2.25$. The national-level multiplier cannot exceed 3, since then the payoff would be too high and no one would die from free-riding. Agents need to consume one unit to maintain health, which is the basic life requirement, denoted as e in the model. Results are insensitive to positive affine transformations of contribution, payoff, consumption, and fitness. When the fitness accumulates to a offspring threshold, which is 40 units, agents give birth to an offspring. The parent will give his offspring 20 units as their initial fitness, which is subtracted from the parent's accumulated fitness. Additionally, agents die "naturally" if their age reach the max age, which is set as 50 in our model.

Upon receiving private information that their fitness level is decreasing, the community level contributors may choose to move to another community. This behavior can be explained by the undesirable living environment whereby individuals are unable to secure their living requirements. The condition for movement is that agents' fitness level exceed the movement cost since no individual would want to move and face immediate death. We set the movement cost five units, which is one quarter of the initial fitness.

3.2 Simulation Results

3.2.1 Baseline Results

1. Classic Public Goods Game

Although the results are as predicted, we have provided simulation results of the classic Public Goods Game in Table 1. In this simulation, there are 360 national contributors and 60 free-riders. Benefiting from the dividends of the public good which the national contributors funded, free-riders initially increase in population size. With this increase, national contributors can no longer reap sufficient dividends to offset their contribution. Their fitness level dropped correspondingly, and thereby, eliminating the entire species. Without the presence of national contributors, free-riders' fitness level decrease at every time period and they will face extinction eventually.

Table 1: Statistics for Classic Public Goods Game

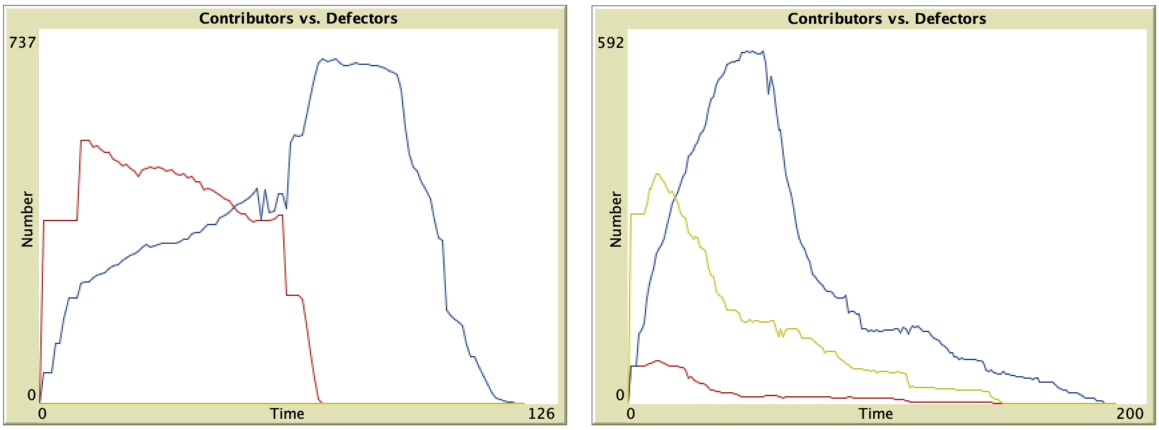
Measure	Free-riders	National Contributors
Max Range	[605, 722]	[505, 562]
Min Range	0	0
Survival Rate	0	0

2. Multi-level Public Goods Game without Mobility

In this simulation, we introduce our model of multi-level public goods and assigned the initial community contributor population to be 300; national contributor population to be 60; and free-rider to be 60. The population distribution can be seen in Figure 3 (b). And in Table 2, we present the range of maximum and minimum population of each type and recorded down their survival rate. Although we do have a 53% survival rate of the community contributors, this data should be interpreted with caution. The initial setting, locations and age of agents are generated at random for every round. Thus, there is a possibility that certain communities are generated to have no free-rider. It is without doubt that such a community or nation can sustain continuous contribution. However, the assumption of no free-rider is unfeasible and not worth analyzing. All in all, community level public good alone is incapable of sustaining contribution.

