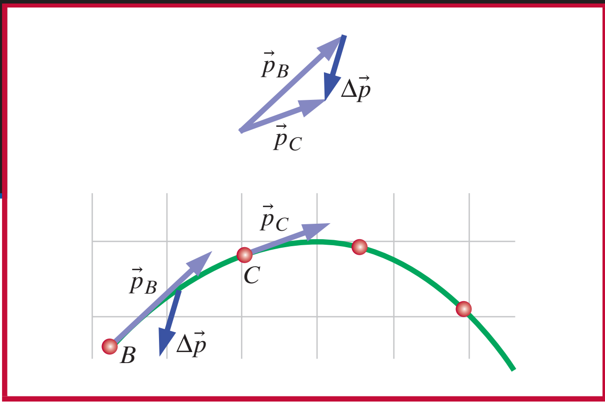
CHAPTER 1

Interactions and Motion



This textbook deals with the nature of matter and its interactions. The variety of phenomena that we will be able to explain and understand is very wide, including the orbit of stars around a black hole, nuclear fusion, and the speed of sound in a solid.

The main goal of this textbook is to have you engage in a process central to science: the attempt to explain in detail a broad range of phenomena using a small set of powerful fundamental principles.

The specific focus is on learning how to model the nature of matter and its interactions in terms of a small set of physical laws that govern all mechanical interactions, and in terms of the atomic structure of matter.

KEY IDEAS

Chapter 1 introduces the notion of interactions between material objects and the changes they produce.

- **Fundamental physics principles apply to all kinds of matter, from galaxies to subatomic particles.**
- **Change is an indication of an interaction.**
- **Position and motion in 3D space can be described precisely by vectors.**
- **The momentum of an object depends on both mass and velocity.**

■ Reading a science textbook efficiently and productively requires "active" reading. To encourage active reading, you will find "Stop and Think" questions, indicated by QUESTION. Trying to answer a question by using what you already know, as well as material you have just read, helps you learn more than if you just read passively. You might cover the rest of the page while you're thinking.

1.1 KINDS OF MATTER

We will deal with material objects of many sizes, from subatomic particles to galaxies. All of these objects have certain things in common.

Atoms and Nuclei

Ordinary matter is made up of tiny atoms. An atom isn't the smallest type of matter, for it is composed of even smaller objects (electrons, protons, and neutrons), but many of the ordinary everyday properties of ordinary matter can be understood in terms of atomic properties and interactions. As you probably know from studying chemistry, atoms have a very small, very dense core, called the nucleus, around which is found a cloud of electrons. The nucleus contains protons and neutrons, collectively called nucleons. Electrons are kept close to the nucleus by electric attraction to the protons (the neutrons hardly interact with the electrons).

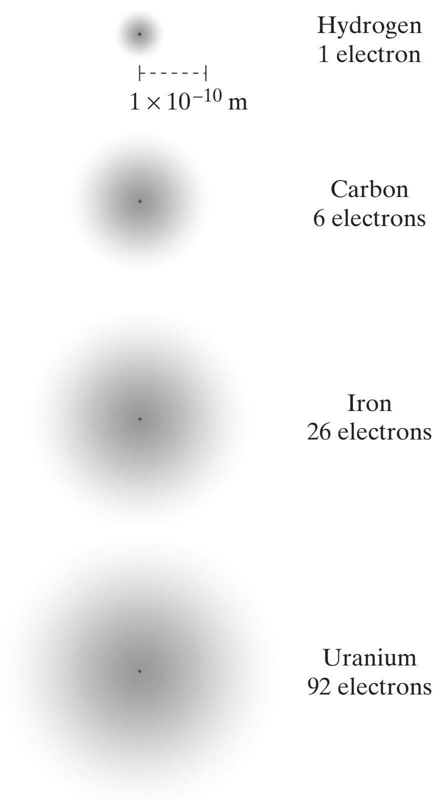


Figure 1.1 Atoms of hydrogen, carbon, iron, and uranium. The gray blur represents the electron cloud surrounding the nucleus. The black dot shows the location of the nucleus. On this scale, however, the nucleus would be much too small to see.

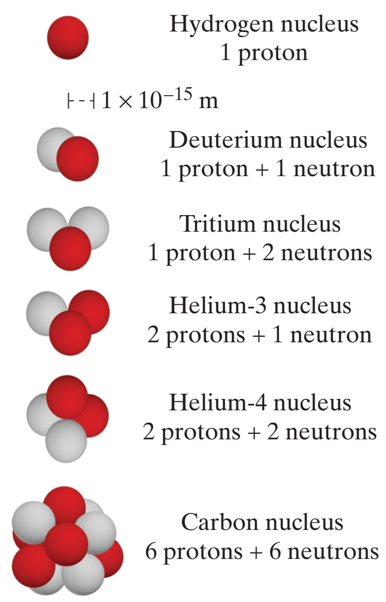


Figure 1.2 Nuclei of hydrogen, helium, and carbon. Note the *very* much smaller scale than in Figure 1.1!

QUESTION Recall your previous studies of chemistry. How many protons and electrons are there in a hydrogen atom? In a helium or carbon atom?

If you don't remember the properties of these atoms, see the periodic table on the inside front cover of this textbook. Hydrogen is the simplest atom, with just one proton and one electron. A helium atom has two protons and two electrons. A carbon atom has six protons and six electrons. Near the other end of the chemical periodic table, a uranium atom has 92 protons and 92 electrons. Figure 1.1 shows the relative sizes of the electron clouds in atoms of several elements but cannot show the nucleus to the same scale; the tiny dot marking the nucleus in the figure is much larger than the actual nucleus.

The radius of the electron cloud for a typical atom is about 1×10^{-10} meter. The reason for this size can be understood using the principles of quantum mechanics, a major development in physics in the early 20th century. The radius of a proton is about 1×10^{-15} meter, very much smaller than the radius of the electron cloud.

Nuclei contain neutrons as well as protons (Figure 1.2). The most common form or "isotope" of hydrogen has no neutrons in the nucleus. However, there exist isotopes of hydrogen with one or two neutrons in the nucleus (in addition to the proton). Hydrogen atoms containing one or two neutrons are called deuterium or tritium. The most common isotope of helium has two neutrons (and two protons) in its nucleus, but a rare isotope has only one neutron; this is called helium-3.

The most common isotope of carbon has six neutrons together with the six protons in the nucleus (carbon-12), whereas carbon-14 with eight neutrons is an isotope that plays an important role in dating archaeological objects.

Near the other end of the periodic table, uranium-235, which can undergo a fission chain reaction, has 92 protons and 143 neutrons, whereas uranium-238, which does not undergo a fission chain reaction, has 92 protons and 146 neutrons.

Molecules and Solids

When atoms come in contact with each other, they may stick to each other ("bond" to each other). Several atoms bonded together can form a molecule—a substance whose physical and chemical properties differ from those of the constituent atoms. For example, water molecules (H_2O) have properties quite different from the properties of hydrogen atoms or oxygen atoms.

An ordinary-sized rigid object made of bound-together atoms and big enough to see and handle is called a solid, such as a bar of aluminum. A new kind of microscope, the scanning tunneling microscope (STM), is able to map the locations of atoms on the surface of a solid, which has provided new techniques for investigating matter at the atomic level. Two such images appear in Figure 1.3. You can see that atoms in a crystalline solid are arranged in a regular three-dimensional array. The arrangement of atoms on the surface depends on the direction along which the crystal is cut. The irregularities in the bottom image reflect "defects," such as missing atoms, in the crystal structure.

Liquids and Gases

When a solid is heated to a higher temperature, the atoms in the solid vibrate more vigorously about their normal positions. If the temperature is raised high enough, this thermal agitation may destroy the rigid structure of the solid. The atoms may become able to slide over each other, in which case the substance is a liquid.

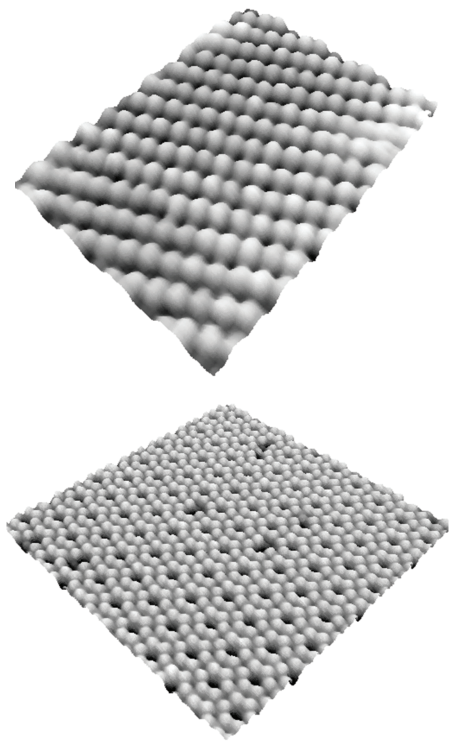


Figure 1.3 Two different surfaces of a crystal of pure silicon. The images were made with a scanning tunneling microscope. Images courtesy of Randall Feenstra, Carnegie Mellon University.

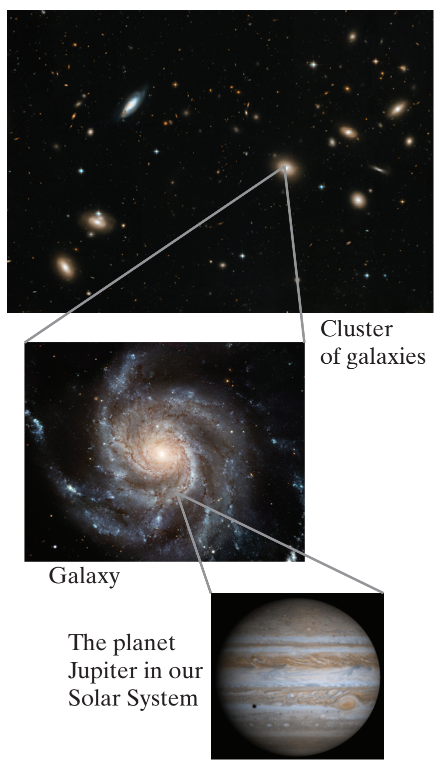


Figure 1.4 Our Solar System exists inside a galaxy, which itself is a member of a cluster of galaxies.

At even higher temperatures the thermal motion of the atoms or molecules may be so large as to break the interatomic or intermolecular bonds completely, and the liquid turns into a gas. In a gas the atoms or molecules are quite free to move around, only occasionally colliding with each other or the walls of their container.

We will learn how to analyze many aspects of the behavior of solids and gases. We won't have much to say about liquids, because their properties are much harder to analyze. Solids are simpler to analyze than liquids because the atoms stay in one place (though with thermal vibration about their usual positions). Gases are simpler to analyze than liquids because between collisions the gas molecules are approximately unaffected by the other molecules. Liquids are the awkward intermediate state, where the atoms move around rather freely but are always in contact with other atoms. This makes the analysis of liquids very complex.

Planets, Stars, Solar Systems, and Galaxies

In our brief survey of the kinds of matter that we will study, we make a giant leap in scale from atoms all the way up to planets and stars, such as our Earth and Sun. We will see that many of the same principles that apply to atoms apply to planets and stars. By making this leap we bypass an important physical science, geology, whose domain of interest includes the formation of mountains and continents. We will study objects that are much bigger than mountains, and we will study objects that are much smaller than mountains, but we don't have time to apply the principles of physics to every important kind of matter.

Our Sun and its accompanying planets constitute our Solar System. It is located in the Milky Way galaxy, a giant rotating disk-shaped system of stars. On a clear dark night you can see a band of light (the Milky Way) coming from the huge number of stars lying in this disk, which you are looking at from a position in the disk, about two-thirds of the way out from the center of the disk. Our galaxy is a member of a cluster of galaxies that move around each other much as the planets of our Solar System move around the Sun (Figure 1.4). The Universe contains many such clusters of galaxies.

1.2 DETECTING INTERACTIONS

Objects made of different kinds of matter interact with each other in various ways: gravitationally, electrically, magnetically, and through nuclear interactions. How can we detect that an interaction has occurred? In this section we consider various kinds of observations that indicate the presence of interactions.

QUESTION Before you read further, take a moment to think about your own ideas of interactions. How can you tell that two objects are interacting with each other?

Change of Direction of Motion

Suppose that you observe a proton moving through a region of outer space, far from almost all other objects. The proton moves along a path like the one shown in Figure 1.5. The arrow indicates the initial direction of the proton's motion, and the "$\times$'s" in the diagram indicate the position of the proton at equal time intervals.

QUESTION Do you see evidence in Figure 1.5 that the proton is interacting with another object?

Evidently a change in direction is a vivid indicator of interactions. If you observe a change in direction of the motion of a proton, you will find another object somewhere that has interacted with this proton.

> **QUESTION** Suppose that the only other object nearby was another proton. What was the approximate initial location of this second proton?

Since two protons repel each other electrically, the second proton must have been located to the right of the bend in the first proton's path.

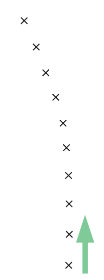


Figure 1.5 A proton moves through space, far from almost all other objects. The initial direction of the proton's motion is upward, as indicated by the arrow. The $\times$'s represent the position of the proton at equal time intervals.

Change of Speed

Suppose that you observe an electron traveling in a straight line through outer space far from almost all other objects (Figure 1.6). The path of the electron is shown as though a camera had taken multiple exposures at equal time intervals.

> **QUESTION** Where is the electron's speed largest? Where is the electron's speed smallest?

The speed is largest at the upper left, where the $\times$'s are farther apart, which means that the electron has moved farthest during the time interval between exposures. The speed is smallest at the bottom right, where the $\times$'s are closer together, which means that the electron has moved the least distance during the time interval between exposures.

> **QUESTION** Suppose that the only other object nearby was another electron. What was the approximate initial location of this other electron?

The other electron must have been located directly just below and to the right of the starting location, since electrons repel each other electrically.

Evidently a change in speed is an indicator of interactions. If you observe a change in speed of an electron, you will find another object somewhere that has interacted with the electron.

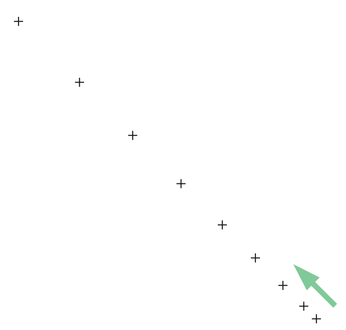


Figure 1.6 An electron moves through space, far from almost all other objects. The initial direction of the electron's motion is upward and to the left, as indicated by the arrow. The $\times$'s represent the position of the electron at equal time intervals.

Change of Velocity: Change of Speed or Direction

In physics, the word "velocity" has a special technical meaning that is different from its meaning in everyday speech. In physics, the quantity called "velocity" denotes a combination of speed and direction. (In contrast, in everyday speech, "speed" and "velocity" are often used as synonyms. In physics and other sciences, however, words have rather precise meanings and there are few synonyms.)

For example, consider an airplane that is flying with a speed of 1000 kilometers/hour in a direction that is due east. We say the velocity is 1000 km/hr, east, where we specify both speed and direction. An airplane flying west with a speed of 1000 km/hr would have the same speed but a different velocity.

We have seen that a change in an object's speed, or a change in the direction of its motion, indicates that the object has interacted with at least one other object. The two indicators of interaction, change of speed and change of direction, can be combined into one compact statement:

> A change of velocity (speed or direction or both) indicates the existence of an interaction.

Pushing forward or back, parallel to the direction of the motion, will make an object speed up or slow down but cannot make it turn. To change the direction of the motion you have to push sideways, perpendicular to the motion.

Figure 1.7 Two successive positions of a particle (indicated by a dot), with arrows indicating the velocity of the particle at each location. The shorter arrow indicates that the speed of the particle at location 2 is less than its speed at location 1.

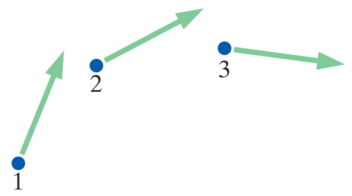


Figure 1.8 Three successive positions of a particle (indicated by a dot), with arrows indicating the velocity of the particle at each location. The arrows are the same length, indicating the same speed, but they point in different directions, indicating a change in direction and therefore a change in velocity.

Figure 1.9 "Uniform motion"—no change in speed or direction.

Diagrams Showing Changes in Velocity

In physics diagrams, the velocity of an object is represented by an arrow: a line with an arrowhead. The tail of the arrow is placed at the location of the object, and the arrow points in the direction of the motion of the object. The length of the arrow is proportional to the speed of the object. Figure 1.7 shows two successive positions of a particle at two different times, with velocity arrows indicating a change in speed of the particle (it's slowing down). Figure 1.8 shows three successive positions of a different particle at three different times, with velocity arrows indicating a change in direction but no change in speed.

We will see a little later in the chapter that velocity is only one example of a physical quantity that has a "magnitude" (an amount or a size) and a direction.

Other examples of such quantities are position relative to an origin in 3D space, force, and magnetic field. Quantities having magnitude and direction can be usefully described as "vectors." Vectors are mathematical quantities that have their own special rules of algebra, similar (but not identical) to the rules of ordinary algebra. Arrows are commonly used in diagrams to denote vector quantities. We will use vectors extensively.

Uniform Motion

Suppose that you observe a rock moving along in outer space far from all other objects. We don't know what made it start moving in the first place; presumably a long time ago an interaction gave it some velocity and it has been coasting through the vacuum of space ever since.

It is an observational fact that such an isolated object moves at constant, unchanging speed, in a straight line. Its velocity does not change (neither its direction nor its speed changes). We call such motion with unchanging velocity "uniform motion" (Figure 1.9).

An Object at Rest

A special case of uniform motion is the case in which an object's speed is zero and remains zero—the object remains at rest. In this case the object's speed is constant (zero) and the direction of motion, while undefined, is not changing.

Uniform Motion Implies No Net Interaction

When we observe an object in uniform motion, we conclude that since its velocity is not changing, either it is not interacting significantly with any other object, or else it is undergoing multiple interactions that cancel each other out. In either case, we can say that there is no "net" (total) interaction.

Exercises

At the end of a section you will usually find short exercises. It is important to work through exercises as you come to them, to make sure that you can apply what you have just read. Simply reading about concepts in physics is not enough—you must be able to use the concepts in answering questions and solving problems. Make a serious attempt to do an exercise before checking the answer at the end of the chapter. This will help you assess your own understanding.

1.X.1 Which of the following do you see moving with constant velocity?

- **(a)** A ship sailing northeast at a speed of 5 meters per second
- **(b)** The Moon orbiting the Earth
- **(c)** A tennis ball traveling across the court after having been hit by a tennis racket
- **(d)** A can of soda sitting on a table
- **(e)** A person riding on a Ferris wheel that is turning at a constant rate

1.X.2 In which of the following situations is there observational evidence for significant interaction between two objects? How can you tell?

- **(a)** A ball bounces off a wall with no change in speed.
- **(b)** A baseball that was hit by a batter flies toward the outfield.
- **(c)** A communications satellite orbits the Earth.

(d) A space probe travels at constant speed toward a distant star.
(e) A charged particle leaves a curving track in a particle detector.

1.3 NEWTON'S FIRST LAW OF MOTION

The basic relationship between change of velocity and interaction is summarized qualitatively by Newton's "first law of motion":

NEWTON'S FIRST LAW OF MOTION

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects.

The words "to the extent" imply that the stronger the interaction, the more change there will be in direction and/or speed. The weaker the interaction, the less change. If there is no net interaction at all, the direction doesn't change and the speed doesn't change (uniform motion). This case can also be called "uniform velocity" or "constant velocity," since velocity refers to both speed and direction. It is important to remember that if an object is not moving at all, its velocity is not changing, so it too may be considered to be in uniform motion.

Newton's first law of motion is only qualitative, because it doesn't give us a way to calculate quantitatively how much change in speed or direction will be produced by a certain amount of interaction, a subject we will take up in the next chapter. Nevertheless, Newton's first law of motion is important in providing a conceptual framework for thinking about the relationship between interaction and motion.

The English physicist Isaac Newton was the first person to state this law clearly. Newton's first law of motion represented a major break with ancient tradition, which assumed that constant pushing was required to keep something moving. This law says something radically different: no interactions at all are needed to keep something moving!

Does Newton's First Law Apply in Everyday Life?

Superficially, Newton's first law of motion may seem at first not to apply to many everyday situations. To push a chair across the floor at constant speed, you have to keep pushing all the time.

