Discrete Random Sequences

I. Random Numbers

A. Generate 1024-point sequences of independent random variables which have the following probability density functions:

1. Uniform (distributed on [0,1]).
2. Gaussian (mean = $m_x = E[x] = 0$ and variance = $\sigma_x^2 = E[(x-m_x)^2] = 1$).

B. Generate and plot histograms corresponding to the two random number sequences created in IA.

C. Using N = 256, generate and plot the autocorrelation sequence estimate $c_{xx}(m)$, m=0, 1, ..., 15 for the two random number sequences created in IA. Include theoretical calculations for the expected values of $c_{xx}(m)$ for the two sequences (ignore bias resulting from 1/N in the expression).

II. Filtering

A. Given the following low-pass FIR filter impulse response:

$$h(n) = \begin{cases} 1, & n=0, 1, \cdots, 7 \\ 0, & \text{otherwise} \end{cases}$$

1. Provide a z-plane description of the FIR filter (i.e. locate its zeros).
2. Augment h(n) with zeros out to NFFT = 256, FFT, and plot the magnitude (dB) response vs. f.

B. Pass the uncorrelated Gaussian random sequence created in IA2 through the filter.

1. Plot a 256-point example of both the input and output sequences.
2. As in IC, estimate and plot the autocorrelation sequence of the filter's output ($c_{yy}(m)$, m = 0, 1, ..., 15). Similarly, estimate and plot the cross-correlation sequence between input and output ($c_{xy}(m)$, m = -15, ..., 0, ..., 15).
3. Include theoretical calculations for the expected values of $c_{xx}(m)$, $c_{yy}(m)$, and $c_{xy}(m)$ (ignore bias resulting from 1/N in the expressions).

Note

For real time series x(n) and y(n), define the auto and cross-correlation estimates for $m \geq 0$ as:

$$c_{xx}(m) = \frac{1}{N}\sum_{n=0}^{N-m-1} x(n)x(n+m) \qquad c_{xx}(-m) = c_{xx}(m)$$

$$c_{xy}(m) = \frac{1}{N}\sum_{n=0}^{N-m-1} x(n)y(n+m) \qquad c_{xy}(-m) = c_{yx}(m)$$