第 2 课时　利用递推公式表示数列

$3 - 2$

$a_2 - a_1 \quad 2(n+1)$

一、填空题

1. 已知数列$\{a_n\}$的前 n 项和为 S_n，若 $S_n = n^2 + 2n$，则 $a_n =$ $2n+1$.

2. 在数列$\{a_n\}$中，已知 $a_1 = 3$，若 $a_n = 2a_{n-1} + 1$，$(n \geq 2, n \in \mathbf{N})$，则 $a_4 =$ 31.

$7, 15, 31$

3. 若数列$\{a_n\}$满足 $a_1 < 0$，$\dfrac{a_{n+2}}{a_{n+1}} = 2$ $(n \in \mathbf{N})$，则数列$\{a_n\}$是 严格减 数列(填"严格增"或"严格减").

4. 数列$\{a_n\}$满足 $a_1 = 2$，$a_{n+1} = a_n + 2n + 2$，则 $a_n =$ n^2+n.

5. 在数列$\{a_n\}$中，若 $a_1 = 1$，$a_{n+1} = a_n + \dfrac{1}{n(n+1)}$，则 a[illegible] 的值为______.

二、选择题

$3, 6, 3, -3, -6, -3, 3$

6. 在数列$\{a_n\}$中，若 $a_1 = 3$，$a_2 = 6$，$a_{n+2} = a_{n+1} - a_n$，则 a_{2021}等于 (B)

A. 6;　　B. -6;　　C. 3;　　D. -3.

7. 在 1，2，3，…，2021 这 2021 个自然数中，将能被 2 除余 1，且被 3 除余 1 的数按从小到大的次序排成一列，构成数列$\{a_n\}$，则 a[illegible] 等于 (　　)

A. 289;　　B. 295;　　C. 301;　　D. 307.

8. 数列$\{a_n\}$定义如下：$a_1 = 1$，当 $n \geq 2$ 时，$a_n = \begin{cases} 1 + a_{\frac{n}{2}}, & n = 2k + 2, k \in \mathbf{N}, \\ \dfrac{1}{a_{n-1}}, & n = 2k + 1, k \in \mathbf{N}. \end{cases}$ 若 $a_n = \dfrac{1}{4}$，则 n 的值等于 (A)

A. 7;　　B. 8;　　C. 9;　　D. 10.

三、解答题

9. 根据数列$\{a_n\}$的递推公式，写出它的前 4 项.

(1) $\begin{cases} a_n = -2a_{n-1} + 1 (n \geq 2, n \in \mathbf{N}), \\ a_1 = 1; \end{cases}$

(2) $\begin{cases} a_{n+2} = 2a_n + a_{n+1} (n \geq 1, n \in \mathbf{N}), \\ a_1 = 1, a_2 = 1. \end{cases}$

2) $\{1, 1, 3, 5\}$

$\cancel{设}$ 令 $b_n = a_n - \frac{1}{3}$

1) $b_n = -2b_{n-1}$

$\therefore b_n = \frac{2}{3} \cdot (-2)^{n-1}$

$\Rightarrow a_n = \frac{2}{3}(-2)^{n-1} + \frac{1}{3}$

$\{1, -1, 3, -5\}$

设 $a_n =$

修正处

$S_n - S_{n-1} = n^2 + 2n - (n-1)^2 - 2(n-1)$

$= \cancel{n^2} + 2n - \cancel{n^2} + 2n - 1$

$- 2n + 2$

$-1, -2, -4$

$a_{n+1} - a_n = 2n + 2$

$a_{n+1} - \cancel{a_1} = 4 + 6 + \cdots + 2(n+1)$

$= \cancel{\frac{(4+2(n+1)) \cdot n}{2}}$

$= \cancel{(n+3)n}$

$a_{n+1} = \frac{(2+2n+4)}{2}(n+1)$

$(n+2)(n+1)$

$(n+1)n$

$1, 2, \frac{1}{2}, 3, \frac{1}{3}, 4,$

$a_n + x = -2(a_{n-1} + x)$

$a_n = -2a_{n-1} - 3x$

$-3x = 1$

$x = -\frac{1}{3}$

$1, 1, 3, 5, 11, 21, 42$

10. 在数列$\{a_n\}$中，已知$a_n=-\frac{1}{a_{n-1}}$，$(n\geqslant 2, n\in\mathbf{N})$.

(1)求证：$a_{n+2}=a_n$；

(2)若$a_1=4$，求a_{20}的值；

(3)若$a_1=1$，求$a_1+a_2+a_3+a_4+a_5+a_6+a_7$的值.

