第 9 章决策论建模

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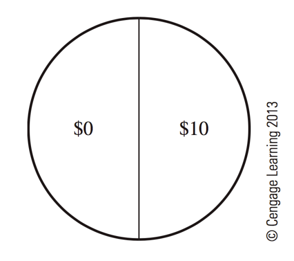
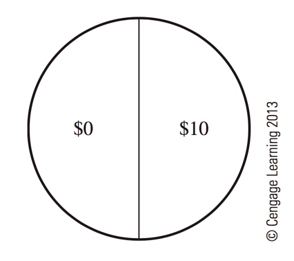
转轮游戏

- 4 美元一次，你会去玩吗？



转轮游戏

- 4 美元一次，你会去玩吗？
- $E = (0 - \$4) \times 0.5 + (\$10 - \$4) \times 0.5 = \$1$



Deal or No Deal?

- 剩下两个盒子，一个 $0.01，一个 $1000000
- Deal: 庄家给你 $400000 报酬
- No Deal: 继续玩

Deal or No Deal?

- 剩下两个盒子，一个 \$0.01，一个 \$1000000
- Deal: 庄家给你 \$400000 报酬
- No Deal: 继续玩
- $E = \$1 \times 0.5 + \$1000000 \times 0.5 \approx \$500000$

Deal or No Deal?

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- Deal: 庄家给你 \$400000 报酬
- No Deal: 继续玩
- $E = \$1 \times 0.5 + \$1000000 \times 0.5 \approx \$500000$
- 你会如何抉择？

修建什么样的工厂？

	Outcomes		
Alternatives	High demand	Moderate demand	Low demand
Build large plant	\$200,000	\$100,000	–\$120,000
Build small plant	\$90,000	\$50,000	–\$20,000
No plant	\$0	\$0	\$0

- 无法估计需求概率该如何决策？
- 高、中、低需求的概率分别是 25%、40%、35% 决策如何改变？
- 这些估计是相关的频率还是专家判断？
- ...

概率和期望值

- 掷出一对骰子，点数之和为 7 算赢
- \$1 玩一次，赢了可以拿回 \$1 并另得 \$5

		Die 2 outcomes					
		1	**2**	**3**	**4**	**5**	**6**
Die 1 outcomes	**1**	2	3	4	5	6	**7**
	2	3	4	5	6	**7**	8
	3	4	5	6	**7**	8	9
	4	5	6	**7**	8	9	10
	5	6	**7**	8	9	10	11
	6	**7**	8	9	10	11	12

概率和期望值

- 掷出一对骰子，点数之和为 7 算赢
- \$1 玩一次，赢了可以拿回 \$1 并另得 \$5
- $E = \$5 \times \frac{1}{6} + (-\$1) \times \frac{5}{6} = 0$

		Die 2 outcomes					
		1	**2**	**3**	**4**	**5**	**6**
Die 1 outcomes	**1**	2	3	4	5	6	**7**
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- 公平游戏

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- 掷出一对骰子，点数之和为 7 算赢
- $1 玩一次，赢了可以拿回 $1 并另得 $5
- $E = \$5 \times \frac{1}{6} + (-\$1) \times \frac{5}{6} = 0$
- 公平游戏
- 如果 $2 玩一次呢？

		Die 2 outcomes					
		1	**2**	**3**	**4**	**5**	**6**
Die 1 outcomes	**1**	2	3	4	5	6	**7**
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人寿保险

- 保险公司将一年期 \$250000 保单卖给 49 岁女性
- 保费 \$550
- 49 岁女性的存活率 0.99791

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- 保费 \$550
- 49 岁女性的存活率 0.99791
- $E = \$550 \times 0.99791 - \$250000 \times (1 - 0.99791) = \$25.201$

高尔夫球场的改建还是新建？

New construction (NC)	Remodeling (R)
Win contract: \$50,000 net profit	Win contract: \$40,000 net profit
Lose contract: –\$1000	Lose contract: –\$500
Probability of award of contract: 20%	Probability of award of contract: 25%

高尔夫球场的改建还是新建？

New construction (NC)	**Remodeling (R)**
Win contract: \$50,000 net profit Lose contract: −\$1000 Probability of award of contract: 20%	Win contract: \$40,000 net profit Lose contract: −\$500 Probability of award of contract: 25%

