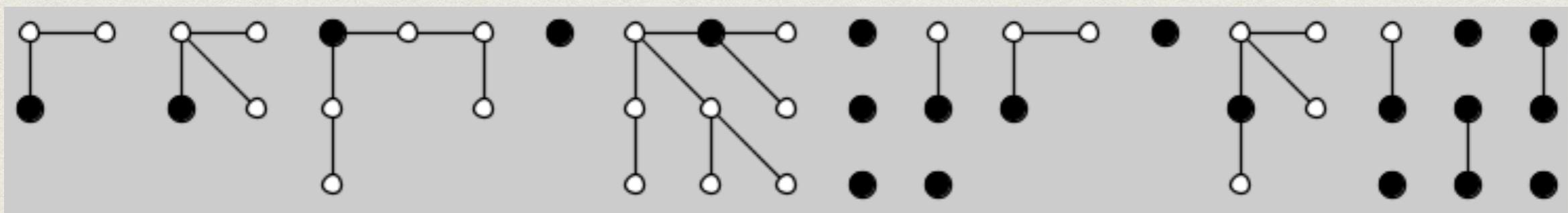


# PRECISE AND CONCISE GRAPHICAL REPRESENTATION OF THE NATURAL NUMBERS

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*Southern Methodist University*

A GRAPHIC IS  
WORTH A  
THOUSAND  
DIGITS



# NAMING NUMBERS

## Cultural

五十

ごじゅう

오십

पचास

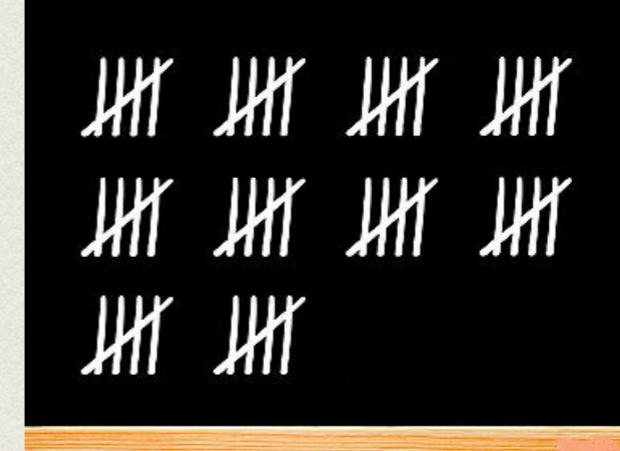
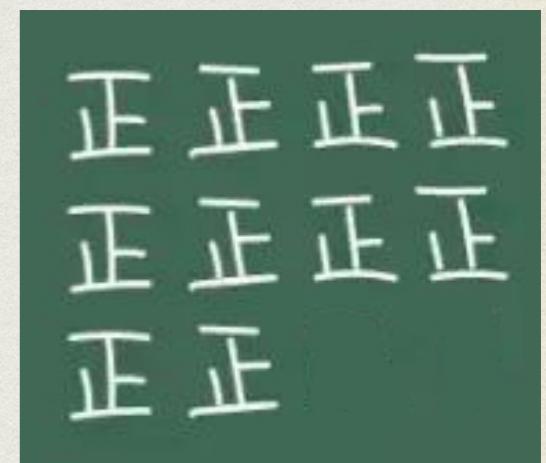
L

50

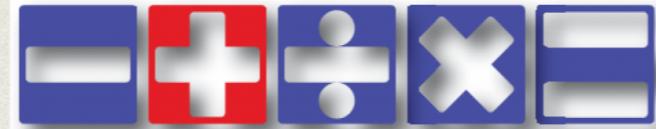
**What's so special about “50”?**

[It's a round number??]

## Natural



五十  
ごじゅう  
오십  
पचास  
L



Why is divisible by 10 so special?

