Bipartite Grid Partitioning of a Random Geometric Graph

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DCOSS 2017, Ottawa, Canada

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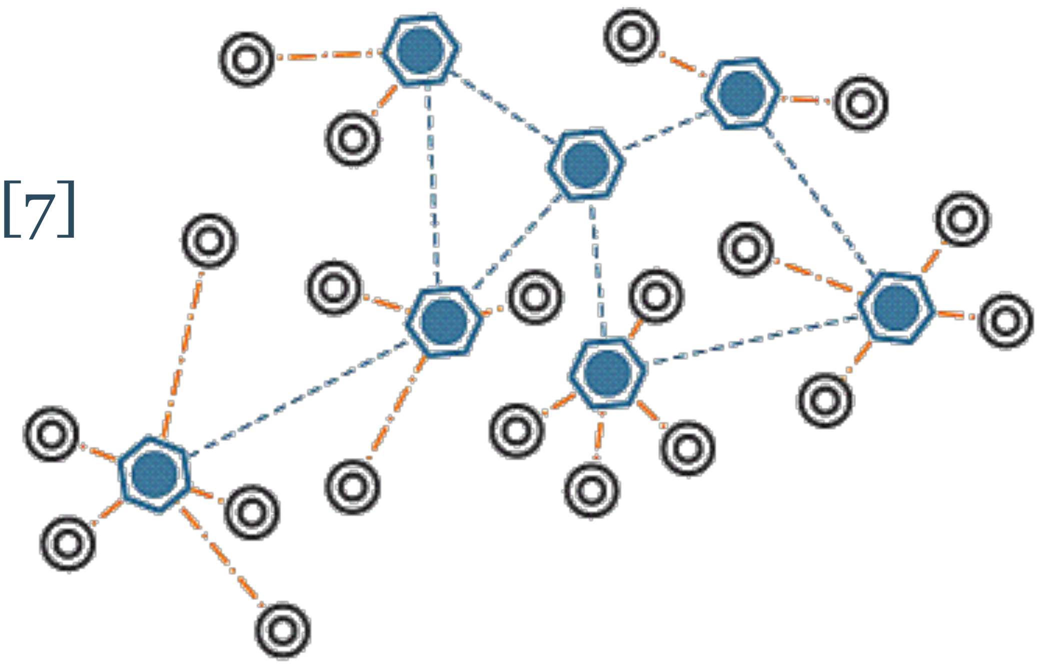
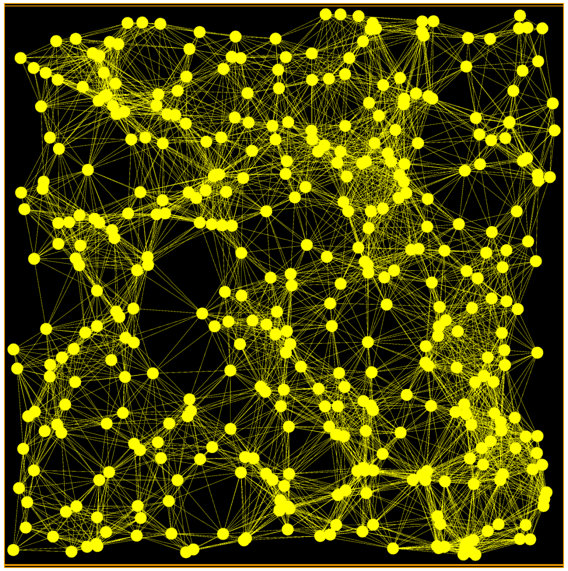
Background

- Wireless Sensor Networks (WSN)
  - Autonomous
  - Low power
  - Inexpensive
  - Large amount
  - Growing Field
    - Outer Space Field [7]



Question

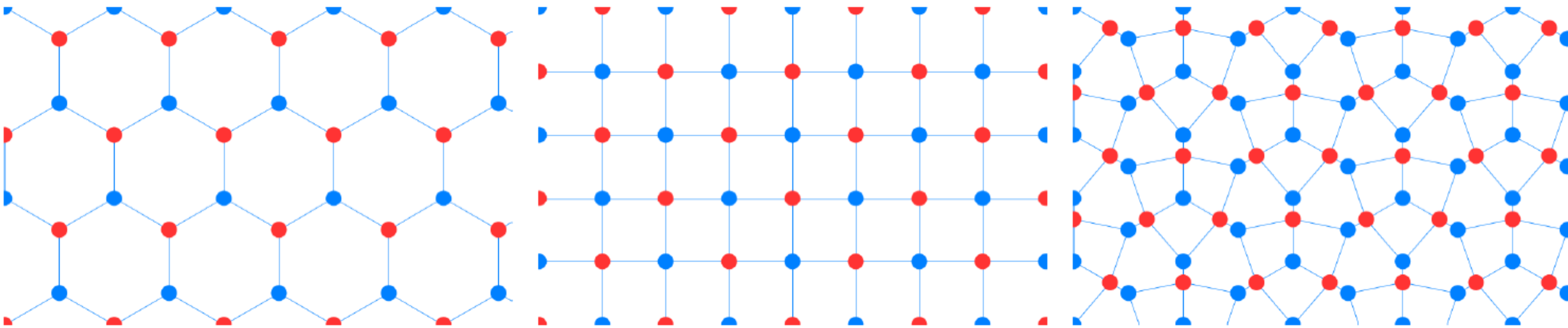
- Can we partition wireless sensors over a region into backbone grids such that:
  - The partition generates small number of grids
  - Each backbone set covers most of the region



Edges: 5038
Max degree: 41
Min degree: 5
Avg. degree: 25.19

Examples

- Manually constructed backbone grid deployments of multiple coverage [2] [3]:



(a) Hexagon — 6-Coverage

(b) Cartesian — 4-Coverage

(c) Bi-regular 3,4 — -Coverage ()

Modeling

- Random Geometric Graph (RGG)
  - A random undirected graph generated on a bounded region by:
    - Deploying n nodes (vertices) at random uniformly and independently
    - Connecting two vertices () iff their distance .
  - For different surfaces and sensor densities, we prefer to specify the parameters and "expected average degree" of the RGG, letting be determined by these specifications.

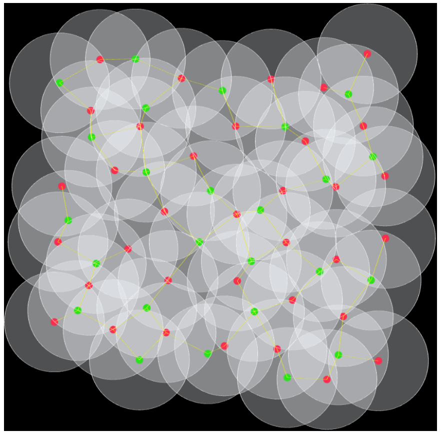
- Reasons [1]
  - Scalability
  - Feasibility
  - Large amount deployment

Goal

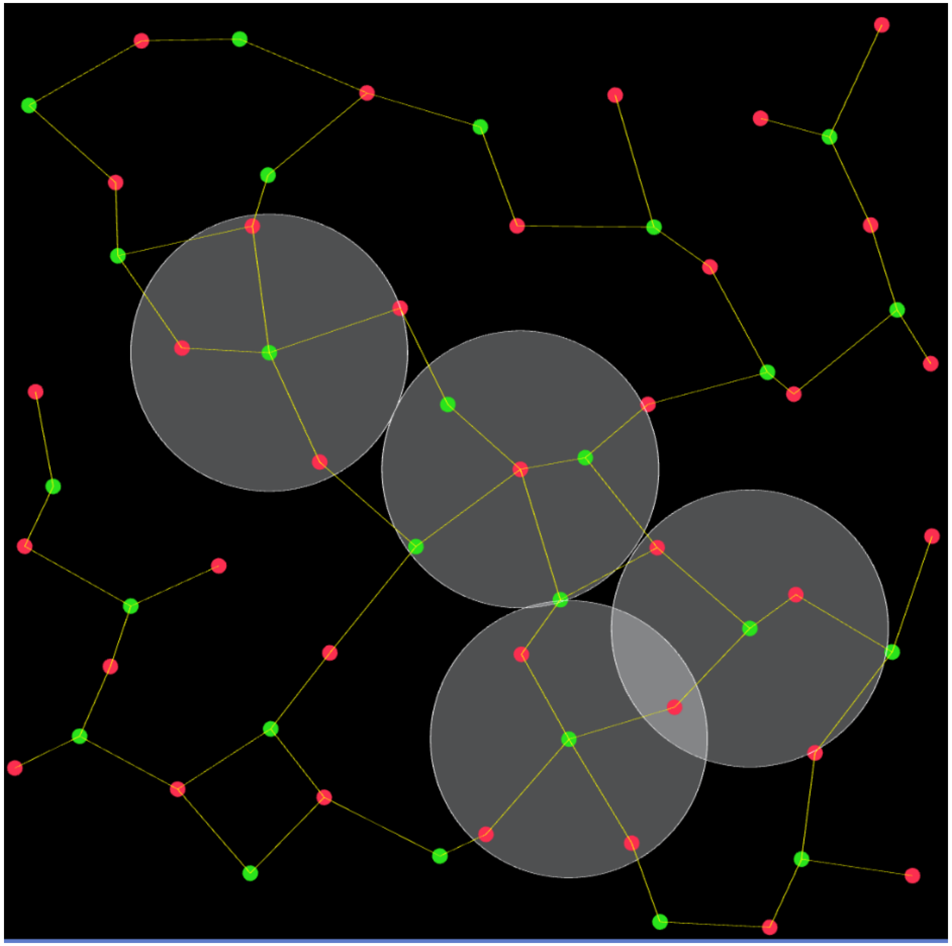
- Partition an RGG into bipartite subgraphs (backbone grids) such that:
  - The partition generates a reasonably small number of bipartite backbone grids
  - The set of bipartite backbone grids comprises a significant majority of the total number of vertices.
  - Each bipartite backbone grid is similarly structured and covers most of the region with at least triple coverage

One Bipartite Backbone Grid Example and its Coverage

Vertices: 61
Edges: 68
Avg. degree: 2.27

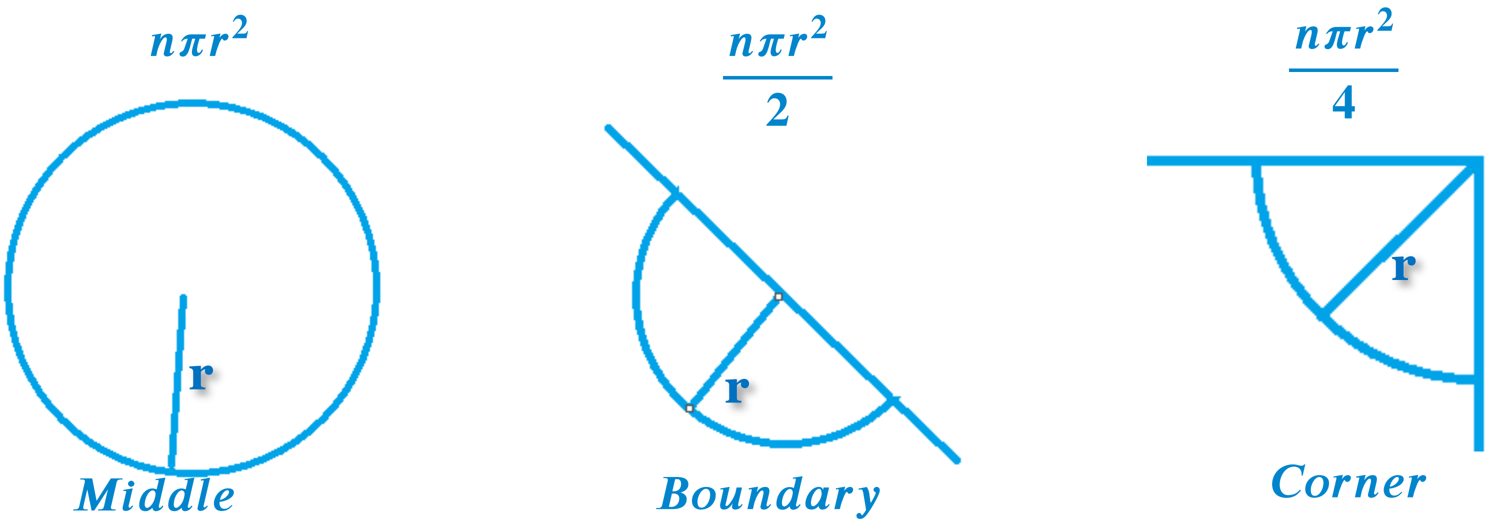


Covers almost all region



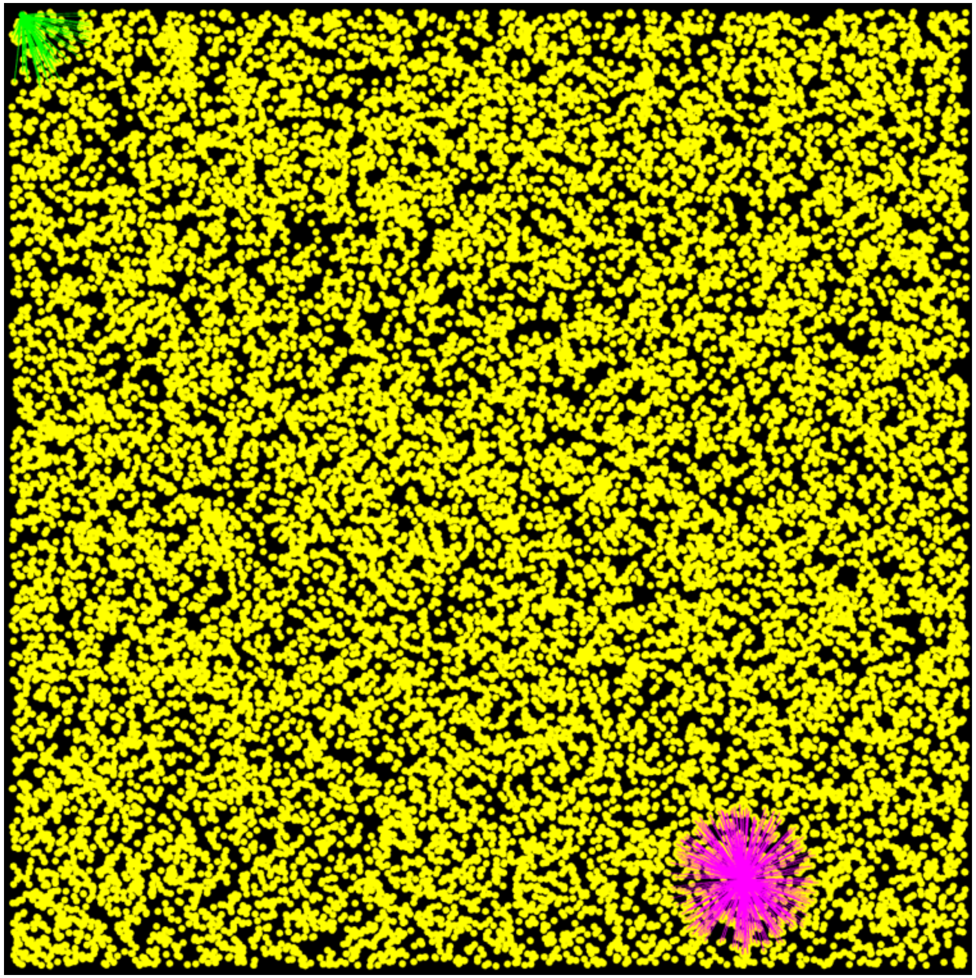
Topologies

- Unit Square, Unit Disk, Unit Sphere and other 3D topology Explorations (All algorithms are topology based only)
- Justification
  - Average degree of vertices in different locations.