QUESTION Doesn't Newton's first law of motion say that the chair should keep moving at constant speed without anyone pushing it? In fact, shouldn't the speed or direction of motion of the chair change due to the interaction with your hands? Does this everyday situation violate Newton's first law of motion? Try to answer these questions before reading farther.

The complicating factor here is that your hands aren't the only objects that are interacting with the chair. The floor also interacts with the chair, in a way that we call friction. If you push just hard enough to compensate exactly for the floor friction, the sum of all the interactions is zero, and the chair moves at constant speed as predicted by Newton's first law. (If you push harder than the floor does, the chair's speed does increase.)

Motion without Friction

It is difficult to observe motion without friction in everyday life, because objects almost always interact with many other objects, including air, flat surfaces, and so on. This explains why it took people such a long time to understand clearly the relationship between interaction and change (Newton was born in 1642).

You may be able to think of situations in which you have seen an object keep moving at constant (or nearly constant) velocity, without being pushed or pulled. One example of a nearly friction-free situation is a hockey puck sliding on ice. The puck slides a long way at nearly constant speed in a straight line (constant velocity) because there is little friction with the ice. An even better example is the uniform motion of an object in outer space, far from all other objects.

1.X.3 Apply Newton's first law to each of the following situations. In which situations can you conclude that the object is undergoing a net interaction with one or more other objects?

(a) A book slides across the table and comes to a stop.
(b) A proton in a particle accelerator moves faster and faster.
(c) A car travels at constant speed around a circular race track.
(d) A spacecraft travels at a constant speed toward a distant star.
(e) A hydrogen atom remains at rest in outer space.

1.X.4 A spaceship far from all other objects uses its rockets to attain a speed of 1×10^4 m/s. The crew then shuts off the power. According to Newton's first law, which of the following statements about the motion of the spaceship after the power is shut off are correct? (Choose all statements that are correct.)

(a) The spaceship will move in a straight line.
(b) The spaceship will travel on a curving path.
(c) The spaceship will enter a circular orbit.
(d) The speed of the spaceship will not change.
(e) The spaceship will gradually slow down.
(f) The spaceship will stop suddenly.

1.4 OTHER INDICATORS OF INTERACTION

Change of Identity

Change of velocity (change of speed and/or direction) is not the only indicator of interactions. Another is change of identity, such as the formation of water (H_2O) from the burning of hydrogen in oxygen. A water molecule behaves very differently from the hydrogen and oxygen atoms of which it is made.

Change of Shape or Configuration

Another indicator of interaction is change of shape or configuration (arrangement of the parts). For example, slowly bend a pen or pencil, then hold it in the bent position. The speed hasn't changed, nor is there a change in the direction of motion (it's not moving!). The pencil has not changed identity. Apparently a change of shape can be evidence for interactions, in this case with your hand.

Other changes in configuration include "phase changes" such as the freezing or boiling of a liquid, brought about by interactions with the surroundings. In different phases (solid, liquid, gas), atoms or molecules are arranged differently. Changes in configuration at the atomic level are another indication of interactions.

Change of Temperature

Another indication of interaction is change of temperature. Place a pot of cold water on a hot stove. As time goes by, a thermometer will indicate a change in the water due to interaction with the hot stove.

Other Indications of Interactions

Is a change of position an indicator of an interaction? That depends. If the change of position occurs simply because a particle is moving at constant speed and direction, then a mere change of position is not an indicator of an interaction, since uniform motion is an indicator of no net interaction.

> **QUESTION** However, if you observe an object at rest in one location, and later you observe it again at rest but in a different location, did an interaction take place?

Yes. You can infer that there must have been an interaction to give the object some velocity to move the object toward the new position, and another interaction to slow the object to a stop in its new position.

In later chapters we will consider interactions involving change of identity, change of shape, and change of temperature, but for now we'll concentrate on interactions that cause a change of velocity (speed and/or direction).

Indirect Evidence for an Additional Interaction

Sometimes there is indirect evidence for an additional interaction. When something doesn't change although you would normally expect a change due to a known interaction, this indicates that another interaction is present. Consider a balloon that hovers motionless in the air despite the downward gravitational pull of the Earth. Evidently there is some other kind of interaction that opposes the gravitational interaction. In this case, interactions with air molecules have the net effect of pushing up on the balloon ("buoyancy"). The lack of change implies that the effect of the air molecules exactly compensates for the gravitational interaction with the Earth.

When you push a chair across the floor and it moves with constant velocity despite your pushing on it (which ought to change its speed), that means that something else must also be interacting with it (the floor).

The stability of the nucleus of an atom is another example of indirect evidence for an additional interaction. The nucleus contains positively charged protons that repel each other electrically, yet the nucleus remains intact. We conclude that there must be some other kind of interaction present, a non-electric attractive interaction that overcomes the electric repulsion. This is evidence for an interaction called the "strong interaction" that acts between protons and neutrons in the nucleus.

Summary: Changes as Indicators of Interactions

Here then are the most common indicators of interactions:

- Change of velocity (change of direction and/or change of speed)
- Change of identity
- Change of shape

- Change of temperature
- Lack of change when change is expected (indirect evidence)

The important point is this: **Interactions cause change.**

In the absence of interactions, there is no change, which is usually uninteresting. The exception is the surprise when nothing changes despite our expectations that something should change. This is indirect evidence for some interaction that we hadn't recognized was present, that more than one interaction is present and the interactions cancel each others' effects.

For the next few chapters we'll concentrate on change of velocity as evidence for an interaction (or lack of change of velocity, which can give indirect evidence for additional interactions).

1.X.5 You slide a coin across the floor, and observe that it travels in a straight line, slowing down and eventually stopping. A sensitive thermometer shows that the coin's temperature increased. What can we conclude? (Choose all statements that are correct.)

(a) Because the coin traveled in a straight line, we conclude that it did not interact with anything.

(b) Because the coin did not change shape, we conclude that it did not interact with anything.

(c) Because the coin slowed down, we conclude that Newton's first law does not apply to objects in everyday life, such as coins.

(d) Because the coin's speed changed, we conclude that it interacted with one or more other objects.

(e) Because the coin got hot, we conclude that it interacted with one or more other objects.

1.5 DESCRIBING THE 3D WORLD: VECTORS

Physical phenomena take place in the 3D world around us. In order to be able to make quantitative predictions and give detailed, quantitative explanations, we need tools for describing precisely the positions and velocities of objects in 3D, and the changes in position and velocity due to interactions. These tools are mathematical entities called 3D "vectors."

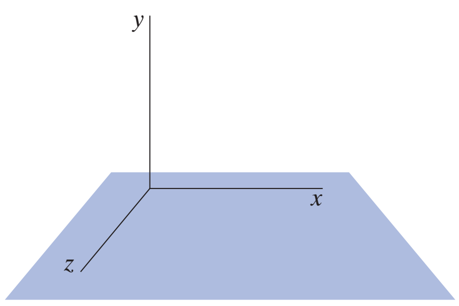


Figure 1.10 Right-handed 3D coordinate system.

3D Coordinates

We will use a 3D coordinate system to specify positions in space and other vector quantities. Usually we will orient the axes of the coordinate system as shown in Figure 1.10: $+x$ axis to the right, $+y$ axis upward, and $+z$ axis coming out of the page, toward you. This is a "right-handed" coordinate system: if you hold the thumb, first, and second fingers of your right hand perpendicular to each other, and align your thumb with the x axis and your first finger with the y axis, your second finger points along the z axis. (In some math and physics textbook discussions of 3D coordinate systems, the x axis points out, the y axis points to the right, and the z axis points up, but we will also use a 2D coordinate system with y up, so it makes sense always to have the y axis point up.)

Basic Properties of Vectors: Magnitude and Direction

A vector is a quantity that has a magnitude and a direction. For example, the velocity of a baseball is a vector quantity. The magnitude of the baseball's velocity is the speed of the baseball—for example, 20 meters/second. The direction of the baseball's velocity is the direction of its motion at a particular instant—for example, "up" or "to the right" or "west" or "in the $+y$ direction."

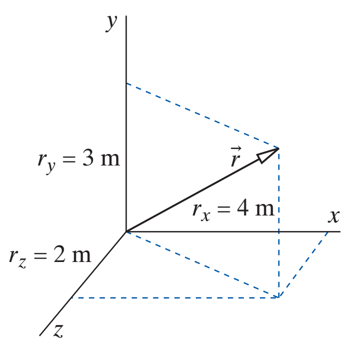


Figure 1.11 A position vector $\vec{r} = \langle 4, 3, 2\rangle$ m and its x, y, and z components.

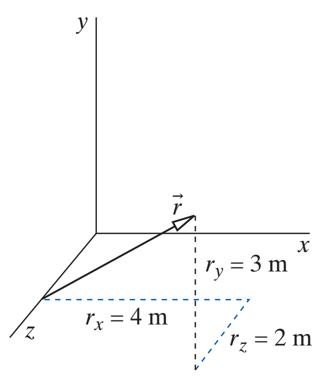


Figure 1.12 The arrow represents the vector $\vec{r} = \langle 4, 3, 2\rangle$ m, drawn with its tail at location $\langle 0, 0, 2\rangle$.

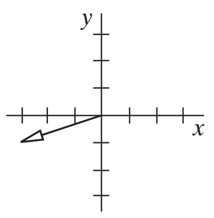


Figure 1.13 The position vector $\langle -3, -1, 0\rangle$, drawn at the origin, in the xy plane. The components of the vector specify the displacement from the tail to the tip. The z axis, which is not shown, comes out of the page, toward you.

A symbol denoting a vector is written with an arrow over it:

$$\vec{v} \text{ is a vector}$$

Position

A position in space can also be considered to be a vector, called a *position vector*, pointing from an origin to that location. Figure 1.11 shows a position vector that might represent your final position if you started at the origin and walked 4 meters along the x axis, then 2 meters parallel to the z axis, then climbed a ladder so you were 3 meters above the ground. Your new position relative to the origin is a vector that can be written like this:

$$\vec{r} = \langle 4, 3, 2\rangle \text{ m}$$
$$x \text{ component } r_x = 4 \text{ m}$$
$$y \text{ component } r_y = 3 \text{ m}$$
$$z \text{ component } r_z = 2 \text{ m}$$

In three dimensions a vector is a triple of numbers $\langle x, y, z\rangle$. Quantities like the position of an object and the velocity of an object can be represented as vectors:

$$\vec{r} = \langle x, y, z\rangle \text{ (a position vector)}$$
$$\vec{r}_1 = \langle 3.2, -9.2, 66.3\rangle \text{ m (a position vector)}$$
$$\vec{v} = \langle v_x, v_y, v_z\rangle \text{ (a velocity vector)}$$
$$\vec{v}_1 = \langle -22.3, 0.4, -19.5\rangle \text{ m/s (a velocity vector)}$$

Components of a Vector

Each of the numbers in the triple is referred to as a *component* of the vector. The x component of the vector $\vec{v}$ is the number v_x. The z component of the vector $\vec{v}_1 = \langle -22.3, 0.4, -19.5\rangle$ m/s is -19.5 m/s. A component such as v_x is not a vector, since it is only one number.

It is important to note that the x component of a vector specifies the difference between the x coordinate of the tail of the vector and the x coordinate of the tip of the vector. It does not give any information about the location of the tail of the vector (compare Figure 1.11 and Figure 1.12).

Drawing Vectors

In Figure 1.11 we represented your position vector relative to the origin graphically by an arrow whose tail is at the origin and whose arrowhead is at your position. The length of the arrow represents the distance from the origin, and the direction of the arrow represents the direction of the vector, which is the direction of a direct path from the initial position to the final position (the "displacement"; by walking and climbing you "displaced" yourself from the origin to your final position).

Since it is difficult to draw a 3D diagram on paper, when working on paper you will usually be asked to draw vectors that all lie in a single plane. Figure 1.13 shows an arrow in the xy plane representing the vector $\langle -3, -1, 0\rangle$.

Vectors and Scalars

A quantity that is represented by a single number is called a *scalar*. A scalar quantity does not have a direction. Examples include the mass of an object, such as 5 kg, or the temperature, such as -20°C. Vectors and scalars are very different entities; a vector can never be equal to a scalar, and a scalar cannot be added to a vector. Scalars can be positive or negative:

$$m = 50\,\text{kg}$$
$$T = -20^\circ\text{C}$$

Although a component of a vector such as v_x is not a vector, it's not a scalar either, despite being only one number. An important property of a true scalar is that its value doesn't change if we orient the xyz coordinate axes differently. Rotating the axes doesn't change an object's mass, or the temperature, but it does change what we mean by the x component of the velocity since the x axis now points in a different direction.

> **1.X.6** How many numbers are needed to specify a 3D position vector?
> **1.X.7** How many numbers are needed to specify a scalar?

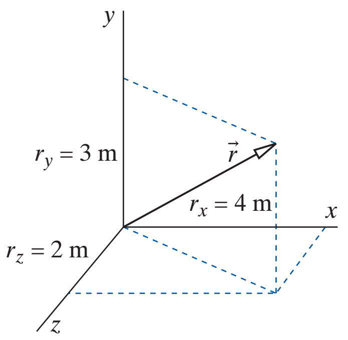


Figure 1.14 A vector representing a displacement from the origin.

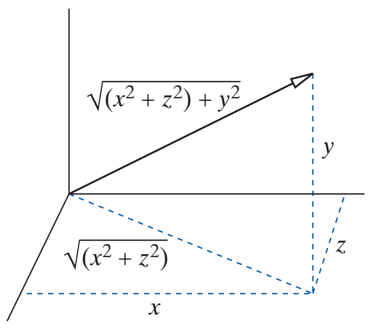


Figure 1.15 The magnitude of a vector is the square root of the sum of the squares of its components (3D version of the Pythagorean theorem).

Magnitude of a Vector

In Figure 1.14 we again show the vector from Figure 1.11, showing your displacement from the origin. Using a 3D extension of the Pythagorean theorem for right triangles (Figure 1.15), the net distance you have moved from the starting point is

$$\sqrt{(4\,\text{m})^2 + (3\,\text{m})^2 + (2\,\text{m})^2} = \sqrt{29}\,\text{m} = \sqrt{5.39}\,\text{m}$$

We say that the *magnitude* $|\vec{r}|$ of the position vector $\vec{r}$ is

$$|\vec{r}| = 5.39\,\text{m}$$

The magnitude of a vector is written either with absolute-value bars around the vector as $|\vec{r}|$, or simply by writing the symbol for the vector without the little arrow above it, r.

The magnitude of a vector can be calculated by taking the square root of the sum of the squares of its components (see Figure 1.15).

MAGNITUDE OF A VECTOR

If the vector $\vec{r} = \langle r_x, r_y, r_z\rangle$ then $|\vec{r}| = \sqrt{r_x^2 + r_y^2 + r_z^2}$ (a scalar).

The magnitude of a vector is always a positive number. The magnitude of a vector is a single number, not a triple of numbers, and it is a scalar, not a vector.

The magnitude of a vector is a true scalar, because its value doesn't change if you rotate the coordinate axes. Rotating the axes changes the individual components, but the length of the arrow representing the vector doesn't change.

Can a *Vector* Be Positive or Negative?

QUESTION Consider the vector $\vec{v} = \langle 8\times 10^6, 0, -2\times 10^7\rangle$ m/s. Is this vector positive? Negative? Zero?

None of these descriptions is appropriate. The x component of this vector is positive, the y component is zero, and the z component is negative. Vectors aren't positive, or negative, or zero. Their components can be positive or negative or zero, but these words just don't mean anything when used with the vector as a whole.

On the other hand, the *magnitude* of a vector such as $|\vec{v}|$ is always positive.

1.X.8 If $\vec{r} = \langle -3, -4, 1 \rangle$ m, find $|\vec{r}|$.

1.X.9 Can the magnitude of a vector be a negative number?

Mathematical Operations Involving Vectors

Although the algebra of vectors is similar to the scalar algebra with which you are very familiar, it is not identical. There are some algebraic operations that cannot be performed on vectors.

Algebraic operations that *are* legal for vectors include the following operations, which we will discuss in this chapter:

- Adding one vector to another vector: $\vec{a} + \vec{w}$
- Subtracting one vector from another vector: $\vec{b} - \vec{d}$
- Finding the magnitude of a vector: $|\vec{r}|$
- Finding a unit vector (a vector of magnitude 1): $\hat{r}$
- Multiplying (or dividing) a vector by a scalar: $3\vec{v}$ or $\vec{w}/2$
- Finding the rate of change of a vector: $\Delta\vec{r}/\Delta t$ or $d\vec{r}/dt$.

In later chapters we will also see that there are two more ways of combining two vectors:

The vector dot product, whose result is a scalar

The vector cross product, whose result is a vector

Operations That Are *Not* Legal for Vectors

Although vector algebra is similar to the ordinary scalar algebra you have used up to now, there are certain operations that are not legal (and not meaningful) for vectors:

A vector cannot be set equal to a scalar.

A vector cannot be added to or subtracted from a scalar.

A vector cannot occur in the denominator of an expression. (Although you can't divide by a vector, note that you can legally divide by the *magnitude* of a vector, which is a scalar.)

1.X.10 The vector $\vec{g} = \langle 2, -7, 3 \rangle$ and the scalar $h = -2$. What is $h + \vec{g}$?

(a) $\langle 0, -9, 1 \rangle$

(b) $\langle 4, -5, 5 \rangle$

(c) $\langle 4, 9, 5 \rangle$

(d) This is a meaningless expression.

1.X.11 Is $4/\langle 6, -7, 4 \rangle$ a meaningful expression? If so, what is its value?

Multiplying a Vector by a Scalar

A vector can be multiplied (or divided) by a scalar. If a vector is multiplied by a scalar, each of the components of the vector is multiplied by the scalar:

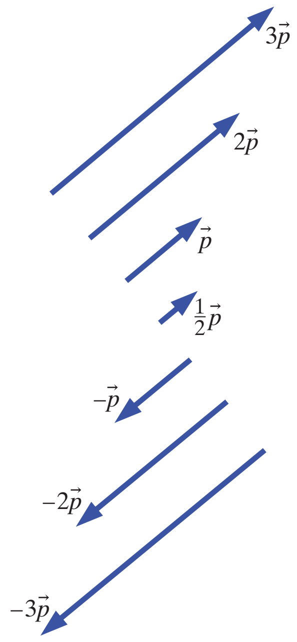


Figure 1.16 Multiplying a vector by a scalar changes the magnitude of the vector. Multiplying by a negative scalar reverses the direction of the vector.

$$\text{If } \vec{r} = \langle x, y, z\rangle,\ \text{then } a\vec{r} = \langle ax, ay, az\rangle$$

$$\text{If } \vec{v} = \langle v_x, v_y, v_z\rangle,\ \text{then } \frac{\vec{v}}{b} = \left\langle \frac{v_x}{b}, \frac{v_y}{b}, \frac{v_z}{b}\right\rangle$$

$$\left\langle\frac{1}{2}\right\rangle\langle 6, -20, 9\rangle = \langle 3, -10, 4.5\rangle$$

Multiplication by a scalar "scales" a vector, keeping its direction the same but making its magnitude larger or smaller (Figure 1.16). Multiplying by a negative scalar reverses the direction of a vector.

1.X.12 The vector $\vec{a} = \langle 0.03, -1.4, 26.0\rangle$ and the scalar $f = -3.0$. What is $f\vec{a}$?
1.X.13 If $\vec{r}\langle 2, -3, 5\rangle$ m, what is $\vec{r}/2$?

Magnitude of a Scalar

You may wonder how to find the magnitude of a quantity like $-3\vec{r}$, which involves the product of a scalar and a vector. This expression can be factored:

$$|-3\vec{r}| = |-3| \cdot |\vec{r}|$$

The magnitude of a scalar is its absolute value, so:

$$|-3\vec{r}| = |-3| \cdot |\vec{r}| = 3\sqrt{r_x^2 + r_y^2 + r_z^2}$$

1.X.14 If $\vec{v} = \langle 2, -3, 5\rangle$ m/s, what is the magnitude of $3\vec{v}$?
1.X.15 How does the magnitude of the vector $-\vec{a}$ compare to the magnitude of the vector $\vec{a}$?

Direction of a Vector: Unit Vectors

One way to describe the direction of a vector is by specifying a *unit vector*. A unit vector is a vector of magnitude 1, pointing in some direction. A unit vector is written with a "hat" (caret) over it instead of an arrow. The unit vector $\hat{a}$ is called "a-hat."