- $E(NC) = (\$50,000) \times 0.2 + (-\$1,000) \times 0.8 = \$9,200$

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- $E(NC) = (\$50,000) \times 0.2 + (-\$1,000) \times 0.8 = \$9,200$
- $E(R) = (\$40,000) \times 0.25 + (-\$500) \times 0.75 = \$9,625$

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- 长期来看，改建现有的高尔夫球场更赚钱

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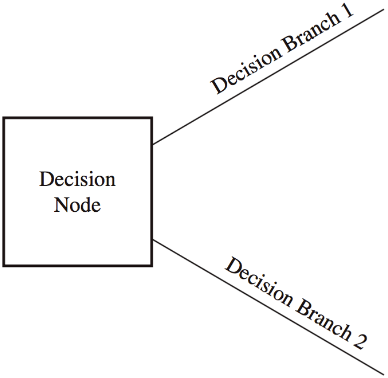
- 长期来看，改建现有的高尔夫球场更赚钱
- 敏感性分析（赢得合同概率、利润）

决策树

■ **Figure 9.3**

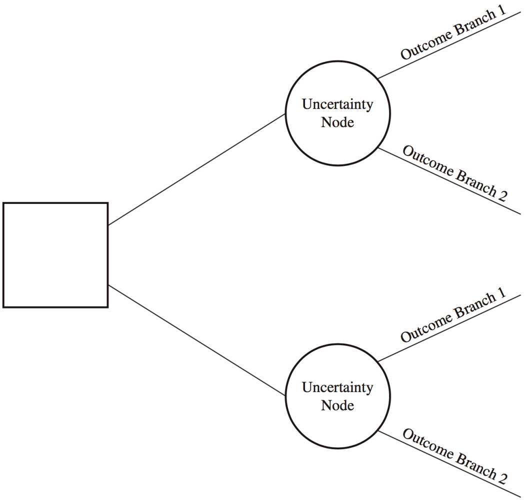
A **decision node** with a **decision branch** for each alternative course of action (strategy)



决策树

■ **Figure 9.4**

An **uncertainty node** reflecting chance (the state of nature) with an **outcome branch** for each possible outcome at that uncertainty node



决策树

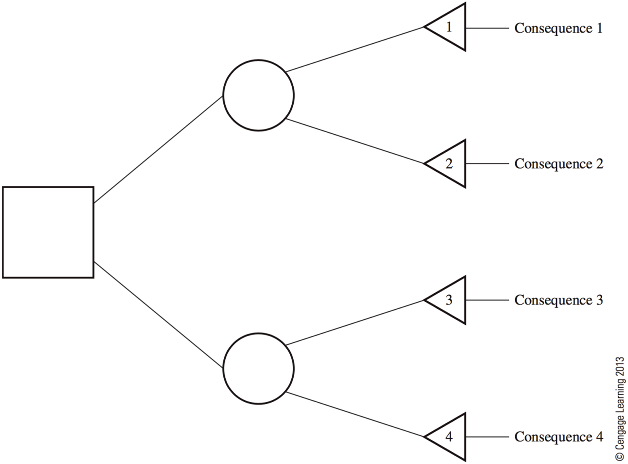


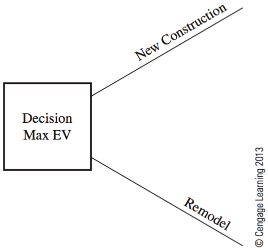
Figure 9.5

A terminal node with a **consequence branch** showing the payoff for that outcome

用决策树选择新建或改建

■ **Figure 9.6**

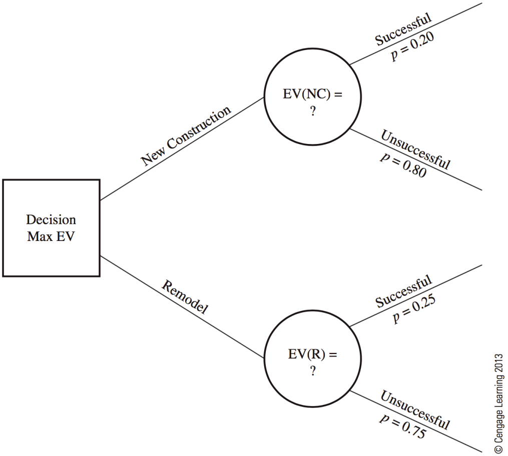
Choose new construction or remodel to maximize expected value