Table 2: Statistics for Multi-level Public Goods Game

Measure	Free-riders	National Contributors	Community Contributors
Max Range	[398, 675]	[60, 91]	[328, 421]
Min Range	0	[0, 5]	[0, 145]
Survival Rate	0	0.67%	53.00%



(a) Classic Public Goods Game (b) Multi-level Public Goods Game

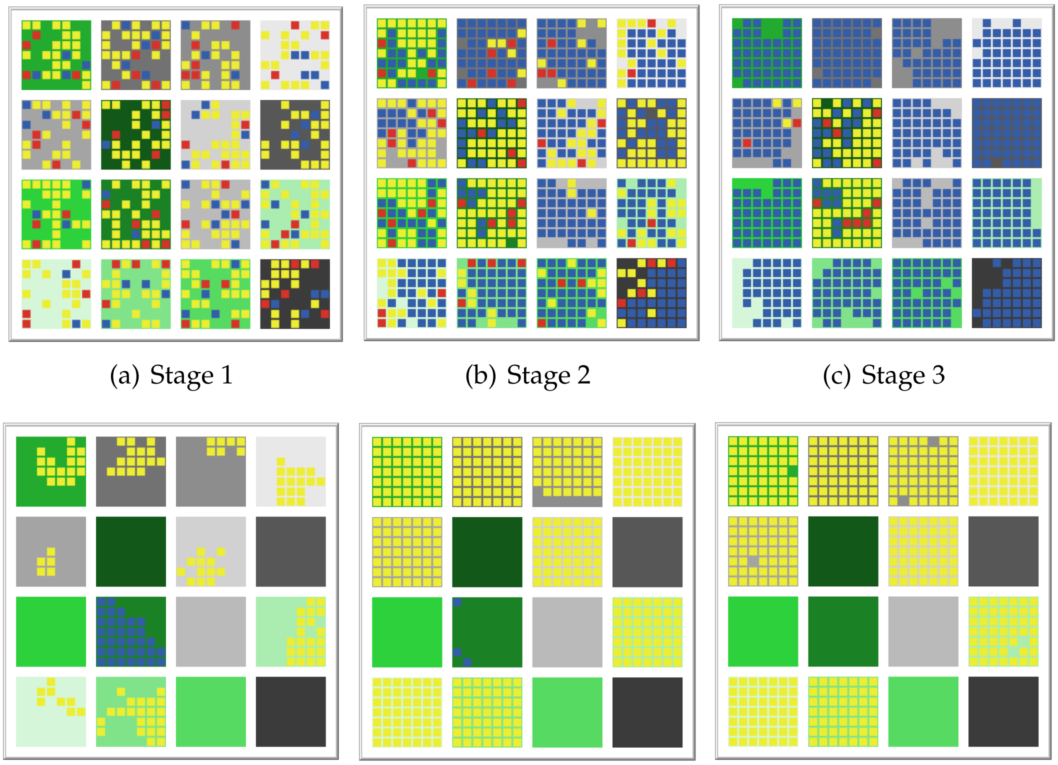
Figure 3: Population Distribution of Baseline Models

3.2.2 Results with full Mobility

The most crucial aspect of our model is mobility. As mentioned in the the previous section, only community contributors possess private information. Furthermore, upon knowing that their fitness level is decreasing, (due to high number of free-riders in the community), community contributors move to another community. We programmed this move to be random, as these contributors do not possess information about the population structure of another community. Thus, there are incidents whereby community contributors moved to another undesirable community and be informed that their fitness level is still decreasing. Then, these contributors will move again in search of a community favourable for survival. From Table 3, we have shown that the survival rate of community contributors is as high as 99.33%. While on the other hand, no free-riders survived.

In Figure 4, we provide visual representation of the 6 stages of process. We begin with the initial setting in stage 1, whereby agents are generated at random locations with a fixed population structure. These agents interact and accumulates payoff which are perceived at fitness level. Those with fitness level above the threshold, give birth to an identical offspring in its surrounding spot. At Stage 2, we can observe an increase in the total number of agents in each community. In certain communities, the number of free-riders way exceed the number of contributors. In such cases, contributors move to other communities. An unfortunate result would be that some contributors are eliminated by the free-riders before reaching a desirable community. The number of contributors decrease sharply and are gathered. This clustering effect is also present in other simulation results (Nowak and May, 1992). This brings us to the result depicted in Stage 3. Next, free-riders living in communities with only free-riders are slowly eliminated, while those in communities with contributors continue to flourish. As their numbers increase, community contributors leave the community again, but onto empty spots where free-riders previously inhabited. With reference to Stage 4, we observe that only free-riders remained in the original community. Their numbers decrease in Stage 5 while the community contributors thrive. Finally, free-riders led to self-destruction in Stage 6 and cooperation can be sustained in this environment without free-riders. The continuous change of population distribution can be seen in Figure 5.