QUESTION Is the vector $\langle 1, 1, 1\rangle$ a unit vector?

The magnitude of $\langle 1, 1, 1\rangle$ is $\sqrt{1^2 + 1^2 + 1^2} = 1.73$, so this is not a unit vector.
The vector $\langle 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\rangle$ is a unit vector, since its magnitude is 1:

$$\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$$

Note that every component of a unit vector must be less than or equal to 1.

In our 3D Cartesian coordinate system, there are three special unit vectors, oriented along the three axes. They are called i-hat, j-hat, and k-hat, and they point along the x, y, and z axes, respectively (Figure 1.17):

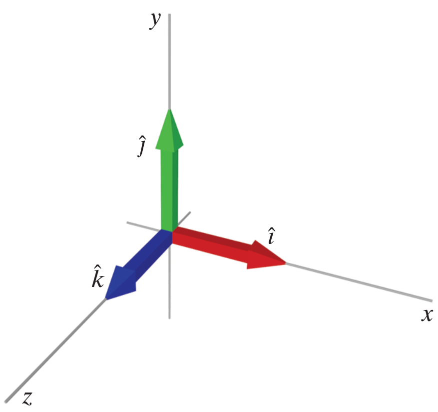


Figure 1.17 The unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$.

$$\hat{\imath} = \langle 1, 0, 0\rangle$$
$$\hat{\jmath} = \langle 0, 1, 0\rangle$$
$$\hat{k} = \langle 0, 0, 1\rangle$$

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One way to express a vector is in terms of these special unit vectors:

$$\langle 0.02, -1.7, 30.0\rangle = 0.02\hat{\imath} + (-1.7)\hat{\jmath} + 30.0\hat{k}$$

We will usually use the $\langle x, y, z\rangle$ form rather than the $\hat{\imath}\hat{\jmath}\hat{k}$ form in this book, because the familiar $\langle x, y, z\rangle$ notation, used in many calculus textbooks, emphasizes that a vector is a single entity.

Not all unit vectors point along an axis, as shown in Figure 1.18. For example, the vectors

$$\hat{g} = \langle 0.5774, 0.5774, 0.5774\rangle \quad \text{and} \quad \hat{F} = \langle 0.424, 0.566, 0.707\rangle$$

are both unit vectors, since the magnitude of each is equal to 1. Note that every component of a unit vector is less than or equal to 1.

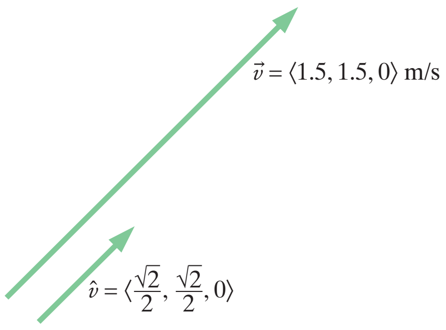

Figure 1.18 The unit vector $\hat{v}$ has the same direction as the vector $\vec{v}$, but its magnitude is 1, and it has no physical units.

Calculating Unit Vectors

Any vector may be factored into the product of a unit vector in the direction of the vector, multiplied by a scalar equal to the magnitude of the vector.

$$\vec{w} = |\vec{w}| \cdot \hat{w}$$

For example, a vector of magnitude 5, aligned with the y axis, could be written as:

$$\langle 0, 5, 0\rangle = 5\langle 0, 1, 0\rangle$$

Therefore, to find a unit vector in the direction of a particular vector, we just divide the vector by its magnitude:

CALCULATING A UNIT VECTOR

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\langle x, y, z\rangle}{\sqrt{(x^2 + y^2 + z^2)}}$$

$$\hat{r} = \left\langle \frac{x}{\sqrt{(x^2 + y^2 + z^2)}}, \frac{y}{\sqrt{(x^2 + y^2 + z^2)}}, \frac{z}{\sqrt{(x^2 + y^2 + z^2)}} \right\rangle$$

EXAMPLE

Unit Vector

If $\vec{v} = \langle -22.3, 0.4, -19.5\rangle$ m/s, then

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle -22.3, 0.4, -19.5\rangle \text{ m/s}}{\sqrt{(-22.3)^2 + (0.4)^2 + (-19.5)^2} \text{ m/s}} = \langle -0.753, 0.0135, -0.658\rangle$$

Remember that to divide a vector by a scalar, you divide each component of the vector by the scalar. The result is a new vector. Note also that a unit vector has no physical units (such as meters per second), because the units in the numerator and denominator cancel.

1.X.16 What is the unit vector in the direction of $\langle 0, 6, 0\rangle$?

1.X.17 What is the unit vector $\hat{a}$ in the direction of $\vec{a}$, where $\vec{a} = \langle 400, 200, -100\rangle$ m/s^2?

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Equality of Vectors

EQUALITY OF VECTORS

A vector is equal to another vector if and only if all the components of the vectors are equal.

$$\vec{w} = \vec{r} \quad \text{means that}$$
$$w_x = r_x \quad \text{and} \quad w_y = r_y \quad \text{and} \quad w_z = r_z$$

The magnitudes and directions of two equal vectors are the same:

$$|\vec{w}| = |\vec{r}| \quad \text{and} \quad \hat{w} = \hat{r}$$

EXAMPLE **Equal Vectors**

$$\vec{r} = \langle 4, 3, 2\rangle$$
$$|\vec{r}| = \sqrt{(4^2 + 3^2 + 2^2)} = 5.39$$
$$\hat{r} = \frac{\langle 4, 3, 2\rangle}{5.39} = \langle 0.742, 0.557, 0.371\rangle$$

If $\vec{w} = \vec{r}$, then

$$\vec{w} = \langle 4, 3, 2\rangle$$
$$|\vec{w}| = 5.39$$
$$\hat{w} = \langle 0.742, 0.557, 0.371\rangle$$

1.X.18 If $\vec{a} = \langle -3, 7, 0.5\rangle$, and $\vec{b} = \vec{a}$, what is the y component of $\vec{b}$?

1.X.19 Consider the vectors $\vec{r}_1$ and $\vec{r}_2$ represented by arrows in Figure 1.19. Are these two vectors equal?

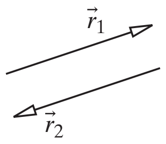

Figure 1.19 Are these two vectors equal? (Exercise 1.X.19.)

Vector Addition

ADDING VECTORS

The sum of two vectors is another vector, obtained by adding the components of the vectors.

$$\vec{A} = \langle A_x, A_y, A_z\rangle$$
$$\vec{B} = \langle B_x, B_y, B_z\rangle$$
$$\vec{A} + \vec{B} = \langle (A_x + B_x), (A_y + B_y), (A_z + B_z)\rangle$$

EXAMPLE **Adding Vectors**

$$\langle 1, 2, 3\rangle + \langle -4, 5, 6\rangle = \langle -3, 7, 9\rangle$$

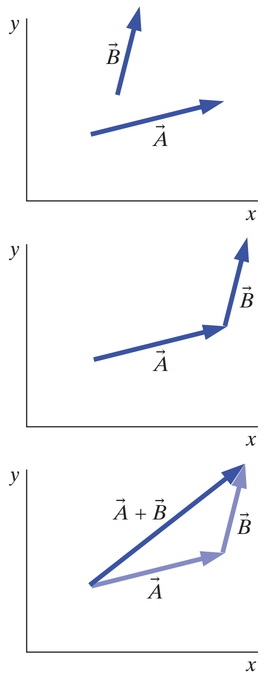

Figure 1.20 The procedure for adding two vectors graphically: draw vectors tip to tail. To add $\vec{A}+\vec{B}$ graphically, move $\vec{B}$ so the tail of $\vec{B}$ is at the tip of $\vec{A}$, then draw a new arrow starting at the tail of $\vec{A}$ and ending at the tip of $\vec{B}$.

Warning: Don't Add Magnitudes!

The magnitude of a vector is *not* in general equal to the sum of the magnitudes of the two original vectors! For example, the magnitude of the vector $\langle 3, 0, 0\rangle$ is 3, and the magnitude of the vector $\langle -2, 0, 0\rangle$ is 2, but the magnitude of the vector $(\langle 3, 0, 0\rangle + \langle -2, 0, 0\rangle)$ is 1, not 5!

Adding Vectors Graphically: Tip to Tail

The sum of two vectors has a geometric interpretation. In Figure 1.20 you first walk along displacement vector $\vec{A}$, followed by walking along displacement vector $\vec{B}$. What is your net displacement vector $\vec{C} = \vec{A}+\vec{B}$? The x component C_x of your net displacement is the sum of A_x and B_x. Similarly, the y component C_y of your net displacement is the sum of A_y and B_y.

GRAPHICAL ADDITION OF VECTORS

To add two vectors $\vec{A}$ and $\vec{B}$ graphically (Figure 1.20):

- Draw the first vector $\vec{A}$.
- Move the second vector $\vec{B}$ (without rotating it) so its tail is located at the *tip* of the first vector.
- Draw a new vector from the tail of vector $\vec{A}$ to the tip of vector $\vec{B}$.

1.X.20 If $\vec{F}_1 = \langle 300, 0, -200\rangle$ and $\vec{F}_2 = \langle 150, -300, 0\rangle$, what is the magnitude of $\vec{F}_1$? What is the magnitude of $\vec{F}_2$? What is the magnitude of $\vec{F}_1+\vec{F}_2$? What is the magnitude of $\vec{F}_1$ plus the magnitude of $\vec{F}_2$? Is $|\vec{F}_1+\vec{F}_2| = |\vec{F}_1| + |\vec{F}_2|$?

1.X.21 $\vec{A} = \langle 3\times10^3, -4\times10^3, -5\times10^3\rangle$ and $\vec{B} = \langle -3\times10^3, 4\times10^3, 5\times10^3\rangle$. Calculate the following:

(a) $\vec{A}+\vec{B}$ **(b)** $|\vec{A}+\vec{B}|$ **(c)** $|\vec{A}|$ **(d)** $|\vec{B}|$ **(e)** $|\vec{A}|+|\vec{B}|$

Vector Subtraction

The difference of two vectors will be very important in this and subsequent chapters. To subtract one vector from another, we subtract the components of the second vector from the components of the first:

$$\vec{A}-\vec{B} = \langle (A_x - B_x), (A_y - B_y), (A_z - B_z)\rangle$$
$$\langle 1, 2, 3\rangle - \langle -4, 5, 6\rangle = \langle 5, -3, -3\rangle$$

1.X.22 If $\vec{F}_1 = \langle 300, 0, -200\rangle$ and $\vec{F}_2 = \langle 150, -300, 0\rangle$, what is the sum $\vec{F}_1+\vec{F}_2$? What is the difference $\vec{F}_1-\vec{F}_2$? What is $\vec{F}_2-\vec{F}_1$?

Subtracting Vectors Graphically: Tail to Tail

To subtract one vector $\vec{B}$ from another vector $\vec{A}$ graphically:

- Draw the first vector $\vec{A}$.
- Move the second vector $\vec{B}$ (without rotating it) so its tail is located at the *tail* of the first vector.
- Draw a new vector from the tip of vector $\vec{B}$ to the tip of vector $\vec{A}$.

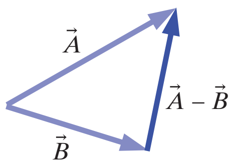

Figure 1.21 The procedure for subtracting vectors graphically: draw vectors tail to tail; draw a new vector from the tip of the second vector to the tip of the first vector.

Note that you can check this algebraically and graphically. As shown in Figure 1.21, since the tail of $\vec{A} - \vec{B}$ is located at the tip of $\vec{B}$, then the vector $\vec{A}$ should be the sum of $\vec{B}$ and $\vec{A} - \vec{B}$, as indeed it is:

$$\vec{B} + (\vec{A} - \vec{B}) = \vec{A}$$

1.X.23 Which of the following statements about the three vectors in Figure 1.22 are correct?
(a) $\vec{s} = \vec{t} - \vec{r}$ **(b)** $\vec{r} = \vec{t} - \vec{s}$ **(c)** $\vec{r} + \vec{t} = \vec{s}$ **(d)** $\vec{s} + \vec{t} = \vec{r}$
(e) $\vec{r} + \vec{s} = \vec{t}$

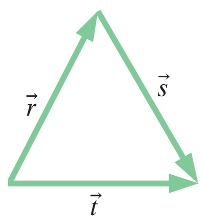

Figure 1.22 Exercise 1.X.23.

Commutativity and Associativity

Vector addition is commutative:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Vector subtraction is *not* commutative:

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

The associative property holds for vector addition and subtraction:

$$(\vec{A} + \vec{B}) - \vec{C} = \vec{A} + (\vec{B} - \vec{C})$$

The Zero Vector

It is convenient to have a compact notation for a vector whose components are all zero. We will use the symbol $\vec{0}$ to denote a zero vector, in order to distinguish it from a scalar quantity that has the value 0.

$$\vec{0} = \langle 0, 0, 0 \rangle$$

For example, the sum of two vectors $\vec{B} + (-\vec{B}) = \vec{0}$.

Change in a Quantity: The Greek Letter Δ

Frequently we will want to calculate the change in a quantity. For example, we may want to know the change in a moving object's position or the change in its velocity during some time interval. The Greek letter Δ (capital delta suggesting "d for difference") is used to denote the change in a quantity (either a scalar or a vector).

We use the subscript i to denote an *initial* value of a quantity, and the subscript f to denote the *final* value of a quantity. If a vector $\vec{r}_i$ denotes the initial position of an object relative to the origin (its position at the beginning of a time interval), and $\vec{r}_f$ denotes the final position of the object, then

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$\Delta\vec{r}$ means "change of $\vec{r}$" or $\vec{r}_f - \vec{r}_i$ (displacement)
Δt means "change of t" or $t_f - t_i$ (time interval)

The symbol Δ (delta) always means "final minus initial," not "initial minus final." For example, when a child's height changes from 1.1 m to 1.2 m, the change is $\Delta y = +0.1$ m, a positive number. If your bank account dropped from

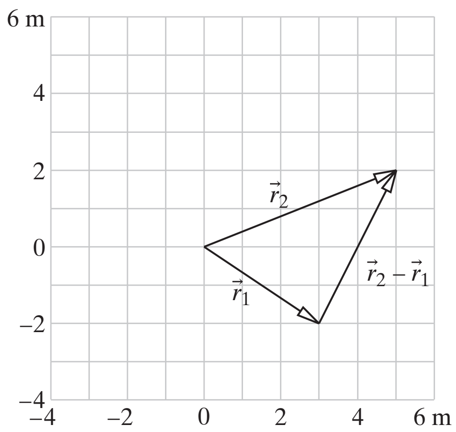

Figure 1.23 Relative position vector.

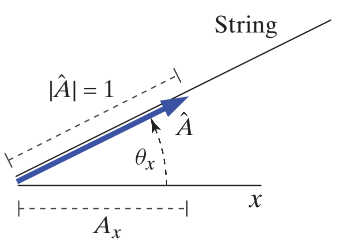

Figure 1.24 A unit vector whose direction is at a known angle from the $+x$ axis.

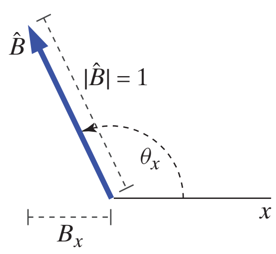

Figure 1.25 A unit vector in the second quadrant from the $+x$ axis.

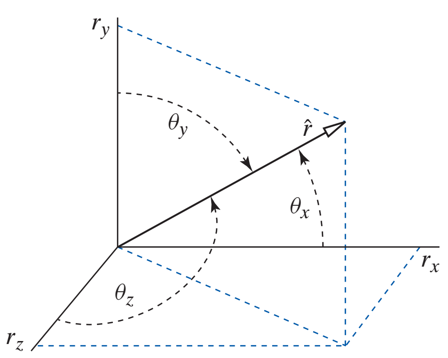

Figure 1.26 A 3D unit vector and its angles to the x, y, and z axes.

\$150 to \$130, what was the change in your balance? Δ (bank account) = −20 dollars.

Relative Position Vectors

Vector subtraction is used to calculate relative position vectors, vectors that represent the position of an object relative to another object. In Figure 1.23, object 1 is at location $\vec{r}_1$ and object 2 is at location $\vec{r}_2$. We want the components of a vector that points from object 1 to object 2. This is the vector obtained by subtraction: $\vec{r}_{2\text{ relative to }1} = \vec{r}_2 - \vec{r}_1$. Note that the form is always "final" minus "initial" in these calculations.

1.X.24 At 10:00 AM you are at location $\langle -3, 2, 5\rangle$ m. By 10:02 AM you have walked to location $\langle 6, 4, 25\rangle$ m.

(a) What is $\Delta\vec{r}$, the change in your position?

(b) What is Δt, the time interval during which your position changed?

1.X.25 A snail is initially at location $\vec{r}_1 = \langle 3, 0, -7\rangle$ m. At a later time the snail has crawled to location $\vec{r}_2 = \langle 2, 0, -8\rangle$ m. What is $\Delta\vec{r}$, the change in the snail's position?

Unit Vectors and Angles

Suppose that a taut string is at an angle θ_x to the $+x$ axis, and we need a unit vector in the direction of the string. Figure 1.24 shows a unit vector $\hat{A}$ pointing along the string. What is the x component of this unit vector? Consider the triangle whose base is A_x and whose hypotenuse is $|\hat{A}| = 1$. From the definition of the cosine of an angle we have this:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A_x}{1}, \text{ so } A_x = \cos\theta_x$$

In Figure 1.24 the angle θ_x is shown in the first quadrant (θ_x less than 90°), but this works for larger angles as well. For example, in Figure 1.25 the angle from the $+x$ axis to a unit vector $\hat{B}$ is in the second quadrant (θ_x greater than 90°) and $\cos\theta_x$ is negative, which corresponds to B_x being negative.

What is true for x is also true for y and z. Figure 1.26 shows a 3D unit vector $\hat{r}$ and indicates the angles between the unit vector and the x, y, and z axes. Evidently we can write

$$\hat{r} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z \rangle$$

These three cosines of the angles between a vector (or unit vector) and the coordinate axes are called the "direction cosines" of the vector. The cosine function is never greater than 1, just as no component of a unit vector can be greater than 1.

A common special case is that of a unit vector lying in the xy plane, with zero z component (Figure 1.27). In this case $\theta_x + \theta_y = 90°$, so that $\cos\theta_y = \cos(90° - \theta_x) = \sin\theta_x$, therefore you can express the cosine of θ_y as the sine of θ_x, which is often convenient. However, in the general 3D case shown in Figure 1.26 there is no such simple relationship among the direction angles or among their cosines.

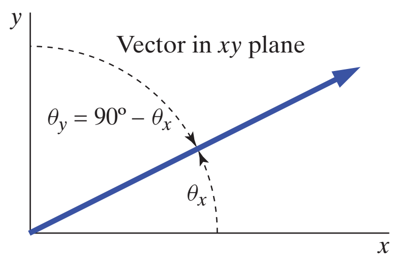

Figure 1.27 If a vector lies in the xy plane, $\cos\theta_y = \sin\theta_x$.

FINDING A UNIT VECTOR FROM ANGLES

To find a unit vector if angles are given:

- Redraw the vector of interest with its tail at the origin, and determine the angles between this vector and the axes.
- Imagine the vector $\langle 1, 0, 0\rangle$, which lies on the $+x$ axis. θ_x is the angle through which you would rotate the vector $\langle 1, 0, 0\rangle$ until its direction matched that of your vector. θ_x is positive, and $\theta_x \le 180^\circ$.
- θ_y is the angle through which you would rotate the vector $\langle 0, 1, 0\rangle$ until its direction matched that of your vector. θ_y is positive, and $\theta_y \le 180^\circ$.
- θ_z is the angle through which you would rotate the vector $\langle 0, 0, 1\rangle$ until its direction matched that of your vector. θ_z is positive, and $\theta_z \le 180^\circ$.

EXAMPLE

From Angle to Unit Vector

A rope lying in the xy plane, pointing up and to the right, supports a climber at an angle of 20° to the vertical (Figure 1.28). What is the unit vector pointing up along the rope?

