Exercise Sheet 2

Discrete Mathematics by Hongfei Fu, 2023.09.14

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1. Show that each of these conditional statements is a tautology.

   a) $[(\neg p) \wedge (p \vee q)] \rightarrow q$
   b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
   c) $[p \wedge (p \rightarrow q)] \rightarrow q$
   d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

   Answer Area:
   a) If q=T, the whole proposition = T
   Else q=F, the whole proposition $\equiv((\neg p) \wedge p) \rightarrow q \equiv$F $\rightarrow q =$ T.
   So the whole proposition is a tautology.

   b) The whole proposition = F $\Leftrightarrow$(p $\rightarrow q) \wedge (q \rightarrow r)$=T and $p \rightarrow r =$ F
   $p \rightarrow r =$ F $\Leftrightarrow$p=T,r=F
   $If q = T$,q $\rightarrow r$=F, else q=F, $p \rightarrow q$=F, both contradict with $(p \rightarrow q) \wedge (q \rightarrow r) =$ T
   So the whole proposition is a tautology.

   c) If q=T, the whole proposition = T
   Else q=F, $[p \wedge (p \rightarrow q)] \rightarrow q \equiv$[p $\wedge(\neg p)] \rightarrow q \equiv$F $\rightarrow q =$ T
   So the whole proposition is a tautology.

   d) If r=T, the whole proposition = T
   Else r=F, $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r \equiv$[(p $\vee q) \wedge (\neg p) \wedge (\neg q)] \rightarrow r \equiv$F $\rightarrow r =$ T
   So the whole proposition is a tautology.

2. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

   Answer Area:

p	q	$\neg q$	p $\leftrightarrow q$	$\neg(p \leftrightarrow q)$	p $\leftrightarrow (\neg q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	F

3. Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

   Answer Area:
   $(p \rightarrow q) \vee (p \rightarrow r) \equiv[(\neg p) \vee q] \vee [(\neg p) \vee r] \equiv(\neg p) \vee (q \vee r) \equiv$p $\rightarrow (q \vee r)$

4. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

   Answer Area:
   If p=F, q=T, r=T, $(p \wedge q) \rightarrow r$=T but $(p \rightarrow r) \wedge (q \rightarrow r)$=F
   So the two propositions are not logically equivalent.

5. Show that the negation $\neg\phi$ of an unsatisfiable compound proposition ϕ is a tautology and the negation $\neg\psi$ of a compound proposition ψ that is a tautology is unsatisfiable.

   Answer Area:
   ϕ is unsatisfiable so it's always false, so $\neg\phi$ is always true. That means $\neg\phi$ is a tautology. Similarly, ψ is a tautology so it's always true, so $\neg\psi$ is always false. That means $\neg\psi$ is unsatisfiable.