Quant II

Lab 6: Instrumental Variable

Junlong Aaron Zhou

March 12, 2021

Outline

- LATE Revisit
- Compliers
- Beyond LATE
- Falsification Tests

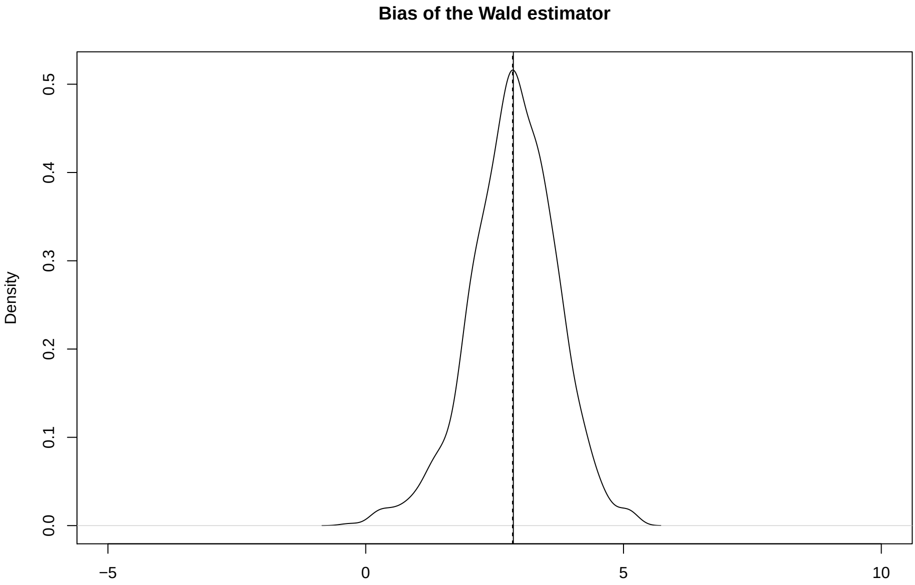
Notice:

- Starting from Sunday, US changes back to daylight saving time.
- Change your clock accordingly!

LATE

- Principal strata: subsets defined by post-treatment variables
- Eg. ATT, LATE, etc.
- To estimate LATE requires four assumptions:
  - Unfoundedness
  - Exclusion restriction
  - Monotonicity (no defiers)
  - First stage

LATE



LATE

```
## The share of always-takers is 20
##  and the estimate is 0.18
## The share of never-takers is 30
##  and the estimate is 0.26
## The share of compliers is 50
##  and the estimate is 0.56
```

Consistent and Asymptotic Unbiased

$$\rho = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(D_i, Z_i)} = \frac{\frac{\text{Cov}(Y_i, Z_i)}{\text{Var}(Z_i)}}{\frac{\text{Cov}(D_i, Z_i)}{\text{Var}(Z_i)}} = \frac{\text{Reduced form}}{\text{First stage}}$$

- 2SLS is consistent and asymptotic unbiased, but biased.
- The key is: $E(\frac{1}{\bar{X}}) \neq \frac{1}{E(X)}$, although $\frac{1}{\bar{X}} \rightarrow \frac{1}{E(X)}$
- Finite sample bias

Consistent and Asymptotic Unbiased

$$\rho = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(D_i, Z_i)} = \frac{\frac{\text{Cov}(Y_i, Z_i)}{\text{Var}(Z_i)}}{\frac{\text{Cov}(D_i, Z_i)}{\text{Var}(Z_i)}} = \frac{\text{Reduced form}}{\text{First stage}}$$

- 2SLS is consistent and asymptotic unbiased, but biased.
- The key is: $E(\frac{1}{\bar{X}}) \neq \frac{1}{E(X)}$, although $\frac{1}{\bar{X}} \rightarrow \frac{1}{E(X)}$
- Finite sample bias
- The asymptotic bias is a function of both violation of exclusion restriction and of strength of first stage.
- One can show for the asymptotic bias to be small you want
  - $\text{cov}(\tilde{Z}_i; \tilde{U}_i)$ to be small
  - $\text{cov}(\tilde{Z}_i; \tilde{D}_i)$ to be large
- With an irrelevant instrument ($F = 0$), the bias is equal to that of OLS (regression of Y on X).
- Weak instrument may cause huge bias: signal is small compared to noise.