ARITH SYMPOSIUM

*From 1st to 26th*

# NAMING NUMBERS

## Cultural

四十九

よんじゅ

사십구

उनचास

XLIX

49

五十

ごじゅう

오십

পচাস

L

50

**Step from 49 to 50**

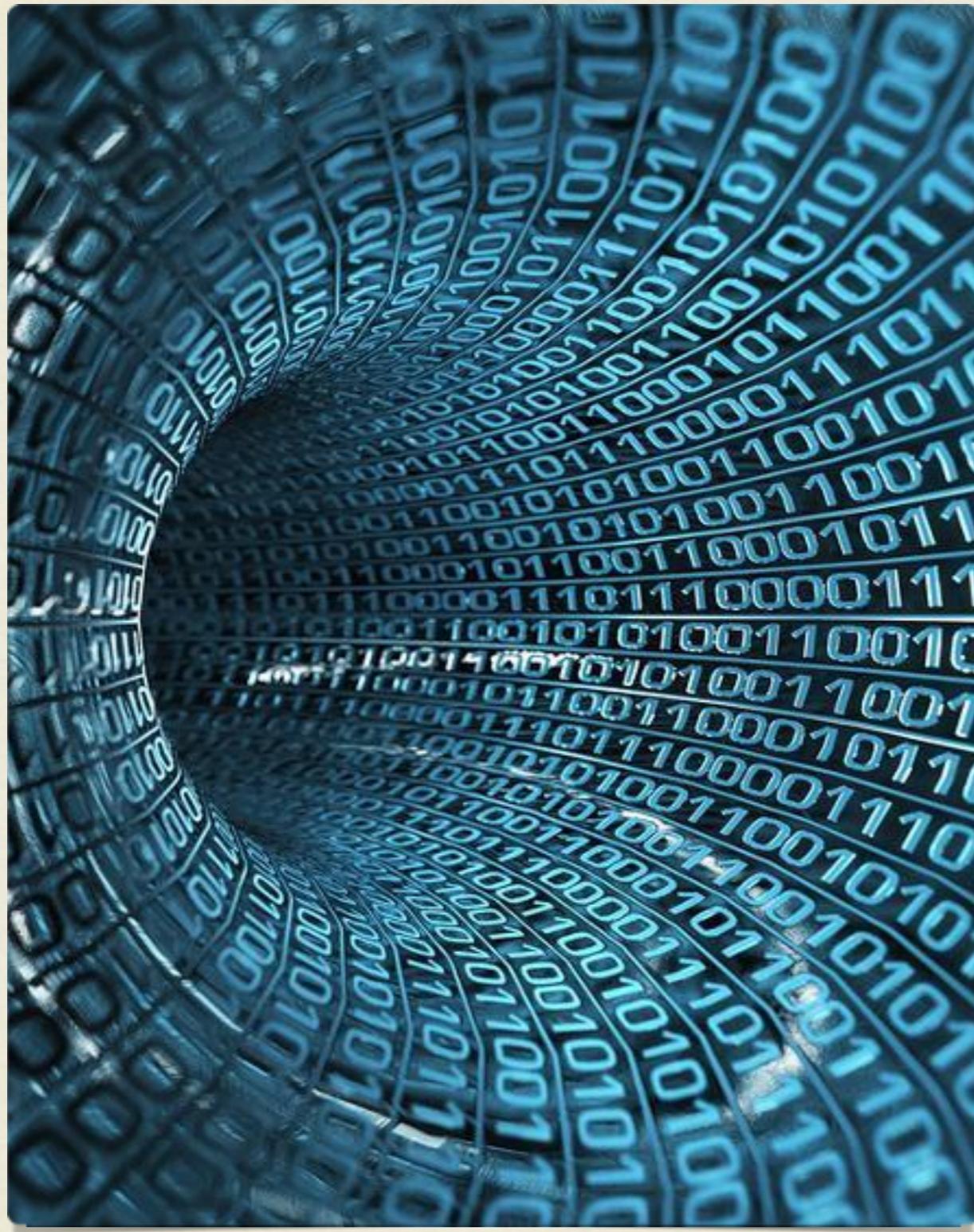
**【Protocol or Obvious??】**

## Natural



正 正 正 正  
正 正 正 正  
正 止





**Digit (bit) strings suggest??**



**See the relations?**



**Feel the music?**

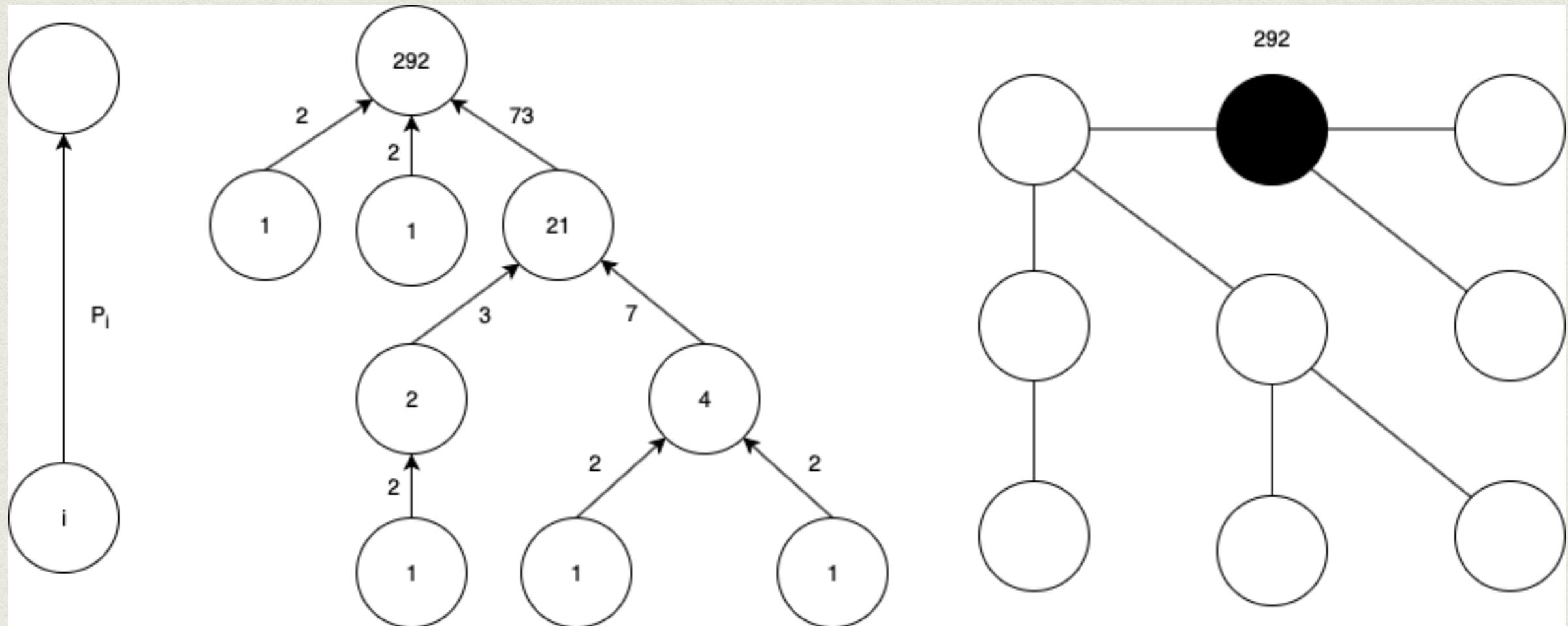
# ROOTED TREES NATURAL NUMBERS **ONE***to***ONE!**

## Fundamentals of Arithmetic

- Theorem: Unique Prime Factorization
- Operation: Counting ( $i$  th prime  $p_i$ )
- Procedure: Recursion (finite stopping rule)

# ONE-TO-ONE CORRESPONDENCE

**A Natural Procedure Over Natural Numbers**



# ROOTED TREES NATURAL NUMBERS **ONE***to***ONE!**

## Fundamentals of Arithmetic

- Theorem: Unique Prime Factorization
- Operation: Counting ( $i$  th prime  $p_i$ )
- Procedure: Recursion (finite stopping rule)

# Let's take a look...

C  
O  
N  
C  
I  
S  
E

Structural-e.g. Digital 7 (linear)

P  
R  
E  
C  
I  
S  
E

Artistic-e.g.Chinese, etc. (2D)

Number Fonts

Integer  $<=>$  One Tree

Rational Fraction  $<=>$  Two Trees

Continued Fraction  $<=>$  Sequence of Trees

Reals by “Best Rational Approximation”

