Quant II

Lab 3: Causal Graph

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February 19, 2021

Today's plan

- Causal Graph Basics
- Causal Graph v.s. Potential Outcome

Causal Graph

- Each data generation process is a distribution $P(X, Y, D, U)$
  - X: observables
  - Y: outcome
  - D: treatment
  - U: unobservables

Causal Graph

- Each data generation process is a distribution $P(X, Y, D, U)$
  - X: observables
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- Causal graph can tell us how the distribution looks like, under the assumptions:
  - There is some directed acyclic graph G representing the relations of causation among the our variables.
  - The Causal Markov condition: The joint distribution of the variables obeys the Markov property on G.
  - Faithfulness: The joint distribution has all of the conditional independence relations implied by the causal Markov property, and only those conditional independence relations.
- The graph also has the noise term but we ignore them for convenience.

Example 1

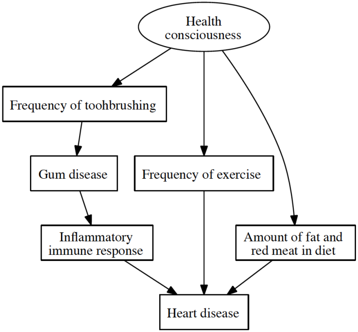

Figure 22.2: Graphical model illustrating hypothetical pathways linking brushing your teeth to not getting heart disease.

Example 1

- We have

$$
\begin{aligned}
& p(\mathit{Yellowteeth}, \mathit{Smoking}, \mathit{Asbestos}, \mathit{Tarinlungs}, \mathit{Cancer}) \\
=& p(\mathit{Smoking})p(\mathit{Asbestos}) \\
& \times p(\mathit{Tarinlungs}|\mathit{Smoking}) \\
& \times p(\mathit{Yellowteeth}|\mathit{Smoking}) \\
& \times p(\mathit{Cancer}|\mathit{Asbestos}, \mathit{Tarinlungs})
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- We can get any conditional density
- but we also know: $p(\mathit{Heartdisease}|\mathit{Brushing} = b) \neq p(\mathit{Heartdisease}|do(\mathit{Brushing} = b))$
- We want to make it happen.

With do calculus

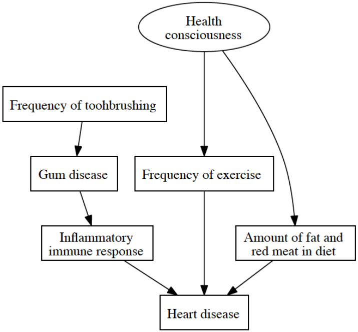


Figure 22.3: The previous graphical model, "surgically" altered to reflect a manipulation (do) of brushing.

Conditioning

- Generally: introducing information about a variable into the analysis by some means (function form is unnecessary).
  - Controlling (e.g. in regression)
  - Subgroup analysis (e.g. restrict analysis to employed women)
  - Sample selection (e.g. only collect data on poor whites)
  - Attrition, censoring, nonresponse (e.g., analyze only respondents or survivors)

Path

- Path does not take the direction into account

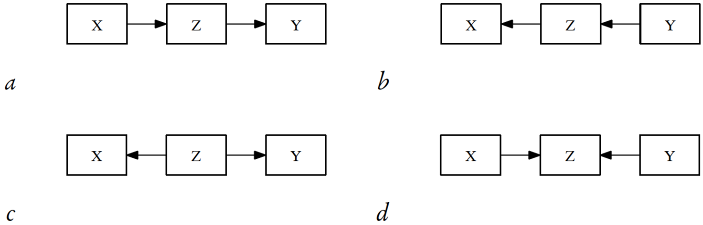
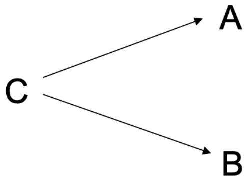


Figure 22.4: Four DAGs for three linked variables. The first two (*a* and *b*) are called **chains**; *c* is a **fork**; *d* is a **collider**. If these were the whole of the graph, we would have $X \not\perp\!\!\!\perp Y$ and $X \perp\!\!\!\perp Y | Z$. For the collider, however, we would have $X \perp\!\!\!\perp Y$ while $X \not\perp\!\!\!\perp Y | Z$.