Example of a Large RGG: G(20000, 0.08)

Edges: 3759504
Max degree: 467
Min degree: 96
Avg. degree: 375.95

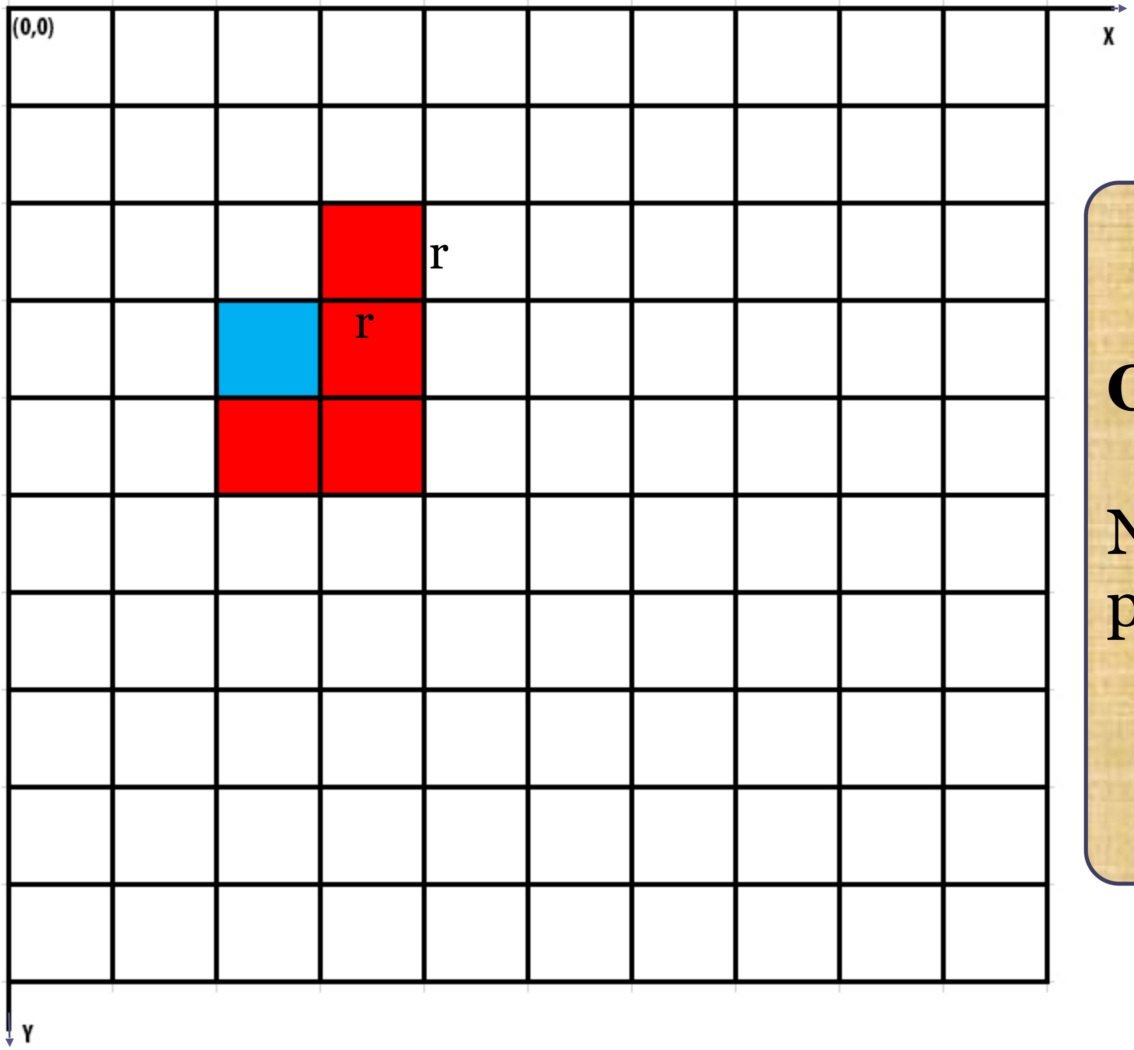


Observations:
Expected Avg. Degree

Degree of vertex on convex hull

implies

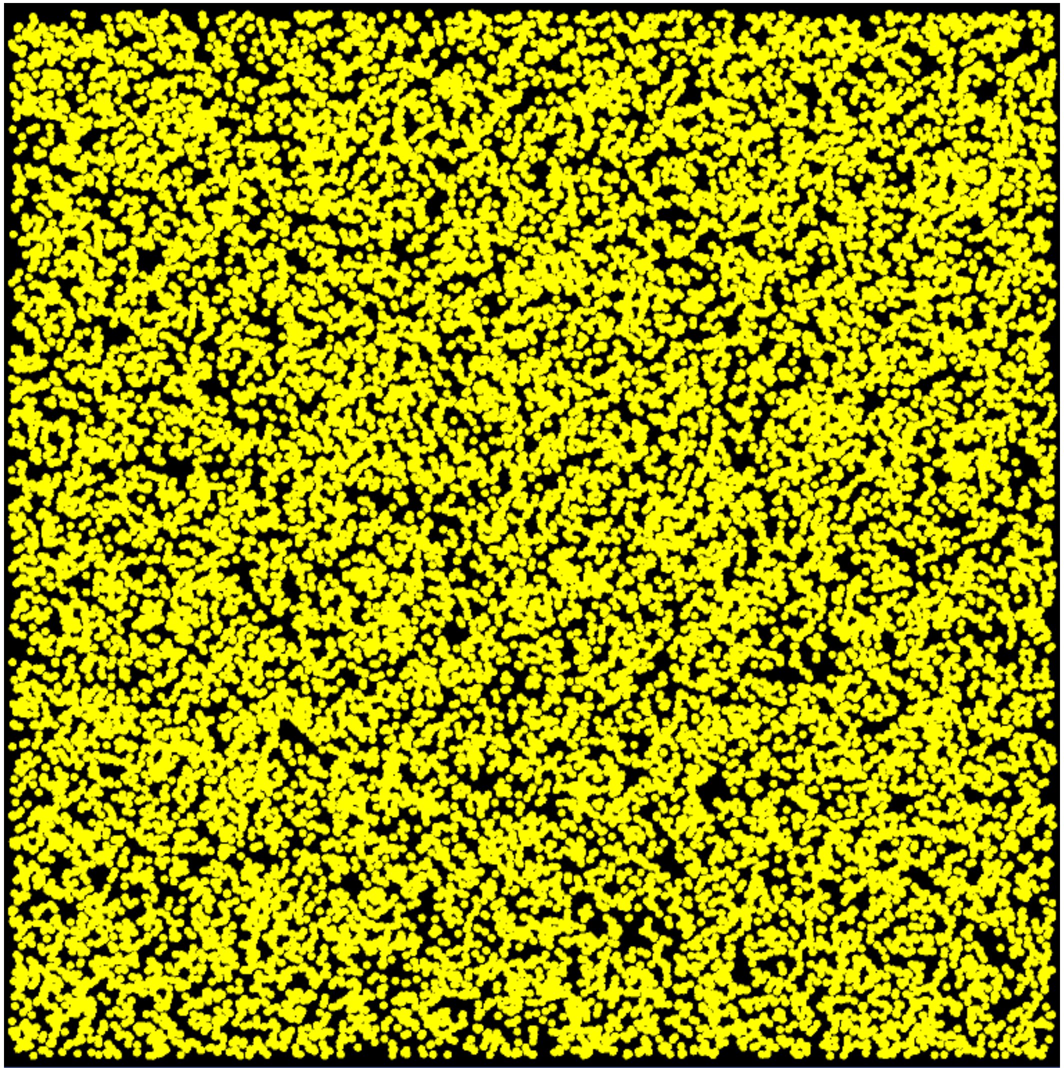
Cell Method Yields Edges in O(m)



Observations:

Number pairs checked per vertex:

Cell method: G(20000, 0.08)



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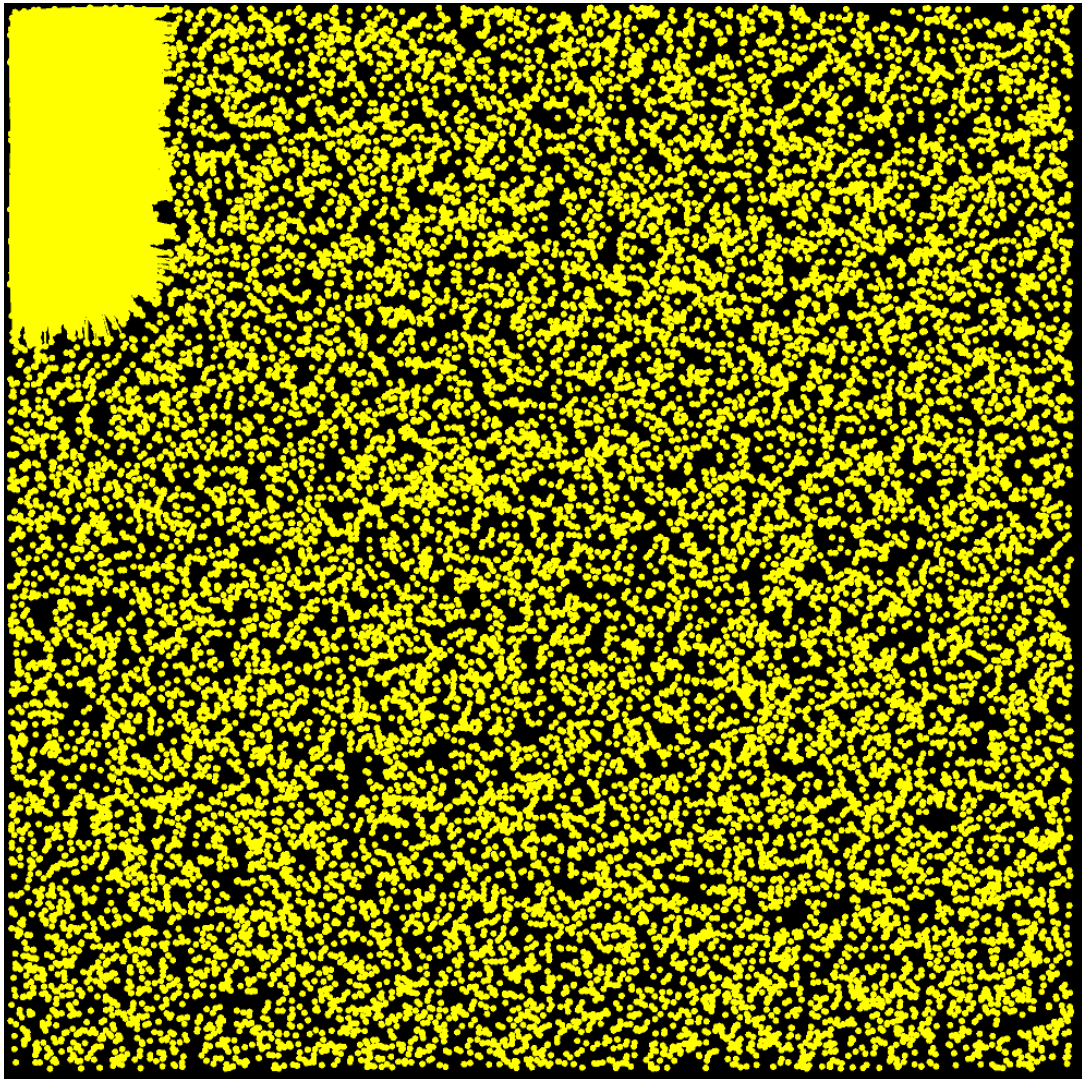
Cell method: G(20000, 0.08)



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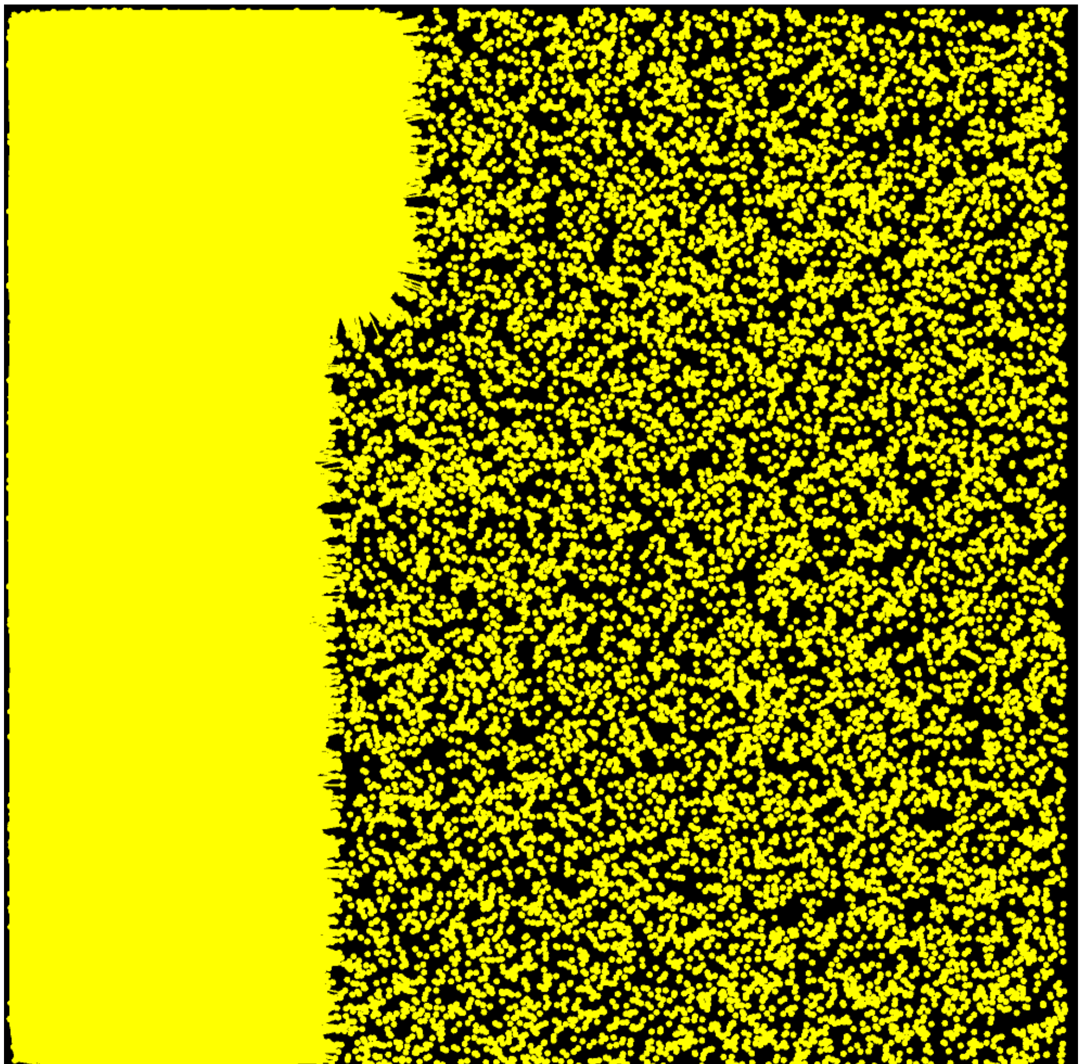

