PARTITIONING RANDOM GEOMETRIC GRAPHS INTO BIPARTITE BACKBONES

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Summary

We investigate the problem of verification and computation all determination of a partition of a random geometric graph (RGG) into k disjoint subgraphs satisfying the following conditions.

All but one of the subgraphs are connected $(1 - \epsilon)$ dominant bipartite (planar) subgraphs of similar size and structure termed "backbones", the other of comparably small size composed of the "noise" in the random distribution.

The verification that such backbone partitions exist employing relatively few dense backbones with little loss to random distribution noise is of interest to the rapidly growing field of wireless sensor networks (WSN's)[6, 5, 1, 4].

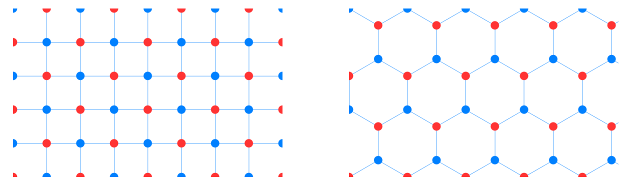
WSN's employ spatially distributed autonomous sensors to monitor physical conditions like sound, temperature, humidity and so on[2, 3, 9]. We use a random geometric graph concept in computer science to model WSNs by placing a random set of points either in a planar region or over the surface of the globe. Our goal is to determine disjoint subsets of the sensors that each can serve as a backbone for monitoring the whole region.

Our algorithm contains two-phase sequential coloring procedures (smallest-last coloring based on smallest-last ordering[7] and relay coloring based on an adaptive relay ordering) which are efficiently used to determine well-connected backbones to achieve the goal.

Extended Abstract

Given numerous randomly placed wireless sensors, how can we organize them into multiple communicating network grids (backbones) each covering the region[8]?

The bipartite planar Cartesian lattice grid with regular degree four and bipartite planar hexagonal (honeycomb) lattice grid with triangular lattice independent sets of regular degree three, see Figure 1 provides idealized placement that can be offset and replicated k times to form k backbones using all vertices. Here the face sizes are four and six.



(a) Cartesian Lattice Grid (b) Hexagonal Lattice Grid

Figure 1: Two lattice grids with face size 4 and 6

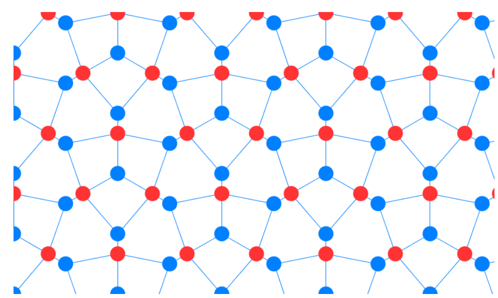


Figure 2: Bi-regular 3,4 Lattice Grid

Our question becomes if points are distributed randomly, can we select at least some minimally distributed grids with similar domination and patterns of face size primarily between four and six (like Figure 2 shows the bi-regular degree 3 and 4 lattice)?

Let a random geometric graph (RGG) denote a graph $G(N, r)$ with vertex set formed by choosing n points in a uniform random manner on the unit square, and introducing an edge between every vertex pair whose Euclidian distance is less than r. Our problem is to partition vertices into k disjoint sets $\{V_1, V_2, ..., V_k\}$ whose induced subgraphs $\langle V_1\rangle, \langle V_2\rangle, ..., \langle V_{k-1}\rangle$ are connected bipartite subgraphs with each part an independent set that dominates all or nearly all N vertices of $G(N, r)$. Let $V_1, V_2, ..., V_{k-1}$ be a partition of a majority of the vertices of $G(N, r)$ into disjoint sets where each set V_i induces a connected bipartite subgraph of $G(N, r)$. Specifically, we shall term $V_1, V_2, ..., V_{k-1}$ a bipartite component partition $BCP(\delta, \epsilon)$ of the random geometric graph $G(N, r)$ if the union of the vertex sets V_i comprise $(1 - \delta)N$ of the vertices and if the induced subgraphs $\langle V_i\rangle$ on average each dominate

$(1-\epsilon)N$ of the vertices. Our goal is to determine such partitions $BCP(\delta, \epsilon)$ for δ and ϵ suitably small, practically for example, with $\delta \approx 1/k$ and $\epsilon < 0.01$.

