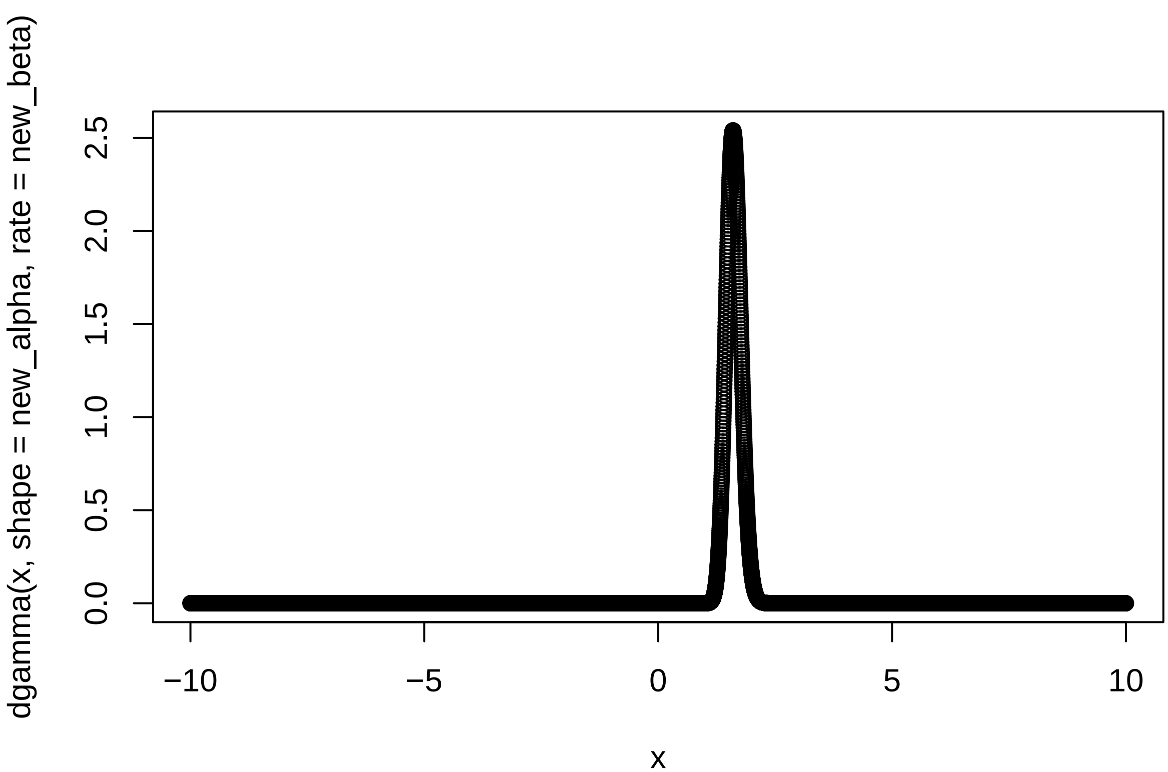
Posterior

For Possion likelihood function

If the $X_1, X_2, ..., X_n$ is a random sample from a Poissson distribution with an unknown value of the mean W. Suppose the prior distribution of W is a gamma distribution with parametes α(shape) andβ (rate). Then the posterior distribution of$W$ is a gamma distribution with parameters$\alpha + \sum_{i=1}^{n} X_i$ and $\beta + n$.

```
n=50
x0<-rpois(n, lambda = 2)
pois_conju<-function(vec, alpha, beta){
 len=length(vec)
 new_alpha=alpha+sum(vec)
 new_beta=beta+len
 cat("Original: Gamma distribution with  shape alpha=%f,  rate beta=%f\n", alpha, beta)
 cat("Now : Gamma distribution with shape alpha'=%f , rate beta'=%f", new_alpha, new_beta)
 x<-x <- seq(-10, 10, length=10000)
 plot(x, dgamma(x, shape = new_alpha, rate = new_beta))
}
pois_conju(x0, alpha=2, beta=15)
```

