TSCS Data Analysis

Ye Wang

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- We then introduce several approaches to fix the problems, such as counterfactual estimation.
- We next review available methods under sequential ignorability.
- We conclude by discussing the idea of manifold learning and how to deal with interference in TSCS data (hopefully!).

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- Now we can relax the assumption along two possible directions (based on your belief).

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- Strict exogeneity: $D_{it} \perp Y_{is}(D_{is}) | \mathbf{X}_i^{1:t}, \mathbf{U}_i^{1:t}$, where $\mathbf{X}_i^{1:t}$ the history of observable confounders and $\mathbf{U}_i^{1:t}$ the history of unobservable confounders.

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- Intuitively, their values do not hinge on the value of the outcome in history.
- We always assume that SUTVA holds and there is no reversal causality (future treatments won't affect past outcomes).

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- You will be vaccinated on Feb. 15 with probability 0.72 if you are an old Asian male who loves tequila and that day's temperature is above zero.
- The dataset available to the analyst only includes each unit's observable attributes, health status, treatment status, and the vaccination date.

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- All the classic methods (regression, weighting, matching, etc.) still apply with some modifications.
- Under strict exogeneity, we have the problem of omitted variables as part of the confounders ($\mathbf{U}_i^{1:T}$) is also unknown.
- To eliminate the influence of the unobservables, we need stronger assumptions on the DGP, such that the values of $\mathbf{U}_i^{1:T}$ can be learned from other variables.

Estimation under strict exogeneity

- Note that the strict exogeneity assumption is satisfied by the following outcome model:

$$Y_{it} = g_{it}(D_{it}, \mathbf{X}_i^{1:t}, \mathbf{U}_i^{1:t}) + \varepsilon_{it}$$
$$\mathbb{E}[\varepsilon_{is} | D_{it}, \mathbf{X}_{it}, \mathbf{U}_{it}] = 0,$$

which is too general for identification.

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- For example, we can assume that $h_{it}(\mathbf{U}_{it}) = \mu + \alpha_i + \xi_t$, then
$$Y_{it} = \mu + \delta_{it} D_{it} + \mathbf{X}_{it}\beta + \alpha_i + \xi_t + \varepsilon_{it}.$$
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- Now, the assumption of strict exogeneity becomes: $\mathbb{E}[\varepsilon_{is} | D_{it}, \mathbf{X}_{it}, \alpha_i, \xi_t] = 0$ for any s.

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Estimation under strict exogeneity

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- We no longer have consistent estimates if treatment effects are heterogeneous.
- There are several solutions to the problem.
- It is easier to fix the problem under the DID setting (Strezhnev, 2017).
- Otherwise, we need to modify the estimand or impose stronger assumptions.

DIDM

- Chaisemartin and D'Haultfœuille (2020) advocate that we should focus only on the instant effects generated by the treatment and estimate

$$\frac{1}{N^*} \sum_{t \geq 2, D_{it} \neq D_{i,t-1}} \delta_{it}$$

where $N^* = \sum_{i=1}^{N} \sum_{t=1}^{T} \mathbf{1}\{t \geq 2, D_{it} \neq D_{i,t-1}\}$.

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- This estimand is well-defined and can be consistently estimated.
- Yet it throws away a ton of information as well thus is not very efficient.
- There are more efficient approaches which require stronger assumptions on model specification.

Counterfactual estimation

- Xu (2017) and Liu, Wang, and Xu (2020) combine factor models with the Neyman-Rubin framework

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- The basic idea is to use untreated observations ($(i, t) \in \mathcal{O}$) to fit a factor model and employ the model to predict $Y_{it}(0)$ for each treated observation ($(i, t) \in \mathcal{M}$).
- Clearly, $\hat{\delta}_{it} = Y_{it} - \hat{Y}_{it}(0)$ and

$$\widehat{ATT} = \frac{1}{|\mathcal{M}|} \sum_{(i,t) \in \mathcal{M}} \hat{\delta}_{it}$$

