Spectral Averaging

The purpose of the following assignment is to investigate the effect of segmenting a time series on estimation of the power spectrum.

Generate a 1024-point time series consisting of a bin centered sinusoid (i.e. $f_k = (1/N)\,k$ cycles/sample for $-N/2 \le k < N/2$) for a FFT of length NFFT = 128 (e.g. f = 0.125 cycles/sample) plus Gaussian white noise with:

$$\mathrm{SNR} = 10\log\frac{A^2}{2\sigma^2} = 0\,\mathrm{dB}.$$

Use $\sigma^2 = 1$ and calculate the appropriate value for A.

A. Plot the resulting time series for N = 128.

B. Using a good window with your FFT (i.e. Hamming, Kaiser-Bessel, etc.), plot the properly normalized power spectrum estimate (dB) for N = NFFT equal to: (1) 128, (2) 256, (3) 512, and (4) 1024.

C. Segment the N = 1024 time series into K records, each of length M = 128. As in B (above), compute the power spectrum estimate for each record, then average the K spectra together and plot the properly normalized smoothed power spectrum estimate (dB) for the following cases:

   1. 0% overlap of records (K=8)
   2. 50% overlap of records (K=15)
   3. 75% overlap of records (K=29)

   Utilize the full dynamic range of your vertical axis to clearly see the effects of averaging on the character of the noise alone regions of the power spectrum. Compare B(1) (no averaging or K=1) and C(1) (K=8). What is the effect of overlapping the data in C(2) and C(3) compared to C(1)?

D. Repeat C(1) using a *coherent* average of both the time series segments and the K-FFT's. Plot the resulting averaged time series, its power spectrum estimate (dB), and the power spectrum estimate (dB) of the coherently averaged FFT's. Comment on what would have happened if you had chosen your sinusoid to be not centered in a FFT bin.

Note Proper normalization of the power spectrum is requested so do not just normalize to 0 dB. Use the appropriate normalization from the power spectrum calculation $(f_s MU)^{-1}$ but note that for this synthetic data case that $f_s = 1$. Recognize that this normalization is appropriate for 2-sided power spectra (positive and negative frequencies).