Coherence and Transfer Function Estimation

Example 6.5. **Physical Illustration of Coherence Measurement.** Consider an airplane flying through a patch of atmospheric turbulence, as illustrated in Figure 6.3. Let the input $x(t)$ be vertical gust velocity in meters/second (m/s) as measured with a probe extending forward of the airplane, and the output $y(t)$ be vertical acceleration in G's measured with an accelerometer at the center of gravity of the airplane. The resulting coherence function and autospectra for actual data of this type are presented in Figures 6.4 and 6.5. In this problem, the spectral data were computed over a frequency range from 0.1 to 4.0 Hz with a resolution bandwidth of 0.05 Hz and a record length of about 10 min.

From Figure 6.4, it is seen that the input gust velocity and output airplane acceleration display a relatively strong coherence of 0.8 to 0.9 over the frequency range from about 0.4 to about 2.0 Hz. Below and above this range, however, the coherence function diminishes. At the lower frequencies, the vertical acceleration of the airplane is increasingly due to maneuver loads induced through the control system by the pilot, rather than due to atmospheric turbulence loads. Hence, the loss of coherence at the lower frequencies probably reflects contributions to the output $y(t)$ from inputs other than the measured input $x(t)$. At the higher frequencies, the low-pass filtering characteristics of the airplane response plus the decaying nature of the input autospectrum cause the output autospectrum to fall off sharply, as indicated in Figure 6.5. On the other hand, the noise floor for the data acquisition and recording equipment generally does not fall off with increasing frequency. Hence, the diminishing coherence at the higher frequencies probably results from the contributions of extraneous measurement noise. This concludes Example 6.5.

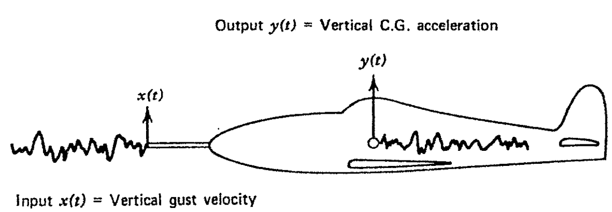


Figure 6.3 Airplane flying through atmospheric turbulence.

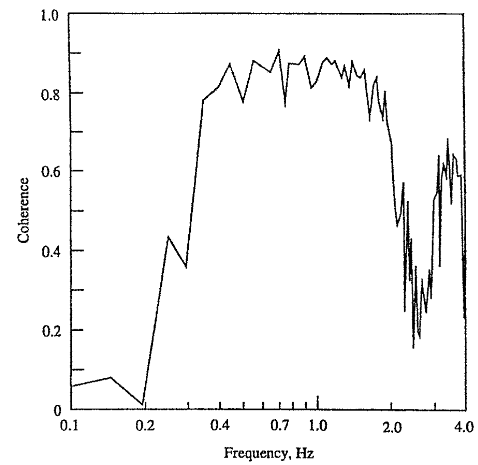


Figure 6.4 Coherence function between gust velocity and response acceleration of airplane. These data resulted from studies funded by the NASA Langley Research Center, Hampton, Virginia, under Contract NAS 1-8538.

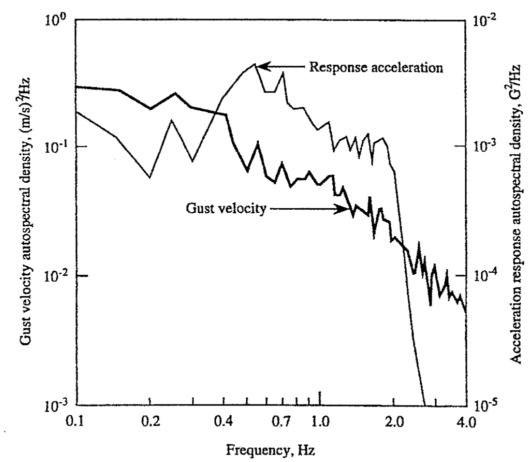


Figure 6.5 Autospectra of gust velocity and response acceleration of airplane. These data resulted from studies funded by the NASA Langley Research Center, Hampton, Virginia, under Contract NAS 1-8538.