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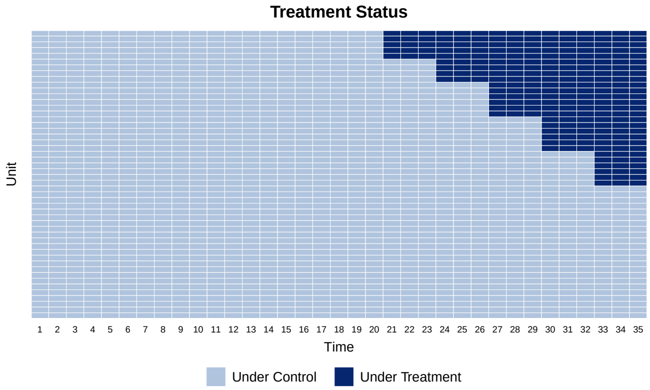

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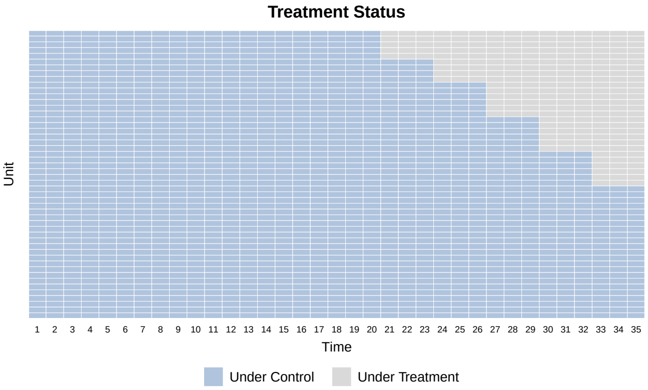

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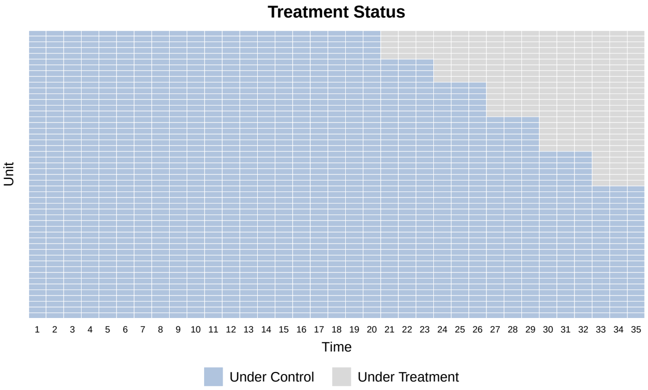

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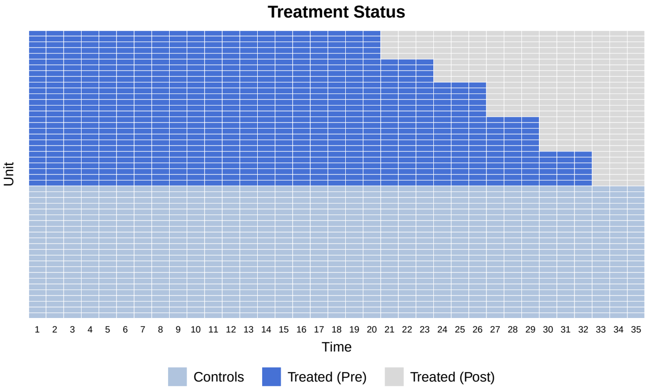

Counterfactual estimation

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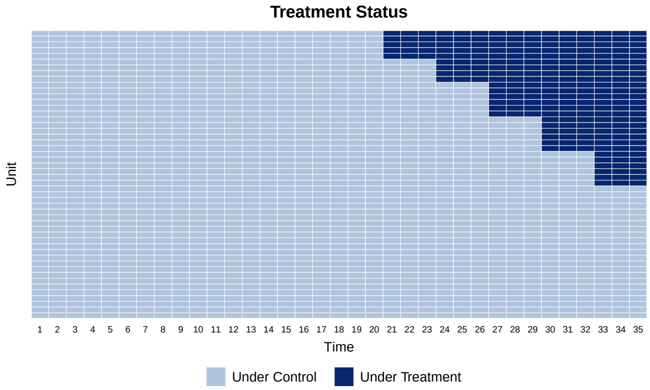
- Estimate ATT by averaging differences between $Y(1)$ and $\hat{Y}(0)$



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  - Observable confounders affect the outcome in a linear and homogeneous manneer.
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- It also requires strict exogeneity, thus excludes temporal interference.

