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Home Task 3

Github Matlab source code: https://github.com/zizowasfy/RRP_Kinematics-and-Jacobian.git

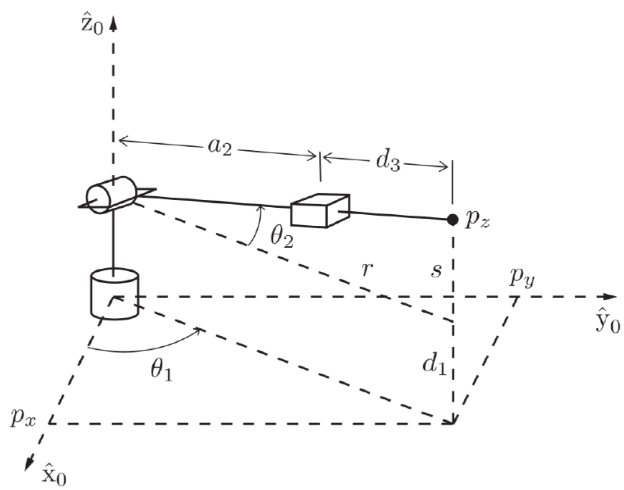


Figure 1: RRP robot.

1) Forward Kinematics

The Homogenous Transformation Matrix of this Robot configuration is:

$$T_H = R_z(\theta_1) \cdot T_z(d_1) \cdot R_y(\theta_2) \cdot T_x(a_2) \cdot T_x(d_3)$$

2) Inverse Kinematics

By knowing the position of $p(x, y, z)$, this part of the Kinematics is solved using analytical solution in finding the unknown joint variables θ_1 , θ_2 , d_3 in terms of the known position.

$$\theta_1 = atan2(y, x)$$

$$\theta_2 = atan2(s, r)$$

Where $s = z - d1$, $r = \sqrt{x^2 + y^2}$

$$d_3 = \sqrt{s^2 + r^2} - a_2$$

Solving these equations yield to 4 solutions; 2 applicable and 2 are not. So, I will present the applicable ones only since the non-applicable solutions compute the value of d3 = -ve value which is considered not reachable because of mechanical limitations and ground constraints.

The following figures are of the FK then the position from the T matrix is extracted and fed to the inverse to test the validity of the IK.

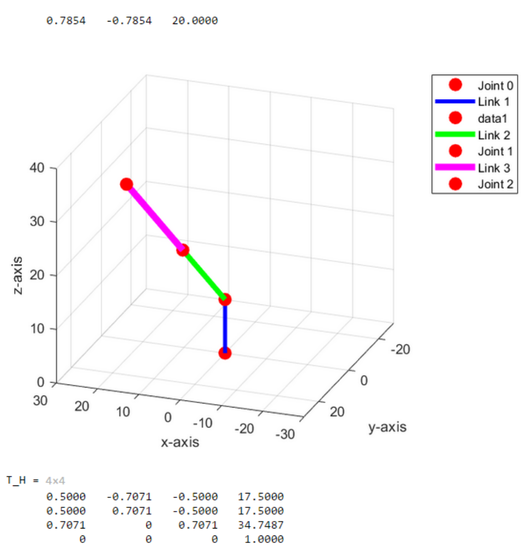


Figure 1: FK Solution

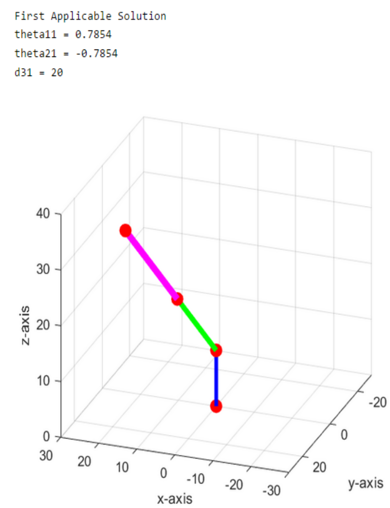


Figure 2: IK 1st Applicable Solution

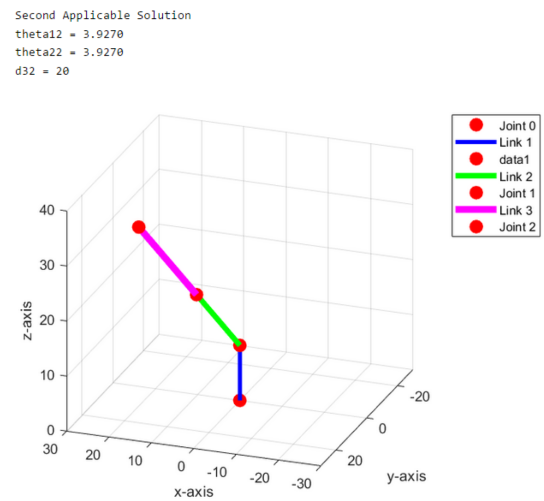


Figure 3: IK 2nd Applicable Solution

3) Jacobian Matrix Computation

i. Classical approach

$$x = \cos(\theta_1)\cos(\theta_2)\ (a_2 + d_3)$$

$$y = \cos(\theta_2)\sin(\theta_1)\ (a_2 + d_3)$$

$$z = d_1 - \sin(\theta_2)\ (a_2 + d_3)$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial d_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial d_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial d_3} \end{bmatrix}$$

ii. Geometric (Skew) approach

$$J_1 = \begin{bmatrix} z_0 \times (o_2 - o_0) \\ z_0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} z_1 \times (o_2 - o_1) \\ z_1 \end{bmatrix} \qquad \cdots$$

