Quant II

Lab 10: Multiple Testing, Missing Data

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Outline

- Multiple Testing
  - Multiple hypothesis
  - Summary Index
- Missing Data
  - Classical Approach
  - Modern Approach

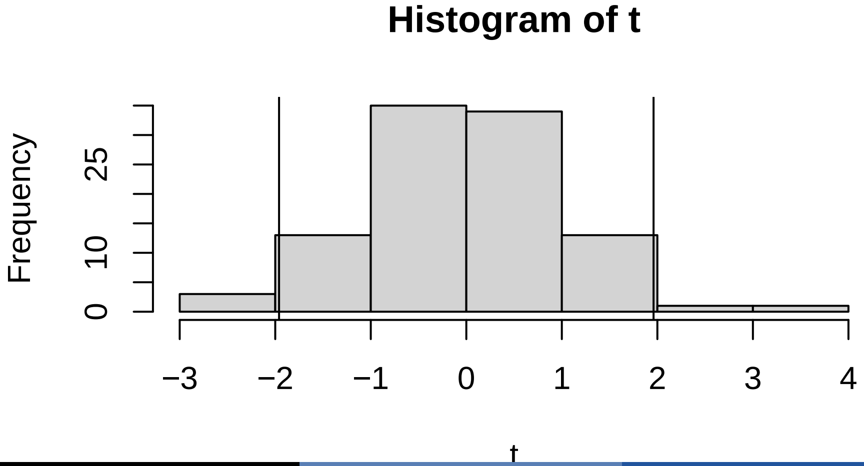
Multiple Hypothesis

- There is a positive probability that you can reject the true null.

```r
B <- 100
t <- sapply(1:B, function(k){
  x <- rnorm(1000)
  y <- rnorm(1000)
  m <- lm(y~x)
  return(lmtest::coeftest(m)[2,3])
})
```

Multiple Hypothesis

```
hist(t)
abline(v=1.96)
abline(v=-1.96)
```



Adjust the p-value

- As discussed in class, we want to adjust the p-value to control the FWER and FDR
- Suppose we have 10 hypothesis.

```
p <- sort(runif(10))
data.frame(p=p,bonferroni=p.adjust(p, method = "bonferroni"),
           fdr=p.adjust(p, method = "fdr"))
```

```
##              p bonferroni      fdr
## 1  0.1270730          1 0.915254
## 2  0.2044053          1 0.915254
## 3  0.2745762          1 0.915254
## 4  0.3957679          1 0.945976
## 5  0.6635761          1 0.945976
## 6  0.6757254          1 0.945976
## 7  0.8855755          1 0.945976
## 8  0.8862800          1 0.945976
```

Adjust the p-value

- This adjustment can be conservative.
- Use bootstrap step-down method discussed in class.

```
# if (!requireNamespace("BiocManager", quietly = TRUE))
#     install.packages("BiocManager")
# BiocManager::install("multtest")
library(multtest)
N <- 1000
x <- matrix(rnorm(N*10),ncol = 10)
beta <- seq(0,2,length.out = 10)
y <- rnorm(N, mean=x%*%beta,sd=1)
m <- MTP(X=t(x),Y=y, B=10000, seed = 1234, test = "lm.YvsXZ",
         typeone = "fwer")
```

```
## running bootstrap...
## iteration = 100 200 300 400 500 600 700 800 900 1000 1100 1
```

Adjust the p-value

```
m@adjp
```

```
##  [1] 1.0000 0.2646 0.0037 0.0000 0.0000 0.0000 0.0000 0.000
```

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- We may want a summary index to capture the ability and to give us an interpretable result.
- We discussed inverse covariance weighting from Anderson (2008) in class.
- The measurement problem is an important topic in social science.

