Linear Time (1 - δ) Partitioning of RGG's into Disjoint (1 - ε) Dominant Planar Bipartite Subgraphs

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Abstract

We describe a linear time algorithm for partitioning an n-vertex, m-edge random geometric graph (RGG) into disjoint connected bipartite planar subgraphs $B_1, B_2, \ldots, B_k$ of substantially the same size where each subgraph dominates $(1-\varepsilon)n$ of the vertices and the subgraphs comprise $(1-\delta)n$ vertices. Our partitioning algorithm employs four procedures each having time complexity $O(n+m)$.

Our first procedure utilizes a cell method to limit the number of vertex pairs to check for adjacency allowing an RGG to be determined in time complexity $O(n+m)$.

Our second procedure employs the linear time smallest last coloring algorithm to determine a set of k-disjoint independent sets $P_1, P_2, \ldots, P_k$ comprising at least half of the n vertices. These k sets form the primary independent sets of each part of the k-part partition, and leave a residual set of about half the vertices for our relay set procedure.

Our third procedure is using a novel adaptive coloring phase for partitioning the residual vertices into k disjoint independent relay sets $R_1, R_2, \ldots, R_k$ each adaptively paired with a primary set. Here each vertex in a greedy manner is paired with high adjacency as well as forming a new independent set, yielding a disjoint bipartite subgraph sequence $P_i \cup R_i$ for $i = 1, 2, \ldots, k$.

These bipartite subgraphs are provably planar subgraphs but not necessarily connected. Our fourth procedure determines the "giant component" of each $P_i \cup R_i$ and deletes sufficient vertices to form the connected bipartite subgraph $B_i \subseteq P_i \cup R_i$ having minimum degree two. The vertices not contained in any B_i are placed in a surplus set S comprising δn vertices.

We implement a version of our bipartite partitioning algorithm experimentally demonstrating that each connected bipartite subgraph of our partition typically dominates $(1-\varepsilon)n$ of the vertices with $\varepsilon < 0.01$ for $n > 1000$. Furthermore the total vertices in the partition comprises $(1-\delta)n$ of the vertices with δ becoming smaller than 0.1 as n grows towards a million. The evidence suggests that $\varepsilon \to 0$ and $\delta \to 0$ as n grows with ε approaching considerably faster than δ.