# STRUCTURAL FONTS

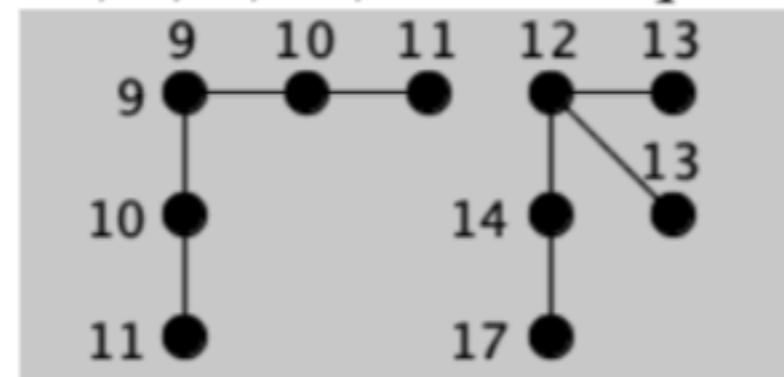
## Decimal Digits vs. Rooted Trees

0 1 2 3 4 5 6 7 8 9

(a) digits in the Digital-7 font



(b) counts 1, 2, 3, ..., 9 in a square grid font



(c) compressed square grid font for selecting counts 9-14 and 17

# FIRST 21 PARTIAL QUOTIENTS

**Everyone looks at**

3.1415926535897931162143382231

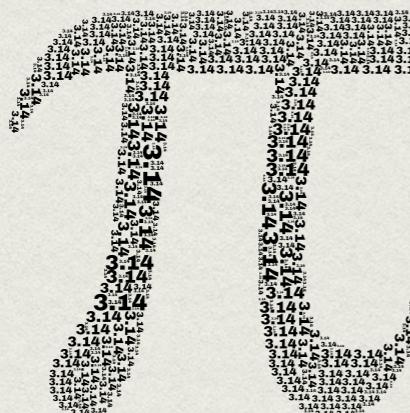
(a) Digital-7 font



(b) square grid font

# RATIONAL FRACTION FORM

Continued Fraction  
(10 partial quotients)