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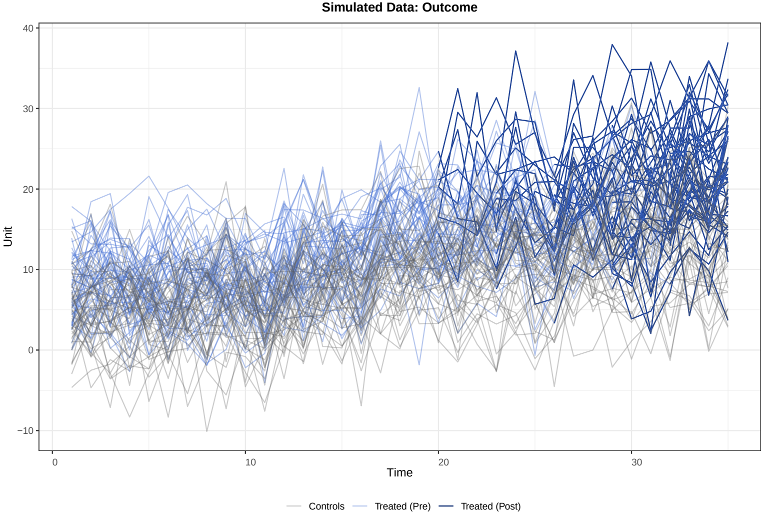
- Liu, Wang, and Xu (2020) provides a set of tools for practitioners to evaluate the identification assumption.
- A plot of dynamic effects.
- A placebo test: estimate treatment effects before the treatment's onset and test their significance.
- An equivalence test: test whether all the pre-treatment ATTs are equal to zero using TOST.

Example

- Let's consider a DGP with two factors (hence FEct will be biased).



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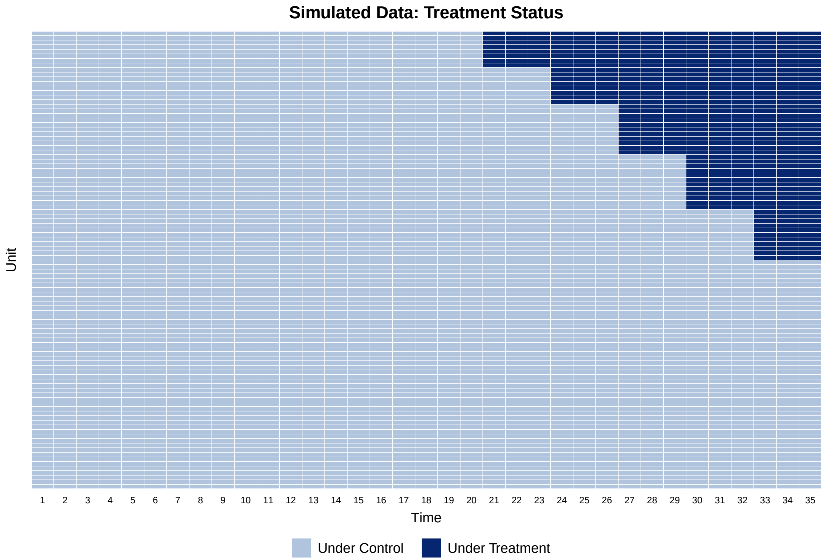
Simulated Data: Treatment Status
Unit
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35
Time
Under Control
Under Treatment

