Coherence and Transfer Function Estimation

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$\leftarrow$ Power Spectrum $G_{yy}(\omega)$

Note: In H/W assignment, r(n) is used as the measurement and y(n) used to denote the linear system output

Note $\gamma^2(\omega) < 1$ when

(1) noise contaminating measurement

(2) system nonlinearity (transfer power from one frequency to another frequency)

(3) other inputs to system

Measured power spectrum $G_{yy}(\omega)$

(1) Component due to system input $\gamma^2(\omega)\, G_{yy}(\omega)$

(2) Component due to additive noise $\left(1-\gamma^2(\omega)\right) G_{yy}(\omega)$

Signal to noise ratio in y(n):

$$SNR = \frac{|H(\omega)|^2\, G_{xx}(\omega)}{G_{nn}(\omega)} = \frac{\gamma^2(\omega)}{1-\gamma^2(\omega)}$$

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Coherence Function Estimate is Biased - Reduce Bias and Variance by Averaging

(1) For $\gamma^2(\omega) = 0$ and $K \geq 32$ (k = # averages)

$$\text{bias}\left[\hat{\gamma}^2(\omega)\right] \simeq \frac{1}{K}$$

$$\text{var}\left[\hat{\gamma}^2(\omega)\right] \simeq \frac{1}{K^2}$$

(2) For $0 < \gamma^2(\omega) \leq 1$ and $K \geq 32$

$$\text{bias}\left[\hat{\gamma}^2(\omega)\right] \simeq \frac{1}{K}\left(1 - \gamma^2(\omega)\right)^2$$

$$\text{var}\left[\hat{\gamma}^2(\omega)\right] \simeq \frac{2}{K}\gamma^2(\omega)\left(1 - \gamma^2(\omega)\right)^2$$

Refs

① C. Carter et al, "Estimation of the magnitude-squared coherence function via overlapped Fast Fourier Transform processing," IEEE Trans. Audio and Electroacoust. AU-21(4): 337-344 (1973).

<u>Note</u> Averaging is critical

$$\hat{\gamma}^2(k) = \frac{\left|\overline{\hat{G}_{yx}(k)}\right|^2}{\overline{\hat{G}_{xx}(k)}\;\overline{\hat{G}_{yy}(k)}} = 1 \text{ when } K = 1$$

K ↑ # averages

② J. Bendat and A. Piersol (1993)

③ J. Bendat and A. Piersol (2000)

④ C. Carter (Proc. IEEE, 1987)

Coherence and Transfer Function Estimation

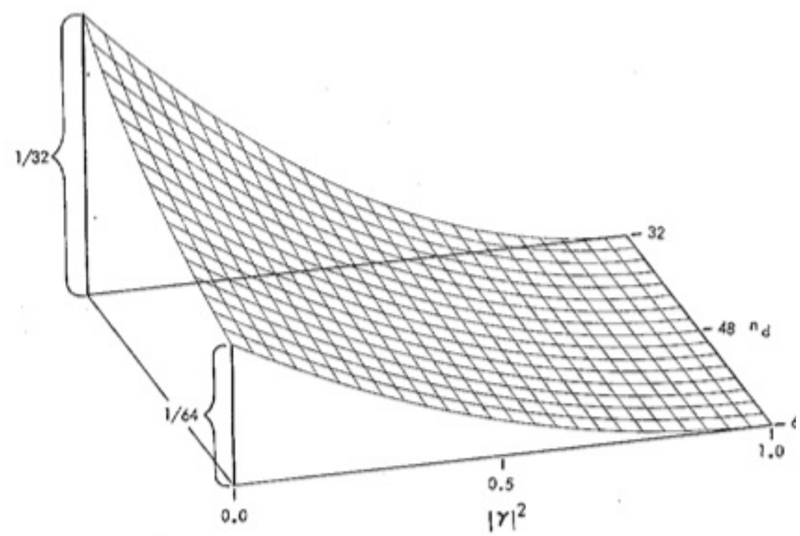


Fig. 5. Bias of $|\hat{\gamma}|^2$ versus $|\gamma|^2$ and n_d.