However, there are strong and unrealistic assumptions in this model. First, we assume that contributors move with probability, $p(move) = 1$ upon being notified. In reality, individuals feel attached to their home and are reluctant to leave even in risky situations (Billig, 2006). Second, there is no movement cost involved. Needless to say, moving from a place to another, regardless of the mode of transportation, requires cost. Hence, it is necessary for us to relax these assumptions to provide a more meaningful analysis.



(a) Stage 1 (b) Stage 2 (c) Stage 3

(d) Stage 4 (e) Stage 5 (f) Stage 6

Figure 4: 6 Stages of the Process

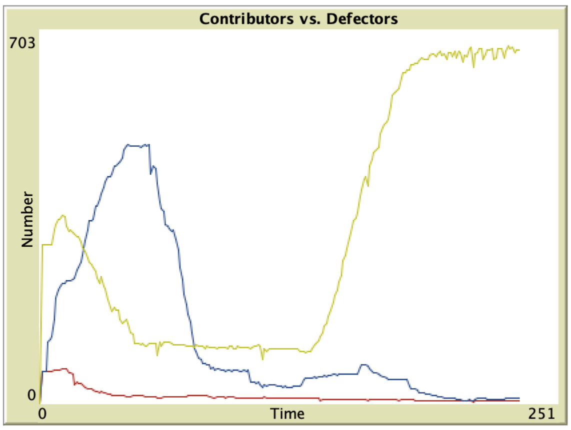


Figure 5: Population Distribution of Mobility

3.2.3 Results with Probabilistic Mobility

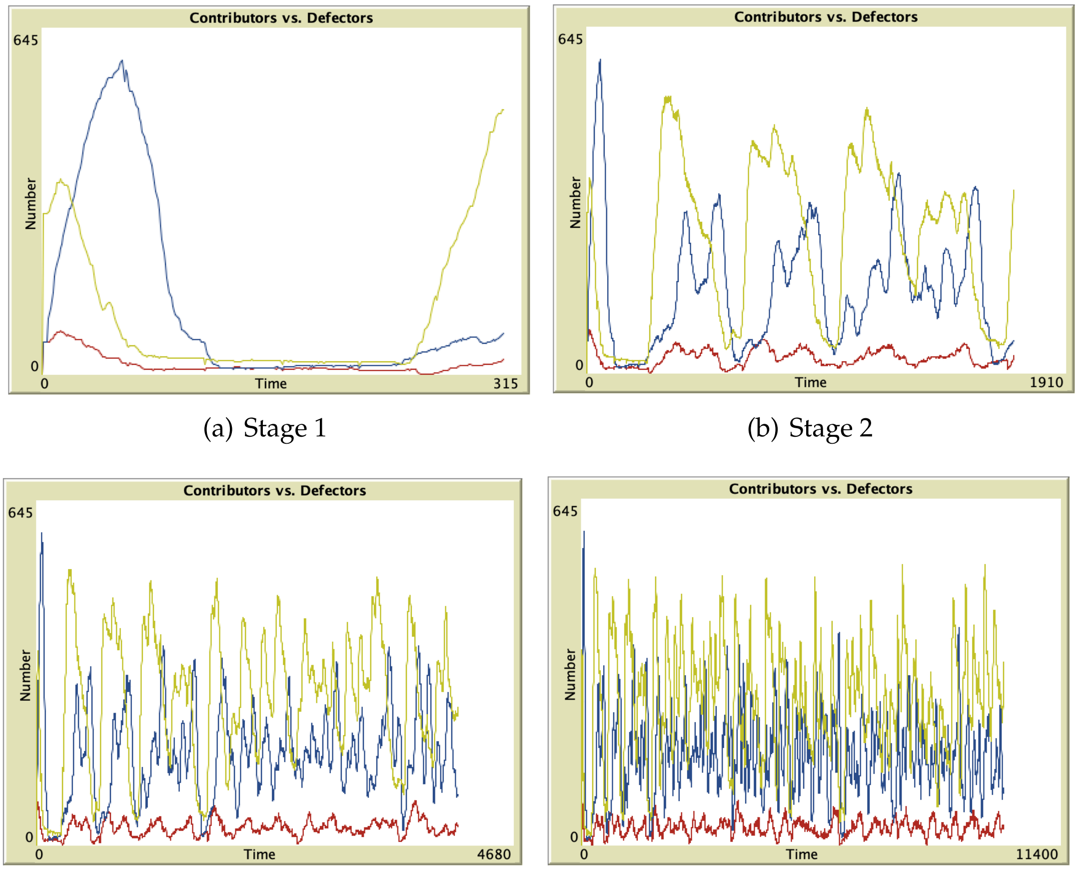
With that, we relax the assumptions mentioned above and assume that agents move with a movement probability $p(move) \in (0, 0.5]$. Simulations are conducted with $p(move) \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$. Furthermore, we defined movement cost to be 5, which will be deducted from the agent at every move. We include an extra command so that these agents will not move when their fitness level is below the movement cost. This is necessary as movement will lead to instant death. Our results are presented in Table 3 and the survival rate of community contributors remain to be higher than 95%. This result provide evidence that despite having low mobility, contribution can still be sustained with the elimination of free-riders. However, there are free-riders who exist around us. This brings us onto our next analysis on mutation.