Solution

Follow the procedure given above for finding a unit vector from angles. In Figure 1.29 we redraw the vector with its tail at the origin, and we determine the angles between the vector and the axes. If we rotate the unit vector $\langle 1, 0, 0\rangle$ from along the $+x$ axis to the vector of interest, we see that we have to rotate through an angle $\theta_x = 70^\circ$. To rotate the unit vector $\langle 0, 1, 0\rangle$ from along the $+y$ axis to the vector of interest, we have to rotate through an angle of $\theta_y = 20^\circ$. The angle from the $+z$ axis to our vector is $\theta_z = 90^\circ$. Therefore the unit vector that points along the rope is this:

$$\langle \cos 70^\circ, \cos 20^\circ, \cos 90^\circ\rangle = \langle 0.342, 0.940, 0\rangle$$

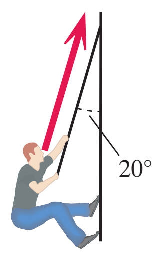

Figure 1.28 A climber supported by a rope.

FURTHER DISCUSSION You may have noticed that the y component of the unit vector can also be calculated as $\sin 70^\circ = 0.940$, and it is often useful to recognize that a vector component can be obtained using sine instead of cosine. There is, however, some advantage always to calculate in terms of direction cosines. This is a method that always works, including in 3D, and that avoids having to decide whether to use a sine or a cosine. Just use the cosine of the angle from the relevant positive axis to the vector.

EXAMPLE

From Unit Vector to Angles

A vector $\vec{r}$ points from the origin to the location $\langle -600, 0, 300\rangle$ m. What is the angle that this vector makes to the x axis? To the y axis? To the z axis?

Solution

$$\hat{r} = \frac{\langle -600, 0, 300\rangle}{\sqrt{(-600)^2 + (0)^2 + (300)^2}\text{ m}} = \langle -0.894, 0, 0.447\rangle$$

But we also know that $\hat{r} = \langle \cos\theta_x, \cos\theta_y, \cos\theta_z\rangle$, so $\cos\theta_x = -0.894$, and the arccosine gives $\theta_x = 153.4^\circ$. Similarly,

$$\cos\theta_y = 0,\ \theta_y = 90^\circ \text{ (which checks; no } y \text{ component)}$$

$$\cos\theta_z = 0.447,\ \theta_z = 63.4^\circ$$

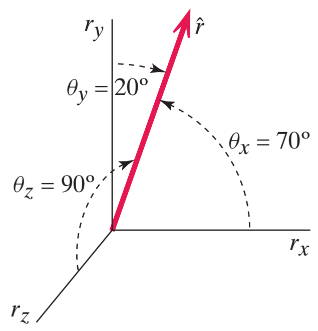

Figure 1.29 Redraw the vector with its tail at the origin. Identify the angles between the positive axes and the vector. In this example the vector lies in the xy plane.

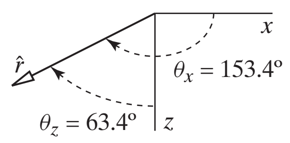

Figure 1.30 Look down on the xz plane. The difference in the two angles is 90°, as it should be.

FURTHER DISCUSSION Looking down on the xz plane in Figure 1.30, you can see that the difference between $\theta_x = 153.4°$ and $\theta_z = 63.4°$ is 90°, as it should be.

1.X.26 A unit vector lies in the xy plane, at an angle of 160° from the $+x$ axis, with a positive y component. What is the unit vector? (It helps to draw a diagram.)

1.X.27 A string runs up and to the left in the xy plane, making an angle of 40° to the vertical. Determine the unit vector that points along the string.

1.6 SI UNITS

In this book we use the SI (Système Internationale) unit system, as is customary in technical work. The SI unit of mass is the kilogram (kg), the unit of distance is the meter (m), and the unit of time is the second (s). In later chapters we will encounter other SI units, such as the newton (N), which is a unit of force.

It is essential to use SI units in physics equations; this may require that you convert from some other unit system to SI units. If mass is known in grams, you need to divide by 1000 and use the mass in kilograms. If a distance is given in centimeters, you need to divide by 100 to convert the distance to meters. If the time is measured in minutes, you need to multiply by 60 to use a time in seconds. A convenient way to do such conversions is to multiply by factors that are equal to 1, such as (1 min)/(60 s) or (100 cm)/(1 m). As an example, consider converting 60 miles per hour to SI units, meters per second. Start with the 60 mi/hr and multiply by factors of 1:

$$\left(60\frac{\cancel{\text{mi}}}{\cancel{\text{hr}}}\right)\left(\frac{1\,\cancel{\text{hr}}}{60\,\cancel{\text{min}}}\right)\left(\frac{1\,\cancel{\text{min}}}{60\,\text{s}}\right)\left(\frac{5280\,\cancel{\text{ft}}}{1\,\cancel{\text{mi}}}\right)\left(\frac{12\,\cancel{\text{in}}}{1\,\cancel{\text{ft}}}\right)\left(\frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}}\right)\left(\frac{1\,\text{m}}{100\,\cancel{\text{cm}}}\right) = 26.8\frac{\text{m}}{\text{s}}$$

Observe how most of the units cancel, leaving final units of m/s.

1.X.28 A snail moved 80 cm (80 centimeters) in 5 minutes. What was its average speed in SI units? Write out the factors as was done above.

1.7 VELOCITY

We use vectors not only to describe the position of an object but also to describe velocity (speed and direction). If we know an object's present speed in meters per second and the object's direction of motion, we can predict where it will be a short time into the future. As we have seen, *change* of velocity is an indication of interaction. We need to be able to work with velocities of objects in 3D, so we need to learn how to use 3D vectors to represent velocities. After learning how to describe velocity in 3D, we will also learn how to describe *change* of velocity, which is related to interactions.

Average Speed

The concept of speed is a familiar one. Speed is a single number, so it is a scalar quantity (speed is the magnitude of velocity). A world-class sprinter can run 100 meters in 10 seconds. We say the sprinter's average speed is

$(100\,\text{m})/(10\,\text{s}) = 10\,\text{m/s}$. In SI units speed is measured in meters per second, abbreviated "m/s."

A car that travels 100 miles in 2 hours has an average speed of (100 mi)/(2 hr) = 50 miles per hour (about 22 m/s). In symbols,

$$v_{\text{avg}} = \frac{d}{t}$$

where v_{avg} is the "average speed," d is the distance the car has traveled, and t is the elapsed time.

There are other useful versions of the basic relationship among average speed, distance, and time. For example,

$$d = v_{\text{avg}} t$$

expresses the fact that if you run 5 m/s for 7 seconds you go 35 meters. Or you can use

$$t = \frac{d}{v_{\text{avg}}}$$

to calculate that to go 3000 miles in an airplane that flies at 600 miles per hour will take 5 hours.

Units

While it is easy to make a mistake in one of the formulas relating speed, time interval, and change in position, it is also easy to catch such a mistake by looking at the units. If you had written $t = v_{\text{avg}}/d$, you would discover that the right-hand side has units of (m/s)/m, or 1/s, not s. Always check units!

Instantaneous Speed Compared to Average Speed

If a car went 70 miles per hour for the first hour and 30 miles an hour for the second hour, it would still go 100 miles in 2 hours, with an average speed of 50 miles per hour. Note that during this 2-hour interval, the car was almost never actually traveling at its average speed of 50 miles per hour.

To find the "instantaneous" speed—the speed of the car at a particular instant—we should observe the short distance the car goes in a very short time, such as a hundredth of a second: If the car moves 0.3 meters in 0.01 s, its instantaneous speed is 30 meters per second.

Vector Velocity

Earlier we calculated vector differences between two different objects. The vector difference $\vec{r}_2 - \vec{r}_1$ represented a *relative* position vector—the position of object 2 relative to object 1 at a particular time. Now we will be concerned with the change of position of *one* object during a time interval, and $\vec{r}_f - \vec{r}_i$ will represent the "displacement" of this single object during the time interval, where $\vec{r}_i$ is the initial 3D position and $\vec{r}_f$ is the final 3D position (note that as with relative position vectors, we always calculate "final minus initial"). Dividing the (vector) displacement by the (scalar) time interval $t_f - t_i$ (final time minus initial time) gives the average (vector) velocity of the object:

DEFINITION: AVERAGE VELOCITY

$$\vec{v}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Another way of writing this expression, using the "Δ" symbol (Greek capital delta, defined in Section 1.5) to represent a change in a quantity, is

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

Remember that this is a compact notation for

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \left\langle \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \right\rangle$$

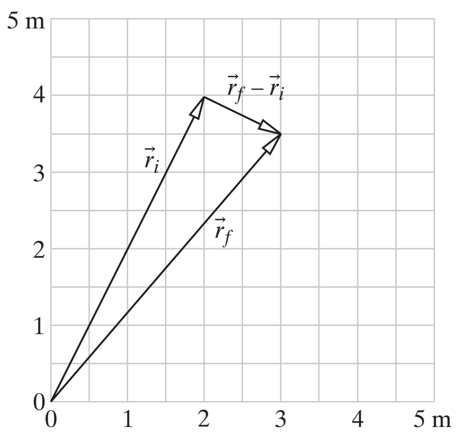

Figure 1.31 The displacement vector points from initial position to final position.

■ Note that the displacement $\vec{r}_f - \vec{r}_i$ refers to the positions of one object (the bee) at two different times, *not* the position of one object relative to a second object at one particular time. However, the vector subtraction is the same kind of operation for either kind of situation.

Determining Average Velocity from Change in Position

Consider a bee in flight (Figure 1.31). At time $t_i = 15$ s after 9:00 AM, the bee's position vector was $\vec{r}_i = \langle 2, 4, 0\rangle$ m. At time $t_f = 15.1$ s after 9:00 AM, the bee's position vector was $\vec{r}_f = \langle 3, 3.5, 0\rangle$ m. On the diagram, we draw and label the vectors $\vec{r}_i$ and $\vec{r}_f$.

Next, on the diagram, we draw and label the vector $\vec{r}_f - \vec{r}_i$, with the tail of the vector at the bee's initial position. One useful way to think about this graphically is to ask yourself what vector needs to be added to the initial vector $\vec{r}_1$ to make the final vector $\vec{r}_f$, since $\vec{r}_f$ can be written in the form $\vec{r}_f = \vec{r}_i + (\vec{r}_f - \vec{r}_i)$.

The vector we just drew, the change in position $\vec{r}_f - \vec{r}_i$, is called the "displacement" of the bee during this time interval. This displacement vector points from the initial position to the final position, and we always calculate displacement as "final minus initial."

We calculate the bee's displacement vector numerically by taking the difference of the two vectors, final minus initial:

$$\vec{r}_f - \vec{r}_i = \langle 3, 3.5, 0\rangle \text{ m} - \langle 2, 4, 0\rangle \text{ m} = \langle 1, -0.5, 0\rangle \text{ m}$$

This numerical result should be consistent with our graphical construction. Look at the components of $\vec{r}_f - \vec{r}_i$ in Figure 1.31. Do you see that this vector has an x component of $+1$ and a y component of -0.5 m? Note that the (vector) displacement $\vec{r}_f - \vec{r}_i$ is in the direction of the bee's motion.

The average velocity of the bee, a vector quantity, is the (vector) displacement $\vec{r}_f - \vec{r}_i$ divided by the (scalar) time interval $t_f - t_i$. Calculate the bee's average velocity:

$$\vec{v}_{avg} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\langle 1, -0.5, 0\rangle \text{ m}}{(15.1 - 15)\text{ s}} = \frac{\langle 1, -0.5, 0\rangle \text{ m}}{0.1 \text{ s}} = \langle 10, -5, 0\rangle \text{ m/s}$$

Since we divided $\vec{r}_f - \vec{r}_i$ by a scalar $(t_f - t_i)$, the average velocity $\vec{v}_{avg}$ points in the direction of the bee's motion, if the bee flew in a straight line.

What is the speed of the bee?

$$\text{speed of bee} = |\vec{v}_{avg}| = \sqrt{10^2 + (-5)^2 + 0^2} \text{ m/s} = 11.18 \text{ m/s}$$

What is the direction of the bee's motion, expressed as a unit vector?

$$\text{direction of bee: } \hat{v}_{avg} = \frac{\vec{v}_{avg}}{|\vec{v}_{avg}|} = \frac{\langle 10, -5, 0\rangle \text{ m/s}}{11.18 \text{ m/s}} = \langle 0.894, -0.447, 0\rangle$$

Note that the "m/s" units cancel; the result is dimensionless. We can check that this really is a unit vector:

$$\sqrt{0.894^2 + (-0.447)^2 + 0^2} = 0.9995$$

This is not quite 1.0 due to rounding the velocity coordinates and speed to three significant figures.

Put the pieces back together and see what we get. The original vector factors into the product of the magnitude times the unit vector:

$$|\vec{v}|\hat{v} = (11.18\,\text{m/s})\langle 0.894, -0.447, 0\rangle = \langle 10, -5, 0\rangle\ \text{m/s}$$

This is the same as the original vector $\vec{v}$.

1.X.29 At a time 0.2 seconds after it has been hit by a tennis racket, a tennis ball is located at $\langle 5, 7, 2\rangle$ m, relative to an origin in one corner of a tennis court. At a time 0.7 seconds after being hit, the ball is located at $\langle 9, 2, 8\rangle$ m.

(a) What is the average velocity of the tennis ball?

(b) What is the average speed of the tennis ball?

(c) What is the unit vector in the direction of the ball's velocity?

1.X.30 A spacecraft is observed to be at a location $\langle 200, 300, -400\rangle$ m relative to an origin located on a nearby asteroid, and 5 seconds later is observed at location $\langle 325, 25, -550\rangle$ m.

(a) What is the average velocity of the spacecraft?

(b) What is the average speed of the spacecraft?

(c) What is the unit vector in the direction of the spacecraft's velocity?

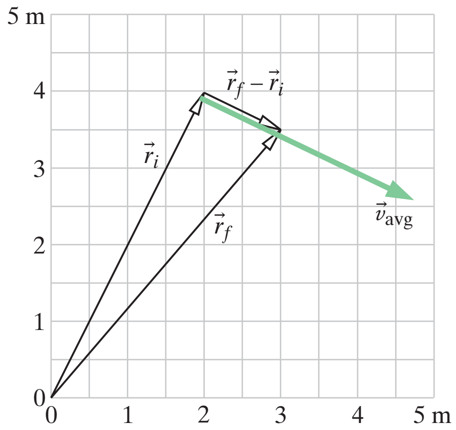

Figure 1.32 Average velocity vector: displacement divided by time interval.

Scaling a Vector to Fit on a Graph

We can plot the average velocity vector on the same graph that we use for showing the vector positions of the bee (Figure 1.32). However, note that velocity has units of meters per second whereas positions have units of meters, so we're mixing apples and oranges.

Moreover, the magnitude of the vector, 11.18 m/s, doesn't fit on a graph that is only 5 units wide (in meters). It is standard practice in such situations to scale down the arrow representing the vector to fit on the graph, preserving the correct direction. In Figure 1.32 we've scaled down the velocity vector by about a factor of 3 to make the arrow fit on the graph. Of course if there is more than one velocity vector we use the same scale factor for all the velocity vectors. The same kind of scaling is used with other physical quantities that are vectors, such as force and momentum, which we will encounter later.

Predicting a New Position

We can rewrite the velocity relationship in the form

$$(\vec{r}_f - \vec{r}_i) = \vec{v}_{\text{avg}}(t_f - t_i)$$

That is, the (vector) displacement of an object is its average (vector) velocity times the time interval. This is just the vector version of the simple notion that if you run at a speed of 7 m/s for 5 s you move a distance of (7 m/s)(5 s) = 35 m, or that a car going 50 miles per hour for 2 hours goes (50 mi/hr)(2 hr) = 100 miles.

Is $(\vec{r}_f - \vec{r}_i) = \vec{v}_{avg}(t_f - t_i)$ a valid vector relation? Yes, multiplying a vector $\vec{v}_{avg}$ times a scalar $(t_f - t_i)$ yields a vector. We make a further rearrangement to obtain a relation for updating the position when we know the velocity:

THE POSITION UPDATE FORMULA

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}(t_f - t_i)$$

or

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

This equation says that if we know the starting position, the average velocity, and the time interval, we can predict the final position. This equation will be important throughout our work.

Using the Position Update Formula

The position update formula $\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\,\Delta t$ is a vector equation, so we can write out its full component form:

$$\langle x_f, y_f, z_f\rangle = \langle x_i, y_i, z_i\rangle + \langle v_{avg,x}, v_{avg,y}, v_{avg,z}\rangle\,\Delta t$$

Because the x component on the left of the equation must equal the x component on the right (and similarly for the y and z components), this compact vector equation represents three separate component equations:

$$x_f = x_i + v_{avg,x}\,\Delta t$$
$$y_f = y_i + v_{avg,y}\,\Delta t$$
$$z_f = z_i + v_{avg,z}\,\Delta t$$

EXAMPLE **Updating the Position of a Ball**

At time $t_i = 12.18$ s after 1:30 PM a ball's position vector is $\vec{r}_i = \langle 20, 8, -12\rangle$ m. The ball's velocity at that moment is $\vec{v} = \langle 9, -4, 6\rangle$ m/s. At time $t_f = 12.21$ s after 1:30 PM, where is the ball, assuming that its velocity hardly changes during this short time interval?

Solution

$$\vec{r}_f = \vec{r}_i + \vec{v}(t_f - t_i) = \langle 20, 8, -12\rangle\text{ m} + (\langle 9, -4, 6\rangle\text{ m/s})(12.21 - 12.18)\text{ s}$$
$$\vec{r}_f = \langle 20, 8, -12\rangle\text{ m} + \langle 0.27, -0.12, 0.18\rangle\text{ m}$$
$$\vec{r}_f = \langle 20.27, 7.88, -11.82\rangle\text{ m}$$

■ Note that if the velocity changes significantly during the time interval, in either magnitude or direction, our prediction for the new position may not be very accurate. In this case the velocity at the initial time could differ significantly from the average velocity during the time interval.

1.X.31 A proton traveling with a velocity of $\langle 3\times10^5, 2\times10^5, -4\times10^5\rangle$ m/s passes the origin at a time 9.0 seconds after a proton detector is turned on. Assuming that the velocity of the proton does not change, what will its position be at time 9.7 seconds?

1.X.32 How long does it take a baseball with velocity $\langle 30, 20, 25\rangle$ m/s to travel from location $\vec{r}_1 = \langle 3, 7, -9\rangle$ m to location $\vec{r}_2 = \langle 18, 17, 3.5\rangle$ m?

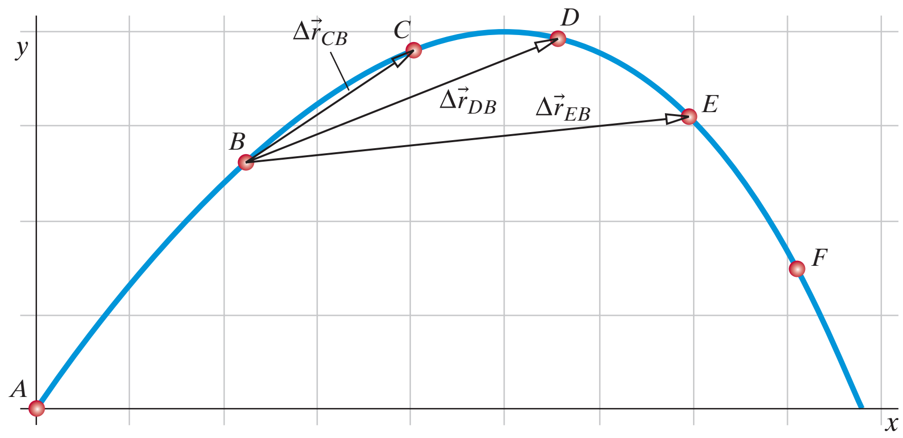

Figure 1.33 The trajectory of a ball through air. The axes represent the x and y distance from the ball's initial location; each square on the grid corresponds to 10 meters. Three different displacements, corresponding to three different time intervals, are indicated by arrows on the diagram.

Instantaneous Velocity

loc.	t (s)	position (m)
A	0.0	$\langle 0, 0, 0\rangle$
B	1.0	$\langle 22.3, 26.1, 0\rangle$
C	2.0	$\langle 40.1, 38.1, 0\rangle$
D	3.0	$\langle 55.5, 39.2, 0\rangle$
E	4.0	$\langle 69.1, 31.0, 0\rangle$
F	5.0	$\langle 80.8, 14.8, 0\rangle$

Figure 1.34 Table showing elapsed time and position of the ball at each location marked by a dot in Figure 1.33.

The curved colored line in Figure 1.33 shows the path of a ball through the air. The colored dots mark the ball's position at time intervals of one second. While the ball is in the air, its velocity is constantly changing, due to interactions with the Earth (gravity) and with the air (air resistance).