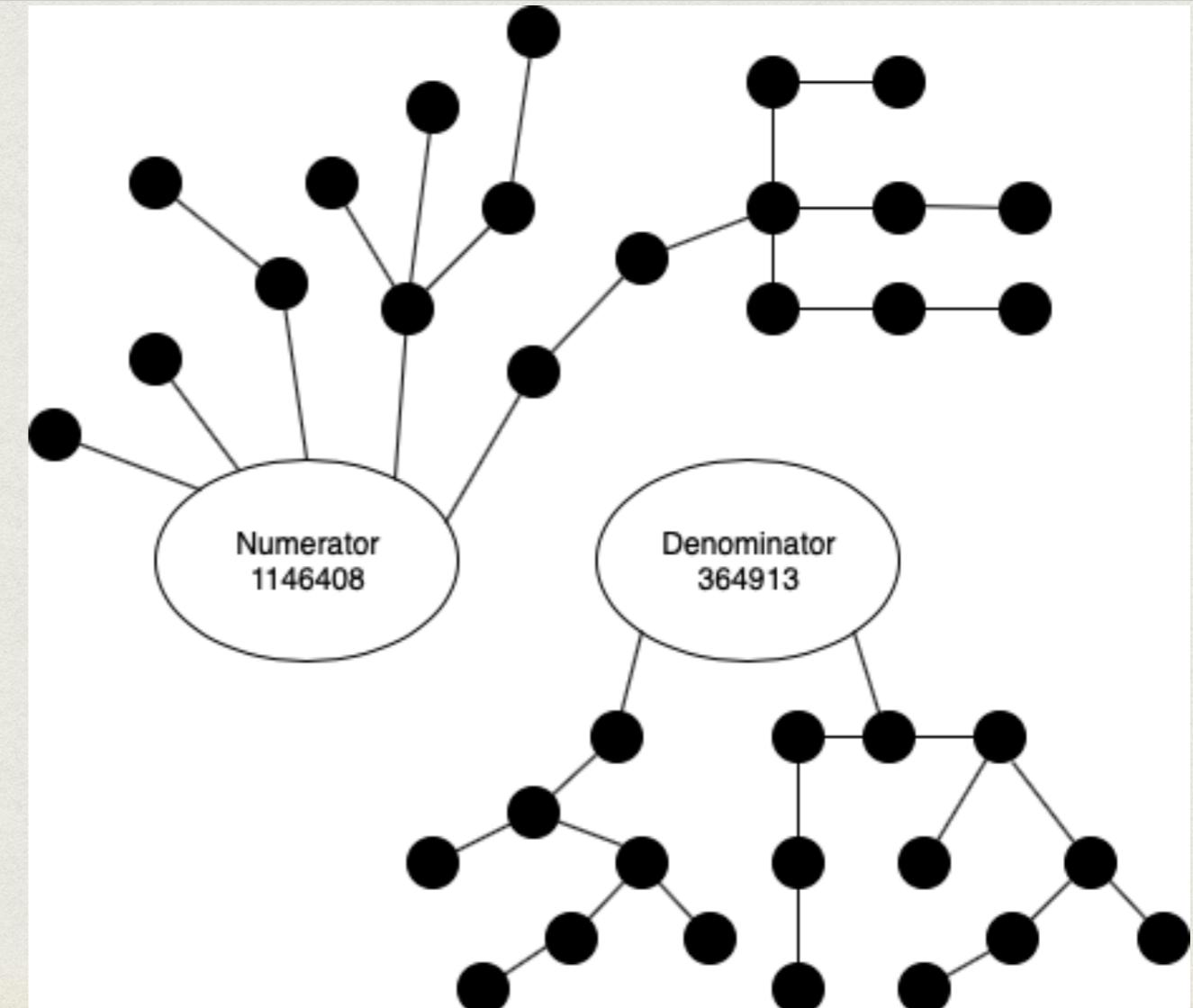
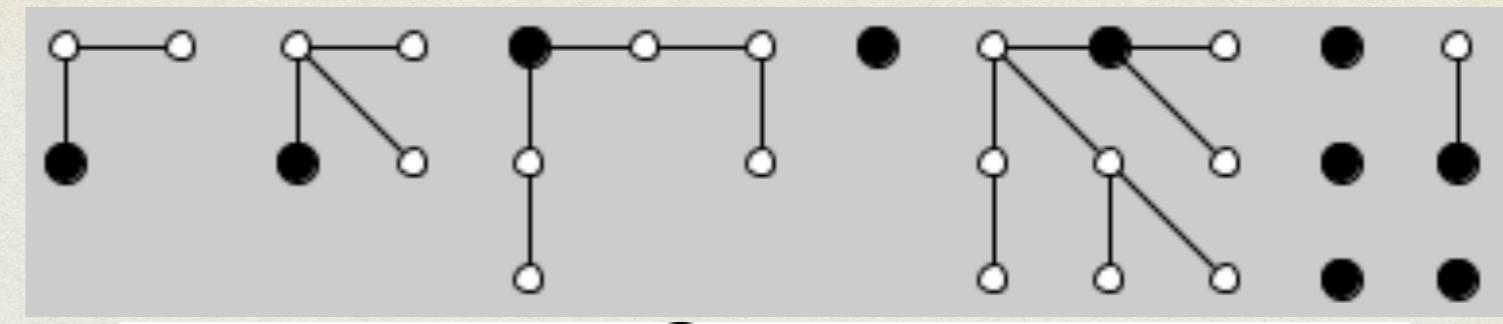


Rational Fraction  
(reduced)