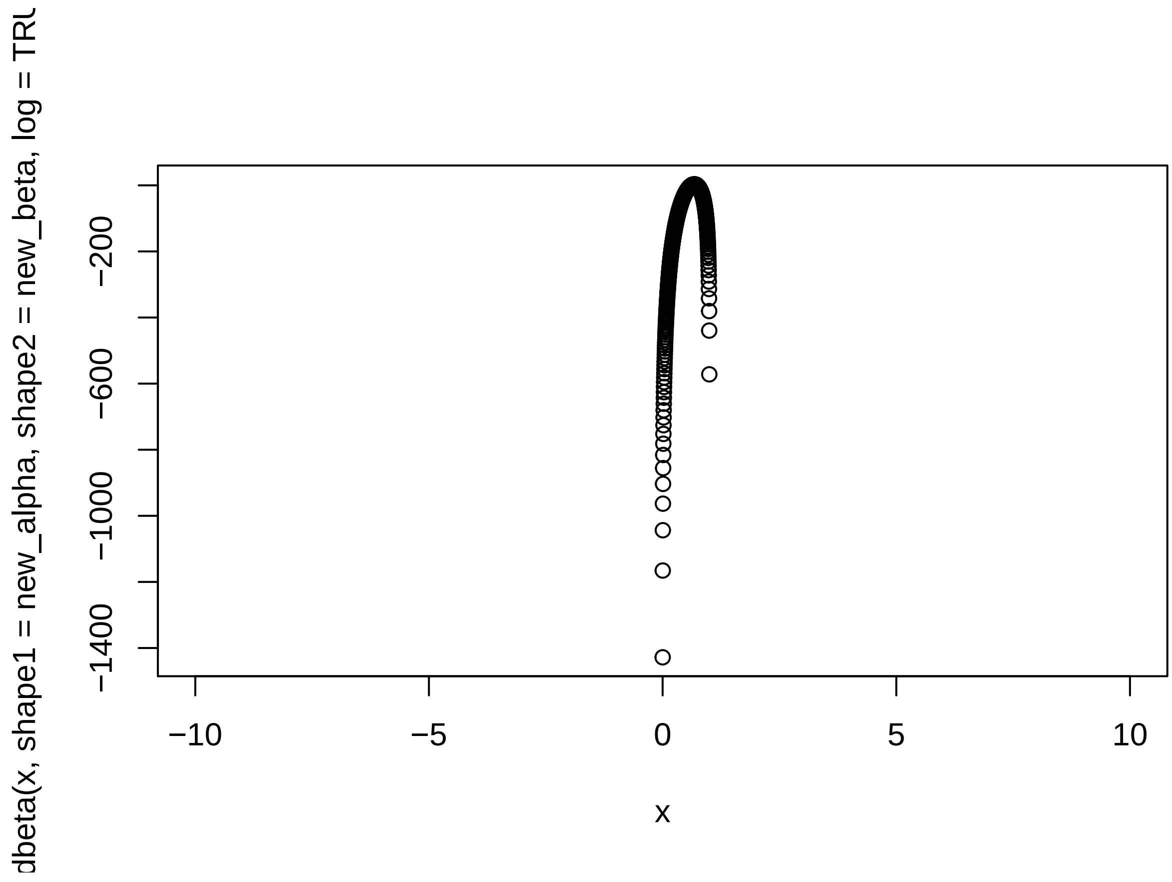
```
## Original: Gamma distribution with  shape alpha=%f,  rate beta=%f
##  2 15Now : Gamma distribution with shape alpha'=%f , rate beta'=%f 105 65
```



For Binomial likelihood function

If the $X_1, X_2, ..., X_n$ is a random sample from a Binomial distribution $B(N, \theta)$, and prior distribution for θ is beta distribution with parametersα and β . The posterior shoule be Beta distribution with parameters $\alpha + \sum_{i=1}^{n} X_i$ and $\beta + nN - \sum_{i=1}^{n} X_i$

```
n1=50
x1<-rbinom(n=n1, size=7, prob=0.7)
binom_conju<-function(vec, alpha, beta, size){
 len=length(vec)
 new_alpha=alpha+sum(vec)
 new_beta=beta+len*size-sum(vec)
 x<-x <- seq(-10, 10, length=10000)
 plot(x, dbeta(x, shape1 = new_alpha, shape2 = new_beta, log = TRUE) )
}
binom_conju(x1, alpha=4.2, beta=.5, size=7)
```