用决策树选择新建或改建

■ **Figure 9.7**

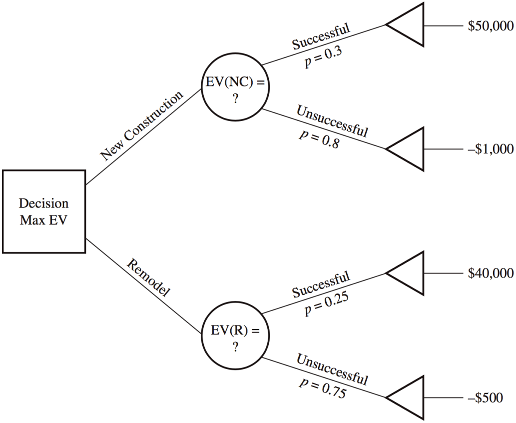
Two uncertainty nodes, each with two outcome branches



用决策树选择新建或改建

■ **Figure 9.8**

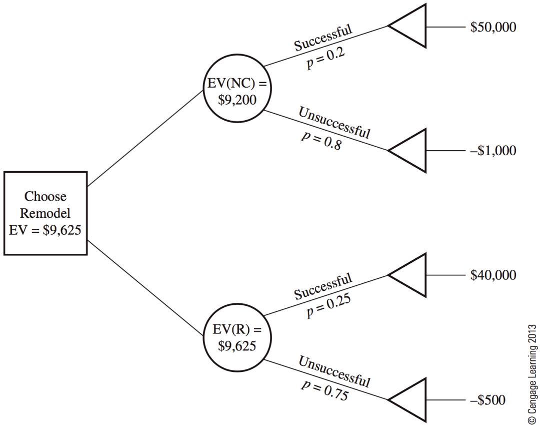
Decision tree for Example 1



用决策树选择新建或改建

Figure 9.9

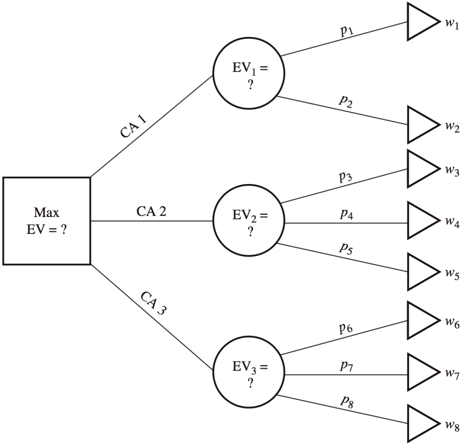
Solution for Example 1



折返方法

Figure 9.10

The fold-back method: Starting with the final decision node, evaluate the expected value of the uncertainty node along each decision branch (course of action). Then, for each decision node, choose the maximum of the corresponding expected values for each course of action emanating from that decision node



Hardware & Lumber 公司的决策

Alternatives	Outcomes		
	High demand	Moderate demand	Low demand
Build large plant	$200,000	$100,000	–$120,000
Build small plant	$90,000	$50,000	–$20,000
No plant	$0	$0	$0

