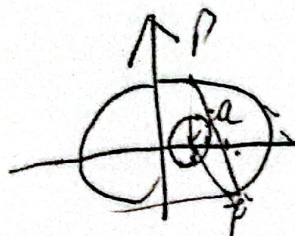
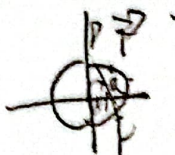


$\frac{1}{3}$



3. 已知  $F$  是椭圆  $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$  的右焦点, 点  $P$  在椭圆  $C$  上, 线段  $PF$  与圆

$(x - \frac{c}{3})^2 + y^2 = \frac{b^2}{9}$  相切于点  $Q$ , (其中  $c$  为椭圆的半焦距), 且  $\overline{PQ} = 2\overline{QF}$  则椭圆  $C$  的离心率等于 (A)

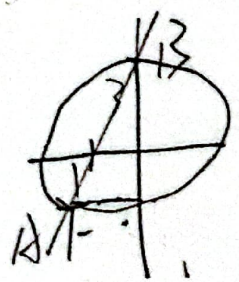
- A.  $\frac{\sqrt{5}}{3}$       B.  $\frac{2}{3}$       C.  $\frac{\sqrt{2}}{2}$       D.  $\frac{1}{2}$

4. 已知椭圆  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$  的右焦点为  $F$ , 过  $F$  点作  $x$  轴的垂线交椭圆于  $A, B$  两点, 若  $\overline{OA} \cdot \overline{OB} = 0$ , 则椭圆的离心率等于 (A)

- A.  $\frac{-1 + \sqrt{5}}{2}$       B.  $\frac{-1 + \sqrt{3}}{2}$       C.  $\frac{1}{2}$       D.  $\frac{-\sqrt{3}}{2}$

5. 过椭圆  $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$  的左焦点  $F$  的直线过  $C$  的上端点  $B$ , 且与椭圆相交于点  $A$ , 若  $\overline{BF} = 3\overline{FA}$ , 则  $C$  的离心率为 (A)

- A.  $\frac{1}{3}$       B.  $\frac{\sqrt{3}}{3}$       C.  $\frac{\sqrt{3}}{2}$       D.  $\frac{\sqrt{2}}{2}$



$$\begin{aligned} \frac{c}{-x} &= \frac{2}{4} \\ -3x &= 4c \\ x &= -\frac{4}{3}c \\ \frac{-y}{b} &= \frac{1}{3} \\ -3y &= b \\ y &= -\frac{1}{3}b \end{aligned}$$

$$\begin{aligned} \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1 \\ \left(\frac{4}{3}c\right)^2 + \frac{(-\frac{1}{3}b)^2}{b^2} &= 1 \\ \frac{16}{9} \cdot \frac{c^2}{a^2} + \frac{1}{9} &= 1 \\ \left(\frac{c}{a}\right)^2 &= \frac{8}{9} \cdot \frac{1}{16} \cdot 1 \end{aligned}$$

$$\begin{aligned} \frac{c}{a} &= (0.6) \\ \frac{c}{a} &= \frac{1}{\sqrt{5}} \end{aligned}$$

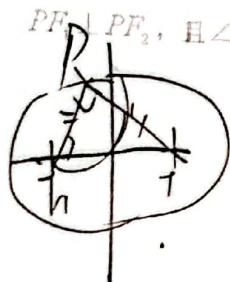
正确  
GM  
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5

6. 设  $F_1, F_2$  是椭圆  $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$  的两个焦点. 若在  $C$  上存在一点  $P$ , 使

$PF_1 \perp PF_2$ , 且  $\angle PF_1F_2 = 45^\circ$ , 则  $C$  的离心率为  $\frac{1}{2}\sqrt{2}$ .



$$b = c$$

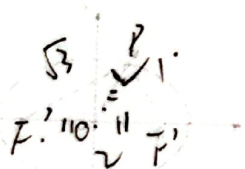
$$a = \sqrt{2}c$$

$$\frac{c}{a} = \frac{1}{2}\sqrt{2}$$

$$\frac{1}{\sin \theta} = \frac{\sqrt{3}}{2}$$

7. 设  $F$  为椭圆  $C: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  的左焦点,  $P$  为  $C$  上第一象限的一点. 若  $\angle FPO = \frac{\pi}{6}$ ,

$|PF| = \sqrt{3}|OF|$ , 则椭圆  $C$  的离心率为  $\frac{\sqrt{3}-1}{2}$ .



$$\frac{|OF|}{\sin \theta} = \frac{|PF|}{\sin \theta}$$

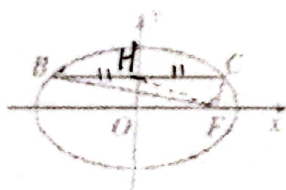
$$\sin \theta = \frac{1}{2}\sqrt{3}$$

$$2a = 1 +$$

$$2a \frac{2c}{2a} = \frac{2}{\sqrt{3}+1}$$

8. 如图, 在平面直角坐标系  $xOy$  中,  $F$  是椭圆  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$  的右焦点, 直线  $y = \frac{b}{2}$

与椭圆交于  $B, C$  两点, 且  $\angle BFC = 90^\circ$ , 则该椭圆的离心率是  $\frac{1}{3}\sqrt{6}$ .



$$y = \frac{1}{2}b$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} = \frac{3}{4}$$

$$x = \pm \frac{\sqrt{3}}{2}a$$

$$B(-\frac{1}{2}\sqrt{3}a, \frac{1}{2}b)$$

$$BF = CF$$

$$\frac{1}{2}\sqrt{3}a = c$$

$$\frac{3}{4}a^2 = c^2 + \frac{1}{4}b^2$$

$$a^2 = b^2 + c^2$$

$$\Rightarrow \frac{1}{4}a^2 = \frac{1}{4}b^2$$

$$a = \sqrt{3}b, a^2 = 3b^2$$

$$\therefore \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{1}{3}}$$

$$= \frac{1}{3}\sqrt{6}$$