$z_0 , z_1, \ldots, z_{n-1}$ $\longrightarrow$ is the column vector in the R matrix of the Transformation matrix that corresponds to the rotated about axis.

$O_0 , O_1 , \ldots\ O_{n-1}$ $\longrightarrow$ is the translation vector in the Transformation matrix

Therefore, the Jacobian equates to:

$$J = (J_1 \quad J_2 \quad \cdots \quad J_n)$$

iii. Numerical approach

$$\frac{\partial}{\partial\theta_1} T_H = \frac{\mathrm{d}}{\mathrm{d}\theta_1} R_z(\theta_1) \cdot T_z(d_1) \cdot R_y(\theta_2) \cdot T_x(a_2) \cdot T_x(d_3) \quad \cdot \begin{bmatrix} R^{-1} & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vdots$$

$$J_{2r} = \text{const} \cdot \frac{\delta R_x(\mathrm{q}2)}{\delta \mathrm{q}2} \cdot \text{const} \cdot \begin{bmatrix} R^{-1} & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\phi_z & \phi_y & \delta x \\ \phi_z & 0 & -\phi_x & \delta y \\ -\phi_y & \phi_x & 0 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \mathrm{J}2r(1,4) \\ \mathrm{J}2r(2,4) \\ \mathrm{J}2r(3,4) \\ \mathrm{J2r}(3,2) \\ \mathrm{J2r}(1,3) \\ \mathrm{J2r}(2,1) \end{bmatrix}$$

4) Singularity Check

Jacobians Can be used also for Singularity Cases in the Robot configuration.

Singularity occurs when one or more row/column are zeros in the Jacobian Matrix. In Zero column case, it means that a particular output parameter $(x\ , y, z, \varphi_x\ ,\ \varphi_y,\ \varphi_z)$

became incontrollable by neither one of the joints. While Zero row means that a particular joint does not have an effect on a certain output parameter.

Taking the determinant of the Jacobian Matrix and making it equal to zero to find all possible values of the joints for singularity cases:

$$|\,J\,| = 0$$

This configuration has 4 singularity cases. $[\theta_2, d_3]$ = $\left[\frac{\pi}{2}, -a_2\right]$ / $\left[\frac{\pi}{2}, a_2\right]$ / $\left[-\frac{\pi}{2}, -a_2\right]$ / $\left[-\frac{\pi}{2}, a_2\right]$