Suppose we ask: What is the velocity of the ball at the precise instant that it reaches location B? This quantity would be called the "instantaneous velocity" of the ball. We can start by approximating the instantaneous velocity of the ball by finding its average velocity over some larger time interval.

The table in Figure 1.34 shows the time and the position of the ball for each location marked by a colored dot in Figure 1.33. We can use these data to calculate the average velocity of the ball over three different intervals, by finding the ball's displacement during each interval, and dividing by the appropriate Δt for that interval:

$$\begin{aligned}
\vec{v}_{EB} &= \frac{\Delta\vec{r}_{EB}}{\Delta t} = \frac{\vec{r}_E - \vec{r}_B}{t_E - t_B} = \frac{(\langle 69.1, 31.0, 0\rangle - \langle 22.3, 26.1, 0\rangle)\,\text{m}}{(4.0 - 1.0)\,\text{s}} \\
&= \langle 15.6, 1.6, 0\rangle \frac{\text{m}}{\text{s}} \\
\vec{v}_{DB} &= \frac{\Delta\vec{r}_{DB}}{\Delta t} = \frac{\vec{r}_D - \vec{r}_B}{t_D - t_B} = \frac{(\langle 55.5, 39.2, 0\rangle - \langle 22.3, 26.1, 0\rangle)\,\text{m}}{(3.0 - 1.0)\,\text{s}} \\
&= \langle 16.6, 6.55, 0\rangle \frac{\text{m}}{\text{s}} \\
\vec{v}_{CB} &= \frac{\Delta\vec{r}_{CB}}{\Delta t} = \frac{\vec{r}_C - \vec{r}_B}{t_C - t_B} = \frac{(\langle 40.1, 38.1, 0\rangle - \langle 22.3, 26.1, 0\rangle)\,\text{m}}{(2.0 - 1.0)\,\text{s}} \\
&= \langle 17.8, 12.0, 0\rangle \frac{\text{m}}{\text{s}}
\end{aligned}$$

Not surprisingly, the average velocities over these different time intervals are not the same, because both the direction of the ball's motion and the speed of the ball were changing continuously during its flight. The three average velocity vectors that we calculated are shown in Figure 1.35.

QUESTION Which of the three average velocity vectors depicted in Figure 1.35 best approximates the instantaneous velocity of the ball at location B?

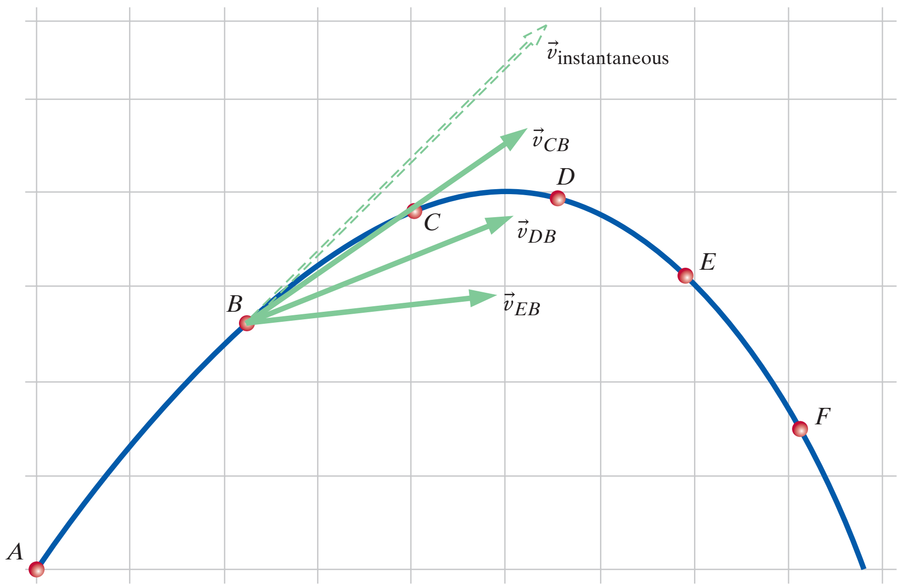

Figure 1.35 The three different average velocity vectors calculated above are shown by three arrows, each with its tail at location B. Note that since the units of velocity are m/s, these arrows use a different scale from the distance scale used for the path of the ball. The three arrows representing average velocities are drawn with their tails at the location of interest. The dashed arrow represents the actual instantaneous velocity of the ball at location B.

Simply by looking at the diagram, we can tell that $\vec{v}_{CB}$ is closest to the actual instantaneous velocity of the ball at location B, because its direction is closest to the direction in which the ball is actually traveling. Because the direction of the instantaneous velocity is the direction in which the ball is moving at a particular instant, the instantaneous velocity is tangent to the ball's path. Of the three average velocity vectors we calculated, $\vec{v}_{CB}$ best approximates a tangent to the path of the ball. Evidently $\vec{v}_{CB}$, the velocity calculated with the shortest time interval, $t_C - t_B$, is the best approximation to the instantaneous velocity at location B. If we used even smaller values of Δt in our calculation of average velocity, such as 0.1 second, or 0.01 second, or 0.001 second, we would presumably have better and better estimates of the actual instantaneous velocity of the object at the instant when it passes location B.

Two important ideas have emerged from this discussion:

- The direction of the instantaneous velocity of an object is tangent to the path of the object's motion.
- Smaller time intervals yield more accurate estimates of instantaneous velocity.

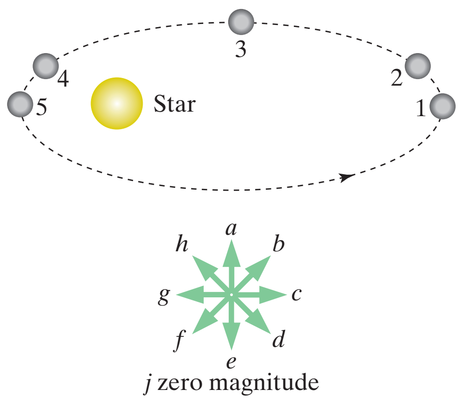

Figure 1.36 A comet goes around a star.

1.X.33 How does average velocity differ from instantaneous velocity?

1.X.34 A comet travels in an elliptical path around a star, in the direction shown in Figure 1.36. Which arrow best indicates the direction of the comet's velocity vector at each of the numbered locations in the orbit?

Connection to Calculus

You may already have learned about derivatives in calculus. The instantaneous velocity is a derivative, the limit of $\Delta\vec{r}/\Delta t$ as the time interval Δt used in the calculation gets closer and closer to zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}, \text{ which is written as } \vec{v} = \frac{d\vec{r}}{dt}$$

In Figure 1.35, the process of taking the limit is illustrated graphically. As smaller values of Δt are used in the calculation, the average velocity vectors approach the limiting value: the actual instantaneous velocity.

■ The rate of change of a vector (the derivative) is itself a vector.

A useful way to see the meaning of the derivative of a vector is to consider the components:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\langle x, y, z\rangle = \left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle = \langle v_x, v_y, v_z\rangle$$

The derivative of the position vector $\vec{r}$ gives components that are the components of the velocity, as we should expect.

Informally, you can think of $d\vec{r}$ as a very small ("infinitesimal") displacement, and dt as a very small ("infinitesimal") time interval. It is as though we had continued the process illustrated in Figure 1.35 to smaller and smaller time intervals, down to an extremely tiny time interval dt with a correspondingly tiny displacement $d\vec{r}$. The ratio of these tiny quantities is the instantaneous velocity.

The ratio of these two tiny quantities need not be small. For example, suppose that an object moves in the x direction a tiny distance of 1×10^{-15} m, the radius of a proton, in a very short time interval of 1×10^{-23} s:

$$\vec{v} = \frac{\langle 1 \times 10^{-15}, 0, 0\rangle \text{ m}}{1 \times 10^{-23} \text{ s}} = \langle 1 \times 10^{8}, 0, 0\rangle \text{ m/s}$$

which is one-third the speed of light (3×10^8 m/s)!

Acceleration

Velocity is the time rate of change of position: $\vec{v} = d\vec{r}/dt$. Similarly, we define "acceleration" as the time rate of change of velocity: $\vec{a} = d\vec{v}/dt$. Acceleration, which is itself a vector quantity, has units of meters per second per second, written as m/s/s or m/s^2.

DEFINITION: ACCELERATION

Instantaneous acceleration is the time rate of change of velocity:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Average acceleration can be calculated from a change in velocity:

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t}$$

The units of acceleration are m/s^2.

If a car traveling in a straight line speeds up from 20 m/s to 26 m/s in 3 seconds, we say that the magnitude of the acceleration is $(26 - 20)/3 = 2$ m/s/s. If you drop a rock, its speed increases 9.8 m/s every second, so its acceleration is 9.8 m/s/s, as long as air resistance is negligible.

1.X.35 Powerful sports cars can go from zero to 25 m/s (about 60 mph) in 5 seconds. What is the magnitude of the acceleration? How does this compare with the acceleration of a falling rock?

1.X.36 Suppose the position of an object at time t is $\langle 3 + 5t, 4t^2, 2t - 6t^3\rangle$. What is the instantaneous velocity at time t? What is the acceleration at time t? What is the instantaneous velocity at time $t = 0$? What is the acceleration at time $t = 0$?

Change of Magnitude and/or Change in Direction

There are two parts to the acceleration, the time rate of change of the velocity $\vec{v} = |\vec{v}|\hat{v}$:

$$\text{rate of change of the magnitude of the velocity (speed) } \frac{d|\vec{v}|}{dt}$$

$$\text{rate of change of the direction of the velocity } \frac{d\hat{v}}{dt}$$

As we'll see in later chapters, these two parts of the acceleration are associated with pushing or pulling parallel to the motion (changing the speed) or perpendicular to the motion (changing the direction).

1.8 MOMENTUM

Newton's first law of motion,

> **An object moves in a straight line and at constant speed except to the extent that it interacts with other objects**

gives us a conceptual connection between interactions and their effects on the motion of objects. However, this law does not allow us to make quantitative (numerical) predictions or explanations—we could not have used this law to predict the exact trajectory of the ball shown in Figure 1.33, and we could not use this law alone to figure out how to send a rocket to the Moon.

In order to make quantitative predictions or explanations of physical phenomena, we need a quantitative measure of interactions and a quantitative measure of effects of those interactions.

Newton's first law of motion does contain the important idea that if there is no interaction, a moving object will continue to move in a straight line, with no change of direction or speed, and an object that is not moving will remain at rest. A quantitative version of this law would provide a means of predicting the motion of an object, or of deducing how it must have moved in the past, if we could list all of its interactions with other objects.

Changes in Velocity

> **QUESTION** What factors make it difficult or easy to change an object's velocity?

You have probably noticed that if two objects have the same velocity but one is much more massive than the other, it is more difficult to change the heavy object's speed or direction. It is easier to stop a baseball traveling at a hundred miles per hour than to stop a car traveling at a hundred miles per hour! It is easier to change the direction of a canoe than to change the direction of a large, massive ship such as the *Titanic* (which couldn't change course quickly enough after the iceberg was spotted).

Momentum Involves Both Mass and Velocity

To take into account both an object's mass and its velocity, we define a vector quantity called "momentum" that involves the product of mass (a scalar) and velocity (a vector). Instead of saying "the stronger the interaction, the bigger the change in the velocity," we now say "the stronger the interaction, the bigger the change in the momentum."

Momentum, a vector quantity, is usually represented by the symbol $\vec{p}$. We might expect that the mathematical expression for momentum would be simply $\vec{p} = m\vec{v}$, and indeed this is almost, but not quite, correct.

Experiments on particles moving at very high speeds, close to the speed of light $c = 3 \times 10^8$ m/s, show that changes in $m\vec{v}$ are not really proportional to the strength of the interactions. As you keep applying a force to a particle near the speed of light, the speed of the particle barely increases, and it is not possible to increase a particle's speed beyond the speed of light.

Through experiments it has been found that changes in the following quantity are proportional to the amount of interaction:

DEFINITION OF MOMENTUM

Momentum is defined as the product of mass times velocity, multiplied by a proportionality factor gamma:

$$\vec{p} = \gamma m\vec{v}$$

The proportionality factor γ (lower-case Greek gamma) is defined as

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

In these equations $\vec{p}$ represents momentum, $\vec{v}$ is the velocity of the object, m is the mass of the object, $|\vec{v}|$ is the magnitude of the object's velocity (the speed), and c is the speed of light (3×10^8 m/s). Momentum has units of kg·m/s. To calculate momentum in these units, you must specify mass in kg and velocity in meters per second.

This is the "relativistic" definition of momentum. Albert Einstein in 1905 in his Special Theory of Relativity predicted that this would be the appropriate definition for momentum at high speeds, a prediction that has been abundantly verified in a wide range of experiments.

EXAMPLE **Momentum of a Fast-Moving Proton**

Suppose that a proton (mass 1.7×10^{-27} kg) is traveling with a velocity of $\langle 2 \times 10^7 \rangle$ m/s. (a) What is the momentum of the proton? (b) What is the magnitude of the momentum of the proton?

Solution

(a) $$|\vec{v}| = \sqrt{(2 \times 10^7)^2 + (1 \times 10^7)^2 + (-3 \times 10^7)^2} = 3.7 \times 10^7 \,\frac{\text{m}}{\text{s}}$$

$$\frac{|\vec{v}|}{c} = \frac{3.7 \times 10^7 \,\frac{\text{m}}{\text{s}}}{3.7 \times 10^8 \,\frac{\text{m}}{\text{s}}} = 0.12$$

$$\gamma = \frac{1}{\sqrt{1-(0.12)^2}} = 1.007$$

$$\vec{p} = (1.007)(1.7 \times 10^{-27}\ \text{kg})\langle 2 \times 10^7, 1 \times 10^7, -3 \times 10^7 \rangle \frac{\text{m}}{\text{s}}$$

$$\vec{p} = \langle 3.4 \times 10^{-20}, 1.7 \times 10^{-20}, -5.1 \times 10^{-20} \rangle \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$|\vec{p}| = \sqrt{(3.4 \times 10^{-20})^2 + (1.7 \times 10^{-20})^2 + (-5.1 \times 10^{-20})^2}\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

(b) $$= 6.4 \times 10^{-20} \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Approximate Expression for Momentum

In the example above, we found that $\gamma = 1.007$. Since in that calculation we used only two significant figures, we could have used the approximation that $\gamma \approx 1.0$ without affecting our answer. Let's examine the expression for γ to see whether we can come up with a guideline for when it is reasonable to use the approximate expression

$$\vec{p} \approx 1 \cdot m\vec{v}$$

Looking at the expression for γ

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{|\vec{v}|}{c}\right)^2}}$$

we see that it depends only on the ratio of the speed of the object to the speed of light (the object's mass doesn't appear in this expression).

If $|\vec{v}|/c$ is a very small number, then $1 - (|\vec{v}|/c)^2 \approx 1 - 0 \approx 1$, so $\gamma \approx 1$.

$\lvert\vec{v}\rvert$ m/s	$\lvert\vec{v}\rvert/c$	γ
0	0	1.0000
3	1×10^{-8}	1.0000
300	1×10^{-6}	1.0000
3×10^{6}	1×10^{-8}	1.0001
3×10^{7}	0.1	1.0050
1.5×10^{8}	0.5	1.1547
2.997×10^{8}	0.999	22.3663
2.9997×10^{8}	0.9999	70.7124
3×10^{8}	1	Infinite! Impossible!

Figure 1.37 Values of γ calculated for some speeds. γ is shown to four decimal places, which is more accuracy than we will usually need.

APPROXIMATION: MOMENTUM AT LOW SPEEDS

$\vec{p} \approx m\vec{v}$ when $|\vec{v}| \ll c$

When $\vec{p} \approx m\vec{v}$, $\vec{v} \approx \vec{p}/m$

Some values of $(|\vec{v}|/c)$ and γ are displayed in Figure 1.37. From this table you can see that even at the very high speed where $|\vec{v}|/c = 0.1$, which means that $|\vec{v}| = 3 \times 10^7$ m/s, the relativistic factor γ is only slightly different from 1. For large-scale objects such as a space rocket, whose speed is typically only about 1×10^4 m/s, we can ignore the factor γ, and momentum is to a good approximation $\vec{p} \approx m\vec{v}$. It is only for high-speed cosmic rays or particles produced in high-speed particle accelerators that we need to use the full relativistic definition for momentum, $\vec{p} = \gamma m\vec{v}$.

From this table you can also get a sense of why it is not possible to exceed the speed of light. As you make a particle go faster and faster, approaching the speed of light, additional increases in the speed become increasingly difficult, because a tiny increase in speed means a huge increase in momentum, requiring huge amounts of interaction. In fact, for the speed to equal the speed of light, the momentum would have to increase to be infinite! There is a cosmic speed limit in the Universe, 3×10^8 m/s.

We will repeatedly emphasize the role of momentum throughout this textbook because of its fundamental importance not only in classical (prequantum) mechanics but also in relativity and quantum mechanics. The use of momentum clarifies the physics analysis of certain complex processes such as collisions, including collisions at speeds approaching the speed of light.

1.X.37 A good sprinter can run 100 meters in 10 seconds. What is the magnitude of the momentum of a sprinter whose mass is 65 kg and who is running at a speed of 10 m/s?

1.X.38 What is the momentum of an electron traveling at a velocity of $\langle 0, 0, -2 \times 10^8 \rangle$ m/s? (Masses of particles are given on the inside back cover of this textbook.) What is the magnitude of the momentum of the electron?

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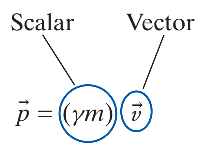

Figure 1.38 The expression for momentum is the product of a scalar times a vector. The scalar factor is positive, so the direction of an object's momentum is the same as the direction of its velocity.

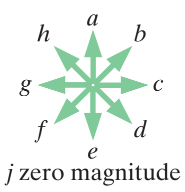

Figure 1.39 A comet's momentum is in the direction of the arrow labeled *b*.

Direction of Momentum

Like velocity, momentum is a vector quantity, so it has a magnitude and a direction. The direction is the same as the direction of the velocity.

QUESTION A leaf is blown by a gust of wind, and at a particular instant is traveling straight upward, in the $+y$ direction. What is the direction of the leaf's momentum?

The mathematical expression for momentum can be looked at as the product of a scalar part times a vector part. Since the mass m must be a positive number, and the factor gamma (γ) must be a positive number, this scalar factor cannot change the direction of the vector (Figure 1.38). Therefore the direction of the leaf's momentum is the same as the direction of its velocity: straight up (the $+y$ direction).

> **1.X.39** A comet's momentum is in the direction of the arrow labeled *b* in Figure 1.39. What arrow best describes the direction of the comet's velocity?

Using Momentum to Update Position

If you know the momentum of an object, you can calculate the change in position of the object over a given time interval. This is straightforward if the object is traveling at a speed low enough that the approximate expression for momentum can be used, since in this case $\vec{v} \approx \vec{p}/m$. For speeds near the speed of light, we must use the exact expression

$$\vec{v} = \frac{\vec{p}/m}{\sqrt{1 + \left(\frac{|\vec{p}|}{mc}\right)^2}}$$

This expression was derived simply by rearranging the equation $\vec{p} = \gamma m \vec{v}$. The detailed derivation is given at the end of the chapter.

EXAMPLE **Displacement of a Fast Proton**

A proton with constant momentum $\langle 0, 0, 2.72 \times 10^{-19}\rangle$ kg · m/s leaves the origin 10.0 seconds after an accelerator experiment is started. What is the location of the proton 2 ns later? (ns = nanosecond = 1×10^{-19} s)

Solution

$$\vec{v} = \frac{\vec{p}/m}{\sqrt{1 + \left(\frac{|\vec{p}|}{mc}\right)^2}} = \frac{\langle 0, 0, 2.72 \times 10^{-19}\rangle \text{ kg} \cdot \text{m/s}}{(1.7 \times 10^{-27}\text{ kg})\sqrt{1 + \left(\frac{2.72 \times 10^{-19}\text{ kg} \cdot \text{m/s}}{(1.7 \times 10^{-27}\text{ kg})(3 \times 10^{8}\text{ m/s})}\right)^2}}$$

$$= \langle 0, 0, 1.4 \times 10^{8}\rangle \text{ m/s}$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\,\Delta t$$

$$= \langle 0, 0, 0\rangle \text{ m} + \langle 0, 0, 1.4 \times 10^{8}\text{ m/s}\rangle(2 \times 10^{-9}\text{ s})$$

$$= \langle 0, 0, 0.28\rangle \text{ m}$$

The proton traveled 28 cm in 2 ns.