Example

```
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```

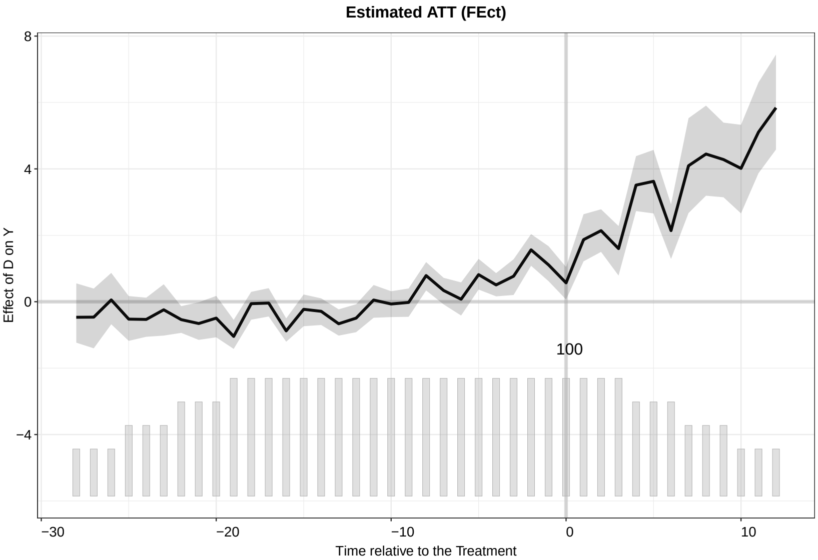

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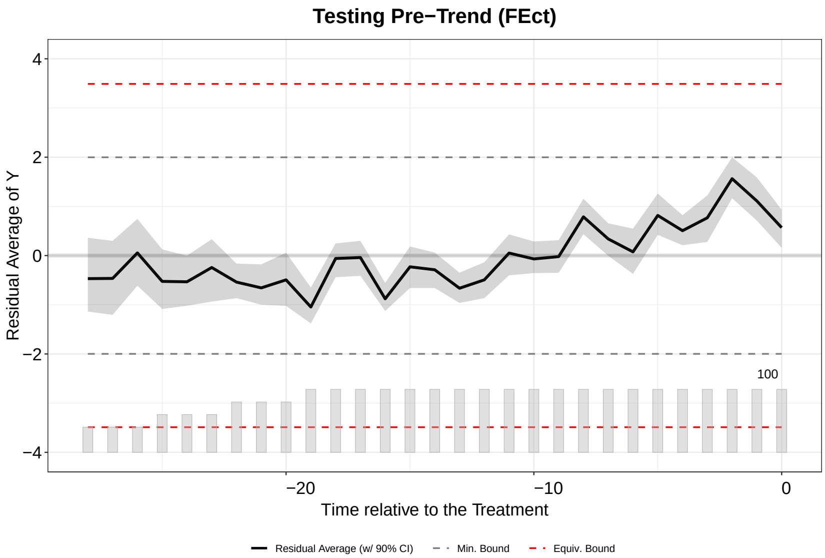

Testing Pre−Trend (FEct)
4
2
0
−2
−4
Residual Average of Y
100
−20
−10
0
Time relative to the Treatment
Residual Average (w/ 90% CI)
Min. Bound
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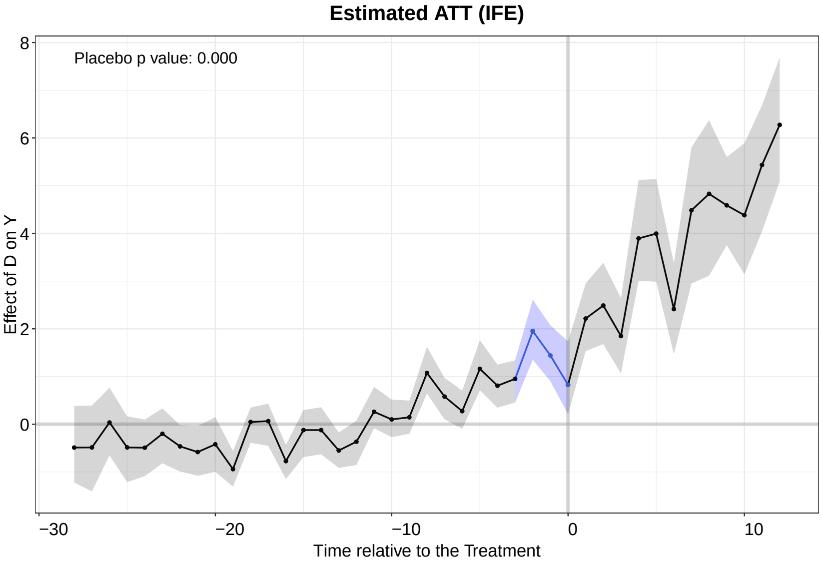

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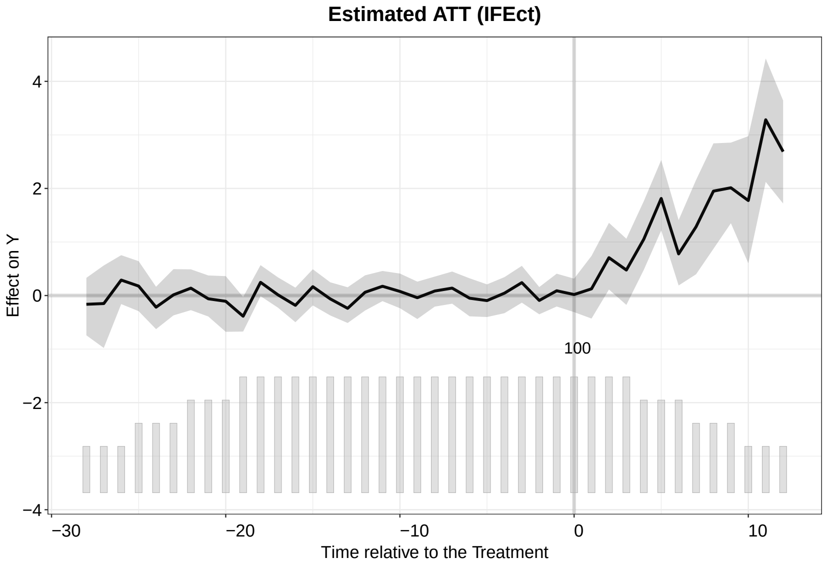

Estimated ATT (IFEct)
Effect on Y
4
2
0
−2
−4
100
−30
−20
−10
0
10
Time relative to the Treatment

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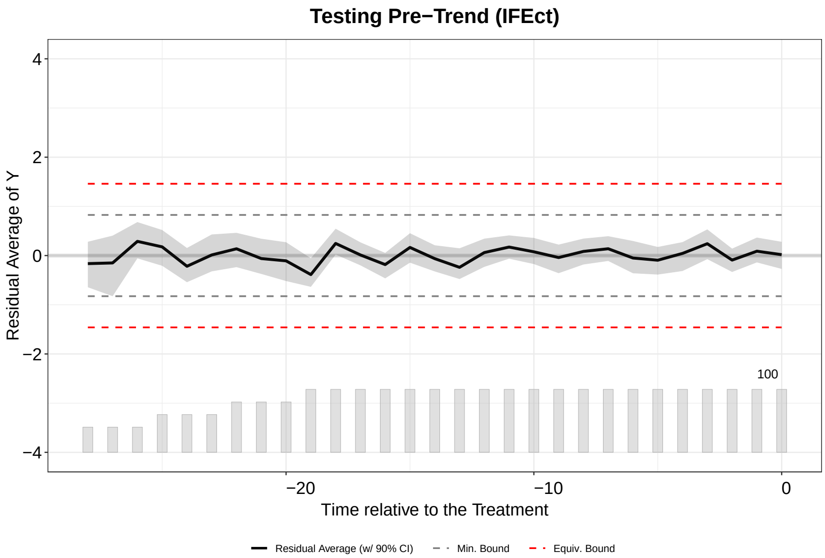

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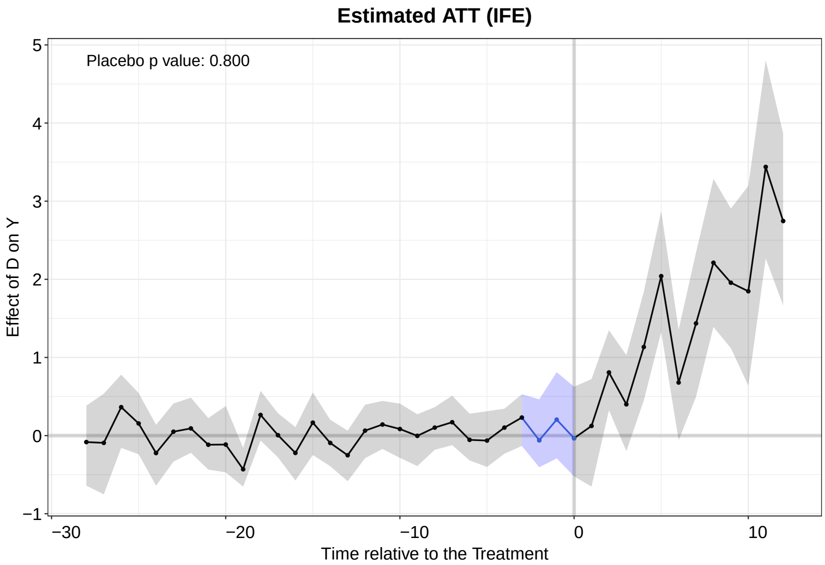

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- The lagged dependent variable does solve a lot of problems, as the time trend is usually the most important confounder.
- Modern approach: IPTW estimators (MSMs) and doubly robust estimators.
- Notice that the propensity score for unit i in period t has the form of $P(D_{it}|\mathbf{Y}_i^{1:(t-1)}, \mathbf{X}_i^{1:t})$.

Weighting under sequential ignorability

- Suppose there is no interference and we are interested in the contemporary effect generated by the treatment:

$$\tau_t = \frac{1}{N}\sum_{i=1}^{N}\tau_{it}, \text{ where } \tau_{it} = Y_{it}(1) - Y_{it}(0)$$

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- It is just another Horvitz-Thompson estimator.