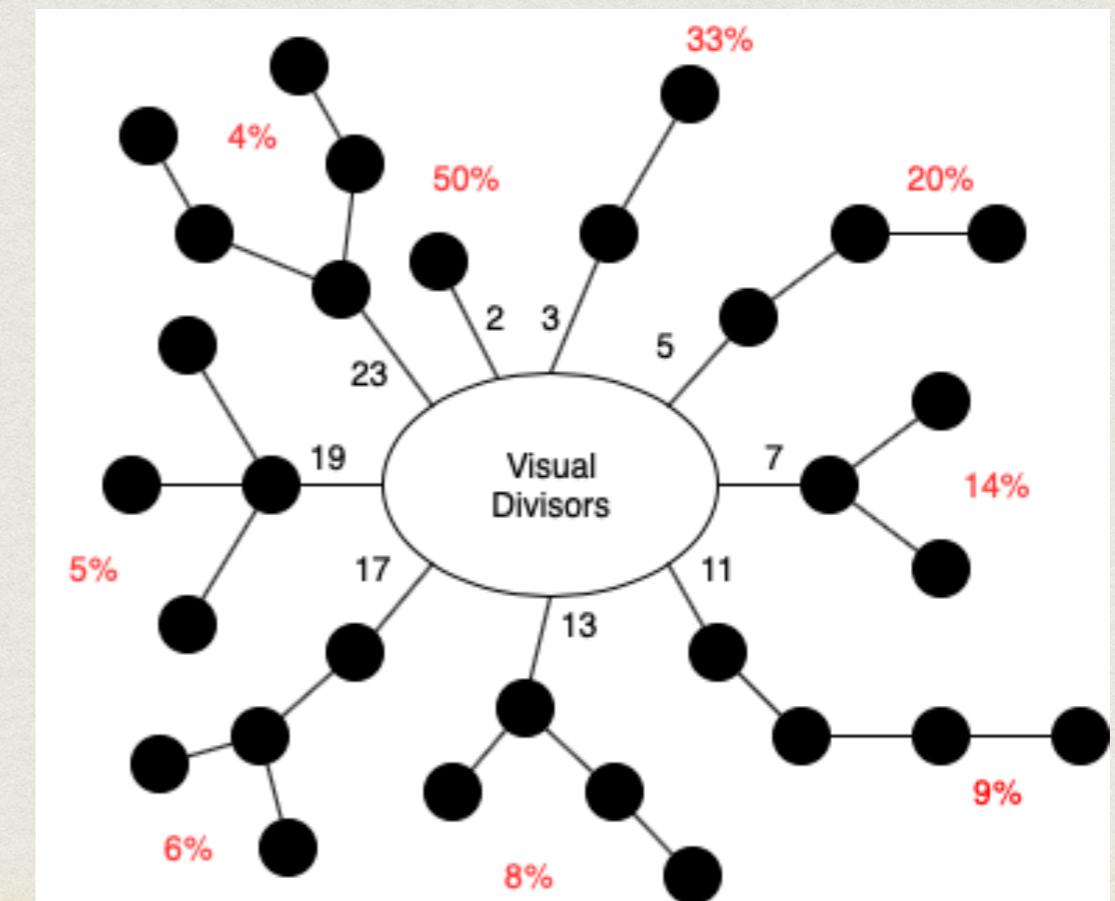
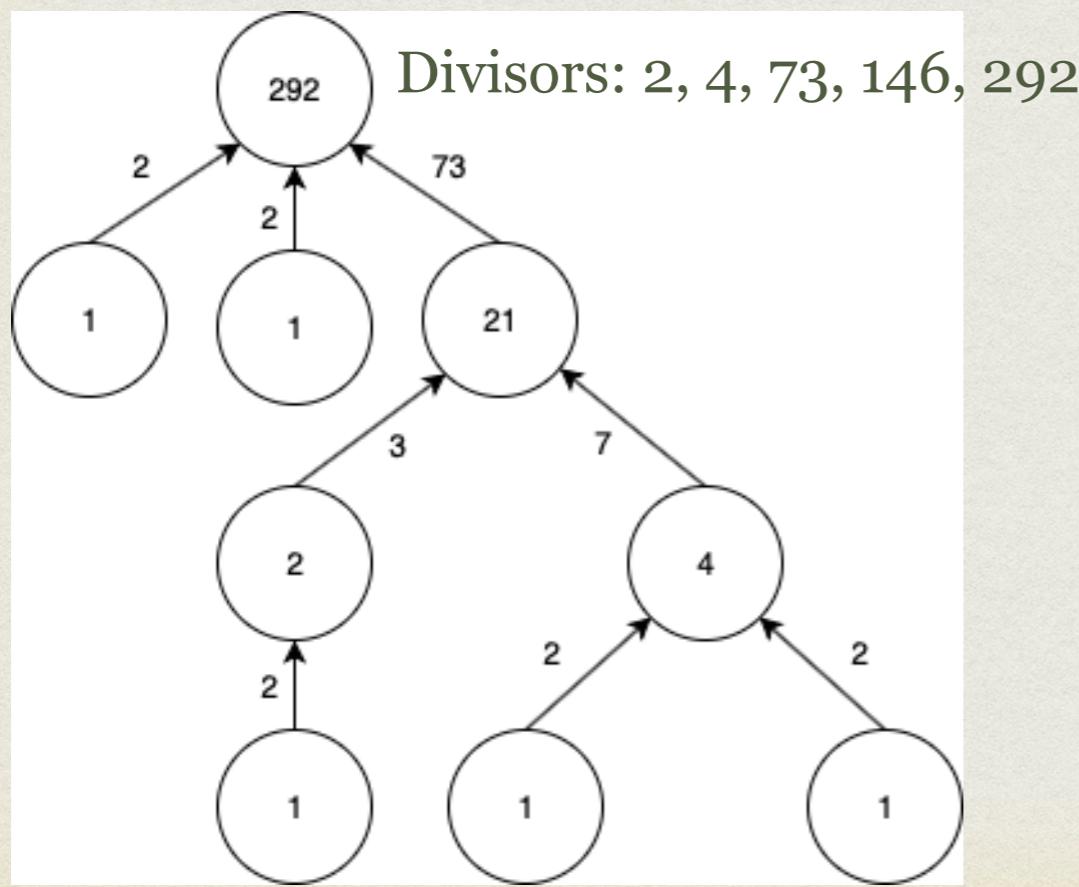