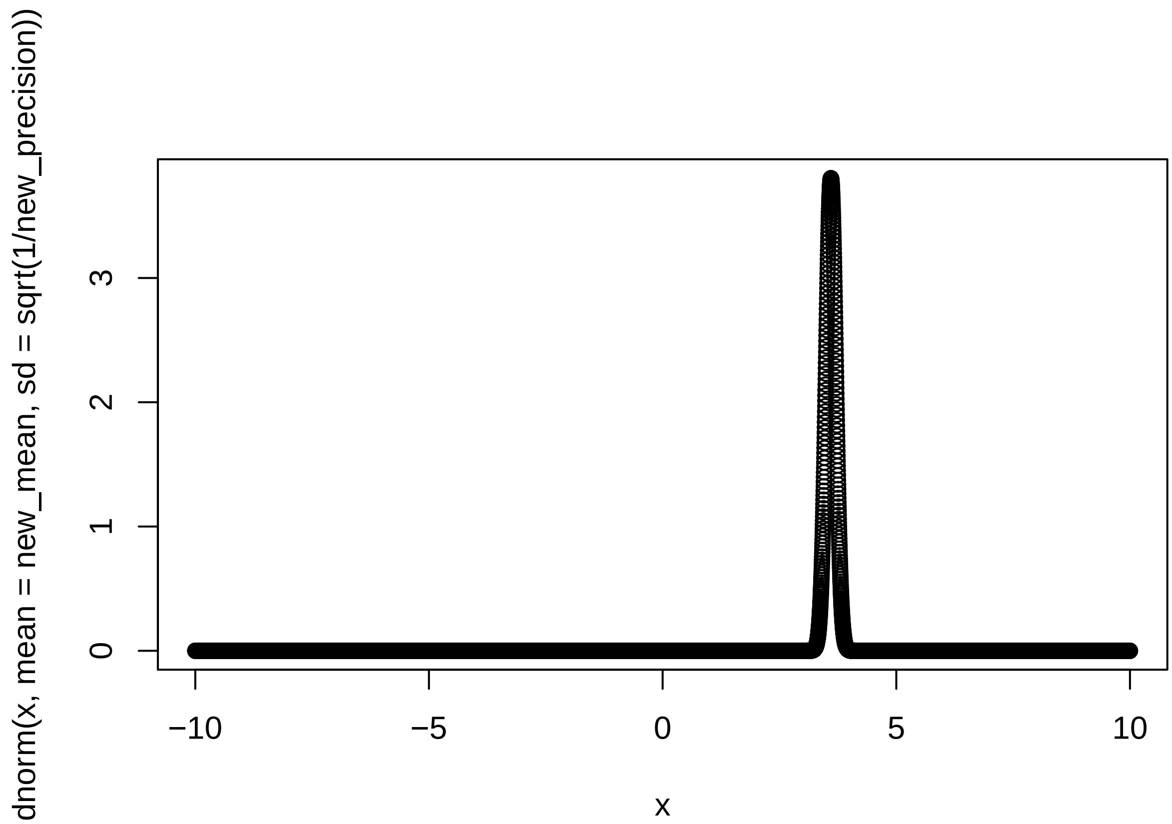
For Normal likelihood function

If the $X_1, X_2, ..., X_n$ is a random sample from a Normal distribution with unknown mean W and a known precision r. Suppose the prior of W is a normal distribution with mean μ and precision τ. Then the posterior distribution of W is a normal distribution with mean

$$\mu' = \frac{\tau\mu + nr\bar{X}}{\tau + nr}$$

and precision $\tau + nr$

```
n2<-50
x2<-rnorm(n2, mean = 4, sd=0.75)
norm_conju<-function(vec, mean, precision, r){
  len=length(vec)
  new_mean=(precision*mean+ len*r*mean(vec))/(precision+n*r)
  new_precision=precision+n*r
  x<-x <- seq(-10, 10, length=10000)
 plot(x, dnorm(x, mean=new_mean, sd=sqrt(1/new_precision)))
}
norm_conju(x2, -11, 2, r=1/0.75^2)
```