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- But when T is large, balancing the entire history can be challenging in practice.
- Hazlett and Xu (2018) suggest that we may instead balance kernels of the history.
- The implicit assumption is that the distribution of the outcome history can be well approximated by its kernels.

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- We search for a group of positive weights $\{w_i\}$ which sum to one such that
$$\frac{1}{|\mathcal{T}|}\sum_{i\in\mathcal{T}}\mathbf{K}_i = \sum_{i\in\mathcal{C}} w_i\mathbf{K}_i$$
where $\mathbf{K}_i = (k(\mathbf{Y}_i, \mathbf{Y}_1), k(\mathbf{Y}_i, \mathbf{Y}_2), \ldots, k(\mathbf{Y}_i, \mathbf{Y}_N))$, $\mathbf{Y}_i$ is the pre-treatment outcome history of unit i, and
$$k(\mathbf{Y}_i, \mathbf{Y}_j) = e^{-||\mathbf{Y}_i - \mathbf{Y}_j||^2/h}.$$

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- Next, we estimate the treatment effect on the treated in each period

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- Kim, Imai, and Wang (2019) show that we can do the same via matching.

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- $S_i = \frac{1}{T}\sum_{t=1}^{T} D_{it}$ is a sufficient statistic for α_i.

Manifold learning

- Arkhangelsky and Imbens (2019) further illustrate that under this assumption, all we need to do is to find a group of weights (again!) such that:

$$\{\hat{w}_{it}\} = \arg\min \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{w}_{it}^2,$$