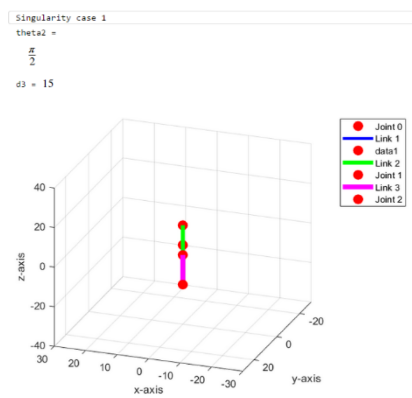


Figure 2: Singularity case 1

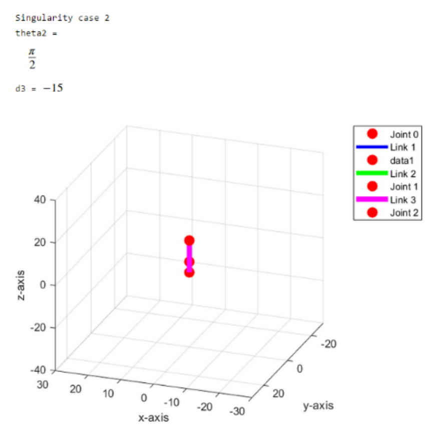


Figure 3: Singularity case 2

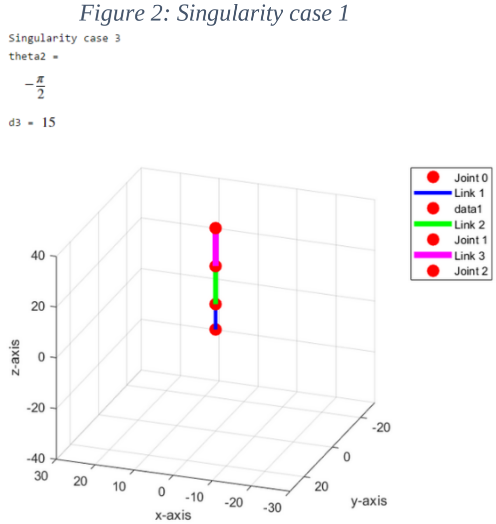


Figure 5 Singularity case 3

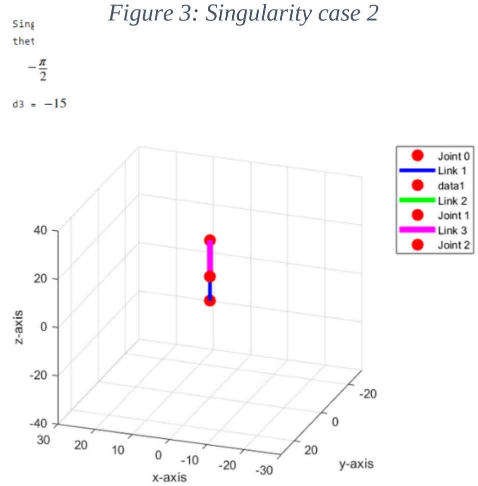


Figure 4 Singularity case 4

5) Tool Frame Velocity

The computed Jacobian matrix can be further used to map from the joints' velocity space to Tool's velocity space.

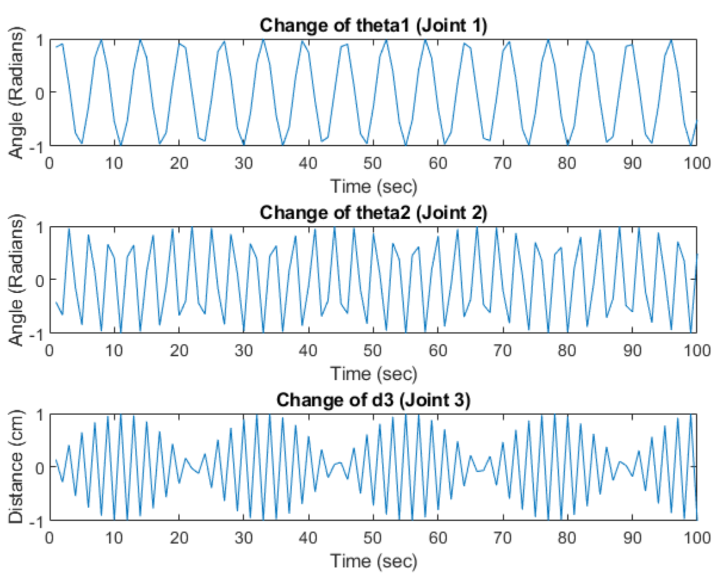
$$q(t) = [\theta_1 \ \theta_2 \ d_3]$$

$$\dot{q}(t) = [\dot{\theta_1} \ \dot{\theta_2} \ \dot{d_3}]$$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = J(\theta_1(t), \theta_2(t), d_3(t)) \cdot \begin{bmatrix} \dot{\theta_1} \\ \dot{\theta_2} \\ \dot{d_3} \end{bmatrix}$$

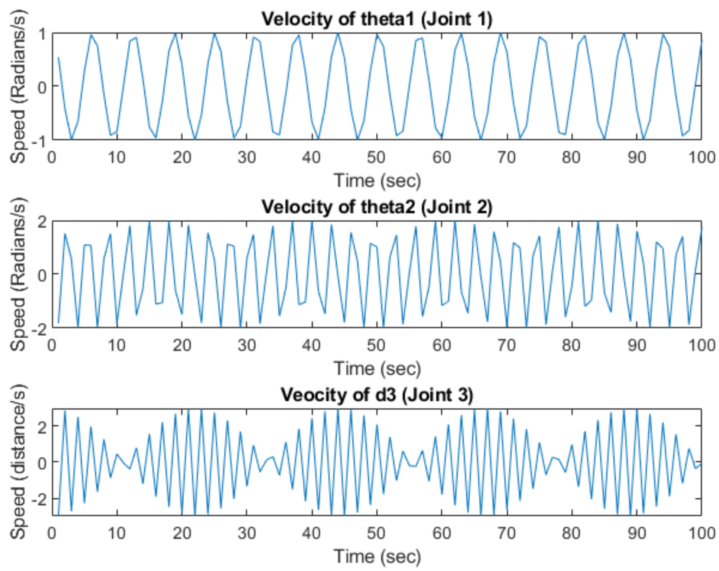
These graphs show graphically how the joints' angles (in radians) changes in time with their given input functions.

Joint 1 $- \theta_1 = \sin(t)$ Joint 2 $- \theta_2 = \cos(2t)$ Joint 3 $- d_3 = \sin(3t)$



The rate of change of the angles of the joints

Joint 1 – $\theta_1 = \cos(t)$ Joint 2 – $\theta_2 = -2\sin(2t)$ Joint 3 – $d_3 = 3\cos(3t)$



Describes the Tool Frame velocity in each component of the cartesian workspace X , Y , Z