Coorelation is not causaltion

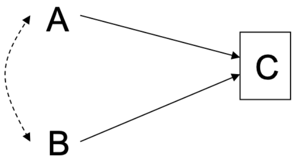
(1) Direct and indirect causation

$A \not\perp\!\!\!\perp B$ and $A \perp\!\!\!\perp B | C$



(2) Common cause confounding

$A \not\perp\!\!\!\perp B$ and $A \perp\!\!\!\perp B | C$

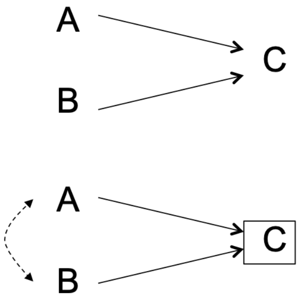


(3) Conditioning on a common effect (“collider”): Selection

$A \perp\!\!\!\perp B$ and $A \not\perp\!\!\!\perp B | C$

$\longleftrightarrow$: non-causal (spurious) association. $\boxed{}$: conditioning.

Coorelation is not causaltion



Pearl's Sprinkler Example

A: It rains

B: The sprinkler is on

C: The lawn is wet

Hollywood Success

A: Good looks

B: Acting skills

C: Fame

Academic Tenure Example

A: Productivity

B: Originality

C: Tenure

In all three examples, conditioning on the collider C induces a spurious association between two variables, A and B, that don't cause each other and share no common cause, i.e. that are marginally independent in the population.

D-separation

- We need some correlation, but it must be causal
- We need to rule out non-causal ones.
- The concept of d-separation ("directional separation", Pearl 1988) subsumes the three structural sources of association and gives them a name.
- d-separation determines which paths transmit association, and which ones don't.

Block and Active

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- We say If $\mathcal{S}$ **blocks** every undirected path from X to Y, then they must be conditionally independent given $\mathcal{S}$.
- An unblocked path is also called **active**.
- A path is active when every variable along the path is active; if even one variable is blocked by $\mathcal{S}$, the whole path is blocked.

Definition

- A variable Z along a path is active, conditioning on $\mathcal{S}$, if
  - Z is a collider along the path, and in $\mathcal{S}$; or,
  - Z is a descendant of a collider, and in $\mathcal{S}$; or
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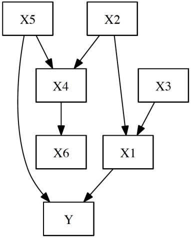
  - Z is not a collider, and not in $\mathcal{S}$.
- Turned around, Z is blocked or de-activated by conditioning on $\mathcal{S}$ if
  - Z is a non-collider and in $\mathcal{S}$; or
  - Z is collider, and neither Z nor any of its descendants is in $\mathcal{S}$

D-separation and Conditional Independence

- In words, $\mathcal{S}$ blocks a path when it blocks the flow of information by conditioning on the middle node in a chain or fork, and doesn't create dependence by conditioning on the middle node in a collider (or the descendant of a collider).
- Only one node in a path must be blocked to block the whole path. When $\mathcal{S}$ blocks all the paths between X and Y, we say it d-separates them.
- A collection of variables U is d-separated from another collection V by $\mathcal{S}$ if every $X \in U$and $Y \in V$ are d-separated.
- In every distribution which obeys the Markov property, d-separation implies conditional independence. If the distribution is also faithful to the graph, then conditional independence also implies d-separation.

Example 2



- Consider Y=Grades in Quant2, X_1=Efforts spent on Quant2, X_2=prior knowledge in statistics, X_3=workload this semester, X_4=Understanding of Neal, X_5=amount learned in Quant1, X_6=grades in Quant1
- Is X_3 and X_5; X_3 and Y independent?
- What should we control?

Short Summary

- "Blocked" (d-separated) paths don't transmit association.
- "Unblocked" (d-connected) paths may transmit association.
- Three blocking criteria (key!!)
  - Conditioning on a non-collider blocks a path
  - Conditioning on a collider, or a descendent of a collider, unblocks a path
  - Not conditioning on a collider leaves a path "naturally" blocked.