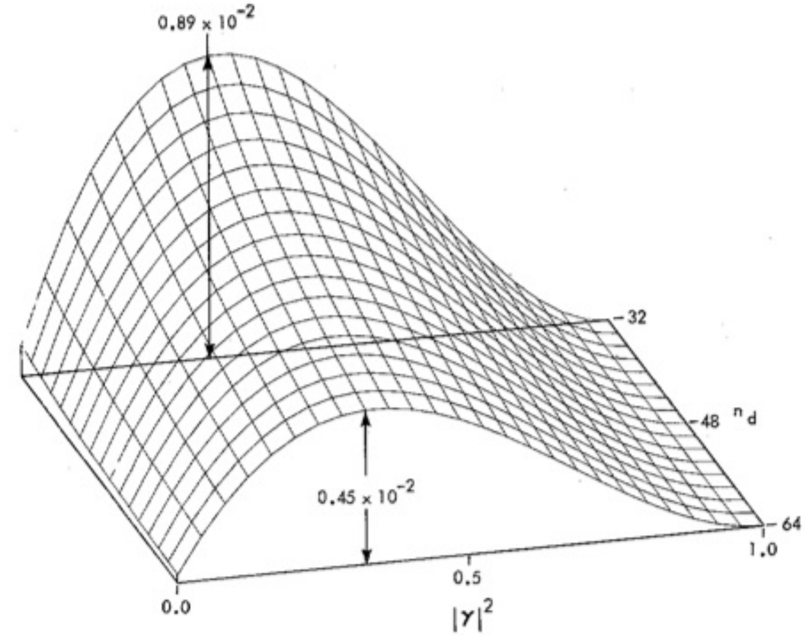


Fig. 6. Variance of $|\hat{\gamma}|^2$ versus $|\gamma|^2$ and n_d.

Note: n_d is the number of independent segments used in computing the MSC.

G.C. Carter, C. Knapp, and A. Nuttall, “Estimation of the magnitude-squared coherence function via overlapped fast Fourier transform processing,” IEEE Trans. Audio Electroacoust. AU-21(4): 337-344 (1973).

Coherence and Transfer Function Estimation

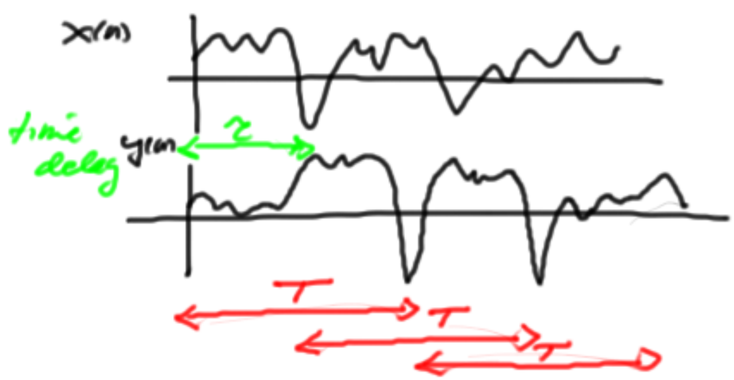
Bias due to time delay between sample records of $x(n)$ and $y(n)$

$$\hat{\gamma}^2(\omega) \simeq \gamma^2(\omega)\left(1 - \frac{\tau}{T}\right)$$

τ = time delay

T = segment length of each FFT

$$x(n) \longrightarrow \boxed{\text{delay } \tau} \longrightarrow y(n)$$



In order to minimize the effect, first compute cross-correlation of $x(n)$ and $y(n)$ then delay/advance $y(n)$ with respect to $x(n)$ to minimize the lag in the peak correlation