With Covariates

- We can identify the LATE for compliers
- What if we have conditional independence of Z_i? Or if we just want to add covariates.

With Covariates

- We can identify the LATE for compliers
- What if we have conditional independence of Z_i? Or if we just want to add covariates.
- Functional form may be misspecified and we need nonparametric or semi-parametric model.

Abadie's (2003) κ

- Recall from the lecture that we can use a weighting scheme to calculate statistics on the compliant population, that comes from Abadie (2003)

$$\kappa = 1 - \frac{D_i(1 - Z_i)}{p(Z_i = 0|X)} - \frac{(1 - D_i)Z_i}{p(Z_i = 1|X)}$$

$$E[\kappa|X] = E[D_1 - D_0|X] = E[D|X, Z = 1] - E[D|X, Z = 0]$$

Abadie's κ

- So, weighting by $\frac{\kappa_i}{E[D_1 - D_0 | X_i]}$ and then computing the usual sample statistics allows you to characterize compliers' attributes.
- Use this in calculating any interesting statistics (means, variance, etc)
- This let's you explore the units composing your LATE.

Abadie's κ

- Moreover, as shown in Theroem 3.1 (Abadie,2003)
- Weighting by κ_i in your object function help to identify LATE with covariates
- For any measurable function $g(Y, D, X)$

$$E(g(Y, D, X)|D_1 > D_0) = \frac{1}{Pr(D_1 > D_0)} E(\kappa g(Y, D, X))$$

Abadie's κ

- Moreover, as shown in Theroem 3.1 (Abadie,2003)
- Weighting by κ_i in your object function help to identify LATE with covariates
- For any measurable function $g(Y, D, X)$

$$E(g(Y, D, X)|D_1 > D_0) = \frac{1}{Pr(D_1 > D_0)} E(\kappa g(Y, D, X))$$

- Therefore one can solve

$$\begin{aligned}
\theta_0 &= \arg\min_{\theta \in \Theta} E[\kappa\{Y - h(D, X; \theta)\}^2] \\
&= \arg\min_{\theta \in \Theta} E[\{E[Y|D, X, D_1 > D_0] - h(D, X; \theta)\}^2 | D_1 > D_0]
\end{aligned}$$

From LATE to ATE

- We can calculate the principal strata score using covariates.
- Although we do not know the strata each observation belongs to, it is still possible to fit a MLE to maximize the probability for the assignment $(\mathbf{D}, \mathbf{Z})$ to occur.
- Based on these scores we can weight LATE to get ATE.
- Aronow and Carnegie (2013); Bisbee et al. (2015); Ding and Lu (2016); Feller et al. (2018)

Aronow and Carnegie (2013): Beyond LATE

- Idea: Complier is determined by observed covariates, but whether you are a complier is unknown.
- We can identify the probability or
  - You are a complier or always taker
    $P_{A,C,i} = Pr(D_1 > D_0 \cup D_0 = 1 | X = x_i) = F(\theta_{A,C} x_i)$, and
  - You are an always-taker conditional you are a complier or always taker.
    $P_{A|A,C,i} = Pr(D_0 = 1 | D_1 > D_0 \cup D_0 = 1, X = x_i) = F(\theta_{A|A,C} x_i)$
  - Compliance score:
    $P_{Ci} = Pr(D_1 > D_0 | X = x_i) = F(\theta_{A,C} x_i)(1 - F(\theta_{A|A,C} x_i))$

