Statistical Analysis of Time Series

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Important Probability Density Functions

① Standardized Normal Distribution

transformation of Gaussian random variable to standardized form

$$z = \frac{x - \mu_x}{\sigma_x} \qquad \text{where} \quad \mu_x = \text{mean}, \quad \sigma_x^2 = \text{variance}$$

$$p(z) = \frac{1}{(2\pi)^{1/2}} e^{-z^2/2}$$

$\Longrightarrow$

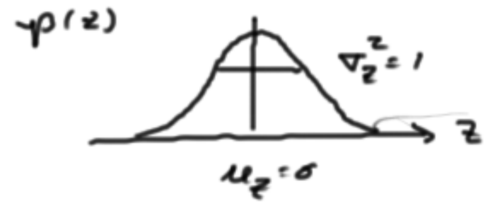


Table A.1

$$P(z_\alpha) = \int_{-\infty}^{z_\alpha} p(z)\,dz = \text{Prob}[z \le z_\alpha] = 1 - \alpha$$

or $\int_{z_\alpha}^{\infty} p(z)\,dz = \text{Prob}[z > z_\alpha] = \alpha$

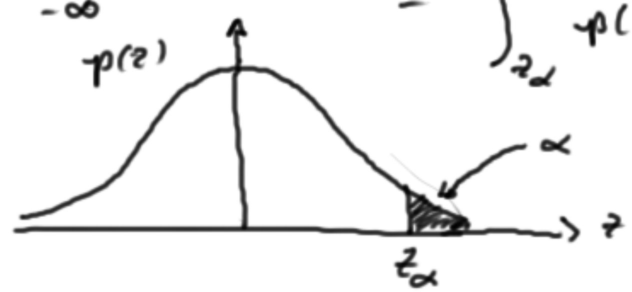


Table A.2

③ Chi-Square Distribution

Let $z_1, z_2, \ldots, z_n$ be n independent random variables each of which has a normal distribution with zero mean and unit variance

Define a new random variable

$$\chi_n^2 = z_1^2 + z_2^2 + \cdots + z_n^2$$

χ_n^2 is the chi-square random variable with n degrees of freedom

Table A.3

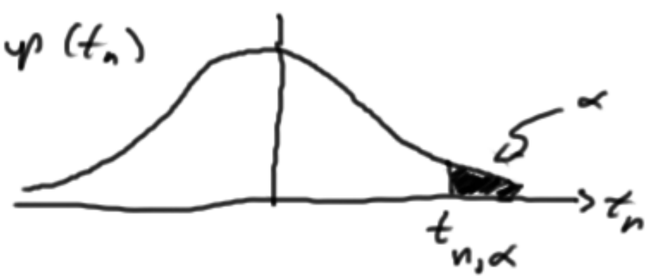
$$\int_{\chi^2_{n,\alpha}}^{\infty} p(\chi_n^2)\, d\chi_n^2 = \text{Prob}\left[\chi_n^2 > \chi^2_{n,\alpha}\right] = \alpha$$

③ Student t Distribution

Let y and z be independent random variables such that y has a χ^2_n distribution and z has a normal distribution with mean $=0$ and variance $=1$.

Define a new random variable

$$t_n = \frac{z}{\sqrt{y/n}}$$ Student t with n degrees of freedom



$$\int_{t_{n,\alpha}}^{\infty} p(t_n)\,dt_n = \mathrm{Prob}\left[t_n > t_{n,\alpha}\right] = \alpha$$

Table A.4

Note: t_n distribution approaches a standardized normal as the number of degrees of freedom becomes large

(4) F Distribution

Let y_1 and y_2 be independent random variables such that y_1 has a χ^2 distribution with n_1 degrees of freedom and y_2 has a χ^2 distribution with n_2 degrees of freedom. Define a new random variable

$$F_{n_1, n_2} = \frac{y_1/n_1}{y_2/n_2}$$

F random variable with n_1 and n_2 degrees of freedom

Table A.5

$$\int_{F_{n_1,n_2,\alpha}}^{\infty} p(F_{n_1,n_2})\, dF_{n_1,n_2} = \text{Prob}\left[F_{n_1,n_2} > F_{n_1,n_2,\alpha}\right] = \alpha$$

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Sampling Distributions

$$\hat{\mu}_x = \bar{x} = \frac{1}{N}\sum_{n=0}^{N-1} x(n) \qquad \hat{\sigma}_x^2 = s^2 = \frac{1}{N-1}\sum_{n=0}^{N-1}\left(x(n)-\bar{x}\right)^2$$

Both $\hat{\mu}_x$ and $\hat{\sigma}_x^2$ are functions of the random sequence $x(n)$ and also are random variables