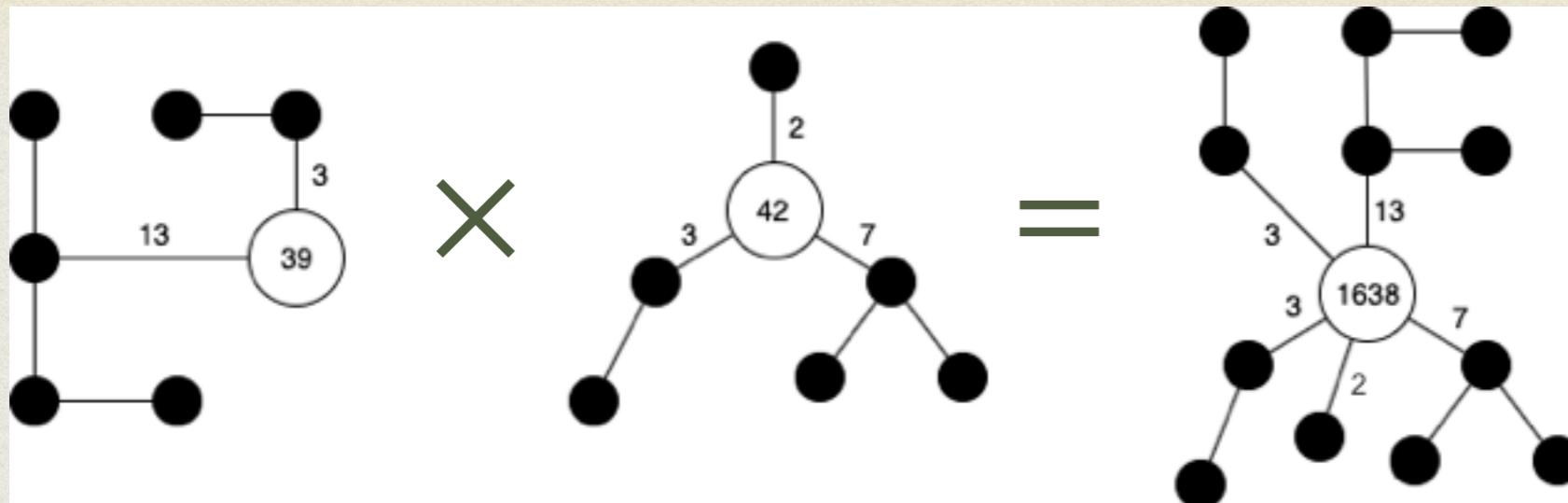
**1146408/364913**