Aronow and Carnegie (2013): Beyond LATE

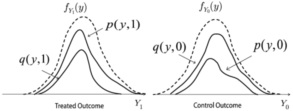
- Idea: Complier is determined by observed covariates, but whether you are a complier is unknown.
- We can identify the probability or
  - You are a complier or always taker $P_{A,C,i} = Pr(D_1 > D_0 \cup D_0 = 1 | X = x_i) = F(\theta_{A,C} x_i)$, and
  - You are an always-taker conditional you are a complier or always taker. $P_{A|A,C,i} = Pr(D_0 = 1 | D_1 > D_0 \cup D_0 = 1, X = x_i) = F(\theta_{A|A,C} x_i)$
  - Compliance score: $P_{Ci} = Pr(D_1 > D_0 | X = x_i) = F(\theta_{A,C} x_i)(1 - F(\theta_{A|A,C} x_i))$
- Then follow the same idea of IPW.
- Hinge on the assumption that complier status is determined by X, so $E(Y_1 - Y_0 | starta, X) = E(Y_1 - Y_0 | X)$.

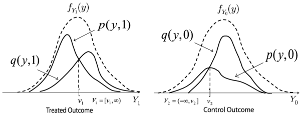
Many instrumental variables

- Conventional approach: GMM
  - Each IV gives a moment condition
  - Use GMM to combine all the conditions
- Modern approach: LASSO
  - The first stage is a prediction problem
  - Hence we can use all kinds of ML algorithms to fit the first stage
  - The concern is whether the approach leads to valid estimates

IV test

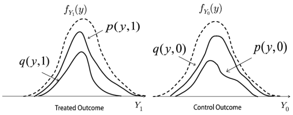
- A recently developed branch of the literature
- The exclusion restriction alone cannot be tested.
- But we can test its combination with the monotonicity assumption.
- $P(y, D = 1|Z = 1) > P(y, D = 1|Z = 0)$ and
  $P(y, D = 0|Z = 1) < P(y, D = 0|Z = 0)$

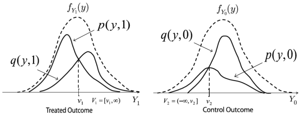




IV test

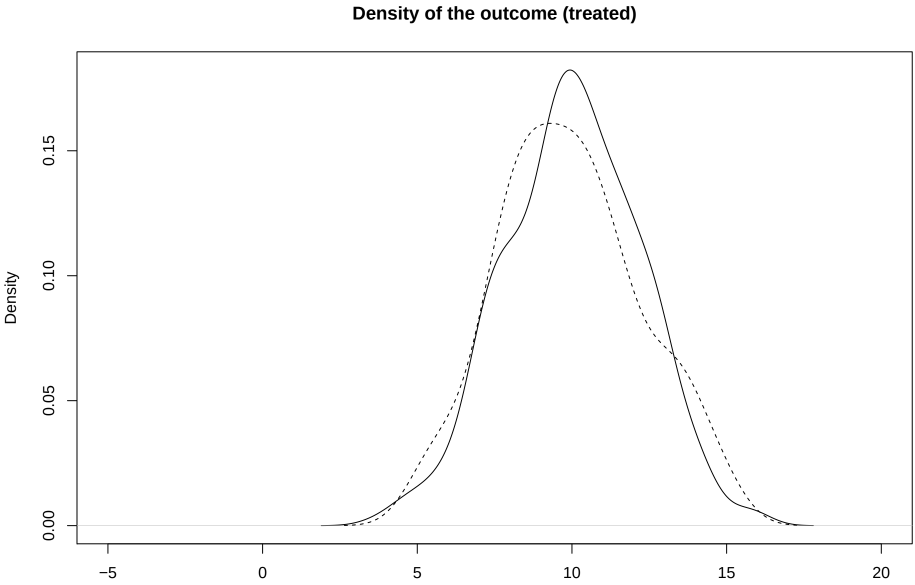
- A recently developed branch of the literature
- The exclusion restriction alone cannot be tested.
- But we can test its combination with the monotonicity assumption.
- $P(y, D = 1|Z = 1) > P(y, D = 1|Z = 0)$ and
  $P(y, D = 0|Z = 1) < P(y, D = 0|Z = 0)$