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EXAMPLE **Displacement of an Ice Skater**

An ice skater whose mass is 50 kg moves with constant momentum $\langle 400, 0, 300\rangle$ kg · m/s. At a particular instant in her skating program she passes location $\langle 0, 0, 3\rangle$ m. What was her location at a time 3 seconds earlier?

Solution

$$\vec{v} \approx \frac{\vec{p}}{m} = \frac{\langle 400, 0, 300\rangle \text{ kg} \cdot \text{m/s}}{50 \text{ kg}} = \langle 8, 0, 6\rangle \text{ m/s}$$

$$(\vec{r}_f - \vec{r}_i) = \vec{v}\,\Delta t$$

$$\vec{r}_i = \vec{r}_f - \vec{v}\,\Delta t$$

$$= \langle 0, 0, 3\rangle \text{ m} - (\langle 8, 0, 6\rangle \text{ m/s})(3 \text{ s})$$

$$= \langle -24, 0, -15\rangle \text{ m}$$

1.X.40 A 4.5 kg bowling ball rolls down an alley with nearly constant momentum of $\langle 0.9, 0, 22.5\rangle$ kg·m/s, starting from the origin. What will be the location of the ball after 2 seconds?

1.X.41 At time $t_1 = 12$ s, a car with mass 1300 kg is located at $\langle 94, 0, 30\rangle$ m and has momentum $\langle 4500, 0, -3000\rangle$ kg·m/s. The car's momentum is not changing. At time $t_2 = 17$ s, what is the position of the car?

1.9 CHANGE OF MOMENTUM

In the next chapter we will introduce "the Momentum Principle," which quantitatively relates change in momentum $\Delta\vec{p}$ to the strength and duration of an interaction. In order to be able to use the Momentum Principle we need to know how to calculate changes in momentum.

Momentum is a vector quantity and proportional to the velocity, so just as was the case with velocity, there are two aspects of momentum that can change: magnitude and direction. A mathematical description of change of momentum must include either a change in the magnitude of the momentum, or a change in the direction of the momentum, or both. As we will see in later chapters, pushing parallel to the motion changes the magnitude of the momentum, but to change the direction of the momentum requires pushing perpendicular (sideways) to the motion.

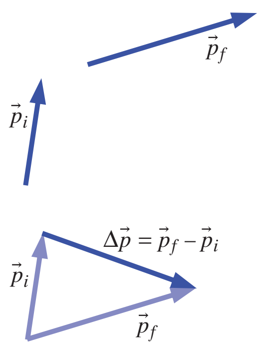


Figure 1.40 Calculation of $\Delta\vec{p}$.

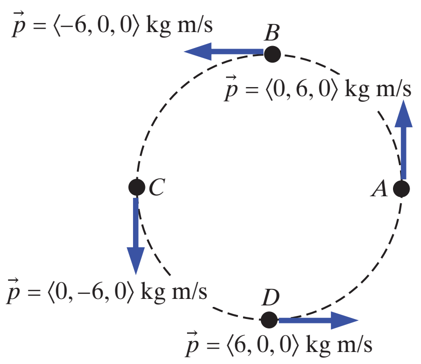


Figure 1.41 The momentum of an object traveling in a circle changes, even if the magnitude of momentum does not change.

Change of the Vector Momentum

The change in the momentum $\Delta\vec{p}$ during a time interval is a vector: $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. This vector expression captures both changes in magnitude and changes in direction. Figure 1.40 is a graphical illustration of a change from an initial momentum $\vec{p}_i$ to a final momentum $\vec{p}_f$. Place the initial and final momentum vectors tail to tail, then draw a vector from initial to final. This is the vector representing $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. This is the same procedure you used to calculate relative position vectors by subtraction, or displacement vectors by subtraction. The rule for subtracting vectors is always the same: Place the vectors tail to tail, then draw from the tip of the initial vector to the tip of the final vector. This resultant vector is "final minus initial."

Change in Magnitude of Momentum

If an object's speed changes (that is, the magnitude of its velocity changes), the magnitude of the object's momentum also changes. Pushing in the direction of the momentum (and velocity) makes the magnitude of the momentum

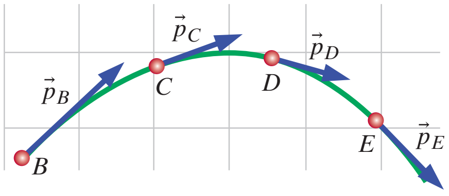


Figure 1.42 A portion of the trajectory of a ball moving through air, subject to gravity and air resistance. The arrows represent the momentum of the ball at the locations indicated by letters.

increase (the speed increases); pushing backwards makes the magnitude decrease (the object slows down).

Change of Direction of Momentum

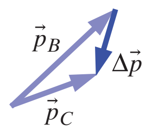
There are various ways to specify a change in the direction of motion. For example, if you used compass directions, you could say that an airplane changed its direction from 30° east of north to 45° east of north: a 15° clockwise change. One can imagine various other schemes, involving other kinds of coordinate systems. The standard way to deal with this is to use vectors, as in Figure 1.41, which shows the changing momentum of an object traveling at constant speed along a circular path.

EXAMPLE

Change in Momentum of a Ball

Figure 1.42 shows a portion of the trajectory of a ball in air, subject to gravity and air resistance. At location B, the ball's momentum is $\vec{p}_B = \langle 3.03, 2.83, 0\rangle$ kg · m/s. At location C, the ball's momentum is $\vec{p}_C = \langle 2.55, 0.97, 0\rangle$ kg · m/s. (a) Find the change in the ball's momentum between these locations, and show it on the diagram. (b) Find the change in the magnitude of the ball's momentum.

Solution



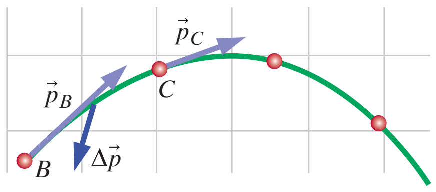


Figure 1.43 Graphical calculation of $\Delta\vec{p}$. The result is also shown superimposed on the ball's path, midway between the initial and final locations.

(a)

$$\begin{aligned}\Delta\vec{p} &= \vec{p}_C - \vec{p}_B = \langle 2.55, 0.97, 0\rangle \text{ kg}\cdot\text{m/s} - \langle 3.03, 2.83, 0\rangle \text{ kg}\cdot\text{m/s}\\ &= \langle -0.48, -1.86, 0\rangle \text{ kg}\cdot\text{m/s}\end{aligned}$$

Both the x and y components of the ball's momentum decreased, so $\Delta\vec{p}$ has negative x and y components. This is consistent with the graphical subtraction shown in Figure 1.43.

(b)

$$|\vec{p}_B| = \sqrt{(3.03)^2 + (2.83)^2 + (0)^2} = 4.15 \text{ kg}\cdot\text{m/s}$$

$$|\vec{p}_C| = \sqrt{(2.55)^2 + (0.97)^2 + (0)^2} = 2.73 \text{ kg}\cdot\text{m/s}$$

FURTHER DISCUSSION The magnitude of the ball's momentum decreased, which means that there must have been a component of gravitational and air resistance forces acting on the ball in a direction opposite to the ball's motion, which slowed it down. The direction of the ball's momentum also changed, as is seen in Figure 1.42, which means that there must have been components of the forces acting perpendicular to the ball's motion.

EXAMPLE

Change in Momentum and in Magnitude of Momentum

Suppose you are driving a 1000 kilogram car at 20 m/s in the $+x$ direction. After making a 180 degree turn, you drive the car at 20 m/s in the $-x$ direction. (20 m/s is about 45 miles per hour or 72 km per hour.) (a) What is the change of magnitude of the momentum of the car $\Delta|\vec{p}|$? (b) What is the magnitude of the change of momentum of the car $|\Delta\vec{p}|$?

Solution

(a)

$$\Delta|\vec{p}| = |\vec{p}_2| - |\vec{p}_1| \approx |m\vec{v}_2| - |m\vec{v}_1| = |m||\vec{v}_2| - |m||\vec{v}_1|$$
$$\Delta|\vec{p}| = (1000\text{ kg})(20\text{ m/s}) - (1000\text{ kg})(20\text{ m/s})$$
$$\Delta|\vec{p}| = 0 \text{ kg}\cdot\text{m/s}$$

(b) $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = 1000\,\text{kg}\langle 20, 0, 0\rangle\,\text{m/s} - 1000\,\text{kg}\langle -20, 0, 0\rangle\,\text{m/s}$

$= \langle 4 \times 10^4, 0, 0\rangle\,\text{kg}\cdot\text{m/s}$

$|\Delta\vec{p}| = 4 \times 10^4\,\text{kg}\cdot\text{m/s}$

So $|\Delta\vec{p}| \neq \Delta|\vec{p}|$.

FURTHER DISCUSSION This calculation shows something important. Change of magnitude, $\Delta|\vec{p}|$, isn't the same as magnitude of change, $|\Delta\vec{p}|$. Here's an even more dramatic example:

QUESTION A ball with momentum $\langle 5, 0, 0\rangle$ kg · m/s rebounds from a wall with nearly the same speed, so its final momentum is approximately $\langle -5, 0, 0\rangle$ kg · m/s. What is $\Delta|\vec{p}|$? What is $|\Delta\vec{p}|$?

The change of the magnitude of the momentum, $\Delta|\vec{p}|$, is zero $(5 - 5 = 0)$, but the magnitude of the change of the momentum, $|\Delta\vec{p}|$, is $|\langle -5, 0, 0\rangle - \langle 5, 0, 0\rangle| = |\langle -10, 0, 0\rangle| = 10$ kg · m/s.

1.X.42 A tennis ball of mass 57 g travels with velocity $\langle 50, 0, 0\rangle$ m/s toward a wall. After bouncing off the wall, the tennis ball is observed to be traveling with velocity $\langle -48, 0, 0\rangle$ m/s.

(a) Draw a diagram showing the initial and final momentum of the tennis ball.

(b) What is the change in the momentum of the tennis ball?

(c) What is the change in the magnitude of the tennis ball's momentum?

1.X.43 The planet Mars has a mass of 6.4×10^{23} kg, and travels in a nearly circular orbit around the Sun, as shown in Figure 1.44. When it is at location *A*, the velocity of Mars is $\langle 0, 0, -2.5 \times 10^4\rangle$ m/s. When it reaches location *B*, the planet's velocity is $\langle -2.5 \times 10^4, 0, 0\rangle$ m/s. We're looking down on the orbit from above the north poles of the Sun and Mars, with $+x$ to the right and $+z$ down the page.

(a) What is $\Delta\vec{p}$, the change in the momentum of Mars between locations *A* and *B*?

(b) On a copy of the diagram in Figure 1.44, draw two arrows representing the momentum of Mars at locations *C* and *D*, paying attention to both the length and direction of each arrow.

(c) What is the direction of the change in the momentum of Mars between locations *C* and *D*? Draw the vector $\Delta\vec{p}$ on your diagram.

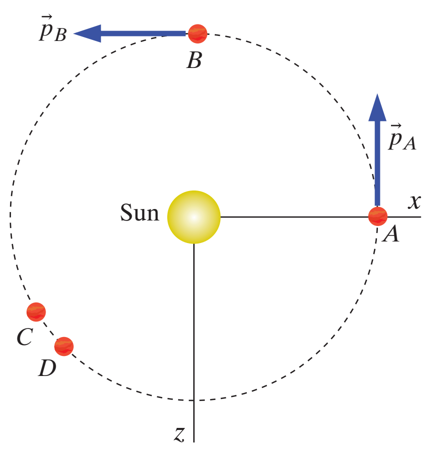


Figure 1.44 The nearly circular orbit of Mars around the Sun, viewed from above the orbital plane ($+x$ to the right, $+z$ down the page). Not to scale: the sizes of the Sun and Mars are exaggerated.

Average Rate of Change of Momentum

The rate of change of the vector position is such an important quantity that it has a special name: "velocity." In Section 1.7 we discussed how to find both the average rate of change of position (average velocity) and the instantaneous rate of change of position (instantaneous velocity) of an object.

The average rate of change of momentum and the instantaneous rate of change of momentum are also extremely important quantities. In some situations, we will only be able to find an average rate of change of momentum:

AVERAGE RATE OF CHANGE OF MOMENTUM

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{t_f - t_i}$$

This quantity is a vector, and points in the direction of $\Delta\vec{p}$.

EXAMPLE

Average Rate of Change of Momentum

If the momentum of a ball changes from $\langle 1, 2, 0 \rangle$ kg · m/s to $\langle 0.5, 0, 0.5 \rangle$ kg · m/s in half a second, the average rate of change of momentum of the ball is

$$\frac{(\langle 0.5, 0, 0.5 \rangle - \langle 1, 2, 0 \rangle)\ \text{kg} \cdot \text{m/s}}{0.5\ \text{s}} = \langle -1, -4, 1 \rangle \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

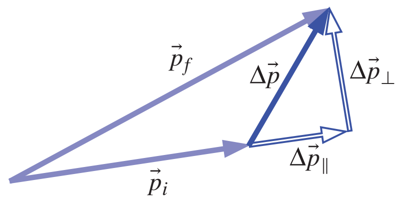


Figure 1.45 The parallel and perpendicular components of the momentum change, $\Delta\vec{p}$.

Parallel and Perpendicular Change

In Figure 1.45 we see the parallel and perpendicular components of a change of momentum, $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. To make a nonzero perpendicular component $\Delta\vec{p}_\perp$, involving a change in direction, requires pushing perpendicular (sideways) to the initial momentum $\vec{p}_i$.

1.X.44 A hockey puck was sliding on ice with momentum $\langle 0, 0, 10 \rangle$ kg · m/s when it was kicked, and the new momentum was $\langle 7, 0, 12 \rangle$ kg · m/s. (a) What was $\Delta\vec{p}_\parallel$? (b) What was $\Delta\vec{p}_\perp$? It helps to draw a diagram.

1.10 *THE PRINCIPLE OF RELATIVITY

Sections marked with a "*" are optional. They provide additional information and context, but later sections of the textbook don't depend critically on them. This optional section deals with some deep issues about the "reference frame" from which you observe motion. Newton's first law of motion only applies in an "inertial reference frame," which we will discuss here in the context of the principle of relativity.

A great variety of experimental observations has led to the establishment of the following principle:

THE PRINCIPLE OF RELATIVITY

Physical laws work in the same way for observers in uniform motion as for observers at rest.

This principle is called "the principle of relativity." (Einstein's extensions of this principle are known as "special relativity" and "general relativity.") Phenomena observed in a room in uniform motion (for example, on a train moving with constant speed on a smooth straight track) obey the same physical laws in the same way as experiments done in a room that is not moving. According to this principle, Newton's first law of motion should be true both for an observer moving at constant velocity and for an observer at rest.

For example, suppose that you're riding in a car moving with constant velocity, and you're looking at a map lying on the dashboard. As far as you're concerned, the map isn't moving, and no interactions are required to hold it still on the dashboard. Someone standing at the side of the road sees the car go by, sees the map moving at a high speed in a straight line, and can see that no interactions are required to hold the map still on the dashboard. Both you and the bystander agree that Newton's first law of motion is obeyed: the bystander sees the map moving with constant velocity in the absence of interactions, and you see the map not moving at all (a zero constant velocity) in the absence of interactions.

On the other hand, if the car suddenly speeds up, it moves out from under the map, which ends up in your lap. To you it looks like "the map sped up in the backwards direction" without any interactions to cause this to happen, which looks like a violation of Newton's first law of motion. The problem is that you're strapped to the car, which is an accelerated reference frame, and Newton's first law of motion applies only to nonaccelerated reference frames, called "inertial" reference frames. Similarly, if the car suddenly turns to the right, moving out from under the map, the map tends to keep going in its original direction, and to you it looks like "the map moved to the left" without any interactions. So a change of speed or a change of direction of the car (your reference frame) leads you to see the map behave in a strange way.

The bystander, who is in an inertial (nonaccelerating) reference frame, doesn't see any violation of Newton's first law of motion. The bystander's reference frame is an inertial frame, and the map behaves in an understandable way, tending to keep moving with unchanged speed and direction when the car changes speed or direction.

The Cosmic Microwave Background

The principle of relativity, and Newton's first law of motion, apply only to observers who have a constant speed and direction (or zero speed) relative to the "cosmic microwave background," which provides the only backdrop and frame of reference with an absolute, universal character. It used to be that the basic reference frame was loosely called "the fixed stars," but stars and galaxies have their own individual motions within the Universe and do not constitute an adequate reference frame with respect to which to measure motion.

The cosmic microwave background is low-intensity electromagnetic radiation with wavelengths in the microwave region, which pervades the Universe, radiating in all directions. Measurements show that our galaxy is moving through this microwave radiation with a large, essentially constant velocity, toward a cluster of a large number of other galaxies. The way we detect our motion relative to the microwave background is through the "Doppler shift" of the frequencies of the microwave radiation, toward higher frequencies in front of us and lower frequencies behind. This is essentially the same phenomenon as that responsible for a fire engine siren sounding at a higher frequency when it is approaching us and a lower frequency when it is moving away from us.

The discovery of the cosmic microwave background provided major support for the "Big Bang" theory of the formation of the Universe. According to the Big Bang theory, the early Universe must have been an extremely hot mixture of charged particles and high-energy, short-wavelength electromagnetic radiation (visible light, x-rays, gamma rays, etc.). Electromagnetic radiation interacts strongly with charged particles, so light could not travel very far without interacting, making the Universe essentially opaque. Also, the Universe was so hot that electrically neutral atoms could not form without the electrons immediately being stripped away again by collisions with other fast-moving particles.

As the Universe expanded, the temperature dropped. Eventually the temperature was low enough for neutral atoms to form. The interaction of electromagnetic radiation with neutral atoms is much weaker than with individual charged particles, so the radiation was now essentially free, dissociated from the matter, and the Universe became transparent. As the Universe continued to expand (the actual space between clumps of matter got bigger), the wavelengths of the electromagnetic radiation got longer, until today this fossil radiation has wavelengths in the relatively low-energy, long-wavelength microwave portion of the electromagnetic spectrum.

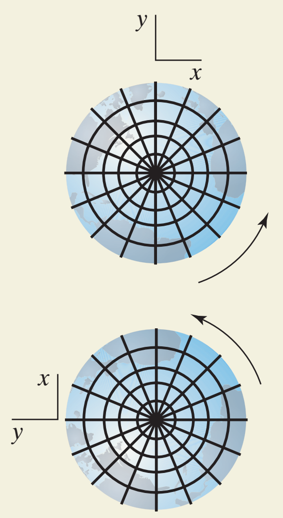


Figure 1.46 Axes tied to the Earth rotate through 90° in a quarter of a day (6 hours).

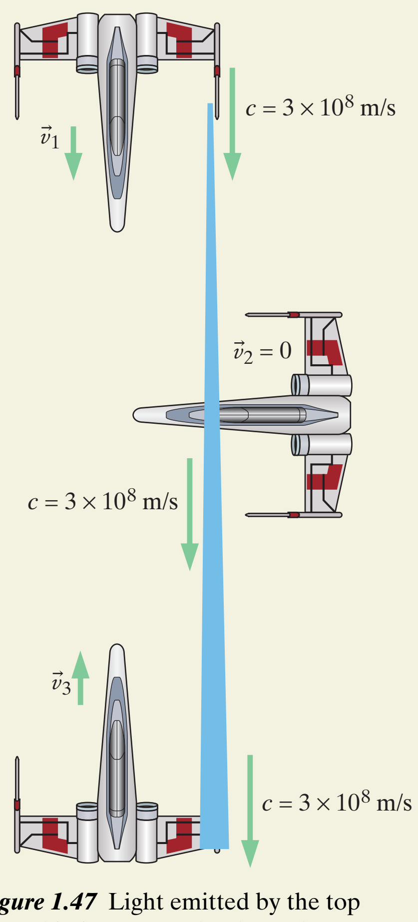


Figure 1.47 Light emitted by the top spaceship is measured to have the same speed by observers in all three ships.

Inertial Frames of Reference

It is an observational fact that in reference frames that are in uniform motion with respect to the cosmic microwave background, far from other objects (so that interactions are negligible), an object maintains uniform motion. Such frames are called "inertial frames" and are reference frames in which Newton's first law of motion is valid. All of these reference frames are equally valid; the cosmic microwave background simply provides a concrete example of such a reference frame.

QUESTION Is the surface of the Earth an inertial frame?