J. Bendat and A. Piersol. Random Data: Analysis and Measurement Procedures (3rd Ed). Wiley (2000).

Coherence and Transfer Function Estimation

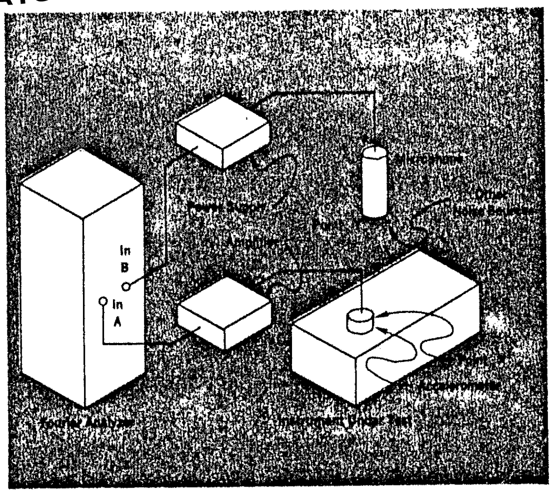


Figure 1—Test setup to determine sources of noise monitored above an electronic instrument case.

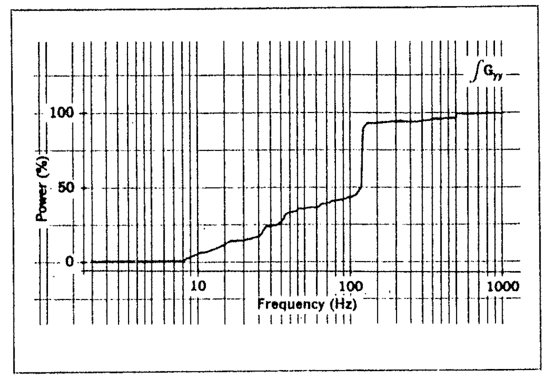


Figure 3—Integral of microphone power spectrum normalized to 100%.

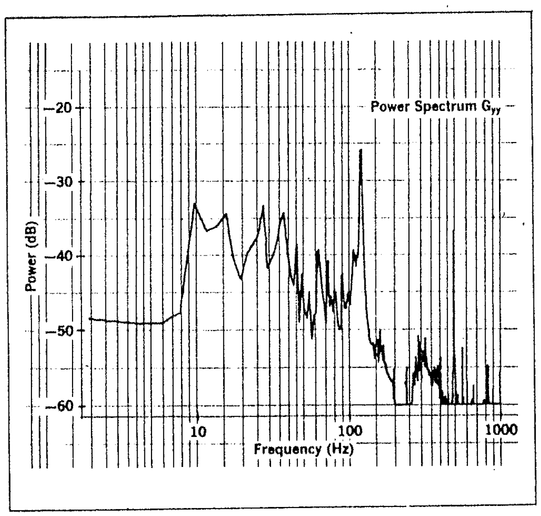


Figure 2—Power spectrum of the noise measured by the microphone shown in Figure 1 (point Y).

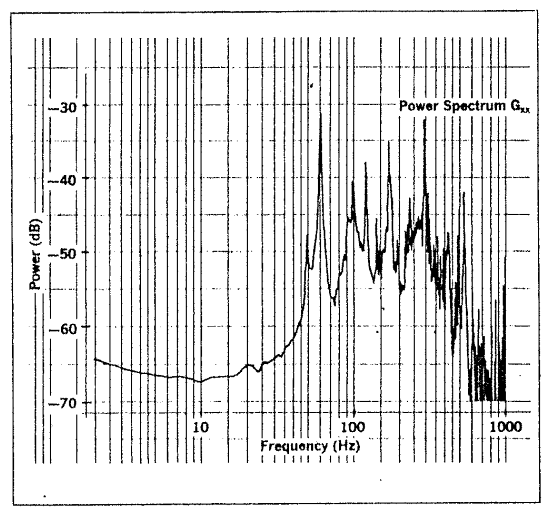


Figure 4—Power spectrum of the vibration measured by the accelerometer shown in Figure 1 (point X).