Hardware & Lumber 公司的决策

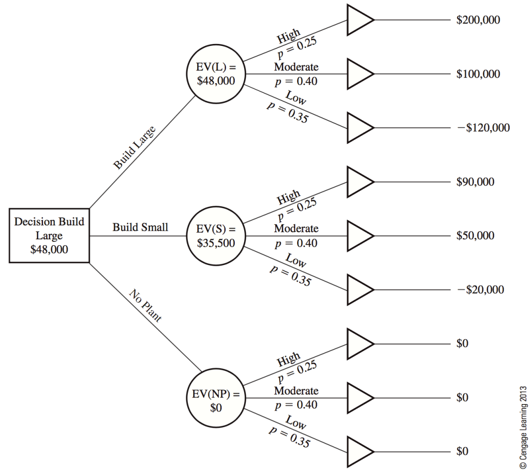


Figure 9.11
Hardware & Lumber tree diagram

地方电视台

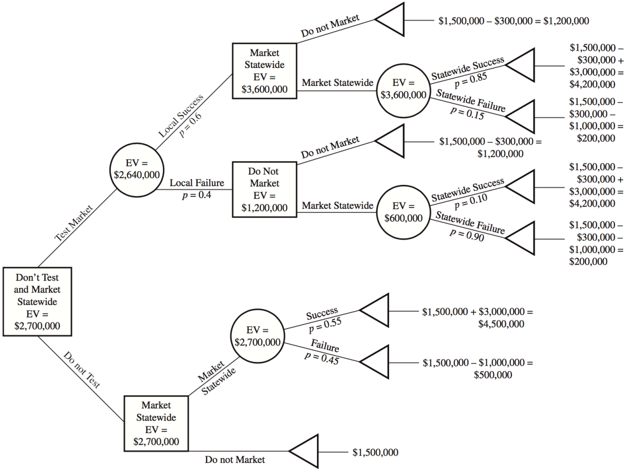
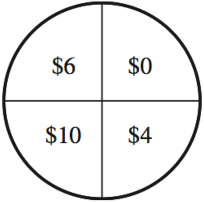


Figure 9.12
Completed Tree Diagram for Example 3

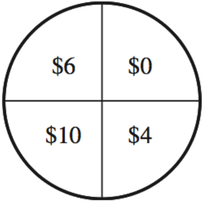
序列决策和条件概率



Payoff	Probability
$0	1 in 4 (25%)
$4	1 in 4 (25%)
$6	1 in 4 (25%)
$10	1 in 4 (25%)

- 每盘游戏可以转 3 次，随时可以停止，寻求最优策略

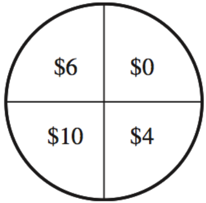
序列决策和条件概率



Payoff	Probability
$0	1 in 4 (25%)
$4	1 in 4 (25%)
$6	1 in 4 (25%)
$10	1 in 4 (25%)

- $E(3) = \$10 \times 0.25 + \$6 \times 0.25 + \$4 \times 0.25 + \$0 \times 0.25 = \$5.00$

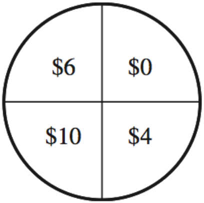
序列决策和条件概率



Payoff	Probability
$0	1 in 4 (25%)
$4	1 in 4 (25%)
$6	1 in 4 (25%)
$10	1 in 4 (25%)

- $E(3) = \$10 \times 0.25 + \$6 \times 0.25 + \$4 \times 0.25 + \$0 \times 0.25 = \$5.00$
- $E(2) = \$10 \times 0.25 + \$6 \times 0.25 + \$5 \times 0.5 = \$6.50$

序列决策和条件概率



Payoff	Probability
$0	1 in 4 (25%)
$4	1 in 4 (25%)
$6	1 in 4 (25%)
$10	1 in 4 (25%)

- $E(3) = \$10 \times 0.25 + \$6 \times 0.25 + \$4 \times 0.25 + \$0 \times 0.25 = \$5.00$
- $E(2) = \$10 \times 0.25 + \$6 \times 0.25 + \$5 \times 0.5 = \$6.50$
- $E(1) = \$10 \times 0.25 + \$6.5 \times 0.75 \approx \$7.375$

序列决策和条件概率

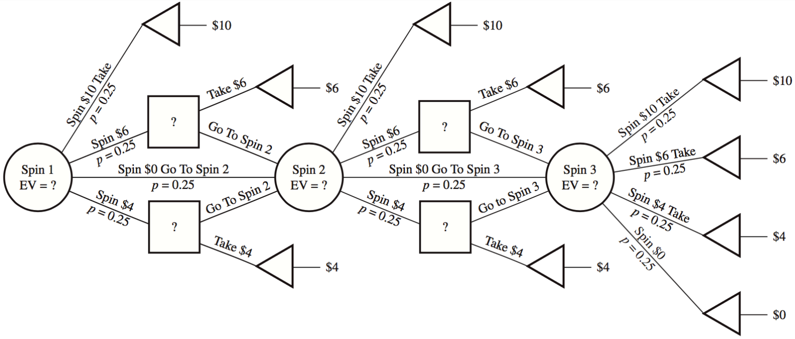
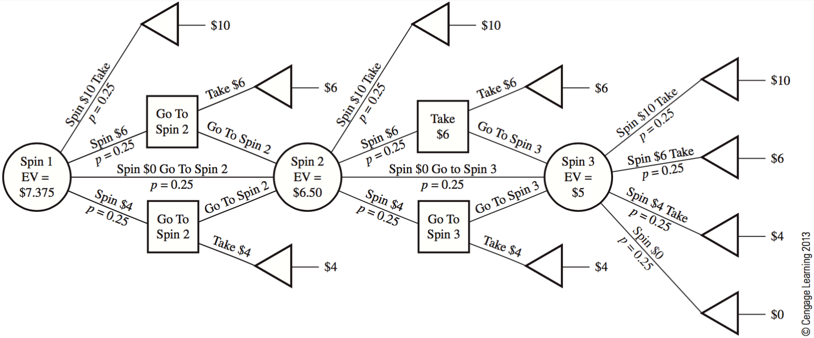


