8.64. A problem that often arises in practice is one in which a distorted signal $y[n]$ is the output that results when a desired signal $x[n]$ has been filtered by an LTI system. We wish to recover the original signal $x[n]$ by processing $y[n]$. In theory, $x[n]$ can be recovered from $y[n]$ by passing $y[n]$ through an inverse filter having a system function equal to the reciprocal of the system function of the distorting filter.

Suppose that the distortion is caused by an FIR filter with impulse response

$$h[n] = \delta[n] - \frac{1}{2}\delta[n - n_0],$$

where n_0 is a positive integer, i.e., the distortion of $x[n]$ takes the form of an echo at delay n_0.

(a) Determine the z-transform $H(z)$ and the N-point DFT $H[k]$ of the impulse response $h[n]$. Assume that $N = 4n_0$.

(b) Let $H_i(z)$ denote the system function of the inverse filter, and let $h_i[n]$ be the corresponding impulse response. Determine $h_i[n]$. Is this an FIR or an IIR filter? What is the duration of $h_i[n]$?

(c) Suppose that we use an FIR filter of length N in an attempt to implement the inverse filter, and let the N-point DFT of the FIR filter be

$$G[k] = 1/H[k], \qquad k = 0, 1, \ldots, N-1.$$

What is the impulse response $g[n]$ of the FIR filter?

(d) It might appear that the FIR filter with DFT $G[k] = 1/H[k]$ implements the inverse filter perfectly. After all, one might argue that the FIR distorting filter has an N-point DFT $H[k]$ and the FIR filter in cascade has an N-point DFT $G[k] = 1/H[k]$, and since $G[k]H[k] = 1$ for all k, we have implemented an all-pass, nondistorting filter. Briefly explain the fallacy in this argument.

(e) Perform the convolution of $g[n]$ with $h[n]$, and thus determine how well the FIR filter with N-point DFT $G[k] = 1/H[k]$ implements the inverse filter.

A. Oppenheim, R. Schafer, and J. Buck. *Discrete-Time Signal Processing*, 2nd *Ed.* Prentice-Hall, (1999). ISBN 0-13-754920-2.