Coherence and Transfer Function Estimation

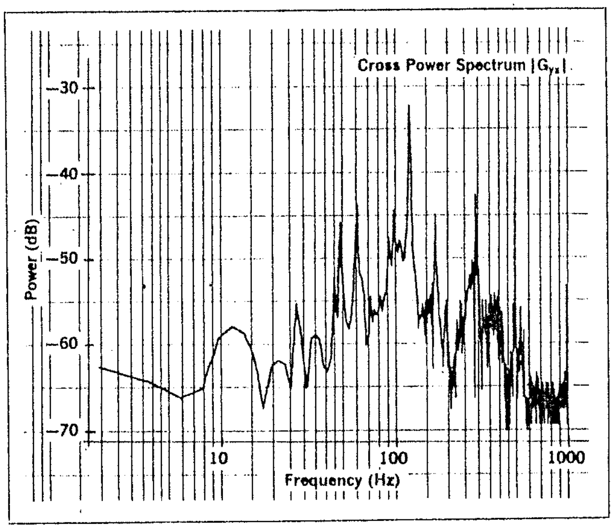


Figure 5—Cross power spectrum of the microphone (Y) and accelerometer (X) signals. The cross spectrum shows the components that are common to X and Y but does not indicate the degree to which the monitored noise is caused by the panel vibration.

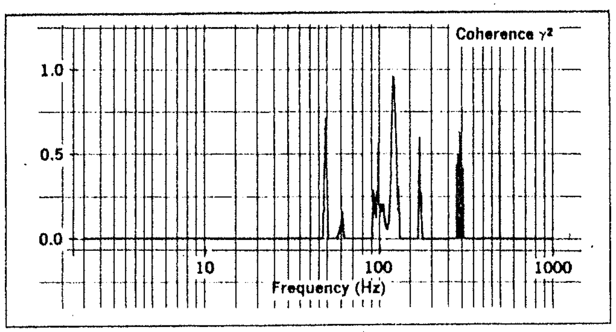


Figure 6—Coherence function of the microphone signal to the accelerometer signal. The coherence function shows the fraction of power at the microphone (Y) which is coherent with the assumed source (X).

