Conventional Power Spectral Estimation

Power Spectral Estimation

Periodogram is an estimate of the power spectrum

$$I_N(\omega) = \sum_{m=-(N-1)}^{N-1} c_{xx}(m)\, e^{-j\omega m} \qquad c_{xx}(m) = \hat{\phi}(m)$$

$$= \frac{1}{N}\left|X(e^{j\omega})\right|^2 \quad \text{where: } X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

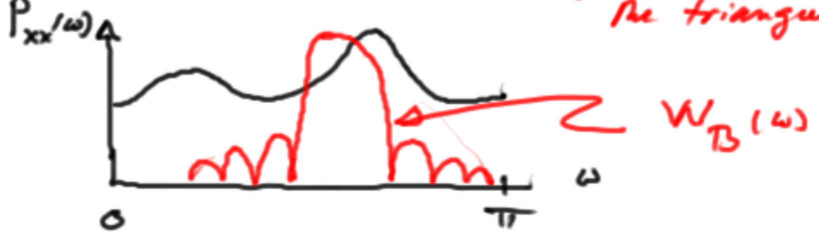
Bias

$$E[I_N(\omega)] = \sum_{m=-(N-1)}^{N-1} \left(\frac{N-|m|}{N}\right)\phi_{xx}(m)\, e^{-j\omega m}$$

triangular window

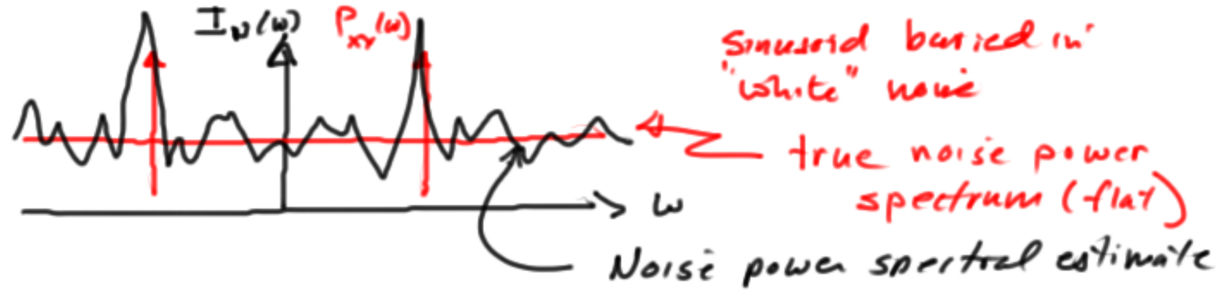
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} P_{xx}(\theta)\, W_B\!\left(e^{j(\omega-\theta)}\right) d\theta$$

Fourier transform of the triangular window



Conventional Power Spectral Estimation

<u>Variance</u>



$$\operatorname{var}[I_N(\omega)] = P_{xx}^2(\omega)\left\{1 + \left(\frac{\sin \omega N}{N \sin \omega}\right)^2\right\}$$

Thus, variance of periodogram does <u>not</u> get smaller as N increases

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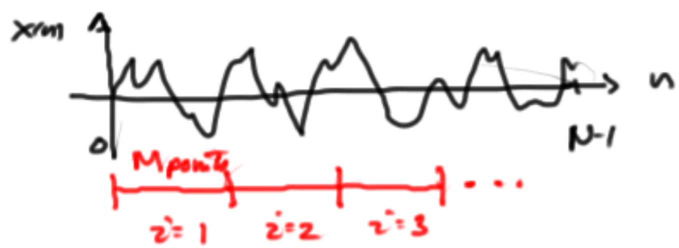
Conventional Power Spectral Estimation

Bartlett's Procedure of Averaging Periodograms



$$B_{xx}(\omega) = \frac{1}{K}\sum_{i=1}^{K} I_M^{(i)}(\omega)$$

$$E[B_{xx}(\omega)] = \frac{1}{K}\sum_{i=1}^{K} E[I_M^{(i)}(\omega)] = E[I_M^{(i)}(\omega)]$$

convolution of $P_{xx}(\omega)$ with the Fourier transform of triangular window $\left(\frac{M-|m|}{M}\right)$

Variance

$$\mathrm{var}[B_{xx}(\omega)] = \frac{1}{K}\,\mathrm{var}[I_M^{(i)}(\omega)]$$

Variance reduction resulting from averaging

$$\simeq \frac{1}{K} P_{xx}^2(\omega)$$

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Conventional Power Spectral Estimation

