Partitioning RGG's Into Disjoint (1 - ε) Dominant Bipartite Subgraphs

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Abstract

We show that a sufficiently large Random Geometric Graph $G(n, r)$ can be efficiently partitioned into disjoint connected bipartite subgraphs such that a sequence B_1, B_2, …, B_k of these subgraphs comprising over 85% of the vertices of $G(n, r)$ have vertex sets that are either dominant sets or (1 - ε) dominating for small ε. These subgraphs are applicable to the problem of backbone partitioning in wireless sensor networks (WSN's) where each backbone is desired to be connected, dominating, and amenable to efficient routing. Bipartite subgraphs of an RGG are provably planar, so deadlock free routing is readily available for these backbone subgraphs. We first employ smallest-last coloring (SL-coloring) and show that the initial k color sets sufficient to include about 50% of the vertices of $G(n, r)$ are about the same size. We then employ an adaptive "relay coloring (RL-coloring)" of the remaining vertices to extract k more independent sets matched with the initial sets as paired "relay sets" to achieve our bipartite subgraph partition.

We provide results from extensive tests for various sizes of RGG's and also for random geometric graphs with vertices on the sphere. For the spherical case we obtain that the average face size in the bipartite subgraphs is generally between five and six, which is a further desirable property for routing when these subgraphs are considered as backbones for WSN's on the surface of the earth.

Keywords: Random Geometric Graphs, Wireless Sensor Networks, Smallest-last coloring, Backbone selection.

1. INTRODUCTION

Let a *random geometric graph* (RGG) denote a graph $G(n, r)$ with vertex set formed by choosing n points in a uniform random manner on the unit square, and introducing an edge between every vertex pair whose Euclidian distance is less than r. Our problem is to partition the majority of vertices into k disjoint sets $\{V_1, V_2, \dots V_k\}$ whose induced subgraphs $\langle V_1\rangle$, $\langle V_2\rangle$, …, $\langle V_k\rangle$ are connected bipartite subgraphs with each part an independent set that dominates all or nearly all n vertices of $G(n, r)$. We desire to create such a partition efficiently so as to be applicable for 10's of thousands of vertices. Regarding uniformity, the partition should yield subgraphs of reasonably similar size and structure. Regarding the total partition size we seek that the bipartite subgraphs collectively include a large majority of the n vertices, e.g. $\frac{\sum_i^k |V_i|}{n} \gg \frac{1}{2}$.

This problem is motivated by the extensive research on wireless sensor networks (WSN's) which are typically modeled by RGG's [1], [2], [3], [4], [5], [6].

The partition $V_1|V_2|\dots|V_k$ with S the residual "surplus" vertices of the RGG, allows that each bipartite subgraph $\langle Vi\rangle$ can serve as a backbone for monitoring essentially the whole region and connectivity allows for messages to be routed through each backbone. To preserve sensor lifetime the monitoring function activity may be rotated through the k backbones. The property that each backbone has two disjoint sets each dominating (1 - ε) vertices of the graph for very small ε (e.g. $\varepsilon < 0.01$) gives high overall monitoring effectiveness to the resulting backbone system.

Previous research investigating fully dominating set partitions has focused on the minimum degree $\delta(G(n, r))$ and attempted to find up to $\delta + 1$ suitable backbones. The minimum degree is problematic due to the boundary effect in RGG's and we avoid this issue by shifting our focus in two ways. First we determine the number of parts k by requiring they collectively include a large majority of the sensors. In a second direction we also look at spherical random geometric graphs $G_s(n, r)$ where n vertices are placed at random on the surface of the unit sphere which supports the important application of sensor backbone formations spanning the globe. Note that spherical $G_s(n, r)$ provides that all vertices have an isomorphic probabilistic environment of adjacent neighbors without any boundary bias.

Our bipartite subgraph partitioning algorithms proceeds in two phases employing a greedy selective coloring algorithm in each phase. In the first phase smallest-last coloring of $G(n, r)$ is determined with k determined so that the first k-color sets, denoted $P_1, P_2, \ldots, P_k$, are chosen so that $\cup P_i$ includes at least $\frac{n}{2}$ vertices termed primary independent sets. In a second carefully crafted coloring phase of the remaining vertices ("relay candidates") we sequentially and in a greedy manner assign each vertex to a relay color set R_i, $1 \le i \le k$, based primarily on the vertex having the greatest number of adjacencies in P_i, also maintaining that each R_i is an independent set. Then the bipartite subgraph on $P_i \cup R_i$, is searched to determine the large component with the occasional smaller components or isolated vertices deleted into the surplus set.