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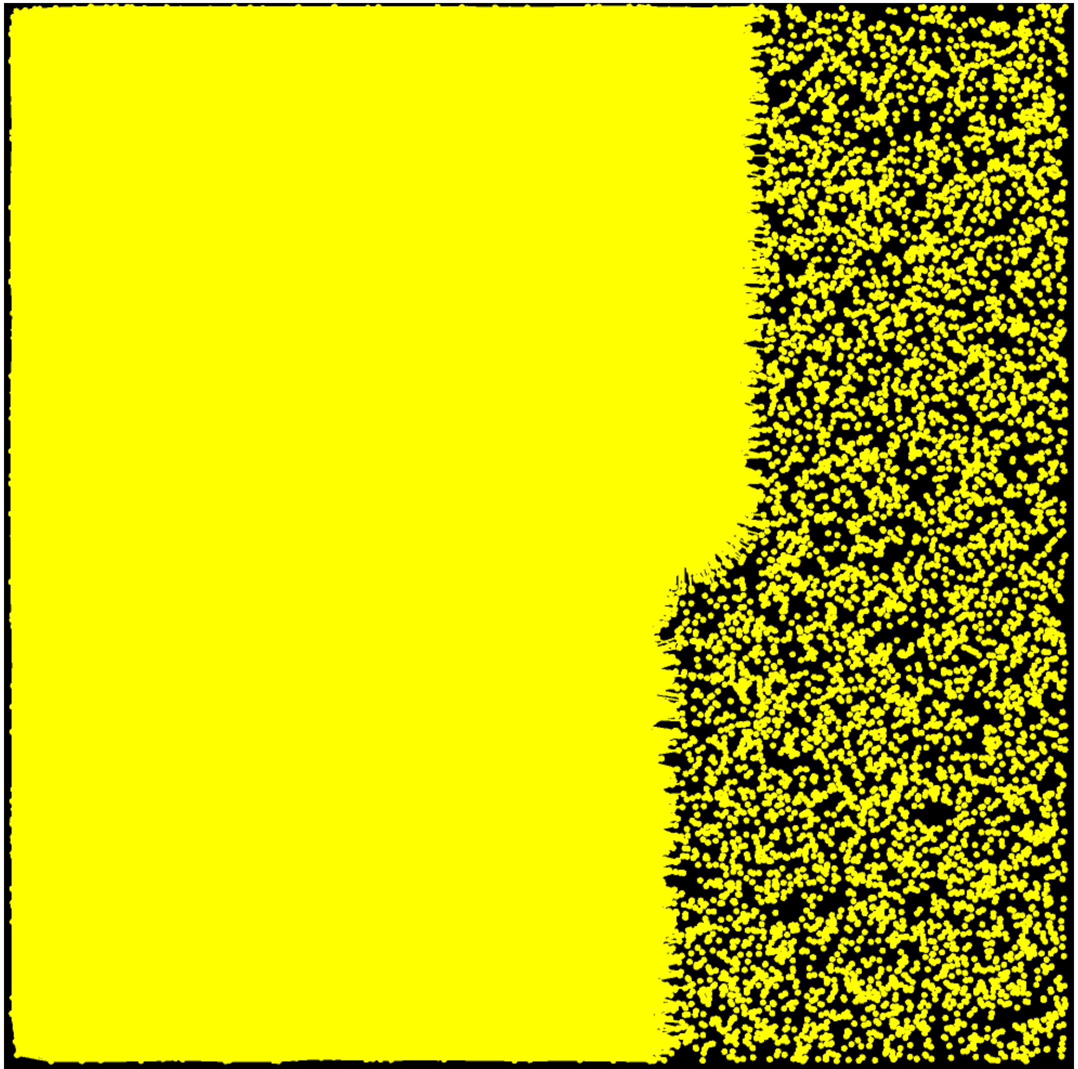

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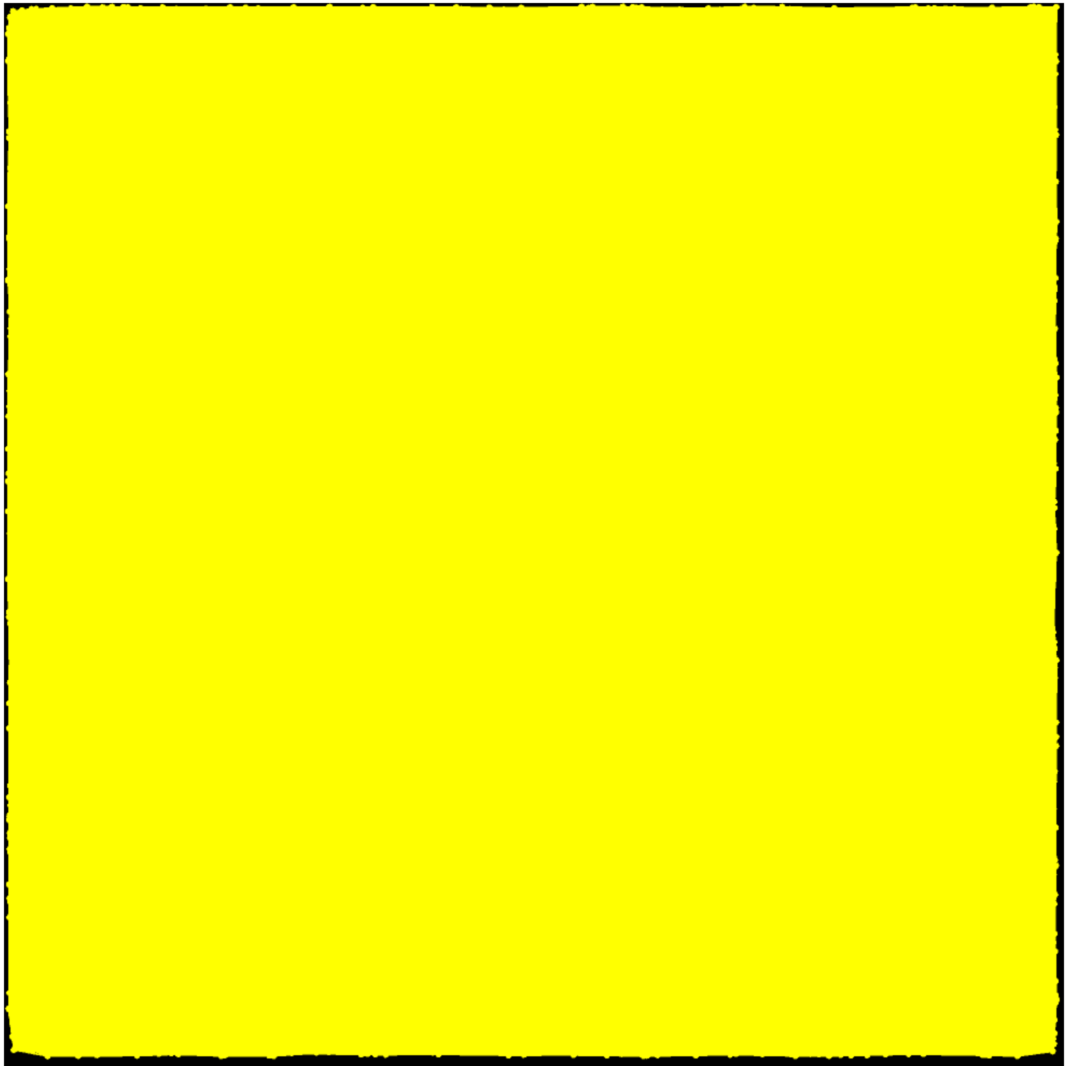


End

Procedure Overview (all linear time)

- Generate an RGG of n vertices, m edges using cell method.
- Determine primary independent sets using smallest-last coloring algorithm [4].
- Determine relay independent sets adaptively paired with primaries using relay coloring algorithm.
- Determine backbone grid in each paired bipartite subgraph.

Smallest-last Coloring Algorithm

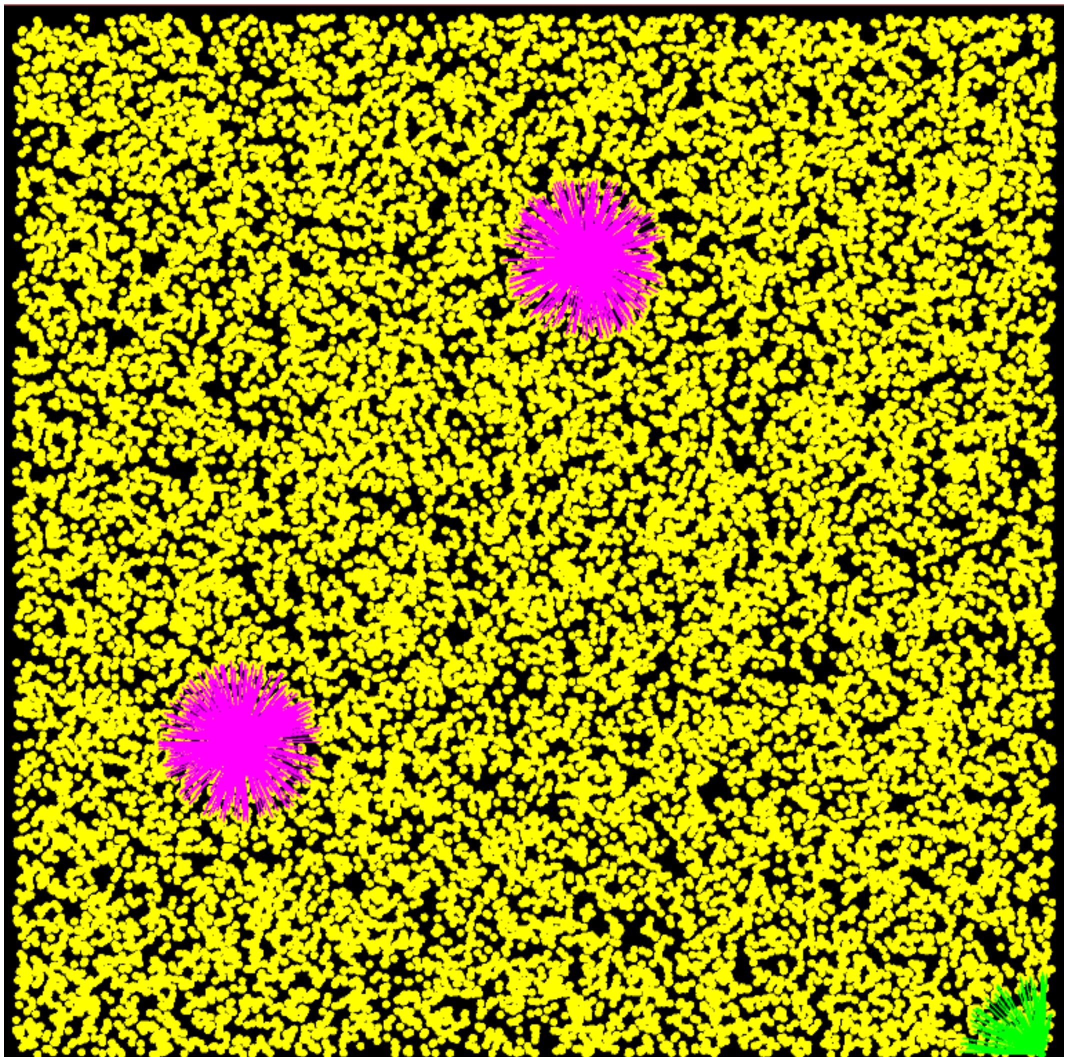
- A sequential coloring algorithm based on vertices in smallest-last ordering.
- Smallest-last ordering has been Proved
  - Efficient (Linear Time) [4]
  - Better than other sequential algorithms [5]

Algorithm 1 Smallest-last Coloring

Input: Graph $G(V, E)$
Output: Vertex Ordering S

```
S ← ∅
while V\S ≠ ∅ do
    append to S the vertex with smallest degree
    in the subgraph induced by V in S
end while
Greedy-Color G\S in inverse order of S
```

Smallest-last Ordering: G(20000,0.08)



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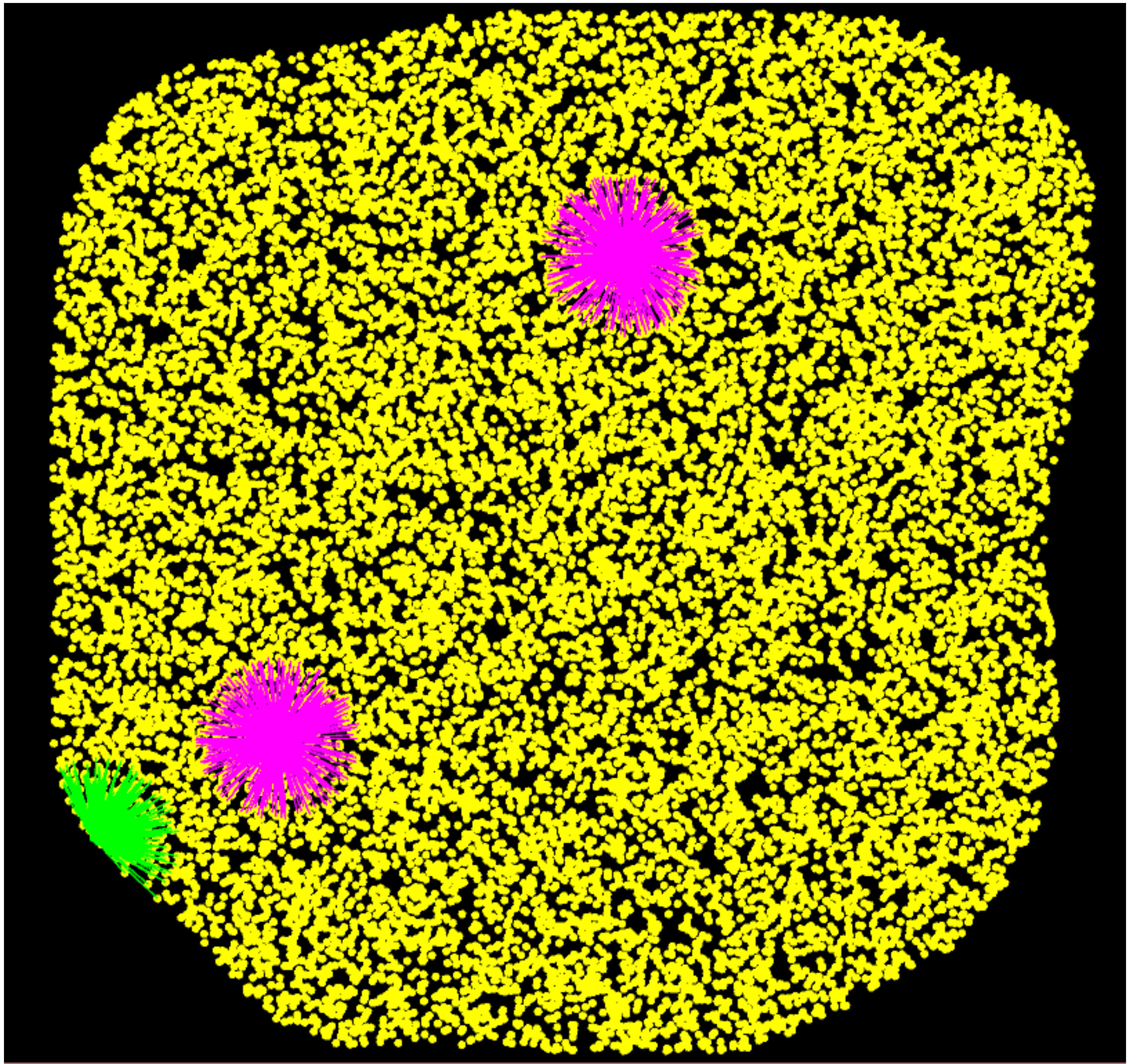
Smallest-last ordering: G(20000,0.08)