Figure 9.13

Decision tree for spinning wheel

序列决策和条件概率



■ **Figure 9.14**

Completed Decisition tree for spinning wheel

再论 Hardware & Lumber 公司的决策

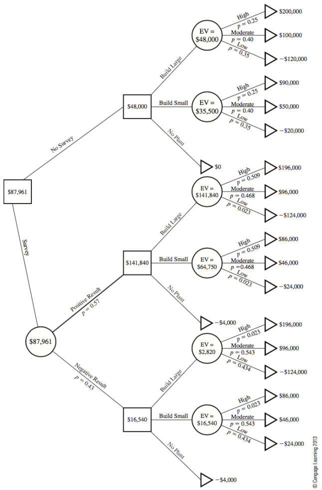


Figure 9.15
Sequential decision tree for the Hardware & Lumber Company

类固醇的检测

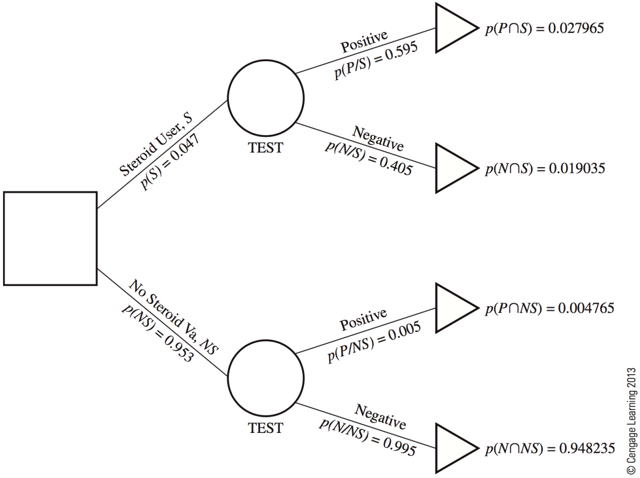


Figure 9.16

Conditional probabilities tree diagram for steroid testing.

利用各种准则的决策

- 风险
- 一次性决策与长期决策

投资与状态

- 初始投资 \$100000 在 5 年后的资金（单位 \$100000)

		Nature of the economy			
		E	**F**	**G**	**H**
Your plan	**A**	2	2	0	1
	B	1	1	1	1
	C	0	4	0	0
	D	1	3	0	0

情形 1：概率已知，最大化期望值

- 最大化期望值准则
- 假设 E、F、G、H 的概率分别是 0.2、0.4、0.3、0.1

情形 1：概率已知，最大化期望值

- 最大化期望值准则
- 假设 E、F、G、H 的概率分别是 0.2、0.4、0.3、0.1
- $E(A) = 1.3$, $E(B) = 1.0$, $E(C) = 1.6$, $E(D) = 1.4$

情形 1：概率已知，最大化期望值

- 最大化期望值准则
- 假设 E、F、G、H 的概率分别是 0.2、0.4、0.3、0.1
- $E(A) = 1.3$, $E(B) = 1.0$, $E(C) = 1.6$, $E(D) = 1.4$
- 不反映包含的风险，但对一次性决策仍然有合适准则的情况

情形 1：概率已知，最大化期望值

- 最大化期望值准则
- 假设 E、F、G、H 的概率分别是 0.2、0.4、0.3、0.1
- $E(A) = 1.3$, $E(B) = 1.0$, $E(C) = 1.6$, $E(D) = 1.4$
- 不反映包含的风险，但对一次性决策仍然有合适准则的情况
- 例如：某人有一次性的机会靠只投掷一次一对骰子的办法去赢 \$1000