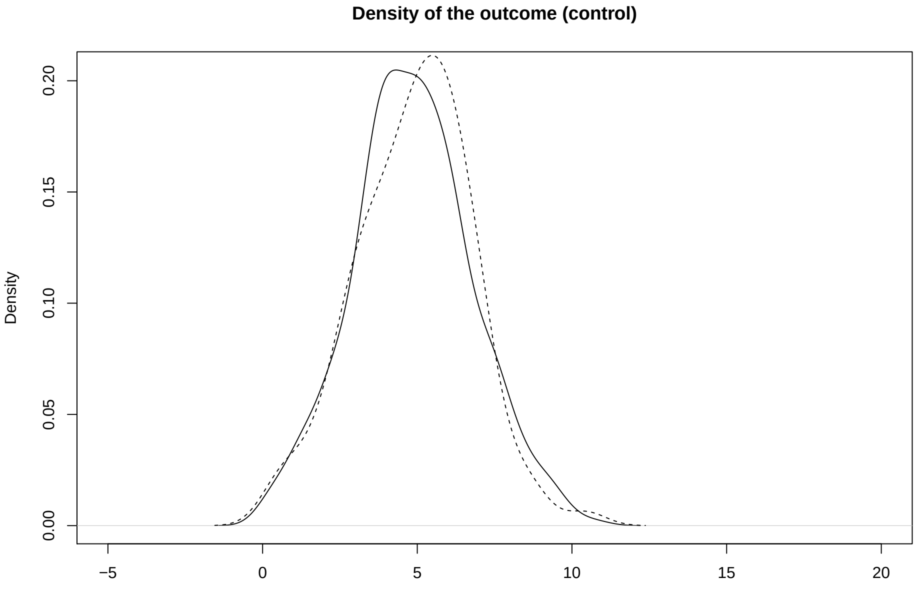



```
D <- D1*Z + D0*(1-Z)
Y <- D*Y1 + (1-D)*Y0
```

IV test


Density of the outcome (treated)
Density
0.00
0.05
0.10
0.15
−5
0
5
10
15
20

IV test

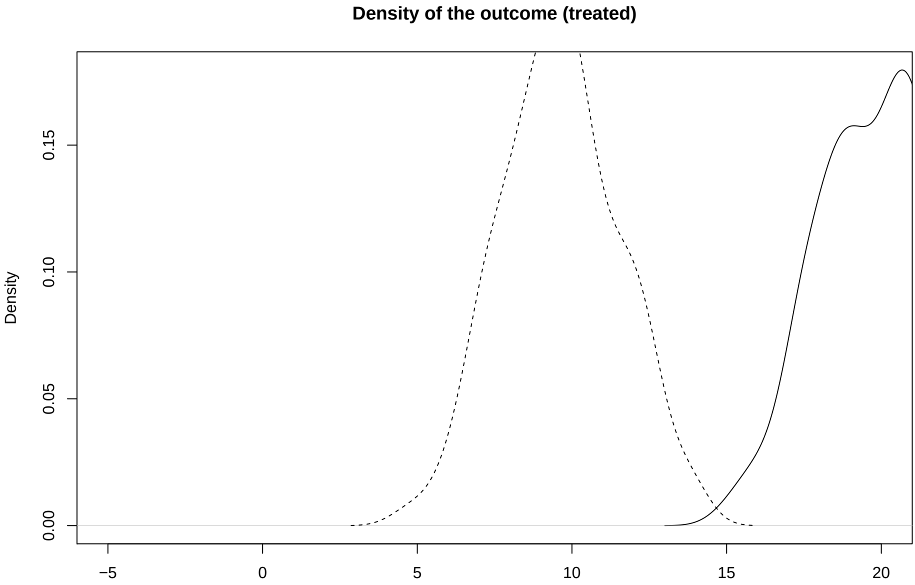

Density of the outcome (control)
Density
0.00
0.05
0.10
0.15
0.20
−5
0
5
10
15
20

IV test

- When exclusion restriction is violated.