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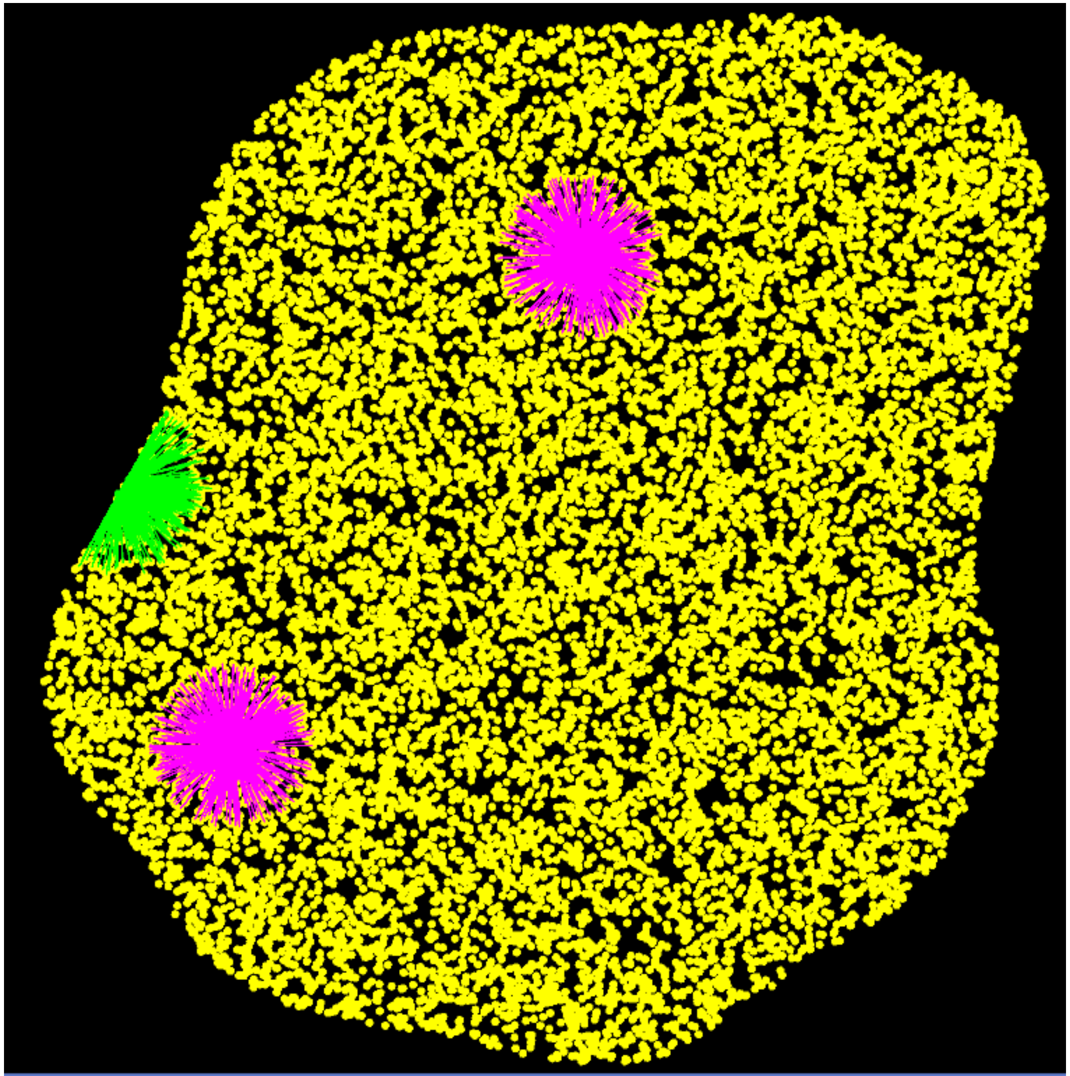

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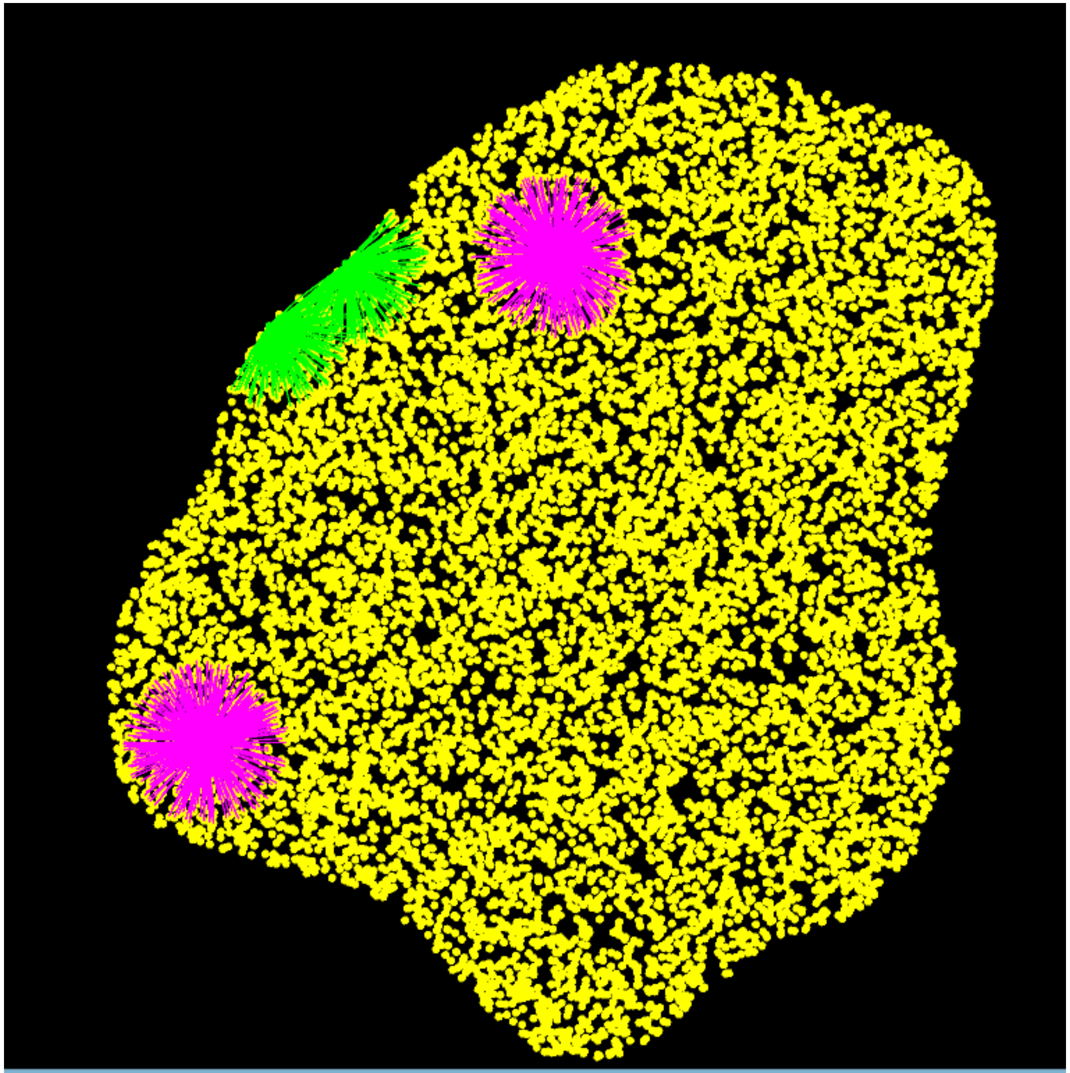

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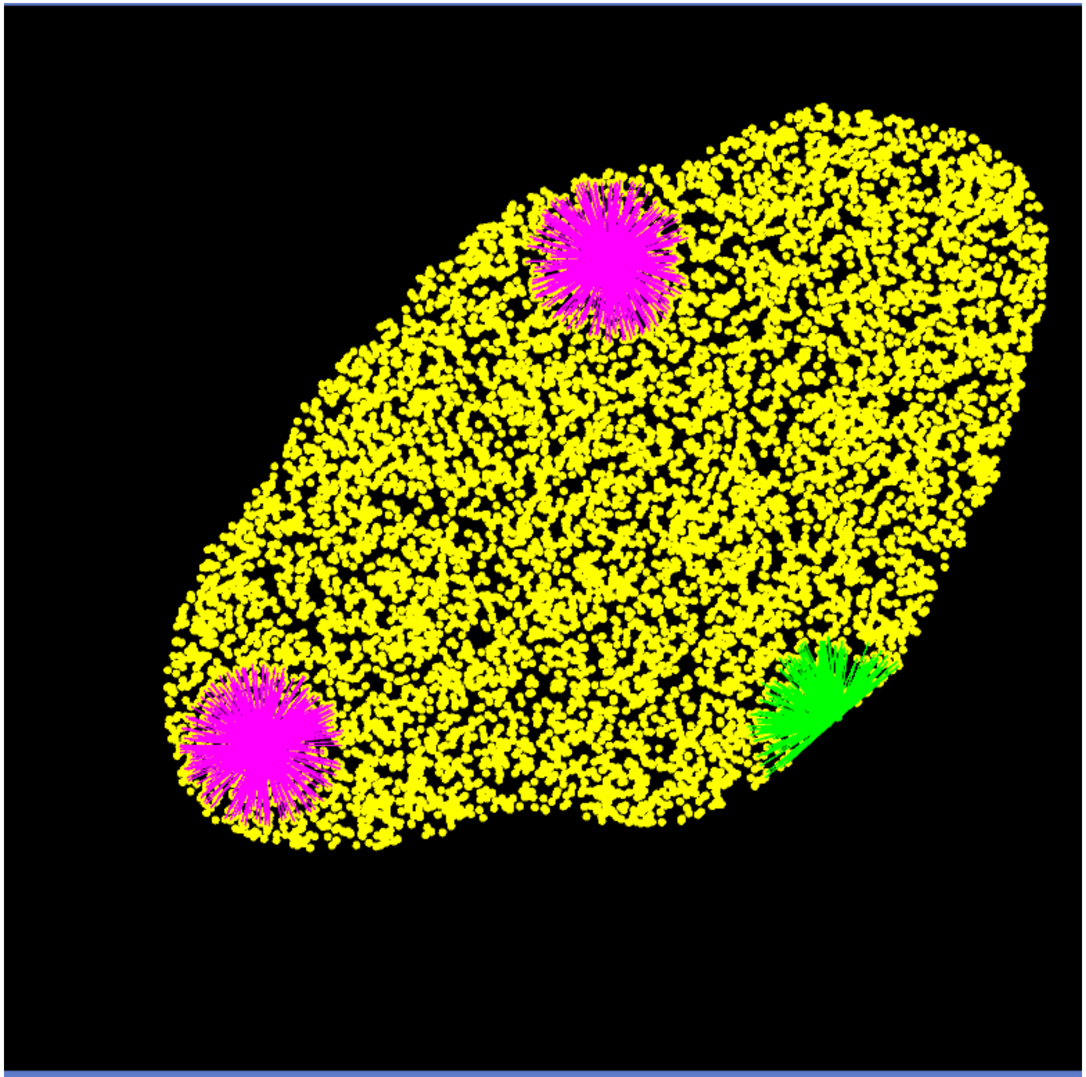
Smallest-last Ordering: G(20000,0.08)



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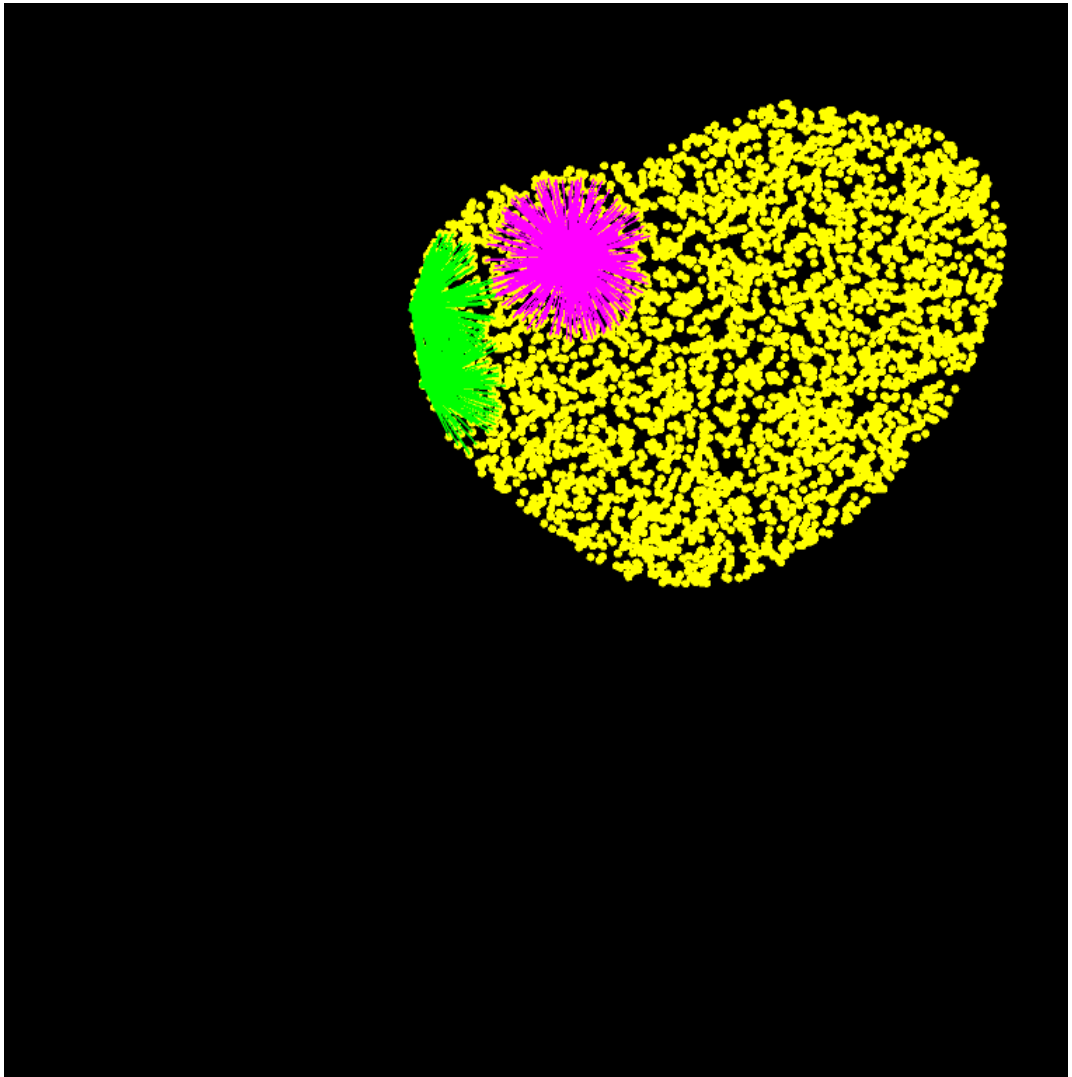