No! The Earth is rotating on its axis, so the velocity of an object sitting on the surface of the Earth is constantly changing direction, as is a coordinate frame tied to the Earth (Figure 1.46). Moreover, the Earth is orbiting the Sun, and the Solar System itself is orbiting the center of our Milky Way galaxy, and our galaxy is moving toward other galaxies. So the motion of an object sitting on the Earth is actually quite complicated and definitely not uniform with respect to the cosmic microwave background.

However, for many purposes the surface of the Earth can be considered to be (approximately) an inertial frame. For example, it takes 6 hours for the rotation of the Earth on its axis to make a 90° change in the direction of the velocity of a "fixed" point. If a process of interest takes only a few minutes, during these few minutes a "fixed" point moves in nearly a straight line at constant speed due to the Earth's rotation, and velocity changes in the process of interest are typically much larger than the very small velocity change of the approximate inertial frame of the Earth's surface.

Similarly, although the Earth is in orbit around the Sun, it takes 365 days to go around once, so for a period of a few days or even weeks the Earth's orbital motion is nearly in a straight line at constant speed. Hence for many purposes the Earth represents an approximately inertial frame despite its motion around the Sun.

The Special Theory of Relativity

Einstein's Special Theory of Relativity (published in 1905) built on the basic principle of relativity but added the conjecture that the speed of a beam of light must be the same as measured by observers in different frames of reference in uniform motion with respect to each other. In Figure 1.47, observers on each spaceship measure the speed of the light c emitted by the ship at the top to be the same ($c = 3 \times 10^8$) m/s, despite the fact that they are moving at different velocities.

This additional condition seems peculiar and has far-reaching consequences. After all, the map on the dashboard of your car has different speeds relative to different observers, depending on the motion of the observer. Yet a wide range of experiments has confirmed Einstein's conjecture: all observers measure the same speed for the same beam of light, $c = 3 \times 10^8$ m/s. (The color of the light is different for the different observers, but the speed is the same.)

On the other hand, if someone on the ship at the top throws a ball or a proton or some other piece of matter, the speed of the object will be different for observers on the three ships; it is only light whose speed is independent of the observer.

Einstein's theory has interesting consequences. For example, it predicts that time will run at different rates in different frames of reference. These predictions have been confirmed by many experiments. These unusual effects are large only at very high speeds (a sizable fraction of the speed of light), which is why we don't normally observe these effects in everyday life, and why we can use nonrelativistic calculations for low-speed phenomena.

However, for the Global Positioning System (GPS) to give adequate accuracy it is necessary to take Einstein's Special Theory of Relativity into account. The atomic clocks on the satellites run slower than our clocks, due to the speed of the satellites, and the difference in clock rate depends on γ. Although γ for the GPS satellites orbiting the Earth is nearly 1, it differs just enough that making the approximation $\gamma \approx 1$ would make the GPS hopelessly inaccurate and useless. It is even necessary to apply corrections based on Einstein's General Theory of Relativity, which correctly predicts an additional change in clock rate due to the gravity of the Earth being weaker at the high altitude of the GPS satellites.

1.X.45 A spaceship at rest with respect to the cosmic microwave background emits a beam of red light. A different spaceship, moving at a speed of 2.5×10^8 m/s toward the first ship, detects the light. Which of the following statements are true for observers on the second ship? (More than one statement may be correct.)

(a) They observe that the light travels at 3×10^8 m/s.
(b) They see light that is not red.
(c) They observe that the light travels at 5.5×10^8 m/s.
(d) They observe that the light travels at 2.5×10^8 m/s.

1.11 *UPDATING POSITION AT HIGH SPEED

If $v \ll c$, $\vec{p} \approx m\vec{v}$ and $\vec{v} \approx \vec{p}/m$. But at high speed it is more complicated to determine the velocity from the (relativistic) momentum. Here is a way to solve for $\vec{v}$ in terms of $\vec{p}$:

$$|\vec{p}| = \frac{1}{\sqrt{1-(|\vec{v}|/c)^2}} m|\vec{v}|$$

Divide by m and square: $$\frac{|\vec{p}|^2}{m^2} = \frac{|\vec{v}|^2}{1-(|\vec{v}|/c)^2}$$

Multiply by $(1-(|\vec{v}|/c)^2)$: $$\frac{|\vec{p}|^2}{m^2} - \left(\frac{|\vec{p}|^2}{m^2c^2}\right)|\vec{v}|^2 = |\vec{v}|^2$$

Collect terms: $$\left(1+\frac{|\vec{p}|^2}{m^2c^2}\right)|\vec{v}|^2 = \frac{|\vec{p}|^2}{m^2}$$

$$|\vec{v}| = \frac{|\vec{p}|/m}{\sqrt{1+\left(\frac{|\vec{p}|}{mc}\right)^2}}$$

The expression above gives the magnitude of $\vec{v}$, in terms of the magnitude of $\vec{p}$. To get an expression for the vector $\vec{v}$, recall that any vector can be factored into its magnitude times a unit vector in the direction of the vector, so

$$\vec{p} = |\vec{p}|\hat{p} \quad \text{and} \quad \vec{v} = |\vec{v}|\hat{v}$$

But since $\vec{p}$ and $\vec{v}$ are in the same direction, $\hat{v} = \hat{p}$, so

$$\vec{v} = |\vec{v}|\hat{p} = \frac{|\vec{p}|/m}{\sqrt{1+\left(\frac{|\vec{p}|}{mc}\right)^2}}\hat{p} = \frac{(|\vec{p}|\hat{p})/m}{\sqrt{1+\left(\frac{|\vec{p}|}{mc}\right)^2}}$$

$$\vec{v} = \frac{\vec{p}/m}{\sqrt{1+\left(\frac{|\vec{p}|}{mc}\right)^2}}$$

THE RELATIVISTIC POSITION UPDATE FORMULA

$$\vec{r}_f = \vec{r}_i + \frac{1}{\sqrt{1+\left(\frac{|\vec{p}|}{mc}\right)^2}}\left(\frac{\vec{p}}{m}\right)\Delta t \text{ (for small } \Delta t\text{)}$$

Note that at low speeds $|\vec{p}| \approx m|\vec{v}|$, and the denominator is

$$\sqrt{1+\left(\frac{|\vec{v}|}{c}\right)^2} \approx 1$$

so the formula becomes the familiar $\vec{r}_f = \vec{r}_i + (\vec{p}/m)\,\Delta t$.

SUMMARY

Interactions (Section 1.2 and Section 1.4)

Interactions are indicated by

- Change of velocity (change of direction and/or change of speed)
- Change of identity
- Change of shape of multiparticle system
- Change of temperature of multiparticle system
- Lack of change when change is expected

Newton's first law of motion (Section 1.3)

An object moves in a straight line and at constant speed, except to the extent that it interacts with other objects.

Vectors (Section 1.5)

A 3D *vector* is a quantity with magnitude and a direction, which can be expressed as a triple $\langle x, y, z\rangle$. A vector is indicated by an arrow: $\vec{r}$.

A *scalar* is a single number.

Legal mathematical operations involving vectors include:

- Adding one vector to another vector
- Subtracting one vector from another vector
- Multiplying or dividing a vector by a scalar
- Finding the magnitude of a vector
- Taking the derivative of a vector

Operations that are *not* legal with vectors include:

- A vector cannot be added to a scalar.
- A vector cannot be set equal to a scalar.
- A vector cannot appear in the denominator (you can't divide by a vector).

A unit vector $\hat{r} = \vec{r}/|\vec{r}|$ has magnitude 1.

A vector can be factored using a unit vector: $\vec{F} = |\vec{F}|\hat{F}$.

Direction cosines: $\hat{r} = \langle\cos\theta_x, \cos\theta_y, \cos\theta_z\rangle$

The symbol Δ

The symbol Δ (delta) means "change of": $\Delta t = t_f - t_i$, $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.

Δ always means "final minus initial."

Velocity and change of position (Section 1.7)

Definition of average velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Velocity is a vector. $\vec{r}$ is the position of an object (a vector). t is the time. Average velocity is equal to the change in position divided by the time elapsed. SI units of velocity are meters per second (m/s).

The position update formula

$$\vec{r}_f = \vec{r}_i + \vec{v}_{\text{avg}}\,\Delta t$$

The final position (vector) is the vector sum of the initial position plus the product of the average velocity and the elapsed time.

Definition of instantaneous velocity

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

The instantaneous velocity is the limiting value of the average velocity as the time elapsed becomes very small.

Velocity in terms of momentum

$$\vec{v} = \frac{\vec{p}/m}{\sqrt{1 + \left(\frac{|\vec{p}|}{mc}\right)^2}} \text{ or } \vec{v} \approx \vec{p}/m \text{ at low speeds}$$

Momentum (Section 1.8)
Definition of momentum

$$\vec{p} = \gamma m \vec{v}$$

where $\gamma = \dfrac{1}{\sqrt{1 - (|\vec{v}|/c)^2}}$ (lower-case Greek gamma)

Momentum (a vector) is the product of the relativistic factor "gamma" (a scalar), mass, and velocity.

Combined into one equation: $\vec{p} = \dfrac{1}{\sqrt{1 - (|\vec{v}|/c)^2}} m\vec{v}$.

Approximation for momentum at low speeds

$$\vec{p} \approx m\vec{v} \text{ at speeds such that } |\vec{v}| \ll c$$

Useful numbers:

Radius of a typical atom: about 1×10^{-10} meter
Radius of a proton or neutron: about 1×10^{-15} meter
Speed of light: 3×10^8 m/s

These and other useful data and conversion factors are given on the inside back cover of the textbook.

EXERCISES AND PROBLEMS

Exercises ("X") are small-scale questions that deal with basic definitions and quantities and require few steps in reasoning. Their purpose is to give you practice in dealing with the basic concepts. Problems ("P") are larger applications that require some thought and may involve several steps in reasoning. The problems give you practice in applying what you are learning.

Section 1.2

1.X.46 Give two examples (other than those discussed in the text) of interactions that may be detected by observing:

(a) Change in velocity
(b) Change in temperature
(c) Change in shape
(d) Change in identity
(e) Lack of change when change is expected

1.X.47 Which of the following observations give conclusive evidence of an interaction? (Choose all that apply.)

(a) Change of velocity, either change of direction or change of speed
(b) Change of shape or configuration without change of velocity
(c) Change of position without change of velocity
(d) Change of identity without change of velocity
(e) Change of temperature without change of velocity. Explain your choice.

1.X.48 Moving objects left the traces labeled *A*–*F* in Figure 1.48 and Figure 1.49. The dots were deposited at equal time intervals (for example, one dot each second). In each case the object starts from the square. Which trajectories show evidence that the moving object was interacting with another object somewhere? If there is evidence of an interaction, what is the evidence?

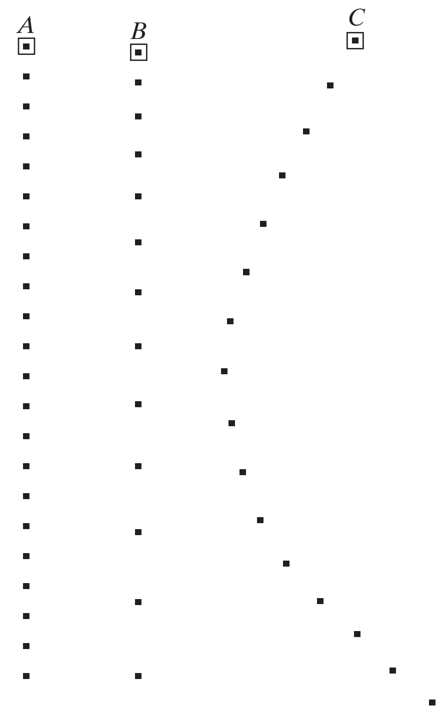


Figure 1.48 Exercise 1.X.48. Also see Figure 1.49.

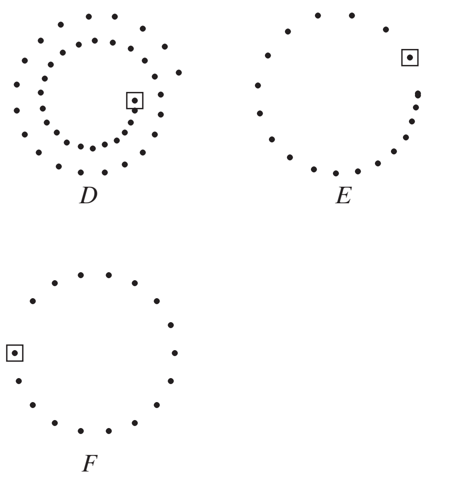


Figure 1.49 Exercise 1.X.48. Also see Figure 1.48.

Section 1.3

1.X.49 Which of the following observers might observe something that appears to violate Newton's first law of motion? Explain why.

- **(a)** A person standing still on a street corner
- **(b)** A person riding on a roller coaster
- **(c)** A passenger on a starship traveling at 0.75*c* toward the nearby star Alpha Centauri
- **(d)** An airplane pilot doing aerobatic loops
- **(e)** A hockey player coasting across the ice

1.X.50 Why do we use a spaceship in outer space, far from other objects, to illustrate Newton's first law? Why not a car or a train? (More than one of the following statements may be correct.)

- **(a)** A car or train touches other objects, and interacts with them.
- **(b)** A car or train can't travel fast enough.
- **(c)** The spaceship has negligible interactions with other objects.
- **(d)** A car or train interacts gravitationally with the Earth.
- **(e)** A spaceship can never experience a gravitational force.

1.X.51 A spaceship far from all other objects uses its thrusters to attain a speed of 1×10^4 m/s. The crew then shuts off the power. According to Newton's first law, what will happen to the motion of the spaceship from then on?

1.X.52 Some science museums have an exhibit called a Bernoulli blower, in which a volleyball hangs suspended in a column of air blown upward by a strong fan. If you saw a ball suspended in the air but didn't know the blower was there, why would Newton's first law suggest that something must be holding the ball up?

1.X.53 Place a ball on a book and walk with the book in uniform motion. Note that you don't really have to do anything to the ball to keep the ball moving with constant velocity (relative to the ground) or to keep the ball at rest (relative to you). Then stop suddenly, or abruptly change your direction or speed. What does Newton's first law of motion predict for the motion of the ball (assuming that the interaction between the ball and the book is small)? Does the ball behave as predicted? It may help to take the point of view of a friend who is standing still, watching you.

Section 1.5

Vector equality; components

1.X.54 Consider a vector $\vec{u} = \langle u_x, u_y, u_z \rangle$, and another vector $\vec{p} = \langle p_x, p_y, p_z \rangle$. If $\vec{u} = \vec{p}$, then which of the following statements must be true? Some, all, or none of the following may be true:

- **(a)** $u_x = p_x$
- **(b)** $u_y = p_y$
- **(c)** $u_z = p_z$
- **(d)** The direction of $\vec{u}$ is the same as the direction of $\vec{p}$.

Vectors and scalars

1.X.55 Does the symbol $\vec{a}$ represent a vector or a scalar?

1.X.56 Which of the following are vectors?

- **(a)** 5 m/s
- **(b)** $\langle -11, 5.4, -33 \rangle$ m
- **(c)** $\vec{r}$
- **(d)** v_z

1.X.57 Does the symbol $|\vec{v}|$ represent a vector or a scalar?

1.X.58 Which of the following are vectors? **(a)** 3.5 **(b)** 0 **(c)** $\langle 0.7, 0.7, 0.7 \rangle$ **(d)** $\langle 0, 2.3, -1 \rangle$ **(e)** -3×10^6 **(f)** $3 \cdot \langle 14, 0, -22 \rangle$

1.X.59 Which of the following are vectors? **(a)** $\vec{r}/2$ **(b)** $|\vec{r}|/2$ **(c)** $\langle r_x, r_y, r_z \rangle$ **(d)** $5 \cdot \vec{r}$

Magnitude of a vector

1.X.60 What is the magnitude of the vector $\vec{v}$, where $\vec{v} = \langle 8 \times 10^6, 0, -2 \times 10^7 \rangle$ m/s?

1.X.61 Figure 1.50 shows several arrows representing vectors in the *xy* plane.

- **(a)** Which vectors have magnitudes equal to the magnitude of $\vec{a}$?
- **(b)** Which vectors are equal to $\vec{a}$?

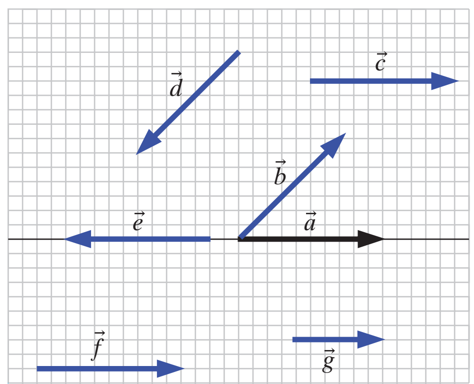


Figure 1.50 Exercise 1.X.61.

1.X.62 In Figure 1.51 three vectors are represented by arrows in the *xy* plane. Each square in the grid represents one meter. For each vector, write out the components of the vector, and calculate the magnitude of the vector.

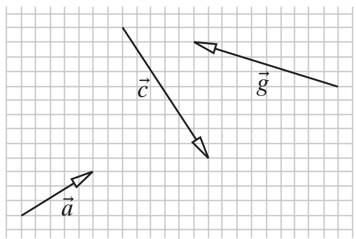


Figure 1.51 Exercise 1.X.62.

Illegal operations with vectors

1.X.63 Is $3 + \langle 2, -3, 5\rangle$ a meaningful expression? If so, what is its value?

Multiplying a vector by a scalar

1.X.64 On a piece of graph paper, draw arrows representing the following vectors. Make sure the tip and tail of each arrow you draw are clearly distinguishable.

(a) Placing the tail of the vector at $\langle 5, 2, 0\rangle$, draw an arrow representing the vector $\vec{p} = \langle -7, 3, 0\rangle$. Label it $\vec{p}$.

(b) Placing the tail of the vector at $\langle -5, 8, 0\rangle$, draw an arrow representing the vector $-\vec{p}$. Label it $-\vec{p}$.

1.X.65 A proton is located at $\vec{r}_p = \langle 2, 6, -3\rangle$ m. An electron is located at $\vec{r}_e = \langle 4, 12, -6\rangle$ m. Which of the following statements are true?

(a) $2\vec{r}_p = \vec{r}_e$ **(b)** $2\hat{r}_p = \hat{r}_e$ **(c)** $|2\vec{r}_p| = |\vec{r}_e|$

1.X.66 The following questions refer to the vectors depicted by arrows in Figure 1.52.

(a) What are the components of the vector $\vec{a}$? (Note that since the vector lies in the xy plane, its z component is zero.)

(b) What are the components of the vector $\vec{b}$?

(c) Is this statement true or false? $\vec{a} = \vec{b}$

(d) What are the components of the vector $\vec{c}$?

(e) Is this statement true or false? $\vec{c} = -\vec{a}$

(f) What are the components of the vector $\vec{d}$?

(g) Is this statement true or false? $\vec{d} = -\vec{c}$

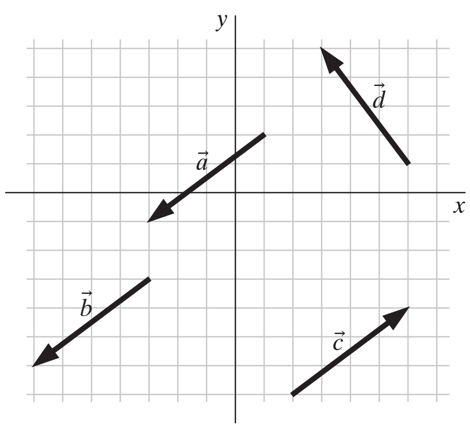


Figure 1.52 Exercise 1.X.66.

1.X.67 **(a)** In Figure 1.53, what are the components of the vector $\vec{d}$?

(b) If $\vec{e} = -\vec{d}$, what are the components of $\vec{e}$?

(c) If the tail of vector $\vec{d}$ were moved to location $\langle -5, -2, 4\rangle$ m, where would the tip of the vector be located?

(d) If the tail of vector $-\vec{d}$ were placed at location $\langle -1, -1, -1\rangle$ m, where would the tip of the vector be located?

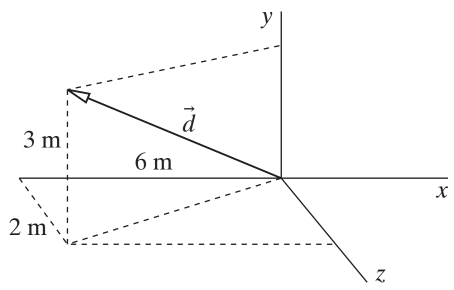


Figure 1.53 Exercise 1.X.67.

