Statistical Tests on Time Series

A. Generate the following N = 1024 point time series:

1. Gaussian white noise with E[x(n)]=0 and var[x(n)] = 1.
2. Add to (A1) a sinusoid $\left(\omega = \frac{\pi}{16}\right)$ such that 10 log (SNR) = 10 log $\left(\frac{A^2}{2\sigma^2}\right)$ = 0 dB.
3. Repeat (A2) with 10 log (SNR) = 9 dB.
4. Gaussian white noise with E[x(n)]=0 and

   var[x(n)] = 1, n = 0, ..., 511
   var[x(n)] = 4, n = 512, ..., 1023.

B. For each of the four time series in A, plot:

1. The N-length time series. Indicate on the plot or figure caption the estimated mean, variance, and standard deviation of the time series.
2. The corresponding histogram (probability density function estimate). Superimpose on the histogram a normal density with mean and variance as estimated in (B1).
3. The smoothed estimate of the power spectrum obtained by averaging K = 15, M = 128-length periodograms with 50% overlap.

C. For each of the four time series in A, apply the following tests:

1. Runs test for stationarity of the sample mean-square estimates (see [1] Examples 4.4 and 7.2).

   Use $\hat{E}_i\left[x^2(n)\right] = \frac{1}{10}\sum_{m=0}^{9} x^2(m+10i)$, i=0..., 99 and

   $$\hat{E}_i\left[x^2(n)\right] \underset{\text{"}-\text{"}}{\overset{\text{"}+\text{"}}{\gtrless}} \hat{E}_i\left[x^2(n)\right]_{median}$$

   A plot of $\hat{E}_i[x^2(n)]$ is interesting. Include a dotted line indicating the median value. Use α=0.05 and note the exact threshold values used for the test statistic.

2. Chi-square goodness of fit test for normality (see [1], Example 4.3). Apply the chi-square test *only* to those time series which the test in (C1) indicates are stationary. Use α=0.05 and note the exact threshold value used for the test statistic. Explain how the chi-square test was implemented. Generate a table similar to that in Example 4.3 [1] to document your results. For N ~ 1000, the number of class intervals K ~ 30.

D. Comment on your results.

References:

[1] J. Bendat and A. Piersol. Random Data: Analysis and Measurement Procedures. NY: Wiley, 1971.

[2] W. Press et. al. Numerical Recipes. Cambridge Univ. Press, 1986.