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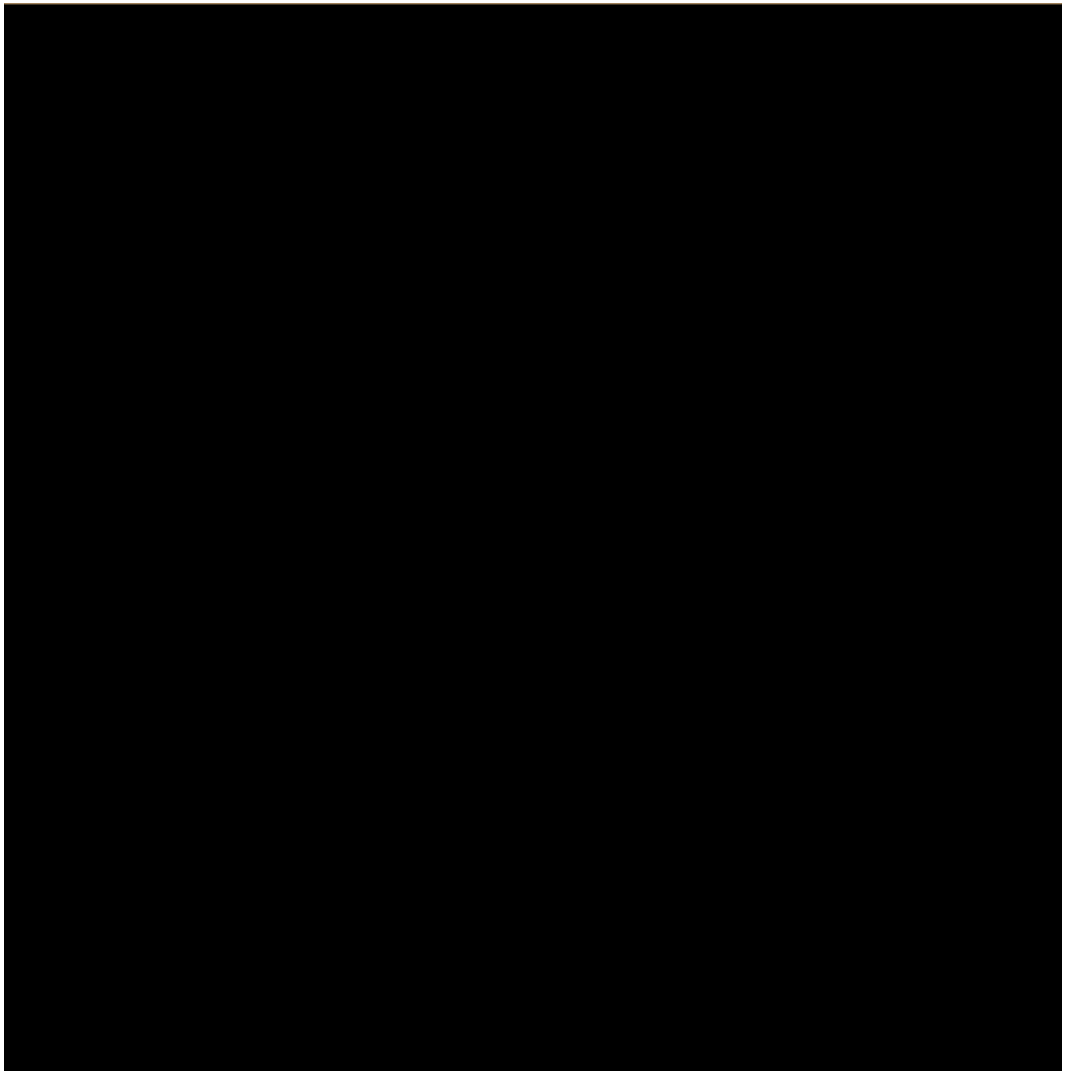
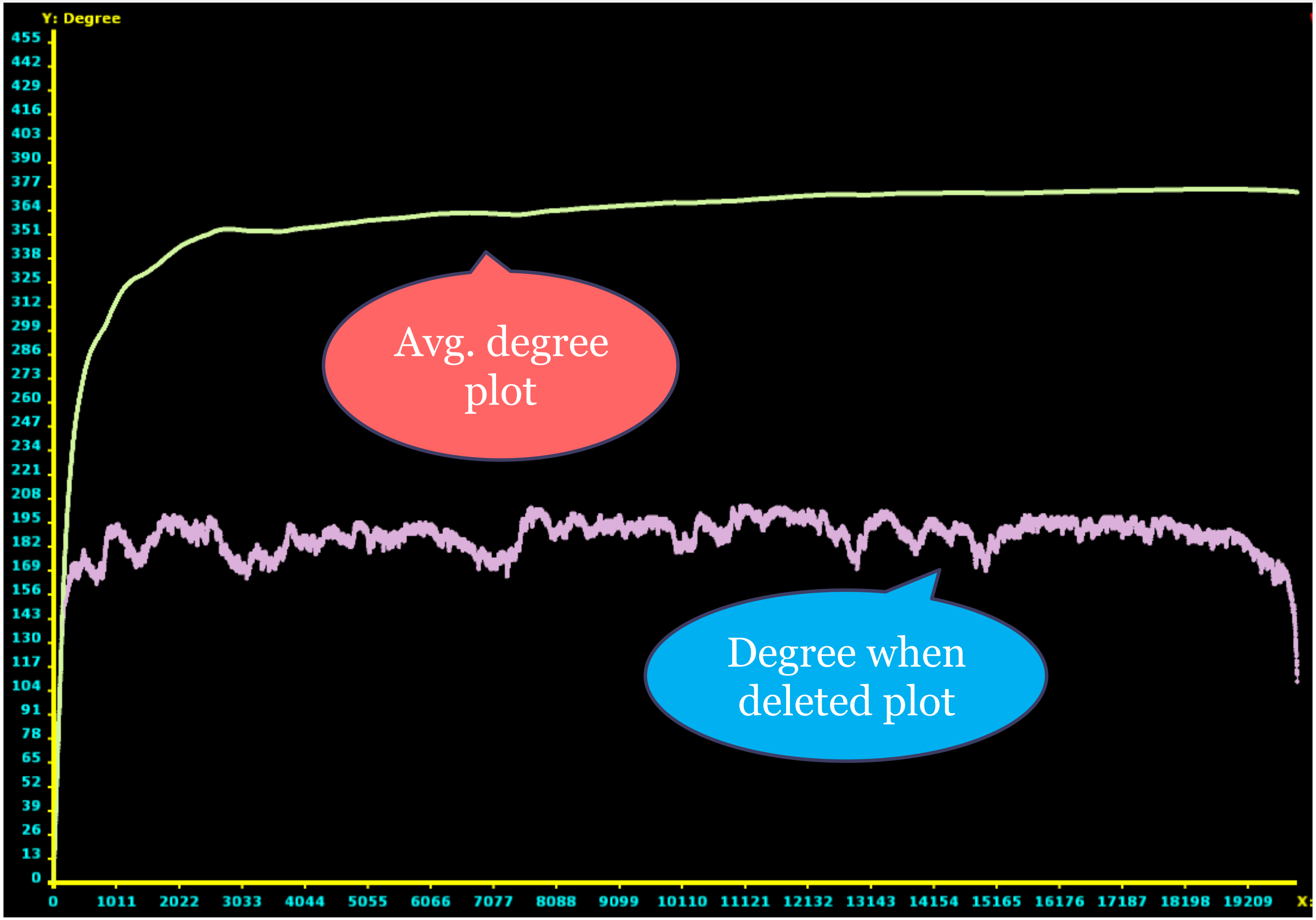
Smallest-last Ordering: G(20000,0.08)



End

Plot: Avg. Degree and Degree when Deleted

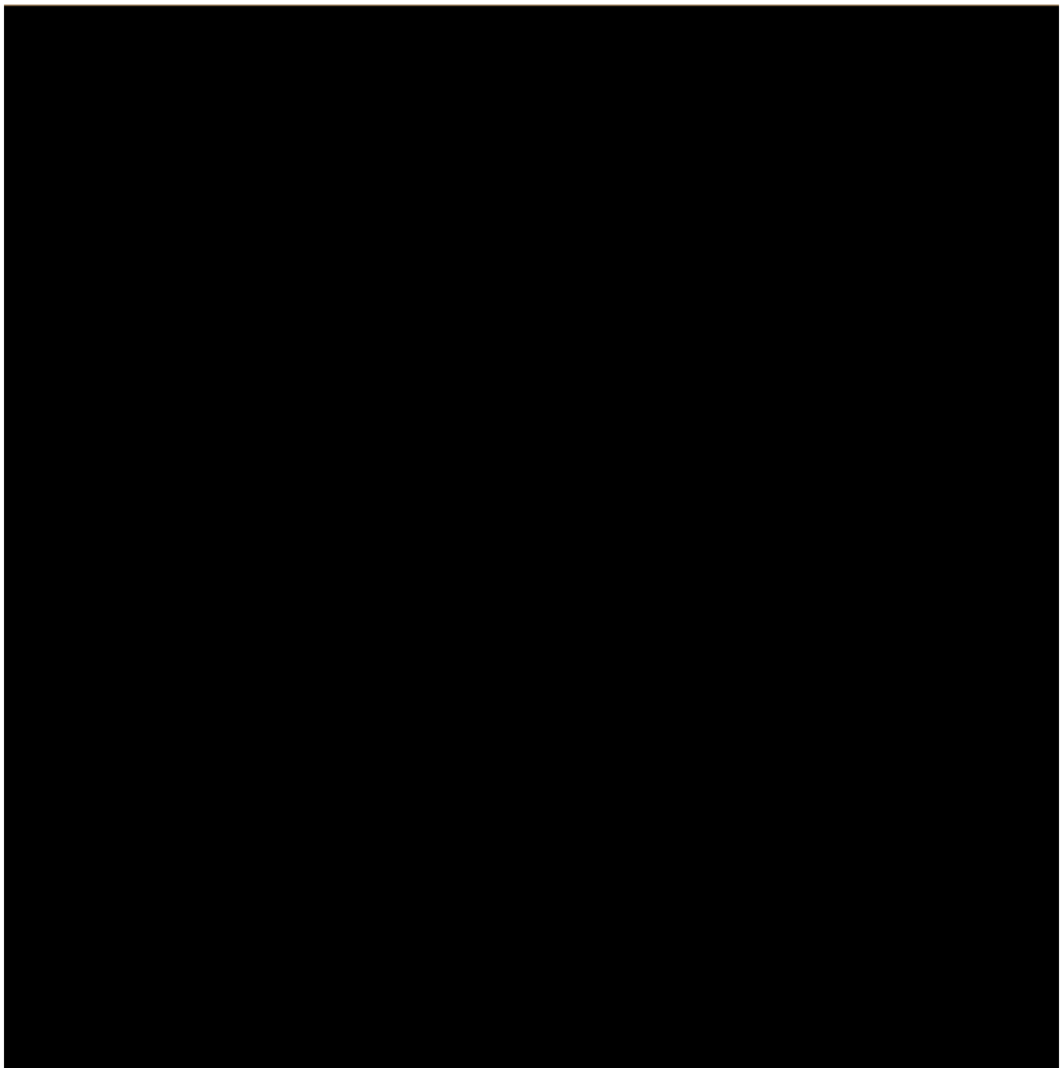


Observations

- Expected deletion degree (Geometry)
- Average "degree when deleted"
- Justification
  - Successive deletion removes all m edges in n steps and each time delete amount on average.

- Claim
  - Degree when deleted plot should be flat.

SL-coloring: G(20,000,0.08)



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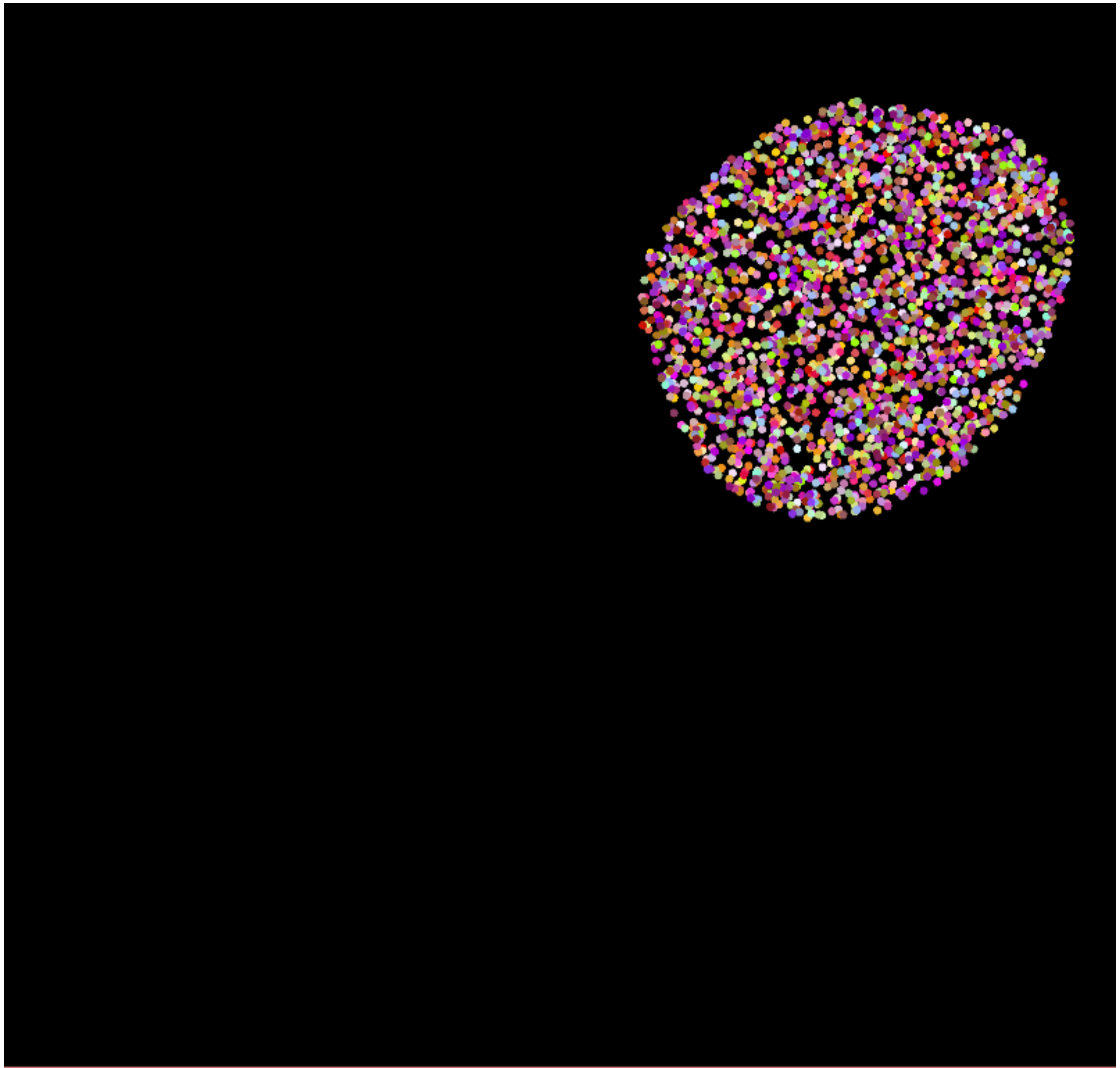
SL-coloring: G(20,000,0.08)



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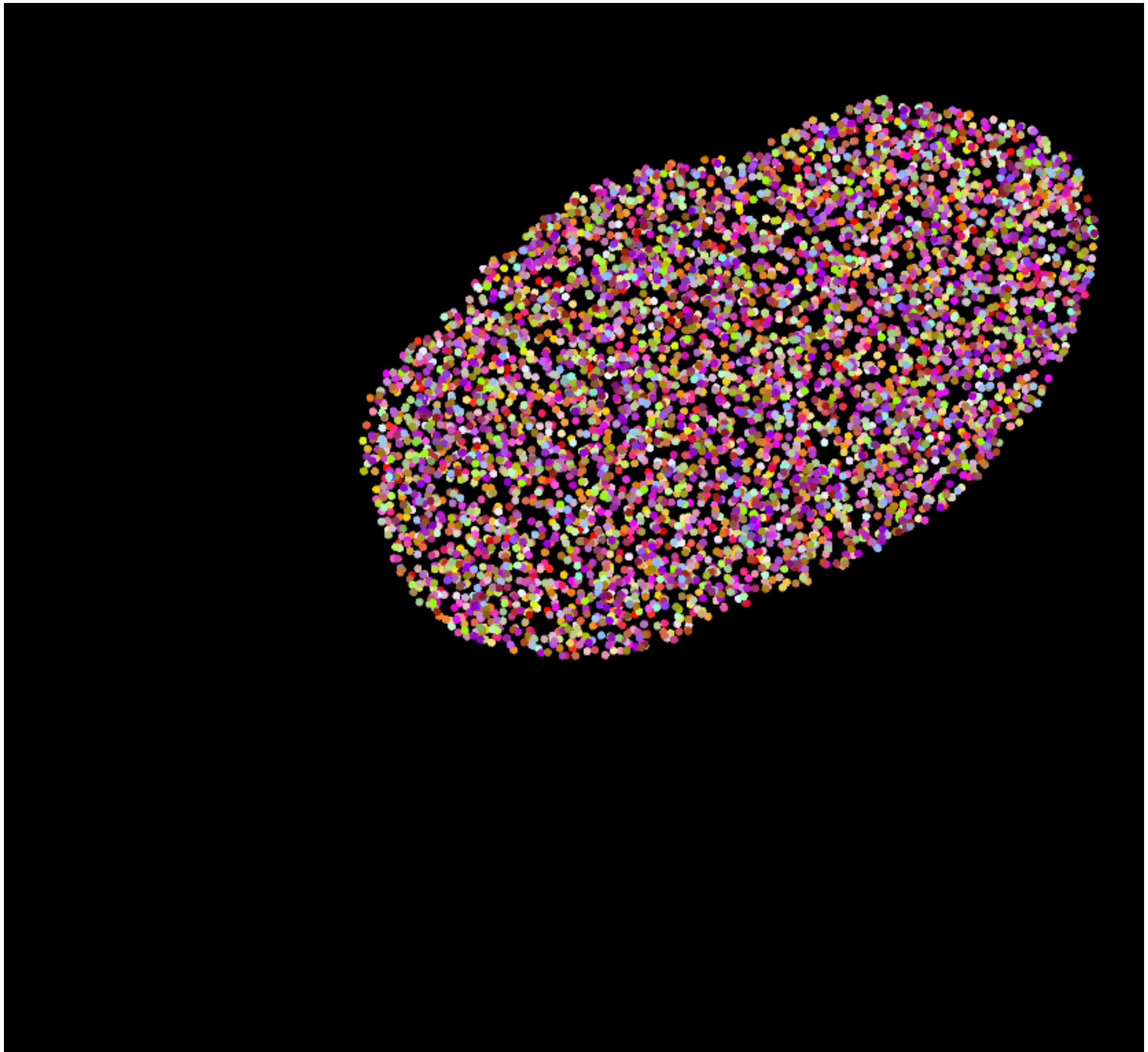
SL-coloring: G(20,000,0.08)