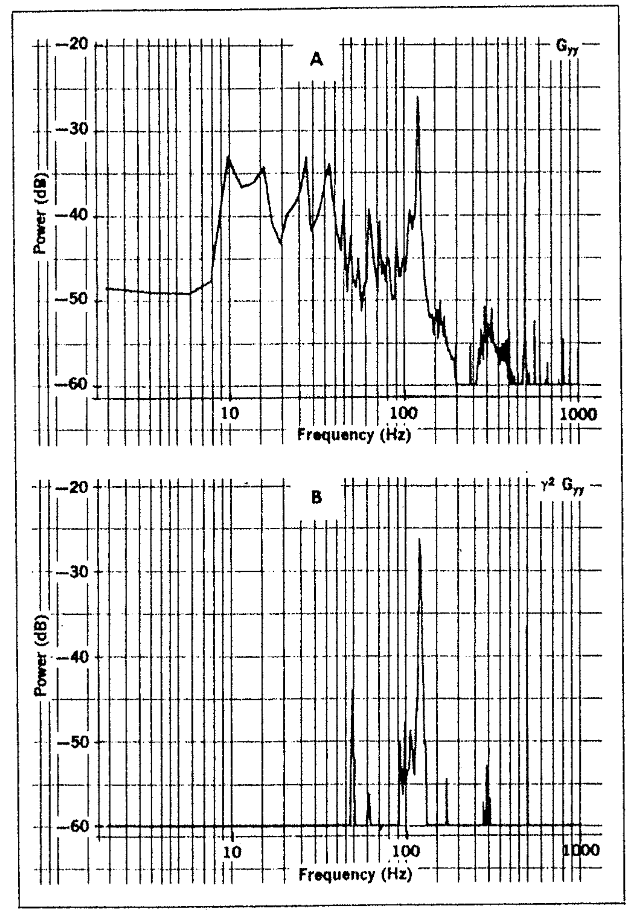


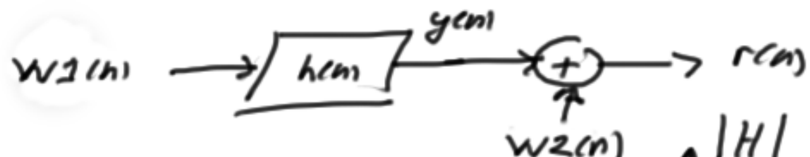
Figure 7—The noise spectrum of the test item is shown above as originally measured (A) and as corrected by the coherence function (B).

P. Roth, "How to use the spectrum and coherence function," Sound and Vibration: 10-14 (Jan. 1971).

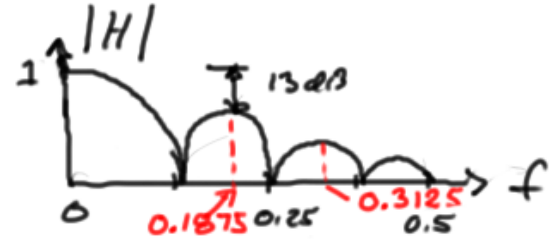
Coherence and Transfer Function Estimation

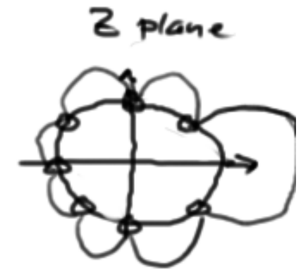
Transfer Function and Coherence Function HW

Note In HW, r(n) is used to represent the measurement time series and y(n) to represent the output of the linear system



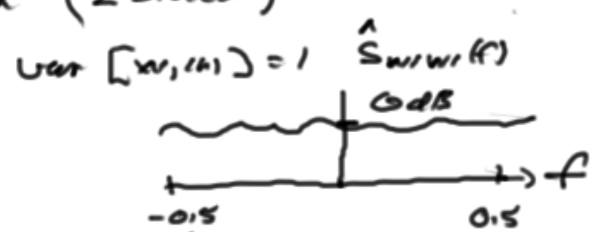
A.

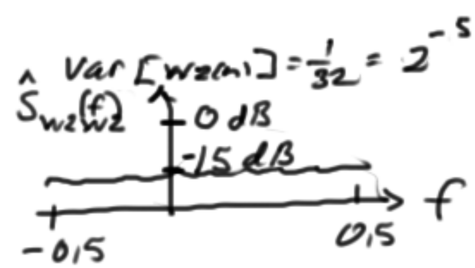


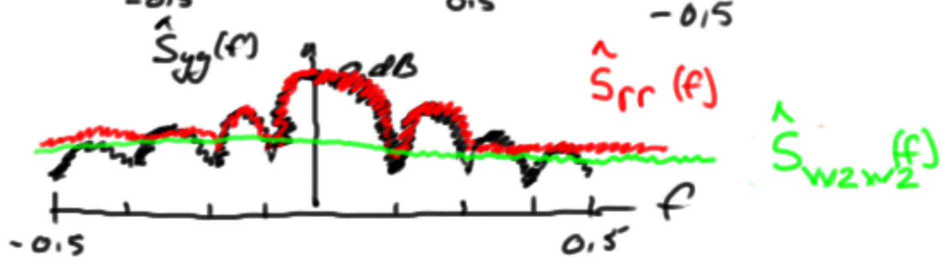


B. Power Spectra (2-sided)

$$\hat{P}_{xx}(f) = \frac{1}{f_s M U} |X(k)|^2$$







MARINE PHYSICAL LABORATORY

Coherence and Transfer Function Estimation

N. Pendergrass, "Coherence function and averaging boost confidence in spectrum measurements," Electronics: 132-137 (14 Sept. 1978).

