NOTE : 12/20

Exercice 1 :

Calcul du determinant de A :

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} = \begin{vmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

$$\det(A) = -1 \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = -1 \left| -1 \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right|$$

$$= -1 \left| -1(0-1) \right|$$

$$= -1 \left| -1 \right)$$

$$\boxed{\det(A) = -1}$$

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Exercice 2 :

1. Calcul de $C = AB$

$$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \times$$

$$\begin{pmatrix} 1 & -10 & 11 \\ -3 & 6 & 5 \\ -6 & 12 & 8 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = 1$$

$$b_1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ 6 \\ 12 \end{pmatrix} = -10$$

$$c_1 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 11 \\ 5 \\ 8 \end{pmatrix} = 11$$

$$a_2 = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = 3 + 3 = 6$$

$$b_2 = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} -10 \\ 6 \\ 12 \end{pmatrix} = -30 + 6 = -24$$

$$c_2 = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 11 \\ 5 \\ 8 \end{pmatrix} = 33 + 5 = 38$$

$$a_3 = \begin{pmatrix} 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = -6$$

$$b_3 = \begin{pmatrix} 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ 6 \\ 12 \end{pmatrix} = -12 + 12 = 0$$

$$c_3 = \begin{pmatrix} 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 11 \\ 5 \\ 8 \end{pmatrix} = -10 + 8 = -2$$

1,5 | 3

$$\boxed{C = \begin{pmatrix} 1 & -10 & 11 \\ \cancel{6} & -24 & 38 \\ -\cancel{6} & 0 & -2 \end{pmatrix}}$$

$$\det(C) = -\begin{vmatrix} 6 & 38 \end{vmatrix} = 1$$

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$$\boxed{\det(C) = \cancel{28}}$$

Exercice 4 :

$$A = \begin{pmatrix} 5 & 1 & 1 & -1 \\ 1 & 5 & 1 & -1 \\ 1 & 1 & 5 & -1 \\ -1 & -1 & -1 & 5 \end{pmatrix}$$

1. Determiner les valeur propre de A

$\cancel{A(\lambda)}$

$$A(\lambda) = \begin{pmatrix} 5\lambda & 1 & 1 & -1 \\ 1 & 5\lambda & 1 & -1 \\ 1 & 1 & 5\lambda & -1 \\ -1 & -1 & -1 & 5\lambda \end{pmatrix}$$

$$= \begin{pmatrix} 5\lambda - 1 & 1 & 1 & -1 \\ 0 & 5\lambda & 1 & -1 \\ 0 & 1 & 5\lambda & -1 \\ 5\lambda - 1 & -1 & -1 & 5\lambda \end{pmatrix}$$

$$= (5\lambda - 1)\begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 5\lambda & 1 & -1 \\ 0 & 1 & 5\lambda & -1 \\ 1 & -1 & -1 & 5\lambda \end{pmatrix}$$

$$= (5\lambda - 1)\begin{pmatrix} 5\lambda & 1 & 1 \\ 1 & 5\lambda & -1 \\ -1 & -1 & 5\lambda \end{pmatrix}$$

$$= (5\lambda - 1)\begin{vmatrix} 5\lambda - 1 & 1 & -1 \\ 0 & 5\lambda & -1 \\ 5\lambda - 1 & -1 & 5\lambda \end{vmatrix}$$

$$= (5\lambda - 1)(5\lambda - 1)\begin{vmatrix} 1 & 1 & -1 \\ 0 & 5\lambda & -1 \\ 1 & -1 & 5\lambda \end{vmatrix}$$

$$= (5\lambda - 1)(5\lambda - 1) - \begin{vmatrix} 1 & -1 \\ -1 & 5\lambda \end{vmatrix}$$

$$= (5\lambda - 1)(5\lambda - 1) - (5\lambda - (1))$$

$$= (5\lambda - 1)(5\lambda - 1) - (5\lambda - 1)$$

$$= 25\lambda^2 - 5\lambda - 5\lambda + 1 - 5\lambda + 1$$

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$$= 25\lambda^2 - 15\lambda + 2$$

$$P(\lambda) = 25\lambda^2 - 15\lambda + 2$$

$$\Delta = 25$$

$$\lambda_1 = \frac{15 + 5}{4} = \frac{20}{4} \quad ; \quad \lambda_2 = \frac{15 - 5}{4} = \frac{10}{4}$$

$$\boxed{\lambda_1 = 5} \quad ; \quad \boxed{\lambda_2 = 5/2}$$

1) $\boxed{Sp_{\mathbb{R}}(A) = \{5 ; 5/2\}}$

2) A admet $|A| = 25$ represente la determinant donc A est diagonalisable

3) On a : $A = P^{-1} \hat{A} P$ or $\hat{A}$

donc $A^n = P^{-1} \hat{A}^n P$.