Table 3: Statistics for Probabilistic Mobility

Measure	Free-riders	National Contributors	Community Contributors
		Probability 1(no movement cost)	
Max Range	[361, 608]	[60, 78]	[335, 735]
Min Range	0	[0, 24]	[0, 260]
Survival Rate	0	29.7%	99.3%
		Probability 0.5	
Max Range	[348, 609]	[63, 81]	[329, 735]
Min Range	0	[0, 21]	[0, 215]
Survival Rate	0	31.3%	98.3%
		Probability 0.4	
Max Range	[339, 590]	[61, 80]	[328, 735]
Min Range	0	[0, 22]	[0, 217]
Survival Rate	0	30.0%	97.7%
		Probability 0.3	
Max Range	[371, 604]	[60, 82]	[339, 735]
Min Range	0	[0, 18]	[0, 214]
Survival Rate	0	30.3%	98.0%
		Probability 0.2	
Max Range	[394, 611]	[62, 82]	[325, 735]
Min Range	0	[0, 14]	[0, 207]
Survival Rate	0	27.0%	95.7%
		Probability 0.1	
Max Range	[367, 670]	[61, 83]	[332, 735]
Min Range	0	[0, 16]	[0, 200]
Survival Rate	0	31.7%	97.0%

3.3 Mutation

Our previous results provide evidence that contribution can be sustained in the long term with even minimal mobility. We note that all free-riders are eliminated



(a) Stage 1

(b) Stage 2

(c) Stage 3

(d) Stage 4

Figure 6: Population Distribution with Mutation

in the process. Some may question how reliable our results are, given the existence of free-riders among us. It may be convenient to comment that we are still in the process of reaching this equilibrium. However, a better explanation would be due to mutation.

In the case of mutation, offspring is born with a strategy which is different from its parent. Going back to the equilibrium outcome of simulation results of mobility whereby only community contributors survive, free-riders and national contributors would be born into these communities. They survive by exploiting the contributors around them and reproduce. Hence, none of the 3 types of agents would be eliminated. It is widely known that individuals do not behave in the same way as their parent. Thus, it is essential that an evolutionary model should be resistant to mutation. To confirm that the strategy which we have initiated to be evolutionary stable, it cannot be invaded by other strategies which are played by a small number of individuals.