Go back to Causality

- **Adjustment Criterion** (Shpitser et al. 2010)
- Total causal effect of D on Y is identifiable if one can condition on ("adjust for") a set of variables $\mathcal{S}$ that
  - blocks all non-causal paths between D and Y
  - without blocking any causal paths between D and Y
- Equivalently: d-separate D and Y along all non-causal paths while leaving all causal paths d-connected)

Backdoor Criterion

- The backdoor criterion is a narrower version of the adjustment criterion that omits some unnecessary conditioning sets.
- **Backdoor Criterion** (Pearl 1995)
- Definition: A set of variables $\mathcal{S}$ satisfies the backdoor criterion relative to an ordered pair of variables (D, Y) in a DAG if:
  - no node in $\mathcal{S}$ is a descendant of D, and
  - $\mathcal{S}$ blocks (d-separates) every path between D and Y that contain an arrow into D (so-called "backdoor paths").
- Theorem: The total causal effect of D on Y is non-parametrically identifiable given $\mathcal{S}$ if $\mathcal{S}$ meets the backdoor criterion.

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- Theorem: The total causal effect of D on Y is non-parametrically identifiable given $\mathcal{S}$ if $\mathcal{S}$ meets the backdoor criterion.
- This looks familiar: we control confounders and no post-treatment variables.
  - Note this is DAG: so no arrows from Y to D
  - Not necessarily confounder itself

Bias

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  - selection bias (we conditional on $selection = 1$)
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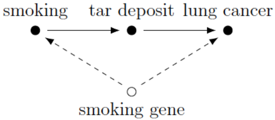
  - selection bias (we conditional on *selection* $= 1$)
  - Bias amplification (one case: we control parents but not confounder)
- post-treatment bias (we block the causal chain)

$$
\begin{aligned}
& E(Y(1)|D=1, X(1)=x) - E(Y(0)|D=0, X(0)=x) \\
=& E(Y(1)|X(1)=x) - E(Y(0)|X(0)-x) \quad \text{by random assignment} \\
=& \int y_1 f_{X(1)}(y_1|x)dy_1 - \int y_0 f_{X(0)}(y_0|x)dy_0 \\
\neq& E(Y(1)-Y(0)|X=x)
\end{aligned}
$$

Post-treatment Variables

- But (only) sometimes it can be useful: **Frontdoor Criterion**



(a) Original Pearl DAG for front-door criterion

- D:smoking; X:tar deposit; Y:lung cancer
- unobserved U: smoking gene
- $P(X|D=d) = P(X|do(d))$, and
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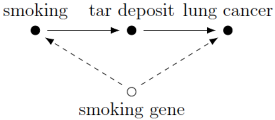

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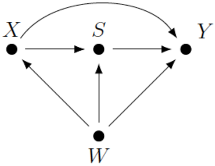
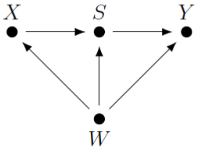
- $P(X|D=d) = P(X|do(d))$, and $P(Y|D=d, X=x) = P(Y|do(x), D=d)$
- $P(Y|do(d)) = \sum_{d'} \sum_{x} P(Y|X=x; D=d')P(d')P(X=x|D=d)$

Post-treatment Variables

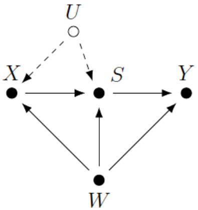
- Different types of post-treatment variables



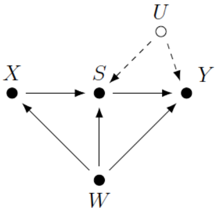
(a) Mediation



(b) Surrogates



(c) Invalid Surrogates



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- Simultaneity
  - by definition, DAG doesn't allow simultaneity
- Conditioning set and confounders (what are they)
- Counterfactuals
  - For PO, you need model assumption
  - For DAG, you need a structrual assumption

Further readings:

- Elwert, Felix. 2013. "Graphical Causal Models." Handbook of Causal Analysis for Social Research.
- Pearl, Judea, 2009. Causality.
- Pearl, Judea, and Dana Mackenzie, 2018, The book of why: the new science of cause and effect.
- Guido W. Imbens, 2020, "Potential Outcome and Directed Acyclic Graph Approaches to Causality: Relevance for Empirical Practice in Economics"