Part1

X	$P(X)$
0	0.300
1	0.700

Y	X	$P(Y\|X)$
0	0	0.300
1	0	0.700
0	1	0.800
1	1	0.200

Z	Y	$P(Z\|Y)$
0	0	0.600
1	0	0.400
0	1	0.800
1	1	0.200

1.

use the product rule:

P(X, Y) = P(Y) * P(X | Y) = P(X) * P(Y | X)

P(X = 0, Y = 0) = P(X = 0) * P(Y=0 | X=0) = 0.3 * 0.3 = 0.9

P(X = 0, Y = 1) = P(X = 0) * P(Y=1 | X=0) = 0.3 * 0.7 = 0.21

P(X = 1, Y = 0) = P(X = 1) * P(Y=0 | X=1) = 0.7 * 0.8 = 0.56

P(X = 1, Y = 1) = P(X = 1) * P(Y=1 | X=1) = 0.7 * 0.2 = 0.14

2.

P(X = 1, Y = 0, Z = 0) = 0.336

use the product rule:

P(X, Z | Y) = P(X | Y) * P(Z | Y)

P(X, Y, Z) = P(Y) * P(X, Z | Y) = P(Y) * P(X | Y) * P(Z | Y) = P(X,Y)

* P(Z | Y)

P(X=0, Y=0, Z=0) = P(X=0, Y=0) * P(Z=0 | Y=0) = 0.9 * 0.6 =0.054

P(X = 0, Y = 0, Z = 1) = 0.036

P(X = 0, Y = 1, Z = 0) = 0.168

P(X = 0, Y = 1, Z = 1) = 0.042

P(X = 1, Y = 0, Z = 0) = 0.336

P(X=1, Y=0, Z=1) = P(X=1, Y=0) * P(Z=1 | Y=0) = 0.56 * 0.4 =0.224

P(X=1, Y=1, Z=0) = P(X=1, Y=1) * P(Z=0 | Y=1) = 0.14 * 0.8 =0.112

P(X=1, Y=1, Z=1) = P(X=1, Y=1) * P(Z=1 | Y=1) = 0.14 * 0.2 =0.028

X	Y	Z	P(X,Y,Z)
0	0	0	0.054
0	0	1	0.036
0	1	0	0.168
0	1	1	0.042
1	0	0	0.336
1	0	1	0.224
1	1	0	0.112

1	1	1	0.028

3.

(i)

- The sum rule: the probability of an event is the sum of all the joint probabilities with another event
  - $P(X = x) = \sum_{y \in \Omega} P(X = x, Y = y)$

P(Z=0) = P(X=1, Y=1, Z=0)+ P(X = 1, Y = 0, Z = 0) + P(X=0, Y=0, Z=0) + P(X=0, Y=0, Z=0)= 0.112 +0.336 +0.168+0.054 = 0.67

P(X = 0, Z = 0) = P(X = 0, Y = 1, Z = 0) + P(X=0, Y=0, Z=0) =0.168+0.054 = 0.222

(ii)

- Independence
  - P(A | B) = P(A)
  - P(B | A) = P(B)
  - P(A, B) = P(A) * P(B)

P(X=0) = P(X = 0, Z = 0) / P(Z=0) = 0.222/0.67 unequal 0.3 which means X and Z are not independent

4.

(i)

P(Z = 1) = P(X = 0, Y = 0, Z = 1)+ P(X = 0, Y = 1, Z = 1)+ P(X=1, Y=0, Z=1)+ P(X=1, Y=1, Z=1)

=0.036+0.042+0.224+0.028 =0.33

P(X = 1, Y = 0|Z = 1) = P(X = 1, Y = 0, Z = 1) / P(Z = 1) = 0.224/0.33 = 0.679

(ii)

P(Y = 0, Z = 0) = P(X=0, Y=0, Z=0)+ P(X = 1, Y = 0, Z = 0)

=0.054+0.336 =0.39

P(X = 0 | Y = 0, Z = 0) = P(X = 0, Y = 0, Z = 0) / P(Y = 0, Z = 0) = 0.054/0.39=0.138

Part2

1.