Welch's Method of Averaging Modified Periodograms

$$B^{W}_{xx}(\omega) = \frac{1}{K}\sum_{i=1}^{K} J_M^{(i)}(\omega)$$

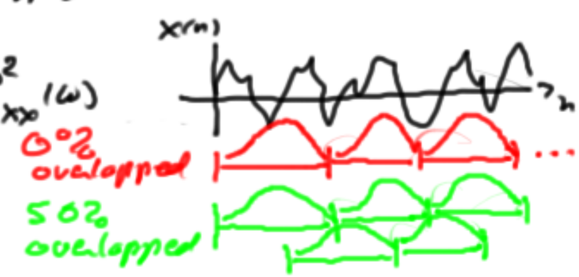
where: $$J_M^{(i)}(\omega) = \frac{1}{MU}\left|\sum_{n=0}^{M-1} w(n)\, x^{(i)}(n)\, e^{-j\omega n}\right|^2$$

and $$U = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n)$$

$$E[B^{W}_{xx}(\omega)] = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_{xx}(\theta)\, W\left(e^{j(\omega-\theta)}\right) d\theta$$

$$W(e^{j\omega}) = \frac{1}{MU}\left|\sum_{n=0}^{M-1} w(n)\, e^{-j\omega n}\right|^2$$

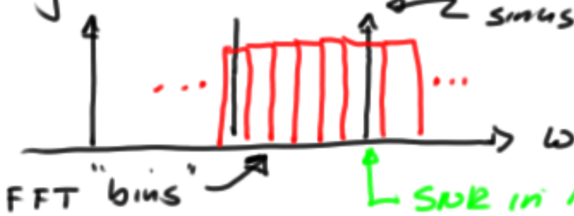
$$\mathrm{var}\left[B^{W}_{xx}(\omega)\right] \simeq \frac{1}{K} P^2_{xx}(\omega)$$



Conventional Power Spectral Estimation

HW #4 - Spectral Averaging

Processing gain of FFT (dB) $\simeq 10 \log_{10} \frac{NFFT}{2}$

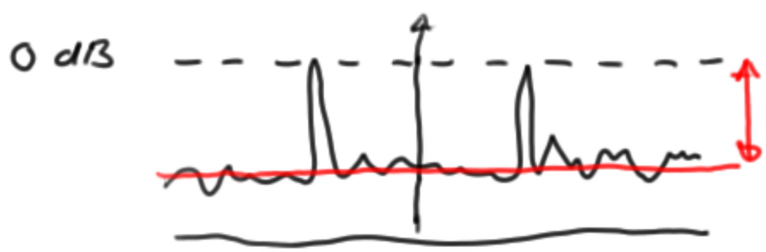


SNR in FFT bin has increased by factor of $\frac{NFFT}{2}$

sinusoid power split into 2 bins
noise power split into N bins

$\therefore$ SNR gain is $\frac{NFFT}{2}$

NFFT	$\frac{NFFT}{2}$	SNR gain
128	$64 = 2^6$	18 dB
256	$128 = 2^7$	21 dB
512	$256 = 2^8$	24 dB
1024	$512 = 2^9$	27 dB



~18 dB for NFFT = 128 when FFT input SNR = 0 dB

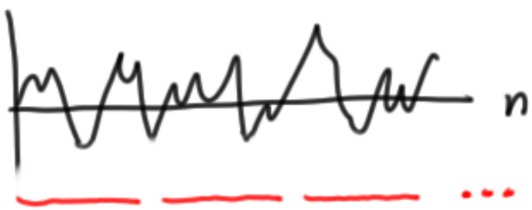
Conventional Power Spectral Estimation

HW #4 - Spectral Averaging

C. Incoherent Average $= \dfrac{1}{K} \sum_{i=1}^{K} \left| X^{(i)}_{(k)} \right|^2$

$M = 128$ point FFTs

$x(n)$

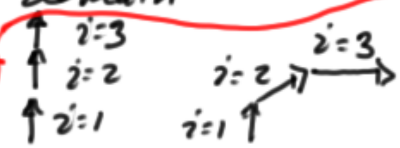


... for 0% overlap

$i=1$ $\quad i=2$

$X^{(1)}_{(k)} \quad X^{(2)}_{(k)}$

frequency domain



bin centered sinusoid

non-bin centered sinusoid

D. Coherent Average $= \dfrac{1}{K} \sum_{i=1}^{K} X^{(i)}_{(k)}$

$$= \sum_{n=0}^{M-1} \left\{ \frac{1}{K} \sum_{i=1}^{K} x^{(i)}_{(n)} \right\} e^{-j\left(\frac{2\pi}{M}\right) nk}$$

to compute power then take $|\cdot|^2$ of result

time domain