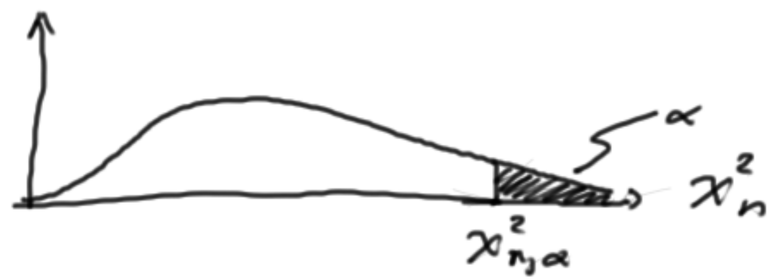
$$z = \frac{\bar{x} - \mu_x}{(\sigma_x^2/N)^{1/2}} \qquad \mathrm{Prob}\left[\bar{x} > \left\{\left(\frac{\sigma_x^2}{N}\right)^{1/2} z_\alpha + \mu_x\right\}\right] = \alpha$$

Standardized normal

$$\chi_n^2 = \frac{n s^2}{\sigma_x^2} \qquad n = N-1$$

$$\mathrm{Prob}\left[s^2 > \frac{\sigma_x^2}{n} \chi^2_{n,\alpha} \right] = \alpha$$

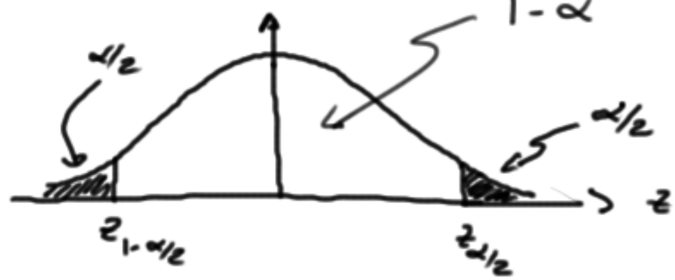
$p(\chi_n^2)$



Confidence Intervals

$$\bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x(n)$$

$$\text{Prob}\left[z_{1-\alpha/2} < \frac{\bar{x} - \mu_x}{(\sigma_x^2/N)^{1/2}} \le z_{\alpha/2} \right] = 1-\alpha$$

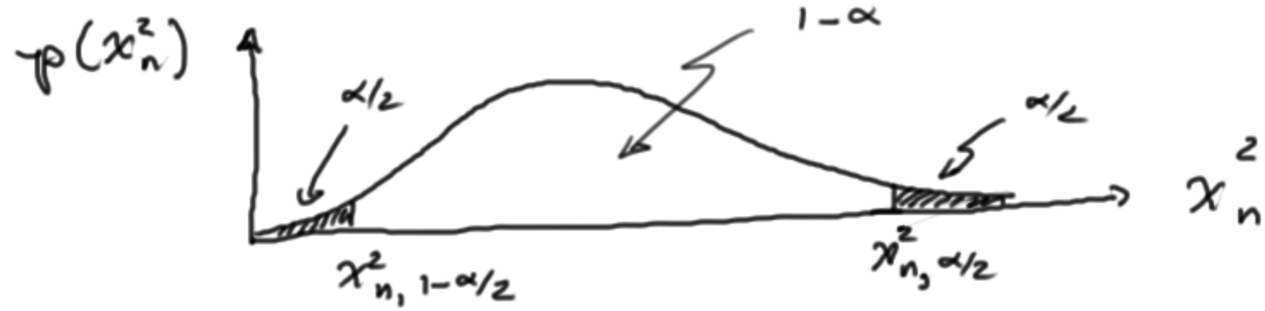


$$\text{Prob}\left[\bar{x} - \left(\frac{\sigma_x^2}{N}\right)^{1/2} z_{\alpha/2} \le \mu_x < \bar{x} + \left(\frac{\sigma_x^2}{N}\right)^{1/2} z_{\alpha/2} \right] = 1-\alpha$$

Confidence Intervals on S^2

$$\chi_n^2 = \frac{n S^2}{\sigma_x^2}$$

$$\text{Prob}\left[\frac{n S^2}{\chi^2_{n,\alpha/2}} \le \sigma_x^2 < \frac{n S^2}{\chi^2_{n,1-\alpha/2}} \right] = 1-\alpha \quad , \quad n = N-1$$



$$\text{Prob}\left[10 \log S^2 + 10 \log\left(\frac{n}{\chi^2_{n,\alpha/2}}\right) \le 10 \log \sigma_x^2 < 10 \log S^2 + 10 \log\left(\frac{n}{\chi^2_{n,1-\alpha/2}}\right) \right]$$

$$= 1-\alpha$$