2. RESULTS

Screenshots of benchmark of RGG G(6400, 0.08) on square model and sphere model:

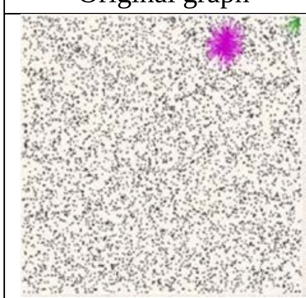
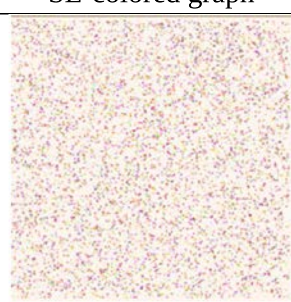
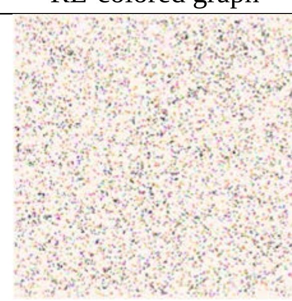
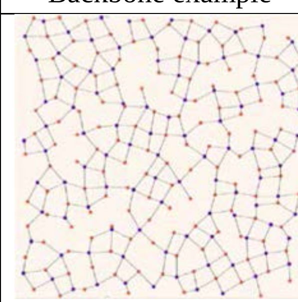
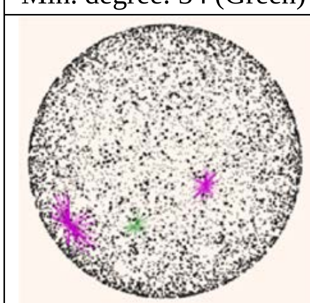
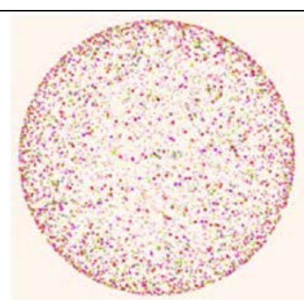
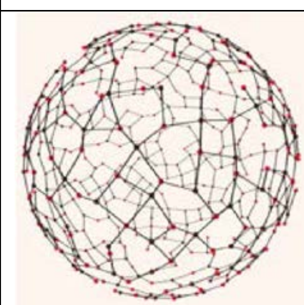
Original graph	SL-colored graph	RL-colored graph	Backbone example
			
Max. degree: 175 (Pink) Min. degree: 34 (Green)	Used colors: 64	Primary/Relay colors:24 Backbones: 24	Components: 1 Dominent: 100%
			
Max. degree: 85 (Pink) Min. degree: 35 (Green)	Used colors: 36	Primary/Relay colors:13 Backbones: 13	Components: 1 Dominent: 100%

Table 1. Simulation results on the unit square model.

Topology	G(3200, r)			G(6400, r)			G(12800, r)		
r	0.06	0.08	0.1	0.06	0.08	0.1	0.06	0.08	0.1
Min. degree	10	14	28	14	37	44	35	63	106
Max. degree	53	94	126	104	164	23	184	312	466
Avg. degree	33.94	59.80	91.83	68.63	119.24	184.82	137.57	240.87	368.06
SL-coloring colors	23	35	50	40	62	92	72	116	157
Backbones (BB)	8	13	19	15	24	35	27	45	67
Avg. BB size	344.25	213.08	145.00	376.93	230.63	155.31	409.59	244.78	163.75
Avg. BB degrees	2.38	2.58	2.72	2.63	2.81	2.88	2.88	2.97	2.99
Avg. BB components	5.88	2.46	1.42	3.33	1.25	1.20	1.48	1.16	1.01
Avg. BB faces	73.00	66.31	54.79	126.47	96.08	70.74	183.74	121.56	83.43
Avg. BB face sizes	12.04	8.60	7.39	8.27	6.82	6.37	6.48	6.02	5.91
Avg. BB dominates	99.97%	99.95%	99.99%	99.97%	99.99%	99.99%	99.99%	99.99%	99.99%

Table 2. Simulation results on the unit sphere model.

Topology	G(6400, r)			G(12800, r)			G(25600, r)		
r	0.06	0.08	0.1	0.06	0.08	0.1	0.06	0.08	0.1
Min. degree	9	20	36	25	52	90	58	114	188
Max. degree	41	60	95	69	113	175	128	215	309
Avg. degree	23.02	40.89	63.86	46.07	81.83	128.00	92.14	163.92	255.96
SL-coloring colors	18	26	37	29	46	65	50	82	115
Backbones (BB)	6	10	14	10	17	25	19	31	47
Avg. BB size	938.17	585.7	409.00	1091.30	659.47	443.36	1189.00	711.10	469.13
Avg. BB degrees	2.22	2.43	2.67	2.57	2.79	2.94	2.82	3.02	3.10
Avg. BB components	29.5	6.2	1.64	4.50	1.82	1.04	1.84	1.10	1.00
Avg. BB faces	135.83	134.5	140.29	321.00	265.35	210.88	490.79	364.71	261.21
Avg. BB face sizes	17.48	11.42	8.02	9.10	7.07	6.27	6.96	5.92	5.59
Avg. BB dominates	99.95%	99.98%	100%	99.99%	100%	100%	100%	100%	100%

3. REFERENCES

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