P(F1 = 1| C = 1) = 0.6730769230769231, P(F1 = 1| C = 0) = 0.36, P(F1 = 0| C = 1) = 0.34615384615384615, P(F1 = 0| C = 0) = 0.6466666666666666

P(F2 = 1| C = 1) = 0.5961538461538461, P(F2 = 1| C = 0) = 0.58, P(F2 = 0| C = 1) = 0.4230769230769231, P(F2 = 0| C = 0) = 0.4266666666666667

P(F3 = 1| C = 1) = 0.46153846153846156, P(F3 = 1| C = 0) = 0.3466666666666667, P(F3 = 0| C = 1) = 0.5576923076923077, P(F3 = 0| C = 0) = 0.66

P(F4 = 1| C = 1) = 0.6153846153846154, P(F4 = 1| C = 0) = 0.4, P(F4 = 0| C = 1) = 0.40384615384615385, P(F4 = 0| C = 0) = 0.6066666666666667

P(F5 = 1| C = 1) = 0.5, P(F5 = 1| C = 0) = 0.34, P(F5 = 0| C = 1) = 0.5192307692307693, P(F5 = 0| C = 0) = 0.6666666666666666

P(F6 = 1| C = 1) = 0.36538461538461536, P(F6 = 1| C = 0) = 0.47333333333333333, P(F6 = 0| C = 1) = 0.6538461538461539, P(F6 = 0| C = 0) = 0.5333333333333333

P(F7 = 1| C = 1) = 0.7884615384615384, P(F7 = 1| C = 0) = 0.5066666666666667, P(F7 = 0| C = 1) = 0.23076923076923078, P(F7 = 0| C = 0) = 0.5

P(F8 = 1| C = 1) = 0.7692307692307693, P(F8 = 1| C = 0) =

0.35333333333333333, P(F8 = 0| C = 1) = 0.25, P(F8 = 0| C = 0) = 0.6533333333333333

P(F9 = 1| C = 1) = 0.34615384615384615, P(F9 = 1| C = 0) = 0.24666666666666667, P(F9 = 0| C = 1) = 0.6730769230769231, P(F9 = 0| C = 0) = 0.76

P(F10 = 1| C = 1) = 0.6730769230769231, P(F10 = 1| C = 0) = 0.29333333333333333, P(F10 = 0| C = 1) = 0.34615384615384615, P(F10 = 0| C = 0) = 0.7133333333333334

P(F11 = 1| C = 1) = 0.6730769230769231, P(F11 = 1| C = 0) = 0.5866666666666667, P(F11 = 0| C = 1) = 0.34615384615384615, P(F11 = 0| C = 0) = 0.42

P(F12 = 1| C = 1) = 0.7884615384615384, P(F12 = 1| C = 0) = 0.34, P(F12 = 0| C = 1) = 0.2307692307692308, P(F12 = 0| C = 0) = 0.6666666666666666

2.

Probability for Spam is 4.563570134160374E-6 , Probability

for non-spam is 4.9542671641766 28E-4, Non_Spam

Probability for Spam is 7.210346867817722E-5 , Probability for non-spam is 4.5438790152493946E-5, Spam

Probability for Spam is 2.337624408575223E-4 , Probability for non-spam is 1.3955766540710917E-4, Spam

Probability for Spam is 7.663585265070243E-6 , Probability for non-spam is 6.466481819829655E-4, Non_Spam

Probability for Spam is 7.7202748511879E-5 , Probability for non-spam is 1.0025583637879425E-4, Non_Spam

Probability for Spam is 7.434338745588349E-5 , Probability for non-spam is 5.026186824410612E-5, Spam

Probability for Spam is 5.124916761793711E-6 , Probability for non-spam is 3.540527014157241E-4, Non_Spam

Probability for Spam is 8.117842746585805E-5 , Probability for non-spam is 4.2357652476948595E-4, Non_Spam

Probability for Spam is 2.3376244085752235E-4 , Probability for non-spam is 4.10847332740555E-5, Spam

Probability for Spam is 2.8353542253659124E-5 , Probability for non-spam is 7.338396327966042E-4, Non_Spam

3. Naive Bayes algorithm for independent feature. The test

results are not all correct that indicate they are not all independent.