Measuring populism (Monkey Cage, Oct 19, 2017)

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- We asked respondents whether they agreed or disagreed with nine statements. Populists should agree that:
  - “Politicians need to follow the will of the people”
  - “The people, and not politicians, should make our most important policy decisions”
  - ...
  - “Regardless of the party in power, all governments just favor the bigwigs.”
- Populists should disagree that:
  - “Politicians care about what people like me think”
  - ...
  - “Those we elect to public office usually try to keep the promises they made during the election”
- We employed a strict definition of populist: Someone who answered all nine questions in the populist direction. We found that 17 percent of Americans are “populists” according to this measure.

Measurement in social sciences

- Multiple measurements (observables) are used to capture the quantity of interest (latent variable).
- How do we approach this problem in a principled way?
  - Aggregation of measurements (dimension reduction)
  - Measurement error
  - Relating latent variables to other observables

Types of latent variable

Observed	Latent	
	Continuous	Categegorical
Continuous	Factor analysis	Latent profiling / mixture models (`mclust`)
Categegorical	Latent traits, IRT (`pscl`)	Latent class analysis (`poLCA`, `BayesLCA`)

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- Suppose you have three variables: College math grade, math GRE, and verbal GRE.
- ICW will give 25% weights to each of the math scores and 50% to the verbal score.
- PCA will generate two (or more) new variables, one for mathematical capability and the other for linguistic capability.
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- Which one is more proper depends on the context.
- Do we want a latent variable to capture the “ability” or other?

Item Response Theory (IRT)

- Say we want to estimate the ideology:
- A spatial model of voting behavior.
- Estimate model from data (structural estimation).
- Item Response Theory (IRT): for $i = 1, ..., N$ legislators (students) you observe $j = 1, ..., n$ binary votes (test answers).

$$y_{ij} \sim Bernoulli(\pi_{ij}), \tag{1}$$
$$\pi_{ij} = \Phi(\beta_j \theta_i - \alpha_j) \tag{2}$$

- θ_i: ideal point (ability)
- α_j: item difficulty
- β_j: item discrimination

Principle Component Analysis (PCA)

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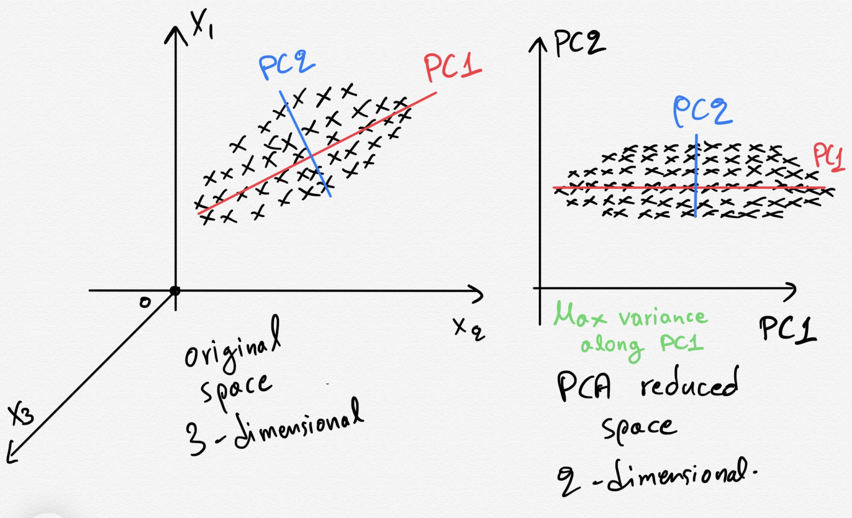
- Idea: extract more information from X as possible.
- Suppose we have $x_1, \dots, x_M$, M features.
- We define the generated $y_1, \dots, y_K$ are K principle component.
- The first principle component explains the most variation, and then the second, etc.
- the principle component are orthogonal to each other.

Principle Component Analysis (PCA)

- Specifically, $y_k = \sum_{j \in \{1,...M\}} \alpha_{j,k} x_j$
- One can show that $\{y_k\}$ is the principle component and:
  - it explains λ_k variation of X, where λ_k is k^{th} large eigen-value of X
  - α_k is k^{th} of X's eigen-vector associated with λ_k.