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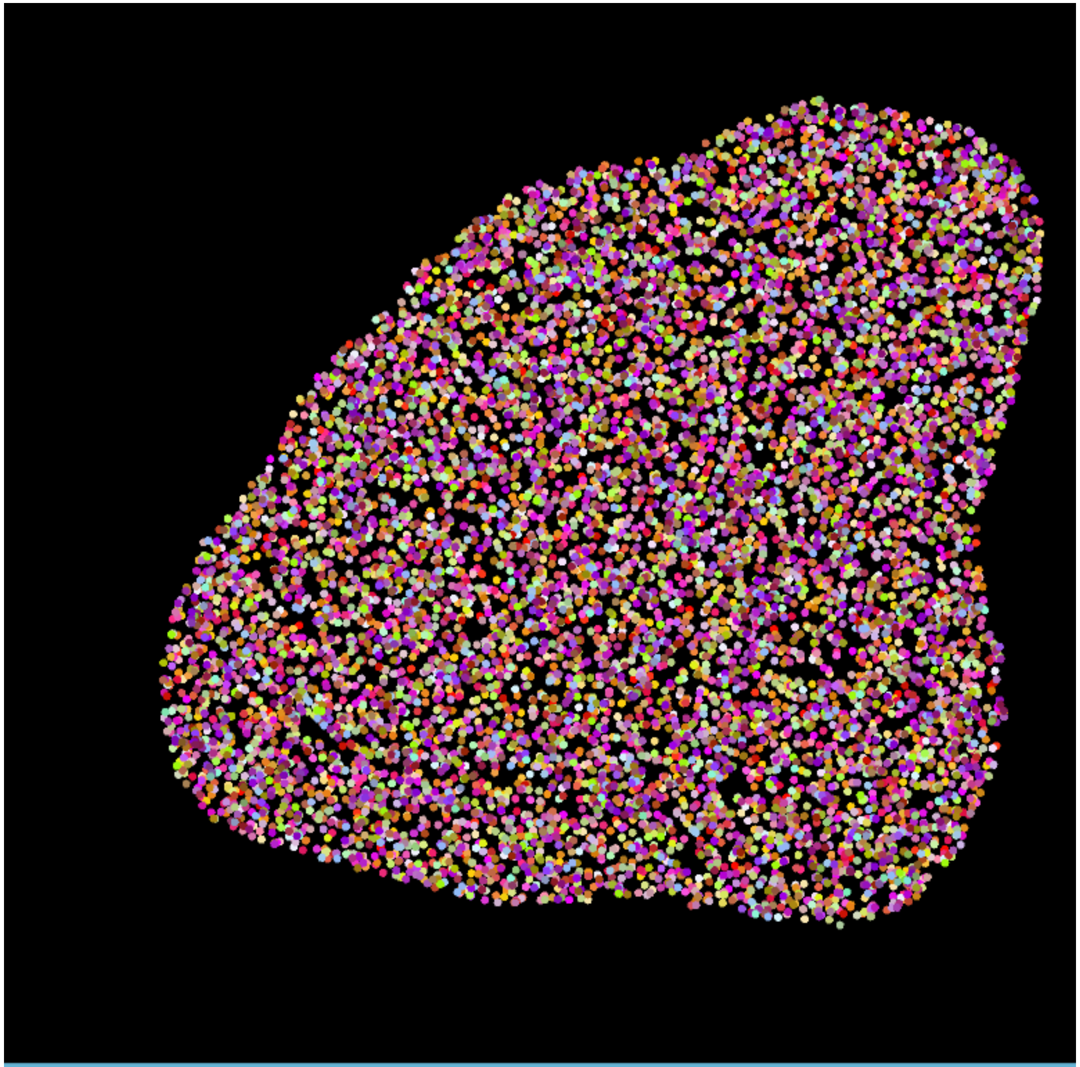
SL-coloring: G(20,000,0.08)



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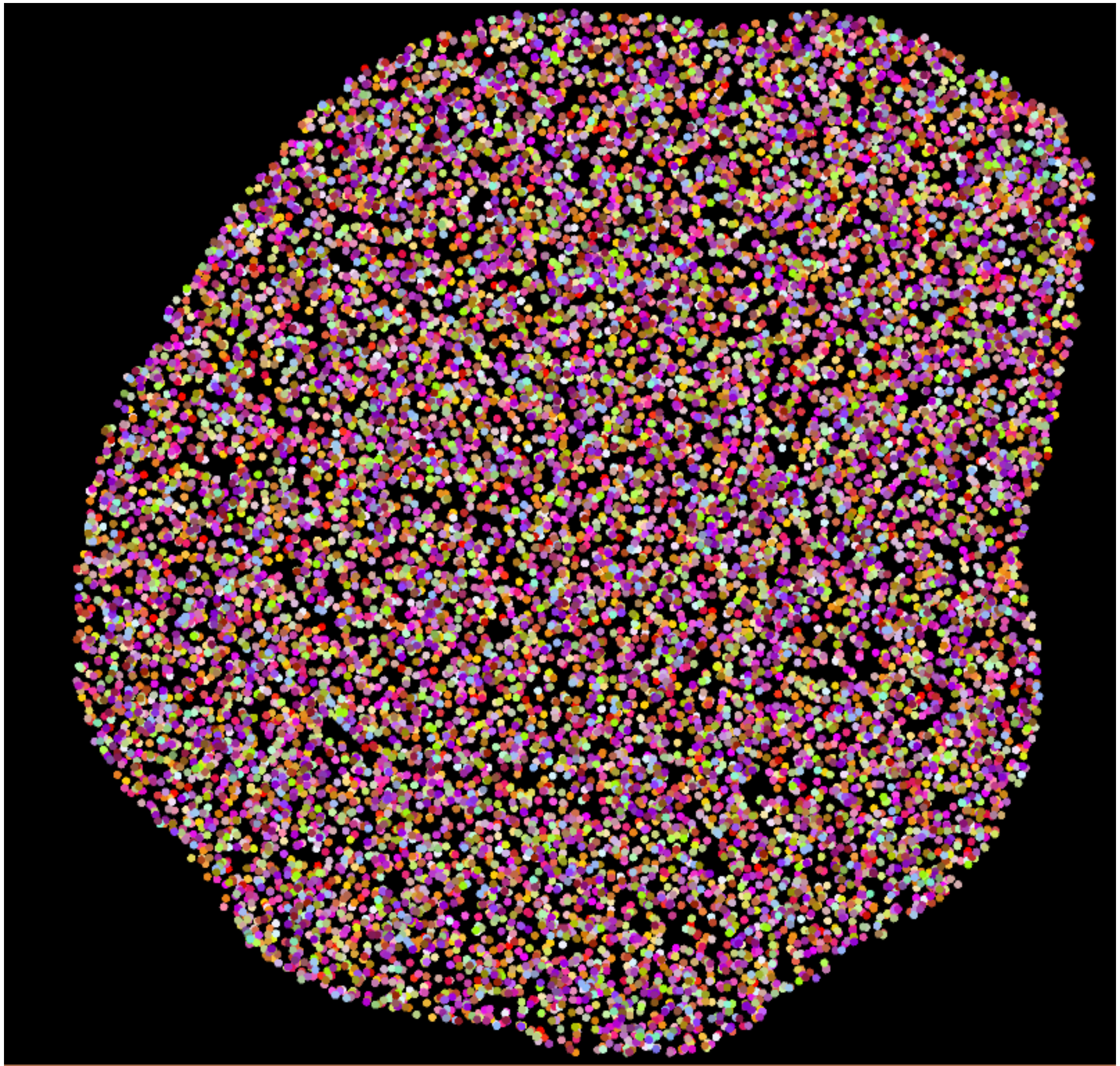
SL-coloring: G(20,000,0.08)



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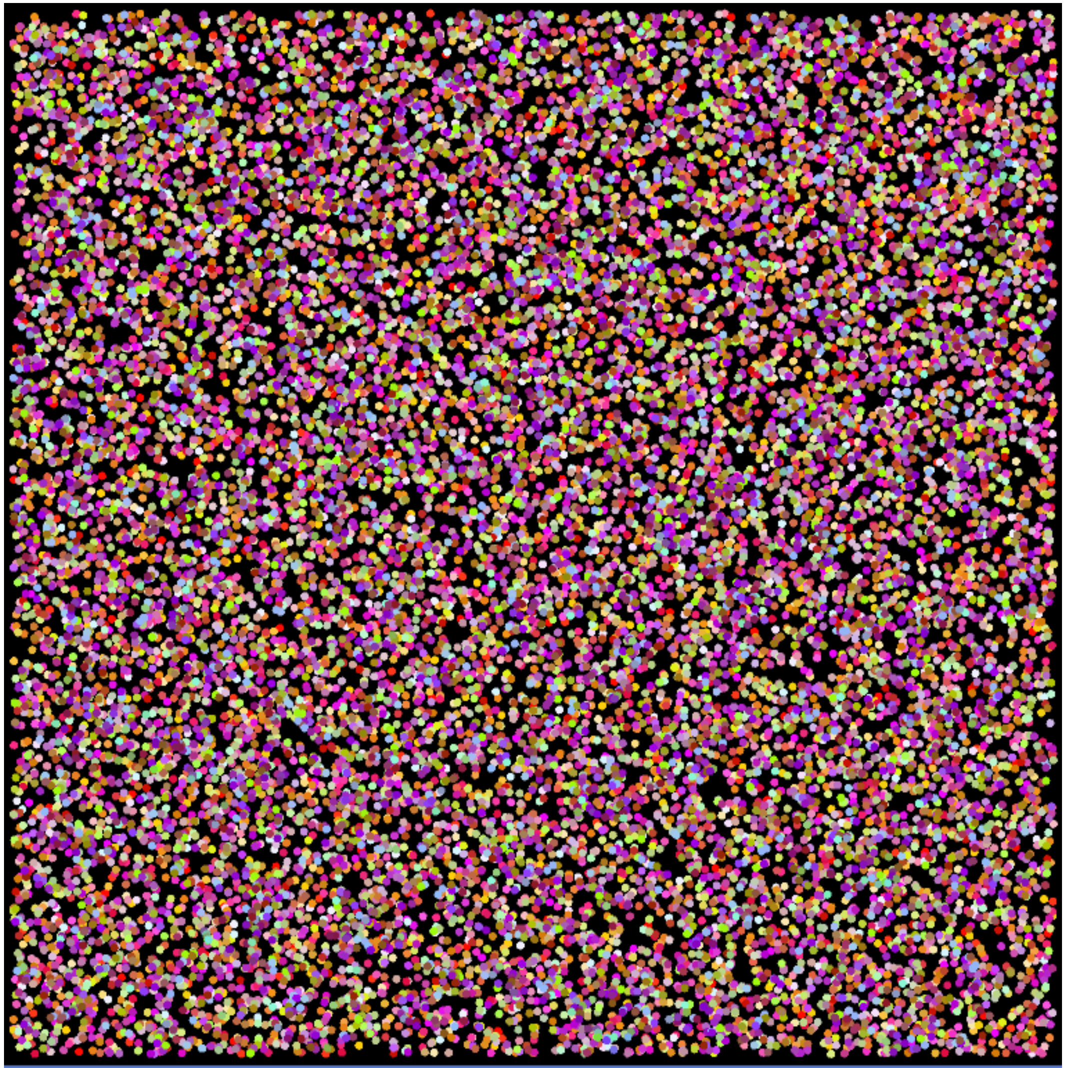
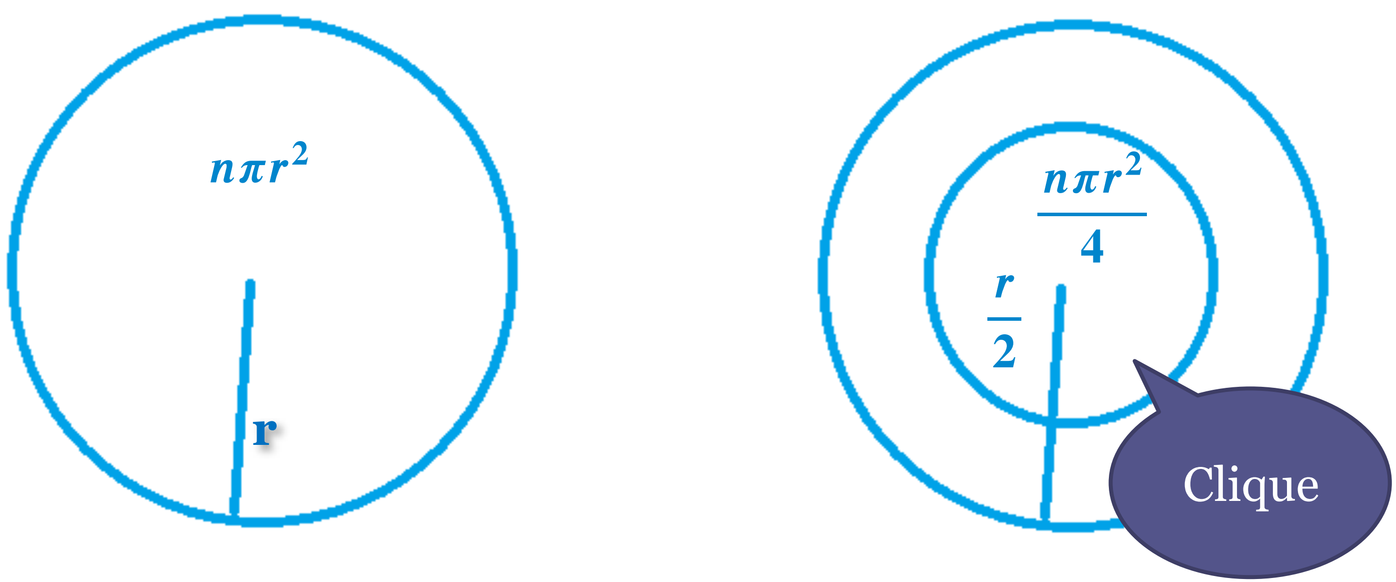
SL-coloring: G(20,000,0.08)