Part3

1.M for meeting, LT for lecture teaching, O for office, L for light, C for computer

M	P(M)
1	0.7
0	0.3

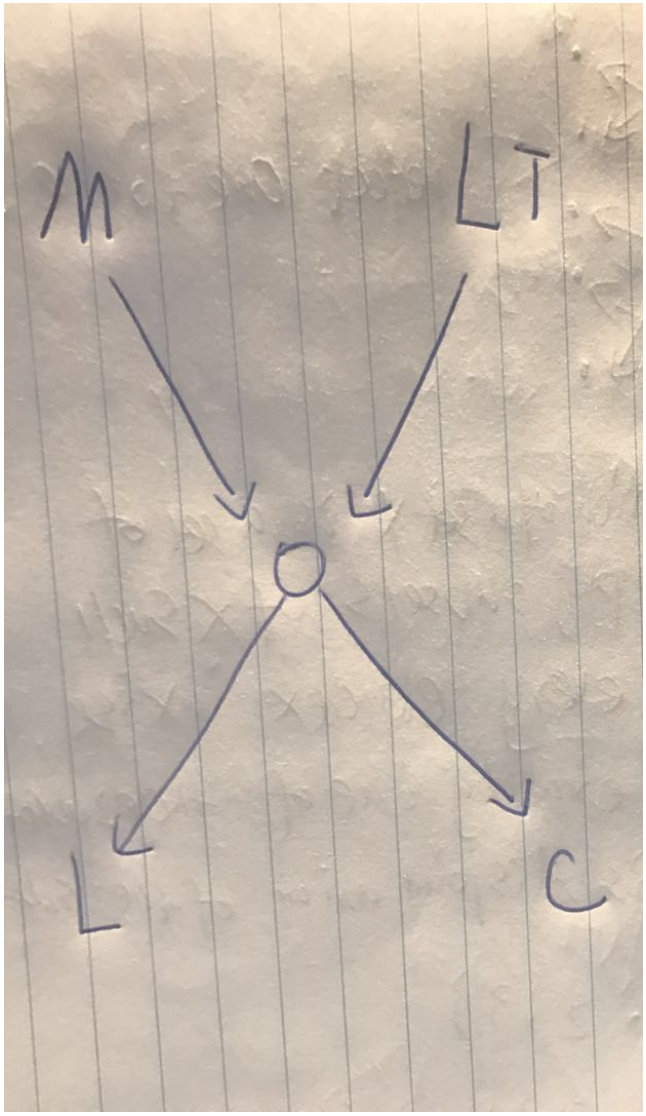
LT	P(LT)
1	0.6
0	0.4

M	LT	O	P(O| M, LT)
1	1	1	0.95
1	1	0	0.05
1	0	1	0.75
1	0	0	0.25
0	1	1	0.8
0	1	0	0.2

0	0	1	0.06
0	0	0	0.94

L	O	P(L|O)
1	1	0.5
0	1	0.5
1	0	0.02
0	0	0.98

C	O	P(C|O)
1	1	0.8
0	1	0.2
1	0	0.2
0	0	0.8



2. depend on above probability graph, half of them is CPT size

2/2 +2/2+8/2+4/4+4/2 =10

3. P(M=0) * P(LT=1) * P(O=1|M=0, LT=1)* P(L=0|O=1)* P(C=1|O=1)

0.3*0.6*0.8*0.5*0.8 = 0.0576

4. P(O|M=1, T=1) + P(O|M=1,T=0) + P(O|M=0,T=1) + P(O|M=0,T=0)

0.95*0.7*0.6+0.75*0.7*0.4+0.8*0.3*0.6+0.06*0.3*0.4=0.7602

5. P(C=1|O=1)*P(L=0|O=1) = 0.8*0.5 = 0.4

6. light and computer log on are dependent, we can not know if Rachels in office. Thus, there is not effect on the students belief that Rachels light is on.