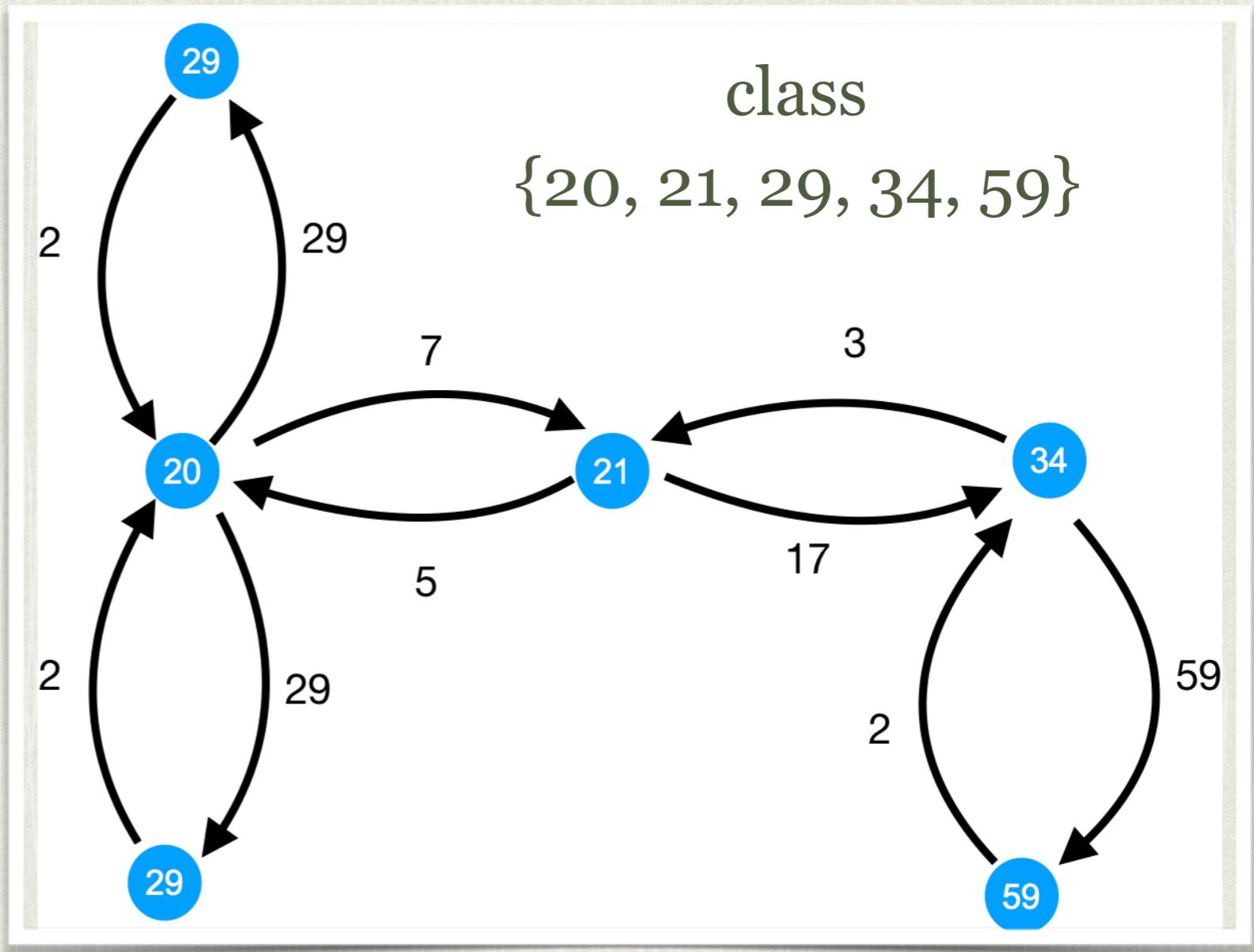
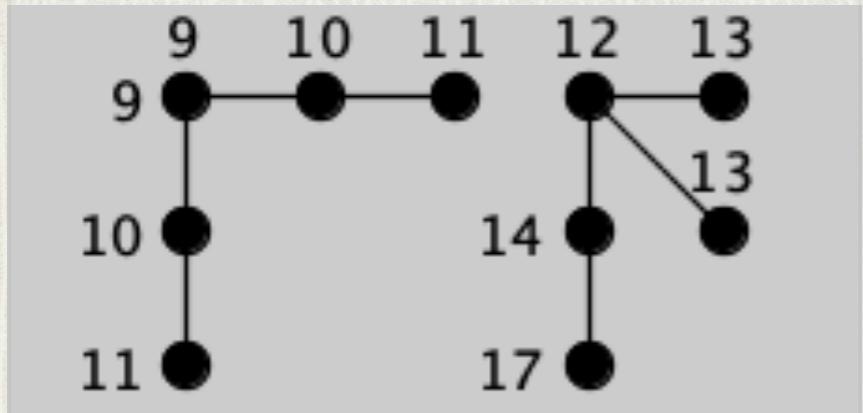
=**3.14159265358...**

**“correct digits”**



# MULTIPLICATION IS VISUAL





# EQUIVALENCE RELATION

$$\{j(p_i)\} \mathbf{R} \{i(p_j)\}$$



First 40 classes

$$\{j(p_i)\} \mathbf{R} \{i(p_j)\}$$