$$\text{s.t. } \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{w}_{it} D_{it} = 1, \frac{1}{N} \sum_{i=1}^{N} \hat{w}_{it} = 0, \frac{1}{T} \sum_{t=1}^{T} \hat{w}_{it} = 0,$$

$$\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{w}_{it} \psi_{it}(S_i) = 0, \hat{w}_{it} D_{it} \geq 0.$$

- Then, the estimate of the ATE,

$$\hat{\tau} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \hat{w}_{it} Y_{it}$$

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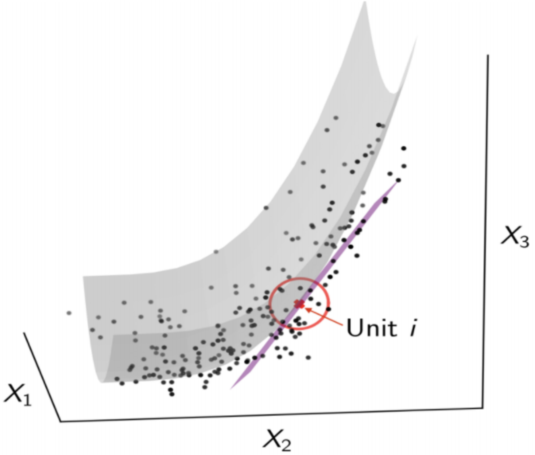
- To estimate λ_i, we first match each unit i to K nearest neighbors.
- Next, we estimate λ_i via PCA on the $K+1$ outcome histories.

Manifold learning

Figure 2: Local Tangent Space Approximation



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- We have to control their influence under either assumption.

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- We can obtain valid estimate for the history's aggregated effect after weighting observations properly.
- Now,

$$P(\mathbf{D}_i^{s:t} = \mathbf{d}^{s:t}) = \prod_{s'=s}^{t} P(D_{is'} | \mathbf{D}_i^{1:(s'-1)}, \mathbf{Y}_i^{1:(s'-1)}, \mathbf{X}_i^{1:s'}).$$

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- Wang (2020) shows that the limit of the DID estimator has no substantive interpretation under spatial interference.
- Intuitively, the control group is contaminated and we cannot eliminate the bias via outcome adjustment.
- Yet under sequential ignorability, we can combine the idea in Aronow, Samii, and Wang (2020) and MSMs to generate consistent estimates for both the direct and the indirect effects.

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