Part4

1(i).

Evidence variables: XRay

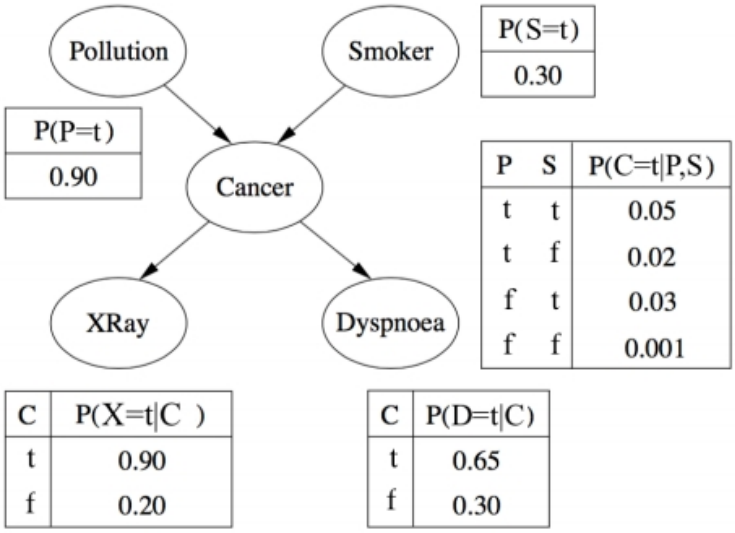
Hidden variables: Smoker, Cancer, Dyspnoea

query variables: Pollution

(ii).

choose c, join p,s,c,x then eliminate c. choose s join p,s,x ,then eliminate s.

(iii).



P(P=t| X=t) = P(P=t) * P(S=t) * P(C=t| P=t, S=t) * P(X=t |C=t)

+ P(P=t) * P(S=f) * P(C=t| P=t, S=f) * P(X=t| C=t)

+ P(P=t) * P(S=t) * P(C=f| P=t, S=t) * P(X=t| C=f)

+ P(P=t) * P(S=f) * P(C=f| P=t, S=f) * P(X=t| C=f) =

0.9*0.3*0.05*0.9+0.9*0.7*0.02*0.9+0.9*0.3*0.95*0.2+0.9*0.7*

0.98*0.2=0.19827

P(P=f| X=t) =

P(P=f) * P(S=t) * P(C=t| P=f, S=t) * P(X=t| C=t)

+ P(P=f) * P(S=f) * P(C=t| P=f, S=f) * P(X=t| C=t)

+ P(P=f) * P(S=t) * P(C=f| P=f, S=t) * P(X=t| C=f)

+ P(P=f) * P(S=f) * P(C=t| P=f, S=f) * P(X=t| C=f) =

0.1*0.3*0.03*0.9+0.1*0.7*0.001*0.9+0.1*0.3*0.97*0.2+0.1*0.

7*0.999*0.2=0.020679

P(p=t|X=t)/(P(p=t|X=t)+P(P=f|x=f))=0.19827/(0.19827+0.02
0679) = 0.906

2.

D and X, they got common cause.

P and X, P is indirect cause of X.

S and X, S is indirect cause of X.

3.

STEP1:

P(S|P) != P(P) P->S

STEP2:

P(C|P,C) != P(C)

P(C|P,S) != P(C|P)

P(C|P,S) = P(C|S) => P->C and S->C

STEP3:

P(X|P,S,C) != P(X)

P(X|P,S,C) = P(X|C) => C->X, no other link

STEP4:

P(D|P,S,C,X) !=P(D)

P(D|P,S,C,X) !=P(D|C)

P(D|P,S,C,X) !=P(D|X)

P(D|P,S,C,X) =P(D|C,X) => C->D,X->D no other link