1) $a_{n+2}=-\frac{1}{a_{n+1}}=-\frac{1}{-\frac{1}{a_n}}=a_n$

2) 又 $a_{20}=4$

3) $S_7=1$

11. 已知在数列$\{a_n\}$中，$a_1=1$，$a_{n+1}=\frac{3a_n}{a_n+3}$$(n\in\mathbf{N}, n\geqslant 1)$，求通项$a_n$.

令 $b_n=\frac{1}{a_n}$ $\Rightarrow a_n=\frac{3}{n+2}$

故 $\frac{1}{a_{n+1}}=\frac{1}{3}+\frac{1}{a_n}$

$b_{n+1}=b_n+\frac{1}{3}$

$b_n=1+\frac{1}{3}(n-1)=\frac{1}{3}n+\frac{2}{3}$

四、能力拓展题

~~12.~~ 若数列$\{a_n\}$及$\{b_n\}$满足$\begin{cases} a_{n+1}=a_n+\frac{1}{3}b_n\,(n\in\mathbf{N}, n\geqslant 1), \\ b_{n+1}=3a_n+b_n+3\,(n\in\mathbf{N}, n\geqslant 1), \end{cases}$且$a_1=1$，$b_1=6$.

(1)证明：$b_n=3a_n+3$$(n\in\mathbf{N}, n\geqslant 1)$；

(2)求数列$\{b_n\}$的通项公式.

修正处

递推公式求通项专题

一、根据递推公式，求数列$\{a_n\}$的通项公式

1、$\begin{cases} a_1 = 1 \\ a_{n+1} = a_n + (n+1) \end{cases}$

$\Rightarrow a_n = n^2$.

$a_{n+1} - a_n = n+1$.

$\Rightarrow a_{n+1} - a_1 = \cancel{n+1}\ 2+3+\cdots+(n+1)$.

$\therefore a_{n+1} = \frac{(1+n+1)\cdot(n+1)}{2} = (n+1)^2$

2、$\begin{cases} a_1 = 1 \\ \frac{a_n}{a_{n-1}} = 3^n \end{cases}$

$\Rightarrow \frac{a_n}{a_1} = 3^n \cdot 3^{n-1} \cdots 3^2 = 3^{2+3+\cdots+\cancel{(n-1)}n} = 3^{\frac{(2+n)\cdot(n-1)}{2}}$

$\Rightarrow a_n = 3^{\frac{n^2+n-2}{2}}$

3、$\begin{cases} a_1 = 3 \\ a_{n+1} = \sqrt{a_n^2+1} \end{cases}$ 且 $a_n > 0$，$n \in N^*$

$\therefore a_n = \sqrt{n+8}$.

$(a_{n+1})^2 = a_n^2 + 1$

令 $b_n = (a_n)^2$，则 $b_{n+1} = a_{n+1}^2$

$b_{n+1} = \cancel{8}\, b_n + 1$

$\Rightarrow b_n = 9 + (n-1)\cdot 1 = n+8$

4、$\begin{cases} a_1 = \frac{1}{2} \\ a_n = \frac{a_{n-1}}{2a_{n-1}+1}, n \ge 2 \end{cases}$

$\therefore a_n = \frac{1}{b_n} = \frac{1}{2n}$

$\Rightarrow \frac{1}{a_n} = \frac{1}{a_{n-1}} + 2$

令 $b_n = \frac{1}{a_n}$

$\Rightarrow b_n = b_{n-1} + 2$

$\Rightarrow b_n = 2 + 2(n-1) = 2n$

5、$\begin{cases} a_1 = 5 \\ a_{n+1} = 2a_n - 3 \end{cases}$

令 $b_n = a_n + x$，则 $b_{n+1} = a_{n+1} + x$

$\Rightarrow a_{n+1} + x = 2(a_n + x) \Rightarrow x = -3$

$\therefore b_n = a_n - 3$.

$\because b_{n+1} = 2b_n$

$\therefore \frac{b_{n+1}}{b_n} = 2$

$\therefore$

$b_n = 2 \cdot 2^{n-1} = 2^n$

$\Rightarrow a_n - 3 = 2^n$

$a_n = 2^n + 3$

6、$\begin{cases} a_1 = 2 \\ a_{n+1} = 2a_n + 2^n \end{cases}$

7、$\begin{cases} a_1 = 3 \\ a_{n+1}^3 = a_n^2 \end{cases}$ 且 $a_n > 0$，$n \in N^*$

$\Rightarrow$

$\log_3 a_{n+1}^3 = \log_3 a_n^2$

$\frac{\log_3 a_{n+1}}{\log_3 a_n} = \frac{2}{3}$

令 $\log_3 a_n = b_n$

$\Rightarrow \frac{b_{n+1}}{b_n} = \frac{2}{3}$

$\Rightarrow b_n = (\frac{2}{3})^{n-1}$

$\Rightarrow a_n = 3^{(\frac{2}{3})^{n-1}}$.