

第 P157
A.1) C

2) 取 $n=1$

$$\text{则 } 1 \times 2 = \frac{1}{2} \times 1(1+1) = 1 \text{ 成立.}$$

取 $n=k$

则当 $n=k+1$ 时, 1

$$\begin{aligned} \text{左} &= 1 \times 2 + 2 \times 3 + \dots + k(k+1) + (k+1)(k+2) \\ &= \frac{1}{2} k^2(k+1) + (k+1)(k+2) \\ &= (k+1)^2(k+2) \end{aligned}$$

= 右

\therefore 成立

3) 取 $n=1$

$$\frac{1}{1 \times 3} = \frac{1}{3} \text{ 成立.}$$

取 $n=k$

则当 $n=k+1$ 时

$$\begin{aligned} \text{左} &= \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \dots + \frac{1}{(2n-1)(2n+1)} + \frac{1}{(2n+1)(2n+3)} \\ &= \frac{n}{2n+1} + \frac{1}{(2n+1)(2n+3)} \\ &= \frac{n(2n+3)+1}{(2n+1)(2n+3)} \\ &= \frac{n+1}{2n+3} \end{aligned}$$

= 右

\therefore 成立

$\sum_{k=1}^{\infty} \frac{1}{k^2}$

$$1. \quad \frac{1}{k^2} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2n-1} - \frac{1}{2n} + \frac{1}{2n+1}$$

$$\frac{1}{k^2} = \frac{1}{(2n-1)^2} + \dots + \frac{1}{2n} + \frac{1}{2n+1}$$

$$2. \quad f(k+1) - f(k) = \frac{1}{2k+2}$$

$$3. \quad \frac{1}{k^2} = \frac{1}{(k+1)^2} + \frac{1}{2k+2}$$