TABLE 1: 90% CONFIDENCE LIMITS FOR RMS AVERAGING

	Number of averages						
	4	8	16	32	64	128	256
Upper limit(dB)	+ 4.7	+ 3.0	+ 2.0	+ 1.4	+ 1.0	+ 0.7	+ 0.5
Lower limit(dB)	− 2.9	− 2.2	− 1.6	− 1.2	− 0.8	− 0.6	− 0.4

TABLE 2: 90% CONFIDENCE LIMITS ON THE MEASUREMENT OF THE AMPLITUDE |H| AND PHASE ϕ OF TRANSFER FUNCTIONS

Measured value of coherence function		Number of averages				
		16	32	64	128	256
0.2	|H| (dB)	+ 5.2, − 14.6	+ 3.8, − 7.1	+ 2.8, − 4.2	+ 2.1, − 2.7	+ 1.5, − 1.8
	ϕ (°)	± 54	± 34	± 23	± 16	± 11
0.3	|H| (dB)	+ 4.2, − 8.4	+ 3.1, − 4.8	+ 2.2, − 3.0	+ 1.6, − 2.0	+ 1.2, − 1.4
	ϕ (°)	± 38	± 25	± 17	± 12	± 8
0.4	|H| (dB)	+ 3.5, − 6.0	+ 2.6, − 3.6	+ 1.8, − 2.3	+ 1.3, − 1.6	+ 1.0, − 1.1
	ϕ (°)	± 30	± 20	± 14	± 10	± 7
0.5	|H| (dB)	+ 3.0, − 4.5	+ 2.1, − 2.8	+ 1.5, − 1.9	+ 1.1, − 1.3	+ 0.8, − 0.9
	ϕ (°)	± 24	±16	± 11	± 8	± 5
0.6	|H| (dB)	+ 2.5, − 3.5	+ 1.8, − 2.2	+ 1.3, − 1.5	+ 0.9, − 1.0	+ 0.7, − 0.7
	ϕ (°)	± 19	± 13	± 9	± 6	± 4
0.7	|H| (dB)	+ 2.1, − 2.7	+ 1.5, − 1.7	+ 1.0, − 1.2	+ 0.7, − 0.8	+ 0.5, − 0.6
	ϕ (°)	± 15	± 10	± 7	± 5	± 4
0.8	|H| (dB)	+ 1.6, − 2.0	+ 1.1, − 1.3	+ 0.8, − 0.9	+ 0.6, − 0.6	+ 0.4, − 0.4
	ϕ (°)	± 12	± 8	± 6	± 4	± 3
0.9	|H| (dB)	+ 1.1, − 1.3	+ 0.8, − 0.8	+ 0.5, − 0.6	+ 0.4, − 0.4	+ 0.3, − 0.3
	ϕ (°)	± 8	± 5	± 4	± 3	± 2

TABLE 3: 90% CONFIDENCE LIMITS ON COHERENCE FUNCTION MEASUREMENTS (MIN, MAX LIMITS)

Measured value of coherence function	Number of averages				
	16	32	64	128	256
0.4	0.15, 0.59	0.23, 0.54	0.28, 0.50	0.32, 0.47	0.34, 0.45
0.5	0.25, 0.67	0.33, 0.63	0.39, 0.59	0.42, 0.57	0.45, 0.55
0.6	0.36, 0.74	0.45, 0.71	0.50, 0.68	0.53, 0.66	0.55, 0.64
0.7	0.50, 0.81	0.57, 0.78	0.61, 0.76	0.64, 0.75	0.66, 0.73
0.8	0.65, 0.88	0.70, 0.86	0.74, 0.84	0.76, 0.83	0.77, 0.82
0.9	0.81, 0.94	0.85, 0.93	0.87, 0.92	0.88, 0.92	0.88, 0.91

Coherence and Transfer Function Estimation

Confidence intervals - see table posted to TED

Refs. Table

N.A. Pendergrass, "Coherence function and averaging boost confidence in spectrum measurements," Electronics, pp. 132-137 (14 Sept 1978)

J. Bendat and A. Piersol (1993)

J. Bendat and A. Piersol (2000)