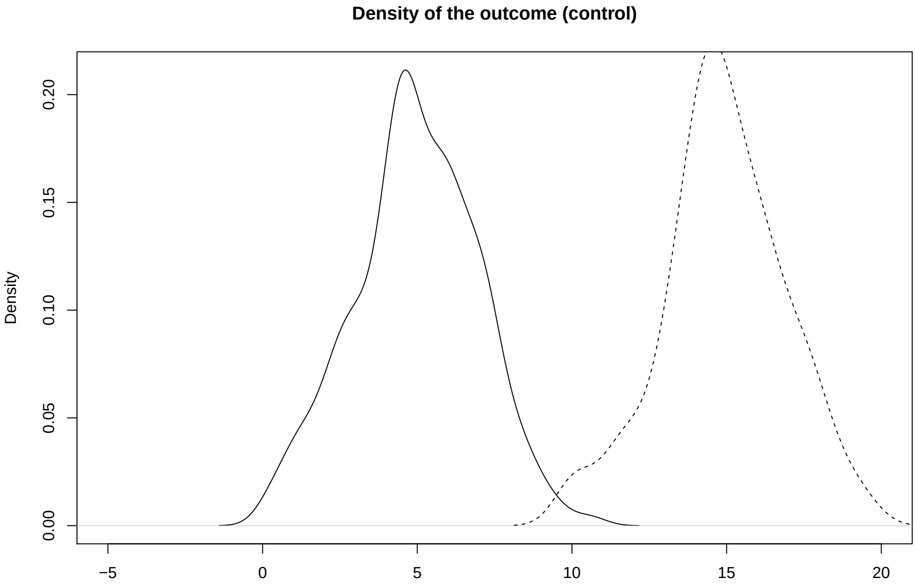
```
D <- D1*Z + D0*(1-Z)
Y <- D*Y1 + (1-D)*Y0 + 10*Z
```

IV test



Density of the outcome (treated)
Density
0.00
0.05
0.10
0.15
−5
0
5
10
15
20

IV test



Density of the outcome (control)
Density
0.00
0.05
0.10
0.15
0.20
−5
0
5
10
15
20

Falsification

- Another approach: following the mechanism.
- Test the reduced form effect of Z_i on Y_i in situations where it is impossible or extremely unlikely that Z_i could affect D_i .
- Because Z_i can't affect D_i, then the exclusion restriction implies that this falsification test should have 0 effect.
- Nunn & Wantchekon (2011): use distance to coast as an instrument for Africans, use distance to the coast in an Asian sample as falsification test

Falsification

TABLE 7—REDUCED FORM RELATIONSHIP BETWEEN THE DISTANCE FROM THE COAST AND TRUST WITHIN AFRICA AND ASIA

	Trust of local government council			
	Afrobarometer sample		Asiabarometer sample	
	(1)	(2)	(3)	(4)
Distance from the coast	0.00039***	0.00031***	−0.00001	0.00001
	(0.00009)	(0.00008)	(0.00010)	(0.00009)
Country fixed effects	Yes	Yes	Yes	Yes
Individual controls	No	Yes	No	Yes
Number of observations	19,913	19,913	5,409	5,409
Number of clusters	185	185	62	62
R^2	0.16	0.18	0.19	0.22

Notes: The table reports OLS estimates. The unit of observation is an individual. The dependent variable in the Asiabarometer sample is the respondent's answer to the question: "How much do you trust your local government?" The categories for the answers are the same in the Asiabarometer as in the Afrobarometer. Standard errors are clustered at the ethnicity level in the Afrobarometer regressions and at the location (city) level in the Asiabarometer and the WVS samples. The individual controls are for age, age squared, a gender indicator, education fixed effects, and religion fixed effects.

***Significant at the 1 percent level.