Matrix Perturbation Theory

The steady state covariance satisfies

$$S = MSM^t + R \; .$$

Suppose M has a parameter θ and we want the derivative of S with respect to θ. If Q is any quantity that depends on θ, write the derivative as

$$\dot{Q} = \frac{d}{d\theta} Q \; .$$

Suppose that M depends on θ but R does not. Differentiate the S equation and use the chain rule. You get

$$\dot{S} - M\dot{S}M^t = -\left(\dot{M}SM^t + MS\dot{M}^t \right)$$

The vectorized version of S is

$$\xi_S = \begin{pmatrix} S_{11} \\ S_{12} \\ S_{22} \end{pmatrix}$$

The vectorized version of the S equation is

$$D\xi_S = \xi_R$$

If S is known, then we can calculate

$$U = \dot{M}SM^t + MS\dot{M}^t$$

The entries of $\dot{S}$ are found by solving

$$D\xi_{\dot{S}} = \xi_U \; .$$

If you look in a book, you might find equations involving the tensor product of matrices, or the symmetric tensor product of matrices. That's a more abstract way to describe the relation between the matrices you call M and D.