Table 4: Statistics for Mutation

	Survival Rate				
Agent Type	Prob. = 0.5	Prob. = 0.4	Prob. = 0.3	Prob. = 0.2	Prob. = 0.1
Free-riders	59.0%	56.3%	50.0%	49.7%	58.3%
National Contributors	58.7%	56.0%	50.0%	49.0%	57.3%
Community Contributors	59.0%	56.3%	50.0%	49.3%	58.0%

For our simulations with mutations, we set the $p(mutate) = 0.1$ and recorded the number of agents playing each strategy at $t = 10000$ for a total of 300 repetitions. In addition, we varied $p(move)$ in a similar way as when we tested for probabilistic mobility.

In Figure 6, we present the population distribution at 4 different stages. At Stage 1, we observe an initial increase in numbers for all three types of agents. However, the increase for free-riders is the largest among all. This observation is explained by the greater increase in fitness level of free-riders. However, when the number of free-riders became too huge, they decrease almost as sharply as how they increased. A similar mechanism to stage 3 and 4 of Figure 4 occurred. Self-destruction eliminated a vast majority of the free-riders and community contributors who moved to communities without free-riders thrive. As their fitness level increase, free-riders may be born into its community. These free-riders leeched off the contributors and start to increase in numbers. This cycle repeats itself and sustain for more than $t = 10000$.

Unfortunately, sustaining cooperation via mobility may fail under mutation. In Table 4, we present our simulation results which show that the survival rate of community contributors drop drastically. We attribute this decrease to our harsh condition that the society must sustain for more than ten-thousand time period. Some results sustain up to 6000 time period but are nevertheless rejected. An insight into why cooperation failed to be established is that with mutation, contributors who moved onto an empty community may give birth to a free-rider before more contributors are born into the society. Hence, when there are many free-riders in every community, community contributors have no safe option to escape. Furthermore, their fitness level reduced rapidly due to the constant search for a desirable community but to no avail. Although the results do not provide sufficient evidence that contributing to a community-level public good is indeed an evolutionary stable strategy, an equilibrium may be reached under higher multiplier, lower number of free-riders and lower movement cost.

Nevertheless, this result warns us that in the situation where there is huge number of free-riders, cooperation is impossible to sustain. However, if we are able to prevent the huge increase in free-riders, the environment is conducive for the survival of all three strategies. Despite the reluctance of of relying on an institution, we may have to turn to governmental efforts to cap the growth of free-riders and ensure the survival of all three types.

4 Summary

Classic Public Goods Game has an unique Nash Equilibrium, which is zero contribution. In order to explore whether contributors with mobility can survive in a Multi-level Public Goods Game, we apply ABM to simulate the model and include probability of mutation. We find that after including mobility into Multilevel Public goods Game, community-level public goods contributors can survive.

In a classic Public Goods Game, our simulation results show that no one survives, which is consistent with the theoretical result. To cope with realistic scenarios, we propose a Multi-level Public Goods Game, where individuals are provided with public goods from both community level and national level. The results of our simulation follows the theoretical prediction, where there is no contribution.

We then test models in which community contributors are mobile. With private information, community contributors know the decreasing tendency of their fitness level, and will look for another community with a better living environment. Under the setting of free mobility, the survival rates of community contributors and free-riders are 99.33% and 0, respectively. In this case, it is observed that the number of contributors drop initially, and contributors gather gradually to form clusters. Then, in communities occupied mainly by free-riders, free-riders are eliminated automatically, because they do not have contributors to exploit. Yet communities in which their main inhabitants are contributors prosper. As the population of contributors increases, contributors move to communities in which free-riders pre-inhabited, and sustain their existence.

Relaxing the assumption of free movement, setting a movement cost and a probability of movement $P(\text{move})$, the survival rates of contributors robustly maintain above 95% as $P(\text{move})$ ranges from 0.1 to 0.5. However, once we relax the assumption of fully inherited strategy and set the probability of mutating at 10%, the simulation results show that, the survival rate of contributors decrease in $P(\text{move})$. This may be due to the fact that the contributors give birth to too many free-riders before the clusters of community-level are formed.

For future extensions, we can vary the movement cost such that it is smaller than the amount to be contributed. With lower movement cost, it is highly likely that we will be able to provide evidence that contribution with mobility is an evolutionary stable strategy. Our model is also extremely flexible and can also be utilized to explain major issues like refugee migration.

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