Our primary result is a linear time algorithm that for sufficiently large N and $k \approx 15$ constructively verifies the existance of a k-part partition with $(k-1)$ subgraphs each forming connected $(1-\epsilon)$ dominant bipartite (planar) subgraphs of similar structure and size $\approx N/k$. More generally, our algorithm provides a tool to analyze and display these bipartite "backbones" both for uniform distributions on the square and on the surface of the sphere. The latter is applicable to WSN's spanning the globe.

Sample Results

Table 1 shows data of RGG's employing the Square Topology and Table 2 shows data of RGG's employing the Sphere Topology. We tested the RGG benchmarks of vertex sizes 8000, 16000, 32000, 64000 and 128000 and all of the graphs are of around average degree 60. "Surplus" denotes the portion of the vertices not partitioning in the backones and is given by $|V_k|$. "Two-core" denotes the connected bipartite subgraph without "tails" (which is the vertices of degree 1), then each vertex in the "two-core" subgraph will have degree larger or equal to 2. The "two-core" subgraph will generate a more well-connected network backbone and further details will be discussed in the following paper of this abstract.

Figure 3 shows some screenshots of benchmarks on both square and sphere topology with $G(16000, 0, 045)$ vertices which indicates the ability of our developed tool. We also believe it is a great method for research via graphical implementation to identify new patterns or features.

Table 1: Data for RGG of Square Topology

N	8000	16000	32000	64000	128000
r	0.049	0.035	0.025	0.017	0.012
Avg. degree	60.01	59.03	60.33	58.99	60.25
Backbones	14	14	14	14	14
Surplus $\lvert V_k \rvert$	871	1679	3368	6255	12422
Avg. backbone size	509.21	1022.93	2045.14	4124.64	8255.57
Avg. backbone avg. degree	2.54	2.55	2.57	2.58	2.59
Avg. backbone face size	9.66	9.64	9.43	9.27	9.19
Avg. two-core face size	7.07	7.04	7.13	7.07	7.08
Avg. backbone dominates	99.63%	97.41%	98.07%	98.68%	98.98%

Table 2: Data for RGG of Sphere Topology

N	8000	16000	32000	64000	128000
r	0.175	0.123	0.087	0.062	0.044
Avg. degree	60.05	60.11	58.98	59.93	60.21
Backbones	15	15	14	14	14
Surplus $\lvert V_k \rvert$	833	1421	2904	5975	11581
Avg. backbone size	477.80	971.93	2078.29	4144.64	8315.64
Avg. backbone avg. degree	2.53	2.53	2.61	2.61	2.62
Avg. backbone face size	10.17	10.38	9.03	8.96	8.82
Avg. two-core face size	7.05	7.13	7.04	7.03	7.00
Avg. backbone dominates	94.39%	96.68%	99.66%	99.45%	99.53%

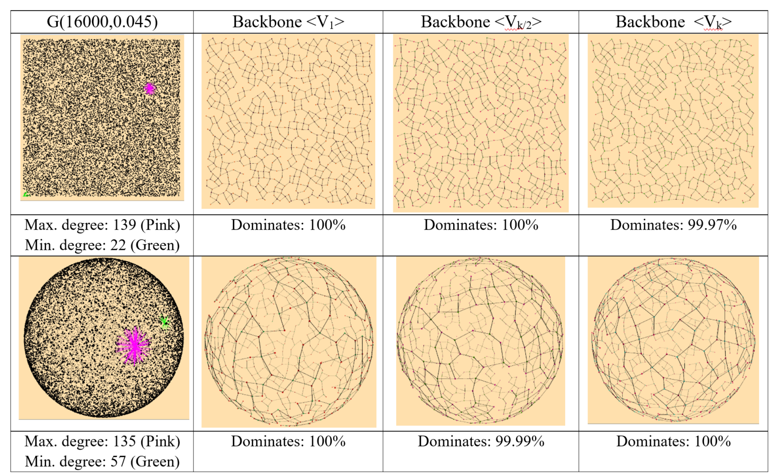


Figure 3: Screenshots of benchmarks on square model and sphere model

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