Principle Component Analysis (PCA)

- As one may know, any linear transformation is a coordinate transformation.



Principle Component Analysis (PCA)

- Easy to implement it in R
- Note we also need to standardize the data.

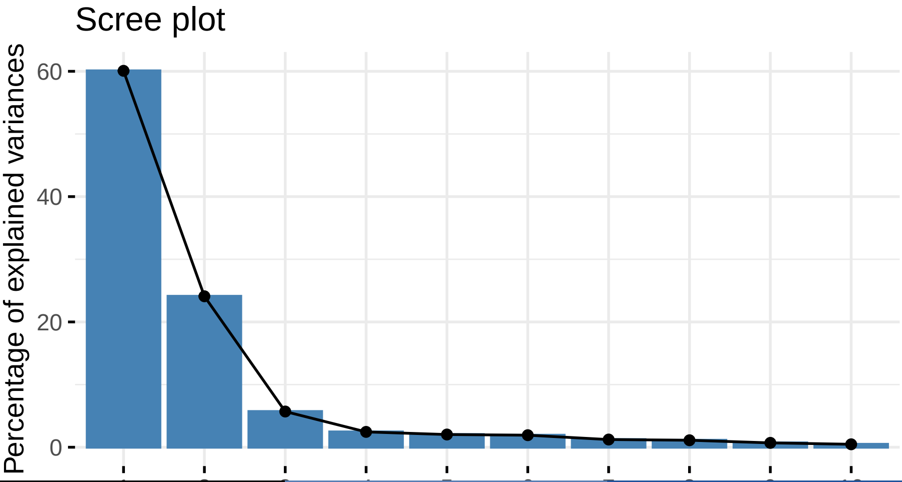
```
summary(pc.cr <- princomp(mtcars, cor = TRUE))
```

```
## Importance of components:
##                           Comp.1    Comp.2     Comp.3     C
## Standard deviation     2.5706809 1.6280258 0.79195787 0.519
## Proportion of Variance 0.6007637 0.2409516 0.05701793 0.024
## Cumulative Proportion  0.6007637 0.8417153 0.89873322 0.923
##                            Comp.6     Comp.7     Comp.8
## Standard deviation     0.45999578 0.36777981 0.35057301 0.2
## Proportion of Variance 0.01923601 0.01229654 0.01117286 0.0
## Cumulative Proportion  0.96279183 0.97508837 0.98626123 0.9
##                             Comp.11
## Standard deviation     0.148473587
## Proportion of Variance 0.002004037
```

Principle Component Analysis (PCA)

```
## Loading required package: ggplot2

## Welcome! Want to learn more? See two factoextra-related boo
```



Missing Data

- Estimation for bounds
  - Manski Bound
  - Lee Bound
- Point estimation
  - Missing data imputation

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Bound Estimation

- We put the least assumption on missing data.
- For example: Lee bound.
  - Assume monotonicity: $S_{1i} \geq S_{0i}, \forall i$ or the other direction.
- Control group: only observe one strata $(S_1 = S_0 = 1)$
- Treated group: observe two strata ($S_1 = S_0 = 1$, and $\{S_1 = 1, S_0 = 0\}$)
- We can use the treated group to construct the bound for $S_1 = S_0 = 1$ people.

Lee Bound

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- Use X to predict the principle strata.
- e.g. educated people join labor market anyway, but low educated people only join market when they get special training.
- we can use education level to construct a tighten bound.

Beyond Lee Bound

- Traditional Lee bound only allows few discrete variables.
- because we need to get two quantities
  - $q = Pr(S_1 = S_0 = 1)$
  - $E(Y|D = 1, y \geq y_q(1))$
- conditional on X causes curse of dimensionality.

Beyond Lee Bound

- Traditional Lee bound only allows few discrete variables.
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- conditional on X causes curse of dimensionality.