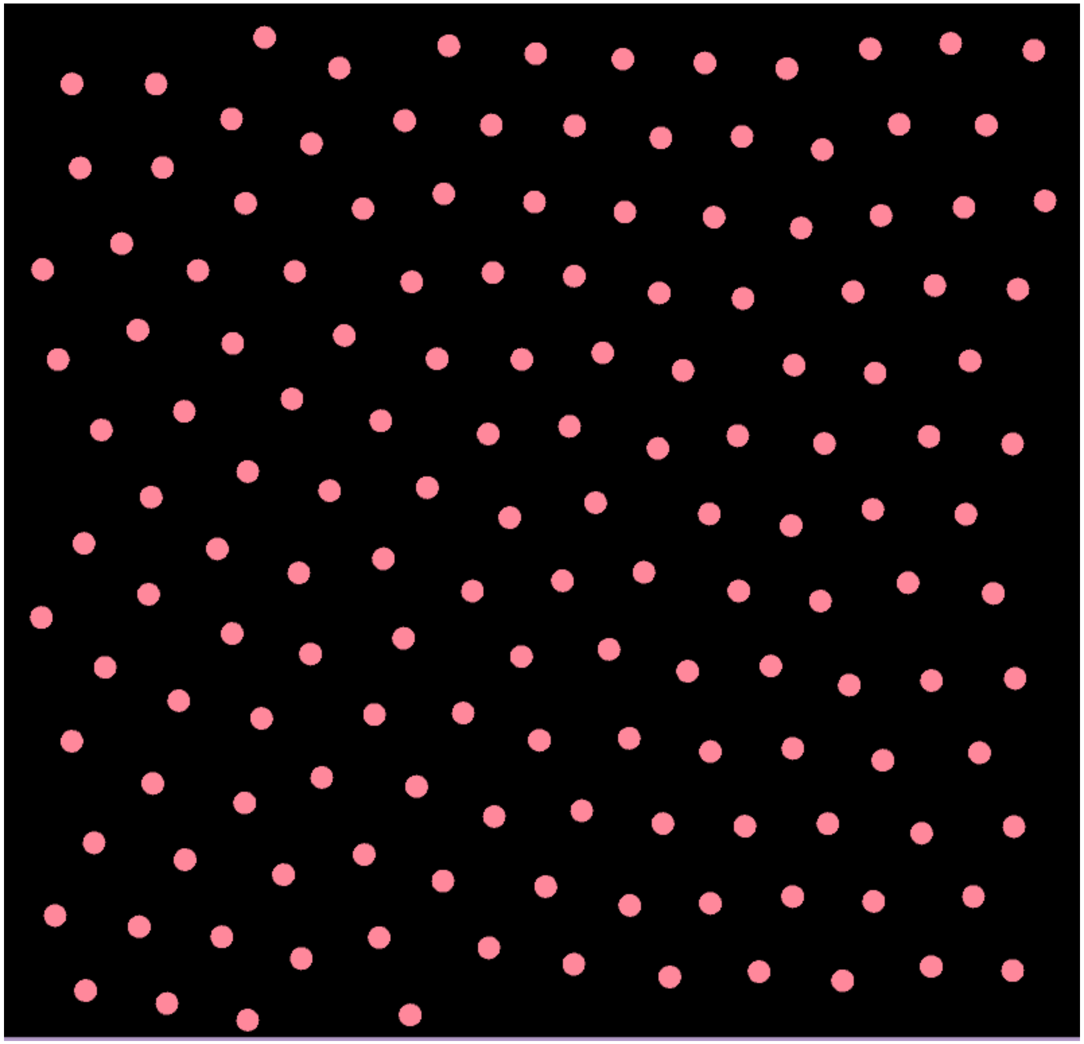
End

Observations

- Upper bound of number of colors should be Max min-degree and the lower bound should be around .



One Independent Set Example



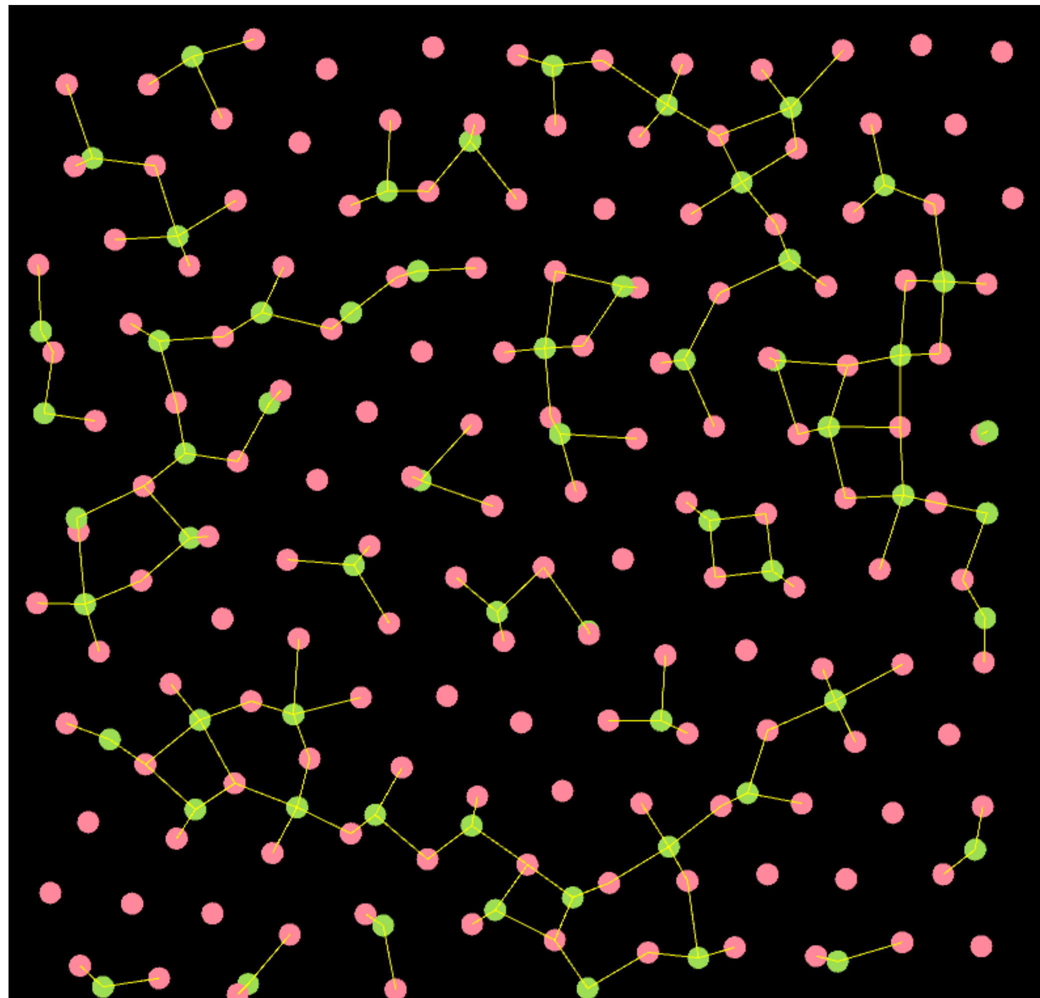
Vertices with same color value forms an independent set.

Combine any two independent sets will get an bipartite sub-graph.

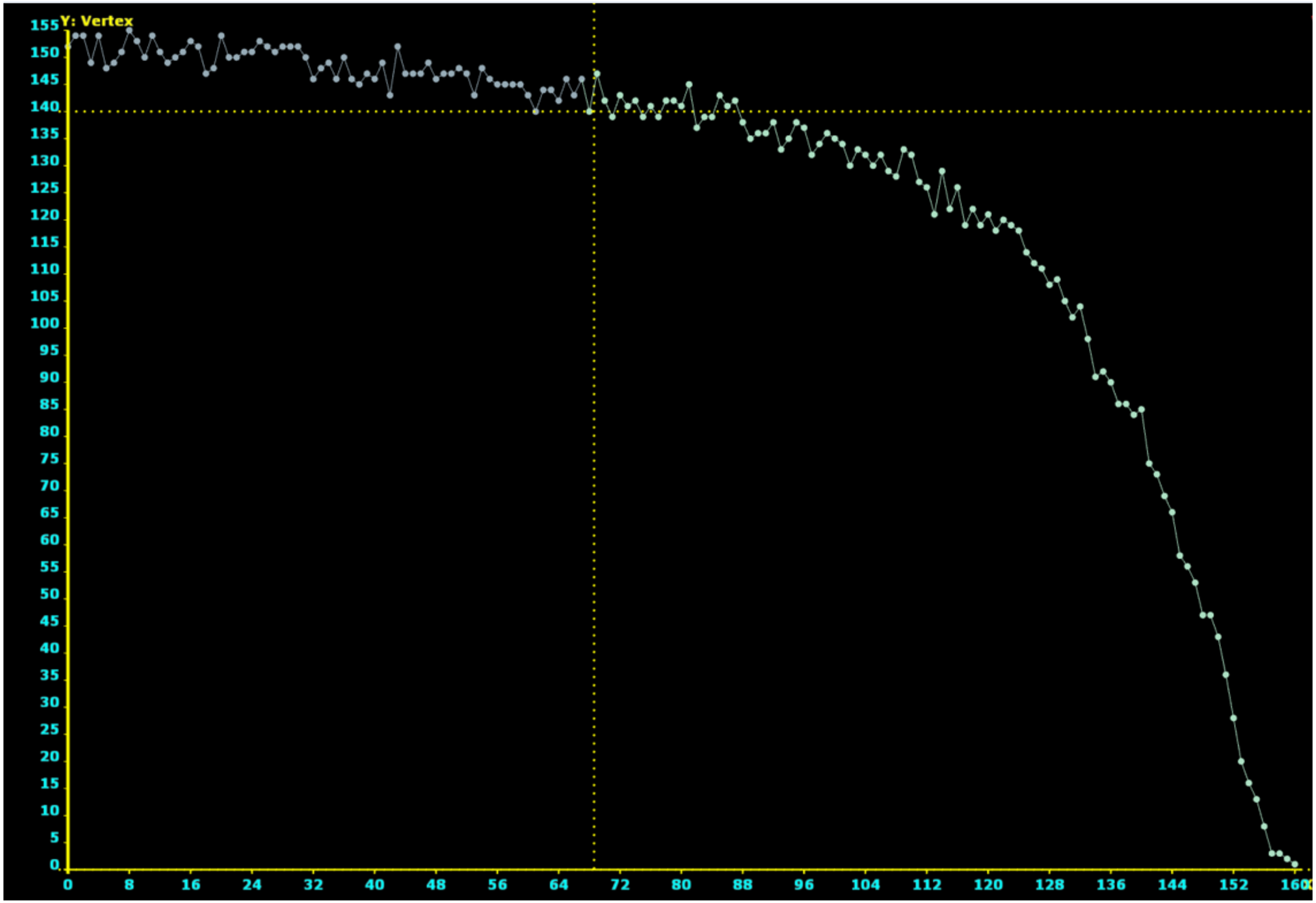
Bipartite Subgraph Pairing problem

"Distort" problem

However, pairing arbitrary independent sets may generate many small components rather than one 'giant' component

