Department of Applied Mathematics and Statistics
COLORADO SCHOOL OF MINES
MATH 531: Theory of Linear Models
Your Name

Homework 06
Due Friday, February 21, 2025

Some background:

Let $\mathbf{z}$ be distributed $MN(0, I_m)$ then we know that $\mathbf{z}^T\mathbf{z} = \sum_{i=1}^m \mathbf{z}_i^2$ is distributed $\chi^2(m)$. – a sum of iid $N(0,1)$ RVs. (We also know that $\chi^2(m)$ is a member of the gamma distribution family with shape $2m$ and scale 2.)

Problem 1 Let M be an $n \times n$ projection matrix and let k be the dimension of the subspace that M projects onto.

(a) For the eigendecomposition $M = UDU^T$ show that the diagonal elements of D must be either 0 or 1.
This is provides an alternative proof that $tr(M) = k$.

(b) Let U be an $n \times n$ orthonormal matrix and $\mathbf{z}$ be distributed $MN(0, I_n)$. Show that $U^T\mathbf{z}$ is also distributed $MN(0, I_n)$.

(c) Let $\mathbf{z}$ be distributed $MN(0, I_n)$ show that $\mathbf{z}^T M\mathbf{z}$ is distributed $\chi^2(k)$.

Answer: (a) We use the fact that...

$$\begin{aligned} XXXXXXXX &= \text{Something else here} \\ XXXXXXXX &= UDU^{\mathrm{T}} \end{aligned}$$

We can see that ...., therefore diagonal elements of D must be either 0 or 1. $\square$

(b) $U^{\mathrm{T}}\mathbf{z}$ satisfies the definition that says $XXXXXX$ $\square$

(c) Comments:
When writing proofs, make sure to say Q.E.D. or $\square$ to complete the proof. This symbol that ends the proof should be the last thing on that homework sub-problem. When writing equations, make sure to align all equal signs. Keep every sub-part of the problem on the same page if possible.

Problem 2 What is $2(3+7)$?

Answer: There are many ways to find this quantity, one way is by distributing first:

$$2(3+7) = 2(3) + 2(7)$$

And then, multiply

$$= 6 + 14$$

Then, add the two numbers 6 and 14 together we get

$$\boxed{2(3+7) = 20}$$

Comments:
It is generally better to start a new problem on a new page. You can make comments while in the aligned equation environment, always box your answers! In aligned envrionments, you have to use `Aboxed{}` instead of `boxed{}` .