- Semenova (2021) proposes use LASSO to construct a better lee bound
- Olma (2021) uses kernel method.
- Samii, Wang, Zhou try to use adaptive kernel to incorporate high-dimensional covariate cases.

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- We need to impute the missing value.
- Missing Completely At Random (MCAR): We are fine.
  - missing is random assigned.
- Missing At Random (MAR): use X to impute the missing values. $P(Missing|X_{obs}) = P(Missing|X_{complete})$

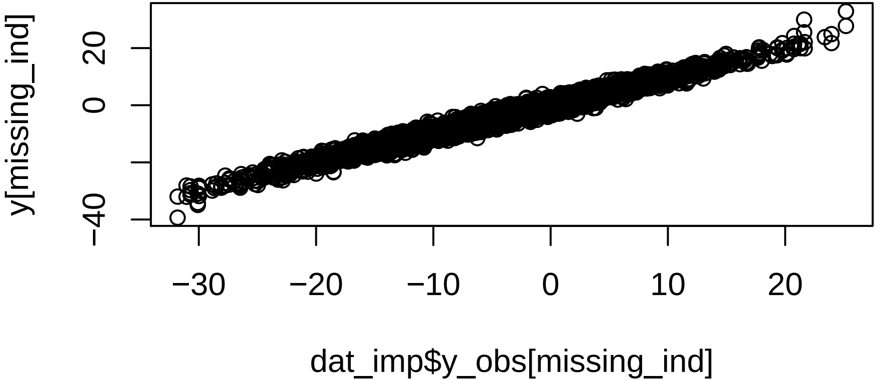
Multiple Imputation using Mice

- Note: multiple imputation is fine when we have enough data.

```
n <- 10000
m <- 10
set.seed(1234)
x <- matrix(rnorm(n*m), nrow=n)
x <- cbind(1,x)
beta <- rnorm(m+1, mean=0, sd=3)
y <- rnorm(n, mean=x%*%beta)
beta_missing <- rnorm(m+1, mean=0, sd=3)
missing <- rbinom(n,size=1, prob= 1/(1+exp(x%*%beta_missing)))
missing_ind <- which(missing==1)
y_obs <- y
y_obs[missing_ind] <- NA
```

Multiple Imputation using Mice

```
dat <- as.data.frame(cbind(y_obs,x))
library(mice)
m1 <- mice(dat, maxit = 100, printFlag = FALSE)
dat_imp <- complete(m1)
plot(y[missing_ind]~ dat_imp$y_obs[missing_ind])
```



Multiple Imputation using Mice

```r
# True beta
beta
```

```
##  [1] -4.2477948  0.9523207  2.1844025 -5.6986484 -3.433510
##  [7]  0.5949779  3.2236872  1.8769698 -2.0990845 -1.189486
```

```r
# with observed data
coef(lm(y[-missing_ind] ~ x[-missing_ind,]))
```

```
##        (Intercept)  x[-missing_ind, ]1  x[-missing_ind, ]
##         -4.2262261                  NA          0.933153
## x[-missing_ind, ]4  x[-missing_ind, ]5  x[-missing_ind, ]
##         -5.7097415          -3.4336913          1.730197
## x[-missing_ind, ]8  x[-missing_ind, ]9 x[-missing_ind, ]1
##          3.2289622           1.8334263         -2.111665
```

```r
# with Imputed data
summary(with(m1,lm(y~x)))
```

Not MAR?

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Not MAR?

- Consider we care about people's attitude towards Trump
- People may refuse to report
- This "not report" is correlated with their underlying attitude towars trump
- Missing is not random.
- $P(Missing|X_{obs}) \neq P(Missing|X_{complete})$
- Liu (2020) proposes a latent factor model a missing not at random case.

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- We can always apply what we learn from basic causal inference to missing data problem
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  - etc.
- Deal with missing data carefully.

Remarks


NOT UNDERSTANDING Your
MISSING DATA
Can
RUIN
the
FOUNDATION
of Your ANALYSIS