

伪造中本聪签名

伪造签名的依据：

- $\sigma = (r, s)$ is valid signature of m with secret key d
- If only the hash of the signed message is required
- Then anyone can forge signature $\sigma' = (r', s')$ for d
- (Anyone can pretend to be someone else)
- Ecdsa verification is to verify:
 - $s^{-1}(eG + rP) = (x', y') = R', r' = x' \bmod n == r ?$
- To forge, choose $u, v \in \mathbb{F}_n^*$
- Compute $R' = (x', y') = uG + vP$
- Choose $r' = x' \bmod n$, to pass verification, we need
 - $s'^{-1}(e'G + r'P) = uG + vP$
 - $s'^{-1}e' = u \bmod n \rightarrow e' = r'uv^{-1} \bmod n$
 - $s'^{-1}r' = v \bmod n \rightarrow s' = r'v^{-1} \bmod n$
- $\sigma' = (r', s')$ is a valid signature of e' with secret key d

伪造签名函数：

```
1 def Pretend(r, s, G, P):
2     u = 3
3     v = 3
4     r_forge = Point_Add(Multi(u, G), Multi(v, P))[0]
5     print(u, v)
6     e_forge = (r_forge * u * multi_inverse(v, mod_value)) % mod_value
7     s_forge = (r_forge * multi_inverse(v, mod_value)) % mod_value
8     if(Verify( r_forge, s_forge, e_forge, G, P)):
9         return (r_forge, s_forge)
```

通过verify函数进行验证。

运行结果：

```
C:\Users\wynne\AppData\Local\Programs\Python\Python39\python.exe F:/practise/Forged_Satoshi_Signature/forged
公钥为 (7, 1)
3 3
伪造通过
伪造的签名 (7, 15)

进程已结束,退出代码0
```

方法二，运用sage能快速伪造

```
sage: F = GF (0xFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFC2F
..... ) #生成一个给定阶数的全局唯一有限域
..... C = EllipticCurve ([F (0), F (7)]) #构造出椭圆曲线
.....
.....
..... G = C.lift_x(0x79BE667EF9DCBBAC55A06295CE870B07029BFCDB2DCE28D959F2815B16F
..... 81798)
..... N = GF (C.order())
..... P = C.lift_x(0x11db93e1dcdb8a016b49840f8c53bc1eb68a382e97b1482ecad7b148
..... a6909a5c) # block 9 coinbase payout key.
.....
..... def forge(c, a=-1): # Create a forged 'ECDSA' (hashless) signature
..... # set a to something other than -1 to be less obvious
..... a = N(a)
..... R = c * G + int(a) * P
..... s = N(int(R.xy()[0])) / a
..... m = N(c) * N(int(R.xy()[0])) / a
..... print ('hash1 = %d'%m)
..... print ('r1 = %d'%(int(R.xy()[0])))
..... print ('s1 = %d'%s)
.....
..... forge(1)
hash1 = 738919280936671005416662050438984412675877586577947667827507163075362860
39192
r1 = 419001611436490948819047799647894665852498056212801375998544468339818754551
45
s1 = 738919280936671005416662050438984412675877586577947667827507163075362860391
92
```

原理是相同的，SAGE运算的更快。SAGE参考代码：<https://www.zuocoin.com/a/news/experience/2018/1119/6754.html>