情形 2：一次性决策，概率未知

拉普拉斯准则 假定未知概率都是相等的

最大最小准则 选取具有最高下限的策略（保守策略）

最大最大准则 乐观策略

乐观系数准则 x(row max) + (1-x)(row min)，乐观、保守相结合

情形 3:“费用”最小化

最小最大准则 选取上限最小的

最小最大缺憾准则 使最大缺憾尽可能小

Regret Matrix

		Nature				
		E	**F**	**G**	**H**	Maximum regret
	A	0	2	1	0	2
You	**B**	1	3	0	0	3
	C	2	0	1	1	2
	D	1	1	1	1	1

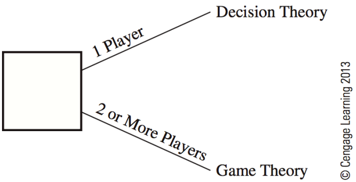
投资策略

Alternatives	Conditions		
Investments	Fast growth	Normal growth	Slow growth
Stocks	$10,000	$6500	–$4000
Bonds	$8000	$6000	$1000
Savings	$5000	$5000	$5000

博弈论

Figure 10.1

Game theory treats outcomes that depend on more than one player



五金连锁店

	Ace	
	Large City	Small City
Large City	60	68
Home Depot	⇑	⇑
Small City	52	60

五金连锁店

		Ace		
		Large City		Small City
Home Depot	Large City	60, 40	⇐	68, 32
		⇑		⇑
	Small City	52, 48	⇐	60, 40

纳什均衡

纳什均衡
纳什均衡是这样一种结果，其中任何一个参与者都不可能通过单方面偏离与该结果想对应的策略获得好处。

混合策略完全冲突博弈：投球手和击球手的较量

	Pitcher		
	Fastball		Curve
Fastball	.400	$\Rightarrow$	.200
Batter	$\Uparrow$		$\Downarrow$
Curve	.100	$\Leftarrow$	.300

博弈论：部分冲突

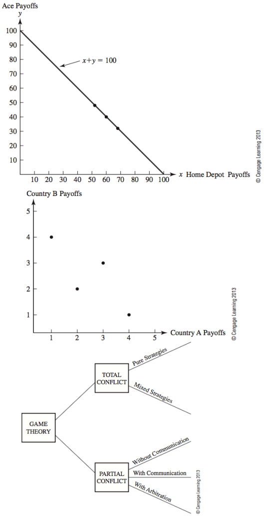
- 囚徒困境

		Country B		
		Disarm		Arm
	Disarm	3,3	$\Rightarrow$	1,4
Country A		$\Downarrow$		$\Downarrow$
	Arm	4,1	$\Rightarrow$	2,2

完全冲突与部分冲突

Figure 10.3

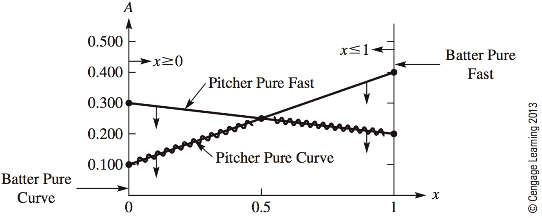
A plot of the payoffs for (a) total conflict and (b) partial conflict. (c) We consider partial conflict games played without communication, with communication, and with arbitration



完全冲突博弈的线性规划模型

■ **Figure 10.11**

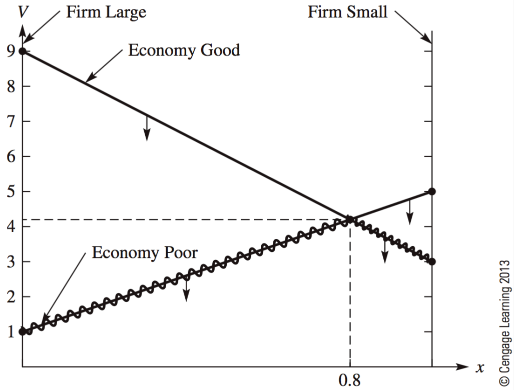
The optimal solution occurs when $x = 0.5$ and $A = .250$



决策论：与大自然博弈

■ **Figure 10.24**

The optimal solution for the firm



2x2 完全冲突博弈的其它简便解法

- 让对手策略对应的期望值相等
- 零头法，也称为 William 方法

部分冲突博弈：经典的两人博弈

- 囚徒困境
- 斗鸡博弈
- 性别战