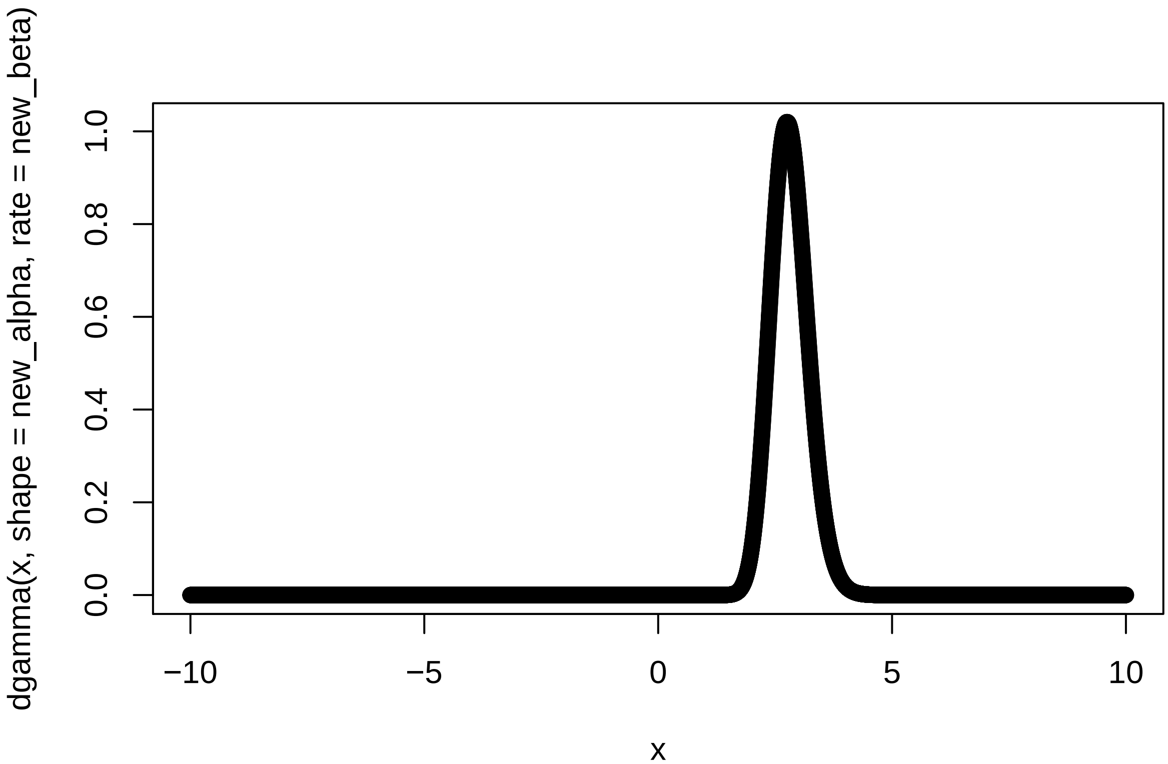


For exponential likelihood function

If the $X_1, X_2, ..., X_n$ is a random sample from a exponential distribution with an unknown value of the parameter W . Supoose the prior for W is a gamma distriution with parameters α(shape) and β(rate) . Then the posterior distribution of W is a gamma distribution with parameters $\alpha + n$ and $\beta + \sum_{i=1}^{n} X_i$

```
n3<-50
x3<-rexp(n3, rate=3)
exp_conju<-function(vec, alpha, beta){
len=length(vec)
new_alpha=alpha+len
```

```
new_beta=beta+sum(vec)
x<-x <- seq(-10, 10, length=10000)
plot(x, dgamma(x, shape=new_alpha, rate = new_beta ))
}
exp_conju(x3, alpha= 0.76, beta=0.2)
```



For Uniform(0, W) likelihood function

If the $X_1, X_2, ..., X_n$ is a random sample from a Uniform(0, W) distribution with an unknown value of the parameter W. Supoose W follows a Pareto distribution with parameters ω_0 and α . Then the posterior distribution is a Pareto distribution with parameters $\omega_0' = max \omega_0, x_1, ..., x_n$ and $\alpha + n$

```
n4<-50
x4<-runif(n3)
pareto<-function(x, eta, theta){
  return ( (theta*eta^(theta))/x^(theta+1) )
}
unif_conju<-function(vec, omega0, alpha){
  len=length(vec)
  omega1<-max(c(omega0, vec))
  new_alpha=alpha+len
  x<-x <- seq(0, 10, length=10000)
  plot(x, log10(pareto(x, omega1, new_alpha) ) )
```

```
}
unif_conju(x4, 1, -49.5)
```