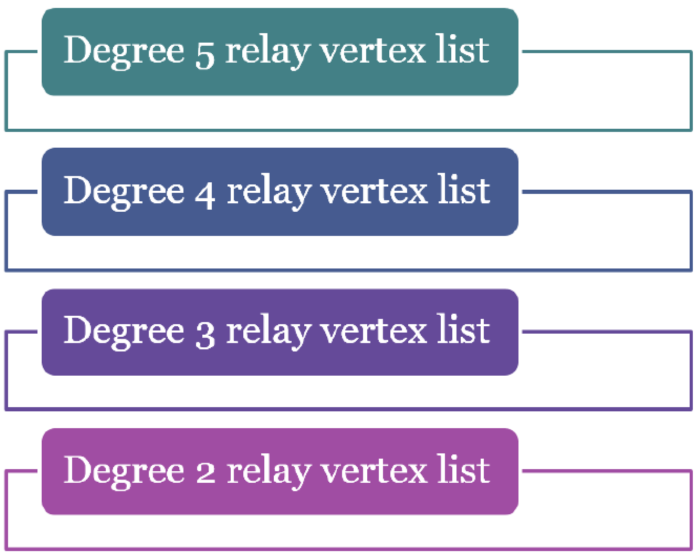


Plot: Color set sizes

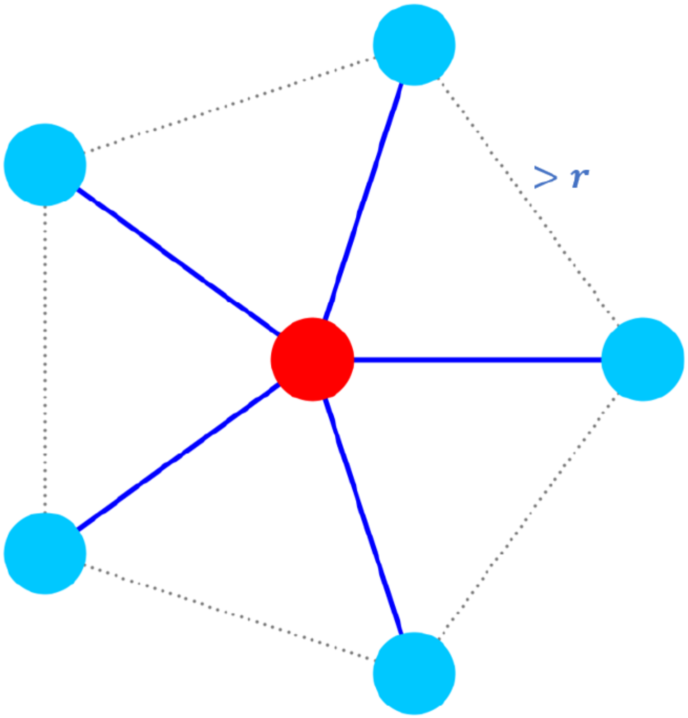


Plot: Color set sizes

Adaptive (Relay) Coloring Algorithm



(a) Relay Degree Lists



(b) Maximum One-color Adjacencies

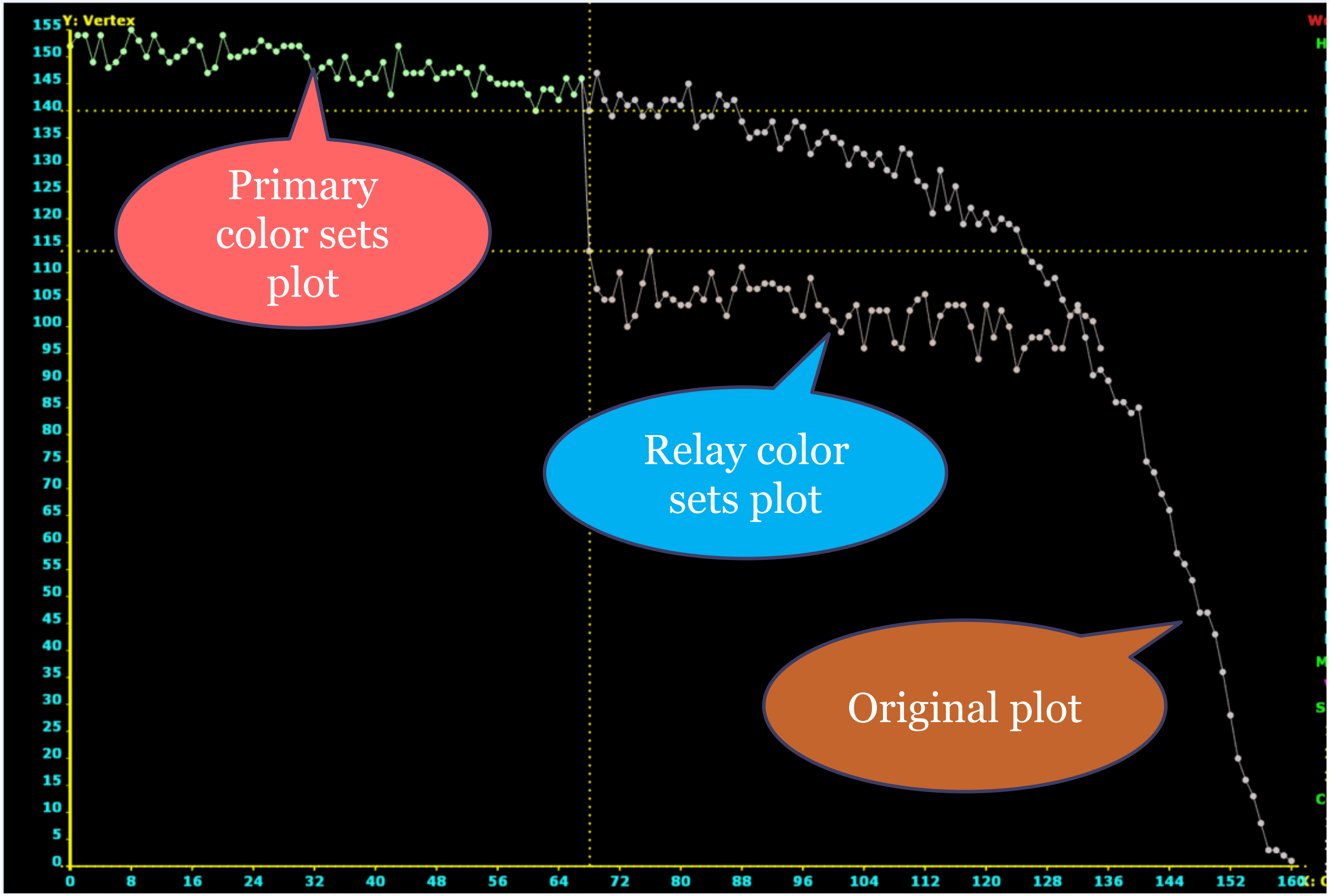
Relay Coloring Algorithm

Algorithm 2 Relay Coloring

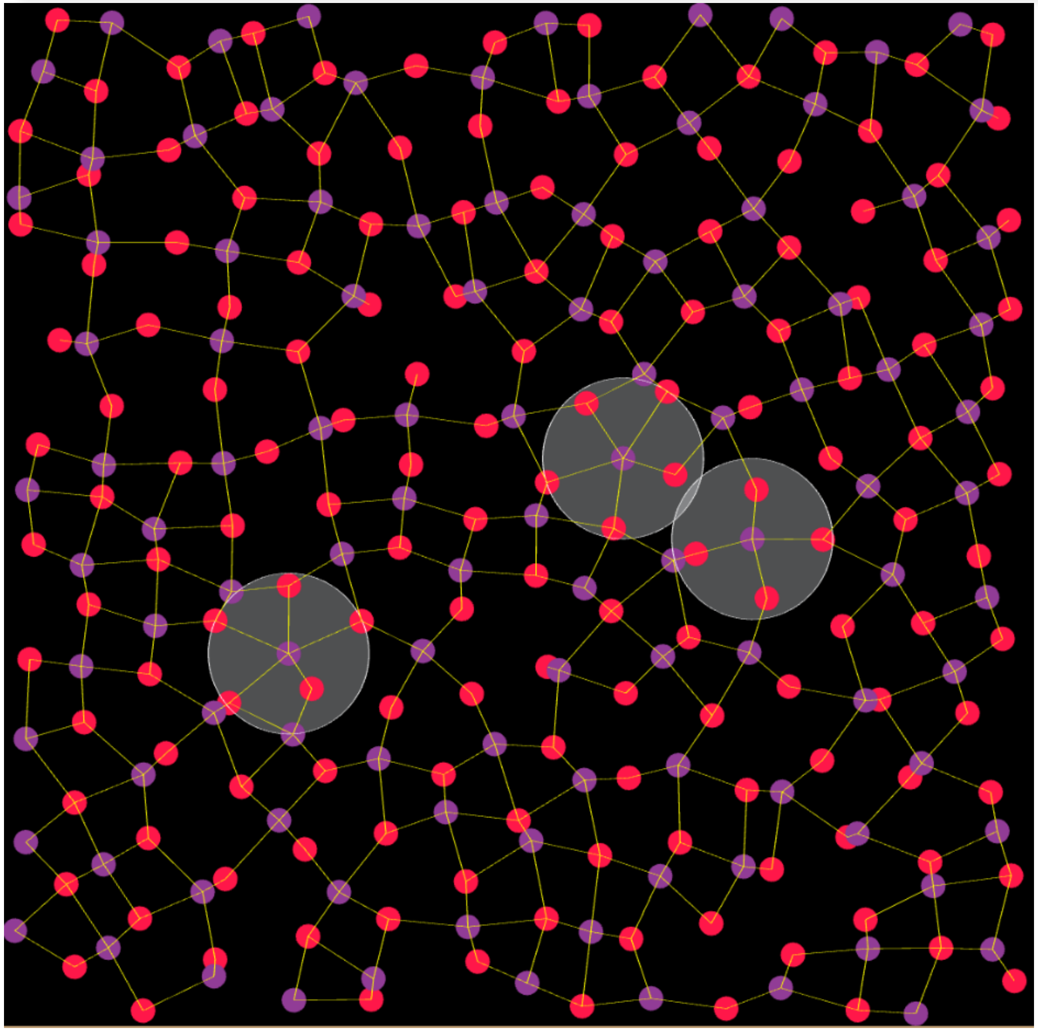
Input: Graph $G(V, E)$ and relay lists $L[i] (2 \le i \le 5)$
Output: Relay colors allocated to vertices in V_r (relay candidates set)

```
for i = 5 to 2 do
    while !L[i].isEmpty() do
        Try to assign a relay color paired with its primary
        neighbor color
        if the relay color is aleady assigned to one of its relay
        neighbour vetices then
            continue
        else
            assign this relay color
        end if
    end while
end for
```

Plot: Color Set Sizes



One Bipartite Subgraph Example

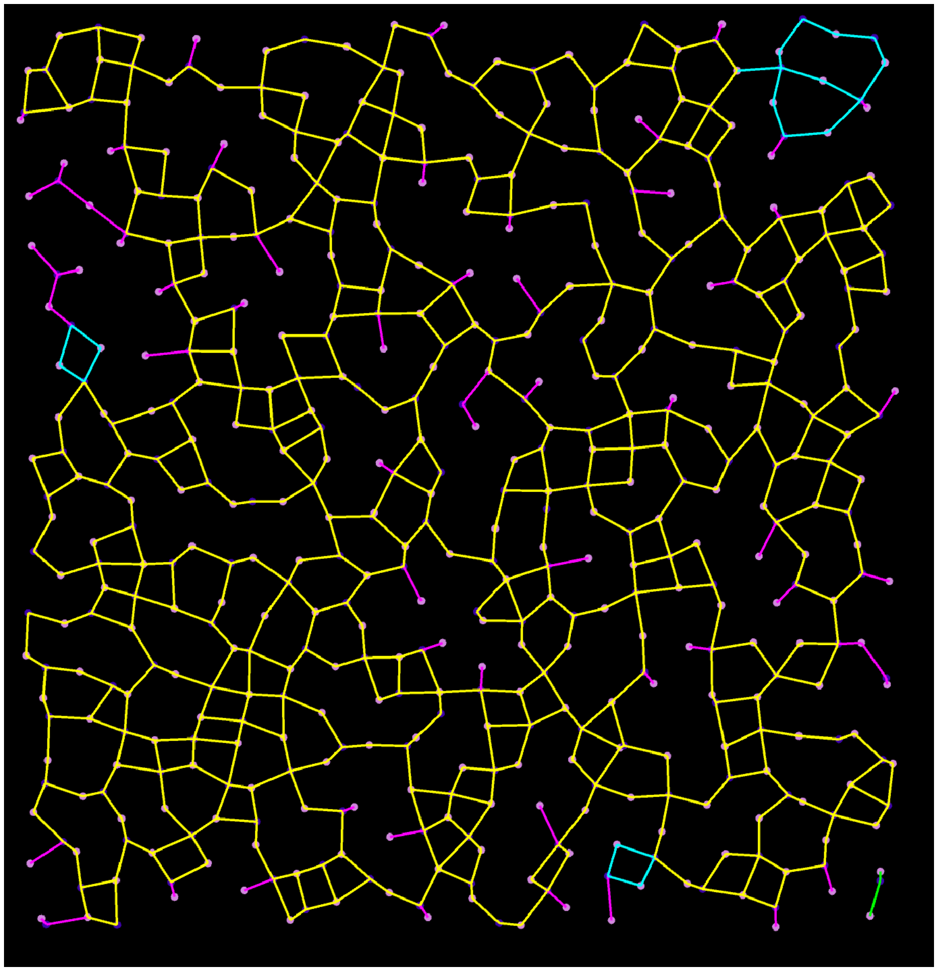


Combining the primary color set and the paired relay color set will generate one bipartite subgraph.

Determine Backbone Grid

- Giant Component Subgraph: Delete minor components (BFS or DFS)
- Two-core Subgraph: Delete vertices of degree 0 and degree 1 (Termed "Tails") (Based on smallest-last ordering)
- Block Subgraph: Delete minor blocks (BFS)

Determine Backbone Grid

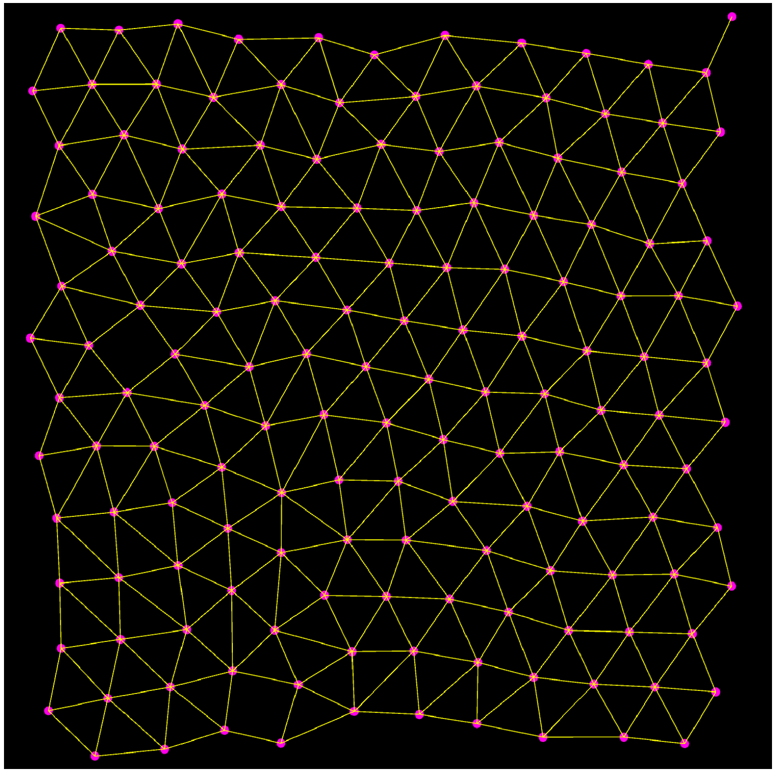


Cyan: Minor blocks
Green: Minor components
Pink: Tails

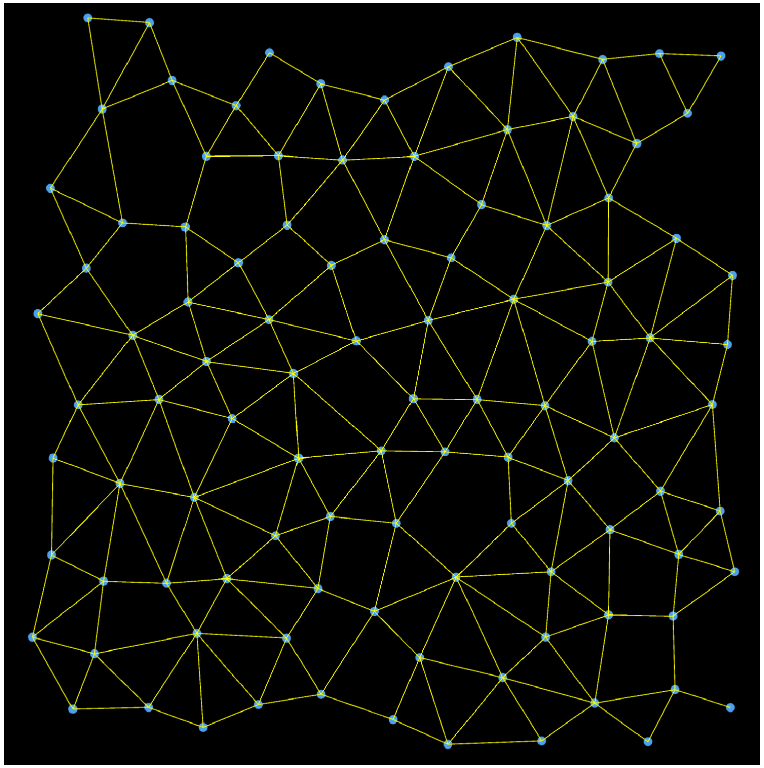
Observations:
For a dense enough graph, it usually generates giant component directly with few or no minor blocks.

Bi-regular Feature(visual triangulation)

- Adding Gabriel rule to independent sets



(a) Primary Set



(b) Relay Set

Benchmark Data Sets

Table I: Unit Square Test Benchmarks

N	8000	16000	32000	64000	128000
r	0.057	0.040	0.028	0.020	0.014
$\overline{d(G)}$	76.52	77.99	78.99	79.62	79.97
Backbones	17	18	18	18	18
Surplus	10.16%	8.01%	8.51%	8.45%	8.51%
$\overline{N(B)}$	422.76	817.67	1626.50	3255.28	6505.89
$\overline{Avg(d(B))}$	2.67	2.64	2.66	2.67	2.69
$\overline{Avg(F(B))}$	8.07	8.49	8.32	8.30	8.08
$\overline{Avg(F(\text{2-core}))}$	6.80	6.84	6.77	6.76	6.72
$\overline{D(B)}$	99.58%	99.12%	99.05%	99.48%	99.37%

Benchmark Data Sets

Table II: Unit Disk Test Benchmarks

N	8000	16000	32000	64000	128000
r	0.101	0.071	0.050	0.036	0.025
$\overline{d(G)}$	77.30	78.86	79.31	79.85	80.17
Backbones	17	18	18	18	18
Surplus	10.29%	8.74%	8.49%	8.12%	8.65%
$\overline{N(B)}$	422.18	811.22	1626.83	3266.94	6496.00
$\overline{Avg(d(B))}$	2.67	2.65	2.66	2.68	2.69
$\overline{Avg(F(B))}$	8.12	8.37	8.34	8.18	8.09
$\overline{Avg(F(\text{2-core}))}$	6.73	6.80	6.75	6.73	6.72
$\overline{D(B)}$	99.78%	99.07%	99.36%	99.67%	99.44%

Benchmark Data Sets

Table III: Unit Sphere Test Benchmarks

N	8000	16000	32000	64000	128000
r	0.201	0.142	0.100	0.071	0.050
$\overline{d(G)}$	81.00	80.91	80.99	80.95	80.98
Backbones	18	18	18	18	18
Surplus	8.66%	8.26%	8.26%	8.37%	8.34%
$\overline{N(B)}$	406.67	815.44	1630.94	3258.06	6517.94
$\overline{Avg(d(B))}$	2.72	2.72	2.71	2.72	2.72
$\overline{Avg(F(B))}$	7.67	7.78	7.78	7.83	7.83
$\overline{Avg(F(\text{2-core}))}$	6.63	6.60	6.64	6.63	6.64
$\overline{D(B)}$	99.86%	99.91%	99.82%	99.86%	99.85%

Conclusions, Questions and Future works

- Advantages
  - Topology only (Avoid geographical variations) and support unlimited topologies.
  - All procedures are efficient (linear time)
  - Algorithms not relying on locations of sensors
- Question: Practical or not?
  - Practical:
    - Great! Where else can I publish my work?
  - Impractical:
    - It's a measurement of constructing dominating backbones in WSN's
    - As an academic research work of Graph Theory, this kind of partitioning exists in RGG.
- Future works: Some empirical observations need theoretical proof.

Visualization and 3D Exploration

Show Time…

Reference

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Thank you!
Merci!

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