High Resolution Spectral Analysis

I. AR Time Series Generation Model

A. Implement in direct form the following all-pole filter $H(z) = \frac{1}{A(z)}$ which is to be excited by white, Gaussian noise where $E[w(n)] = 0$ and $\mathrm{var}[w(n)] = 1$

$$A(z) = \prod_{i=1}^{4} \left(1 - z_i \, z^{-1}\right)\left(1 - z_i^{*} z^{-1}\right) = \sum_{i=0}^{8} a_i \, z^{-i}$$

$$z_i = r_i e^{j\theta_i}$$

i	θ_i	r_i
1	$\pi/8$	0.9896
2	$2\pi/8$	0.9843
3	$3\pi/8$	0.9780
4	$4\pi/8$	0.9686

If instability appears to be a problem due to the direct form implementation, move the theoretical pole locations inward slightly.

B. Make a Z-plane pole-zero plot for H(z).

C. Zero fill $\{a_0, \ldots, a_8\}$out to N-1, compute a N = 256 - point FFT, and plot $10 \log\left(\frac{1}{|A(k)|^2}\right)$, $k = 0, \ldots, \frac{N}{2} - 1$.

D. Plot the first 256 points of the impulse response h(n), compute a N = 256 point FFT, and plot $10 \log |H(k)|^2$, $k = 0, \ldots, \frac{N}{2} - 1$.

II. Time Series Generation

A. Pass w(n) through h(n) and retain $\{ x(n) \mid n = 1024, \ldots, 1279 \}$ (i.e. 256 points) (w(n) must be at least 1280 points long).

B. Compute a N = 256-point FFT of the retained x(n) and plot $10 \log |X(k)|^2$, $k = 0, \ldots, \frac{N}{2} - 1$.

C. Break the retained x(n) into 32 point, 50% overlapped segments. Zero fill each segment out to N-1, compute the N = 256 point FFT of each segment, and plot 10 log $|X(k)|^2_{avg}$, k = 0, ..., $\frac{N}{2} - 1$.

III. Autocorrelation Method of Linear Prediction

A. Using the N = 256-point retained time series x(n), make estimates of the inverse filter for orders p = 2, 8, and 14. Make sure that the time series is windowed prior to computation of the autocorrelation function used in estimating the inverse filter.

1. Zero fill $\{1, \hat{a}_1, ..., \hat{a}_p\}$ out to N-1, compute a N = 256-point FFT, and plot 10 log $\left(\frac{1}{|\hat{A}(k)|^2}\right)$, k = 0, ..., $\frac{N}{2} - 1$.

2. Plot the zero locations of the inverse filter.

B. Using the N = 256-point retained time series x(n), determine E_p for inverse filter orders p = 2, 4, 6, 8, 10, 12, and 14.

1. Plot Ep *vs.* p *where* $Ep = \sum_{n=0}^{N-1} e_p^2(n)$

C. Using only the first 32 points of the retained time series x(n), repeat IIB (zero fill out to N-1) and III A and B.

D. Comment on your results.

Note:

All frequency domain plots should be annotated in terms of f (0–0.5 cycles/sample) not k (FFT bin index).