1.X.68 What is the result of multiplying the vector $\vec{a}$ by the scalar f, where $\vec{a} = \langle 0.02, -1.7, 30.0\rangle$ and $f = 2.0$?

1.X.69 **(a)** On a piece of graph paper, draw the vector $\vec{f} = \langle -2, 4, 0\rangle$, putting the tail of the vector at $\langle -3, 0, 0\rangle$. Label the vector $\vec{f}$.

(b) Calculate the vector $2\vec{f}$, and draw this vector on the graph, putting its tail at $\langle -3, -3, 0\rangle$, so you can compare it to the original vector. Label the vector $2\vec{f}$.

(c) How does the magnitude of $2\vec{f}$ compare to the magnitude of $\vec{f}$?

(d) How does the direction of $2\vec{f}$ compare to the direction of $\vec{f}$?

(e) Calculate the vector $\vec{f}/2$, and draw this vector on the graph, putting its tail at $\langle -3, -6, 0\rangle$, so you can compare it to the other vectors. Label the vector $\vec{f}/2$.

(f) How does the magnitude of $\vec{f}/2$ compare to the magnitude of $\vec{f}$?

(g) How does the direction of $\vec{f}/2$ compare to the direction of $\vec{f}$?

(h) Does multiplying a vector by a scalar change the magnitude of the vector?

(i) The vector $a(\vec{f})$ has a magnitude three times as great as that of $\vec{f}$, and its direction is opposite to the direction of $\vec{f}$. What is the value of the scalar factor a?

Unit vectors

1.X.70 What is the unit vector in the direction of $\langle -300, 0, 0\rangle$?

1.X.71 What is the unit vector in the direction of $\langle 2, 2, 2\rangle$? What is the unit vector in the direction of $\langle 3, 3, 3\rangle$?

1.X.72 Write the vector $\vec{a} = \langle 400, 200, -100\rangle$ m/s^2 as the product $|\vec{a}| \cdot \hat{a}$.

1.X.73 **(a)** On a piece of graph paper, draw the vector $\vec{g} = \langle 4, 7, 0\rangle$ m. Put the tail of the vector at the origin.

(b) Calculate the magnitude of $\vec{g}$.

(c) Calculate $\hat{g}$, the unit vector pointing in the direction of $\vec{g}$.

(d) On the graph, draw $\hat{g}$. Put the tail of the vector at $\langle 1, 0, 0\rangle$ m so you can compare $\hat{g}$ and $\vec{g}$.

(e) Calculate the product of the magnitude $|\vec{g}|$ times the unit vector $\hat{g}$, $(|\vec{g}|)(\hat{g})$.

1.X.74 A proton is located at $\langle 3\times 10^{-10}, -3\times 10^{-10}, 8\times 10^{-10}\rangle$ m.

(a) What is $\vec{r}$, the vector from the origin to the location of the proton?

(b) What is $|\vec{r}|$?

(c) What is $\hat{r}$, the unit vector in the direction of $\vec{r}$?

1.X.75 Write each of these vectors as the product of the magnitude of the vector and the appropriate unit vector:
(a) $\langle 0, 0, 9.5\rangle$ **(b)** $\langle 0, -679, 0\rangle$
(c) $\langle 3.5\times 10^{-3}, 0, -3.5\times 10^{-3}\rangle$ **(d)** $\langle 4\times 10^{6}, -6\times 10^{6}, 3\times 10^{6}\rangle$

1.X.76 A cube is 3 cm on a side, with one corner at the origin. What is the unit vector pointing from the origin to the diagonally opposite corner at location $\langle 3, 3, 3\rangle$ cm? What is the angle from this diagonal to one of the adjacent edges of the cube?

1.X.77 Which of the following are unit vectors? (Numerical values are given to only 3 significant figures.)

(a) $\langle 0, 0, -1\rangle$
(b) $\langle 0.5, 0.5, 0\rangle$
(c) $\langle 0.333, 0.333, 0.333\rangle$
(d) $\langle 0.9, 0, 0.1\rangle$
(e) $\langle 0, 3, 0\rangle$
(f) $\langle 1, -1, 1\rangle$
(g) $\langle 0.577, 0.577, 0.577\rangle$
(h) $\langle 0.949, 0, -0.316\rangle$

1.X.78 Two vectors, $\vec{f}$ and $\vec{g}$, are equal: $\vec{f} = \vec{g}$. Which of the following statements are true?

(a) $\hat{f} = \hat{g}$
(b) $g_x = f_x$
(c) $f_z = g_z$
(d) The directions of $\vec{f}$ and $\vec{g}$ may be different.
(e) The magnitudes of $\vec{f}$ and $\vec{g}$ may be different.

Vector addition and subtraction

1.X.79 Imagine that you have a baseball and a tennis ball at different locations. The center of the baseball is at $\langle 3, 5, 0\rangle$ m, and the center of the tennis ball is at $\langle -3, -1, 0\rangle$ m. On a piece of graph paper, do the following:

(a) Draw dots at the locations of the center of the baseball and the center of the tennis ball.
(b) Draw the position vector of the baseball, which is an arrow whose tail is at the origin and whose tip is at the location of the baseball. Label this position vector $\vec{B}$. Clearly show the tip and tail of each arrow.
(c) Complete this equation: $\vec{B}$ =< ___, ___, ___ > m.
(d) Draw the position vector of the tennis ball. Label it $\vec{T}$.
(e) Complete this equation: $\vec{T}$ =< ___, ___, ___ > m.
(f) Draw the relative position vector for the tennis ball relative to the baseball. The tail of this vector is at the center of the baseball, and the tip of the vector is at the center of the tennis ball. Label this relative position vector $\vec{r}$.
(g) Complete the following equation by reading the coordinates of $\vec{r}$ from your graph: $\vec{r}$ =< ___, ___, ___ > m.
(h) Calculate this difference: $\vec{T}-\vec{B}$ =< ___, ___, ___ > m.
(i) Is the following statement true? $\vec{r} = \vec{T} - \vec{B}$______.
(j) Write two other equations relating the vectors $\vec{B}$, $\vec{T}$, and $\vec{r}$.
(k) Calculate the magnitudes of the vectors $\vec{B}$, $\vec{T}$, and $\vec{r}$.
(l) Calculate the difference of the magnitudes $|\vec{T}| - |\vec{B}|$.
(m) Does $|\vec{T}| - |\vec{B}| = |\vec{T} - \vec{B}|$?

Change of a vector; relative position

1.X.80 In Figure 1.54, $\vec{r}_1 = \langle 3, -2, 0\rangle$ m and $\vec{r}_2 = \langle 5, 2, 0\rangle$ m. Calculate the position of object 2 relative to object 1, as a relative position vector. Before checking the answer, see whether your answer is consistent with the appearance of the vector $\vec{r}_{2\text{ relative to }1} = \vec{r}_2 - \vec{r}_1$ shown in the diagram. What is the position of object 1 relative to object 2, as a vector?

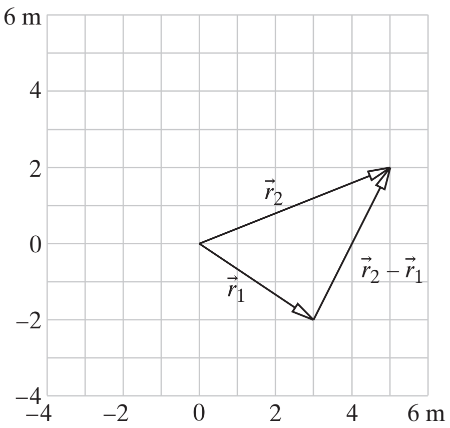


Figure 1.54 Exercise 1.X.80.

1.X.81 **(a)** What is the vector whose tail is at $\langle 9.5, 7, 0\rangle$ m and whose head is at $\langle 4, -13, 0\rangle$ m?
(b) What is the magnitude of this vector?

1.X.82 A man is standing on the roof of a building with his head at the position $\langle 12, 30, 13\rangle$ m. He sees the top of a tree, which is at the position $\langle -25, 35, 43\rangle$ m.

(a) What is the relative position vector that points from the man's head to the top of the tree?
(b) What is the distance from the man's head to the top of the tree?

1.X.83 A star is located at $\langle 6\times 10^{10}, 8\times 10^{10}, 6\times 10^{10}\rangle$ m. A planet is located at $\langle -4\times 10^{10}, -9\times 10^{10}, 6\times 10^{10}\rangle$ m.

(a) What is the vector $\vec{r}_{sp}$ pointing from the star to the planet?
(b) What is the vector $\vec{r}_{ps}$ pointing from the planet to the star?

1.X.84 A planet is located at $\langle -1\times 10^{10}, 8\times 10^{10}, -3\times 10^{10}\rangle$. A star is located at $\langle 6\times 10^{10}, -5\times 10^{10}, 1\times 10^{10}\rangle$.

(a) What is $\vec{r}_{sp}$, the vector from the star to the planet?
(b) What is the magnitude of $\vec{r}_{sp}$?
(c) What is $\hat{r}_{sp}$, the unit vector (vector with magnitude 1) in the direction of $\vec{r}_{sp}$?

1.X.85 A proton is located at $\langle x_p, y_p, z_p\rangle$. An electron is located at $\langle x_e, y_e, z_e\rangle$. What is the vector pointing from the electron to the proton? What is the vector pointing from the proton to the electron?

Section 1.7

1.X.86 In the expression $\Delta\vec{r}/\Delta t$, what is the meaning of $\Delta\vec{r}$? What is the meaning of Δt?

1.X.87 A "slow" neutron produced in a nuclear reactor travels from location $\langle 0.2, -0.05, 0.1\rangle$ m to location $\langle -0.202, 0.054, 0.098\rangle$ m in 2 microseconds ($1\mu s = 1\times 10^{-6}$ s).

(a) What is the average velocity of the neutron?

(b) What is the average speed of the neutron?

1.X.88 The position of a baseball relative to home plate changes from $\langle 15, 8, -3\rangle$ m to $\langle 20, 6, -1\rangle$ m in 0.1 second. As a vector, write the average velocity of the baseball during this time interval.

1.P.89 A spacecraft traveling at a velocity of $\langle -20, -90, 40\rangle$ m/s is observed to be at a location $\langle 200, 300, -500\rangle$ m relative to an origin located on a nearby asteroid. At a later time the spacecraft is at location $\langle -380, -2310, 660\rangle$ m.

(a) How long did it take the spacecraft to travel between these locations?

(b) How far did the spacecraft travel?

(c) What is the speed of the spacecraft?

(d) What is the unit vector in the direction of the spacecraft's velocity?

1.X.90 Start with the definition of average velocity and derive the position update formula from it. Show all steps in the derivation.

1.X.91 At time $t_1 = 12$ s, a car is located at $\langle 84, 78, 24\rangle$ m and has velocity $\langle 4, 0, -3\rangle$ m/s. At time $t_2 = 18$ s, what is the position of the car? (The velocity is constant in magnitude and direction during this time interval.)

1.X.92 At a certain instant a ball passes location $\langle 7, 21, -17\rangle$ m. In the next 3 seconds, the ball's average velocity is $\langle -11, 42, 11\rangle$ m/s. At the end of this 3 second time interval, what is the height y of the ball?

1.X.93 Here are the positions at three different times for a bee in flight (a bee's top speed is about 7 m/s).

Time	6.3 s	6.8 s	7.3 s
Position	$\langle -3.5, 9.4, 0\rangle$ m	$\langle -1.3, 6.2, 0\rangle$ m	$\langle 0.5, 1.7, 0\rangle$ m

(a) Between 6.3 s and 6.8 s, what was the bee's average velocity? Be careful with signs.

(b) Between 6.3 s and 7.3 s, what was the bee's average velocity? Be careful with signs.

(c) Of the two average velocities you calculated, which is the best estimate of the bee's instantaneous velocity at time 6.3 s?

(d) Using the best information available, what was the displacement of the bee during the time interval from 6.3 s to 6.33 s?

1.X.94 The position of a golf ball relative to the tee changes from $\langle 50, 20, 30\rangle$ m to $\langle 53, 18, 31\rangle$ m in 0.1 second. As a vector, write the velocity of the golf ball during this short time interval.

1.X.95 The crew of a stationary spacecraft observe an asteroid whose mass is 4×10^{17} kg. Taking the location of the spacecraft as the origin, the asteroid is observed to be at location $\langle -3 \times 10^3, -4 \times 10^3, 8 \times 10^3\rangle$ m at a time 18.4 seconds after lunchtime. At a time 21.4 seconds after lunchtime, the asteroid is observed to be at location $\langle -1.4 \times 10^3, -6.2 \times 10^3, 9.7 \times 10^3\rangle$ m. Assuming that the velocity of the asteroid does not change during this time interval, calculate the vector velocity $\vec{v}$ of the asteroid.

1.X.96 An electron passes location $\langle 0.02, 0.04, -0.06\rangle$ m, and $2\,\mu$s later is detected at location $\langle 0.02, 1.84, -0.86\rangle$ m (1 microsecond is 1×10^{-6} s).

(a) What is the average velocity of the electron?

(b) If the electron continues to travel at this average velocity, where will it be in another $5\,\mu$s?

1.P.97 Figure 1.55 shows the trajectory of a ball traveling through the air, affected by both gravity and air resistance. Here are the positions of the ball at several successive times:

location	t(s)	position (m)
A	0.0	$\langle 0, 0, 0\rangle$
B	1.0	$\langle 22.3, 26.1, 0\rangle$
C	2.0	$\langle 40.1, 38.1, 0\rangle$

(a) What is the average velocity of the ball as it travels between location A and location B?

(b) If the ball continued to travel at the same average velocity during the next second, where would it be at the end of that second? (That is, where would it be at time $t = 2$ seconds?)

(c) How does your prediction from part (b) compare to the actual position of the ball at $t = 2$ seconds (location C)? If the predicted and observed locations of the ball are different, explain why.

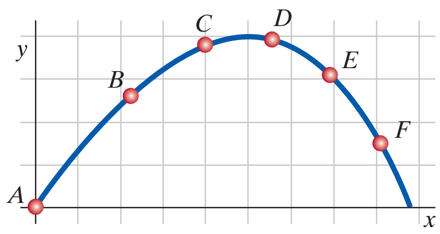


Figure 1.55 Problem 1.P.97.

1.P.98 At 6 seconds after 3:00, a butterfly is observed leaving a flower whose location is $\langle 6, -3, 10\rangle$ m relative to an origin on top of a nearby tree. The butterfly flies until 10 seconds after 3:00, when it alights on a different flower whose location is $\langle 6.8, -4.2, 11.2\rangle$ m relative to the same origin. What was the location of the butterfly at a time 8.5 seconds after 3:00? What assumption did you have to make in calculating this location?

Section 1.8

1.X.99 Which of the following statements about the velocity and momentum of an object are correct?

(a) The momentum of an object is always in the same direction as its velocity.

(b) The momentum of an object can be either in the same direction as its velocity or in the opposite direction.

(c) The momentum of an object is perpendicular to its velocity.

(d) The direction of an object's momentum is not related to the direction of its velocity.

(e) The direction of an object's momentum is tangent to its path.

1.X.100 In which of these situations is it reasonable to use the approximate formula for the momentum of an object, instead of the full relativistically correct formula?

(a) A car traveling on an interstate highway
(b) A commercial jet airliner flying between New York and Seattle
(c) A neutron traveling at 2700 meters per second
(d) A proton in outer space traveling at 2×10^8 m/s
(e) An electron in a television tube traveling 3×10^6 m/s

1.X.101 Answer the following questions about the factor γ (gamma) in the full relativistic formula for momentum:
(a) Is γ a scalar or a vector quantity?
(b) What is the minimum possible value of γ?
(c) Does γ reach its minimum value when an object's speed is high or low?
(d) Is there a maximum possible value for γ?
(e) Does γ become large when an object's speed is high or low?
(f) Does the approximation $\gamma \approx 1$ apply when an object's speed is low or when it is high?

1.X.102 A baseball has a mass of 0.155 kg. A professional pitcher throws a baseball 90 miles per hour, which is 40 m/s. What is the magnitude of the momentum of the pitched baseball?

1.X.103 A proton in an accelerator attains a speed of $0.88c$. What is the magnitude of the momentum of the proton?

1.X.104 A hockey puck with a mass of 0.4 kg has a velocity of $\langle 38, 0, -27 \rangle$ m/s. What is the magnitude of its momentum, $|\vec{p}|$?

1.X.105 A baseball has a mass of about 155 g. What is the magnitude of the momentum of a baseball thrown at a speed of 100 miles per hour? (Note that you need to convert mass to kilograms and speed to meters/second. See the inside back cover of the textbook for conversion factors.)

1.X.106 What is the magnitude (in kg · m/s) of the momentum of a 1000 kg airplane traveling at a speed of 500 miles per hour? (Note that you need to convert speed to meters per second.)

1.X.107 If a particle has momentum $\vec{p} = \langle 4, -5, 2 \rangle$ kg · m/s, what is the magnitude $|\vec{p}|$ of its momentum?

1.X.108 An electron with a speed of $0.95c$ is emitted by a supernova, where c is the speed of light. What is the magnitude of the momentum of this electron?

1.X.109 A "cosmic-ray" proton hits the upper atmosphere with a speed $0.9999c$, where c is the speed of light. What is the magnitude of the momentum of this proton? Note that $|\vec{v}|/c = 0.9999$; you don't actually need to calculate the speed $|\vec{v}|$.

1.X.110 A proton in an accelerator is traveling at a speed of $0.99c$. (Masses of particles are given on the inside back cover of this textbook.)
(a) If you use the approximate nonrelativistic formula for the magnitude of momentum of the proton, what answer do you get?
(b) What is the magnitude of the correct relativistic momentum of the proton?
(c) The approximate value (the answer to part a) is significantly too low. What is the ratio of magnitudes you calculated (correct/approximate)? Such speeds are attained in particle accelerators.

1.X.111 An object with mass 1.6 kg has momentum $\langle 0, 0, 4 \rangle$ kg · m/s.
(a) What is the magnitude of the momentum?
(b) What is the unit vector corresponding to the momentum?
(c) What is the speed of the object?

1.X.112 When the speed of a particle is close to the speed of light, the factor γ, the ratio of the correct relativistic momentum $\gamma m\vec{v}$ to the approximate nonrelativistic momentum $m\vec{v}$, is quite large. Such speeds are attained in particle accelerators, and at these speeds the approximate nonrelativistic formula for momentum is a very poor approximation. Calculate γ for the case where $|\vec{v}|/c = 0.9996$.

1.X.113 An electron travels at speed $|\vec{v}| = 0.996c$, where $c = 3 \times 10^8$ m/s is the speed of light. The electron travels in the direction given by the unit vector $\hat{v} = \langle 0.655, -0.492, -0.573 \rangle$. The mass of an electron is 9×10^{-31} kg.
(a) What is the value of $\gamma = 1/\sqrt{1-(|\vec{v}|/c)^2}$? You can simplify the calculation if you notice that $(|\vec{v}|/c)^2 = (0.996)^2$.
(b) What is the speed of the electron?
(c) What is the magnitude of the electron's momentum?
(d) What is the vector momentum of the electron? Remember that any vector can be "factored" into its magnitude times its unit vector, so that $\vec{v} = |\vec{v}|\hat{v}$.

1.X.114 If p/m is $0.85c$, what is v in terms of c?

Section 1.9

1.X.115 A tennis ball of mass m traveling with velocity $\langle v_x, 0, 0 \rangle$ hits a wall and rebounds with velocity $\langle -v_x, 0, 0 \rangle$.
(a) What was the change in momentum of the tennis ball?
(b) What was the change in the magnitude of the momentum of the tennis ball?

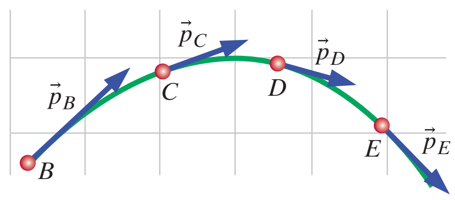


Figure 1.56 Problem 1.P.117.

1.X.116 A 50 kg child is riding on a carousel (merry-go-round) at a constant speed of 5 m/s. What is the magnitude of the change in the child's momentum $|\Delta\vec{p}|$ in going all the way around (360°)? In going halfway around (180°)? Draw a diagram showing the initial vector momentum and the final vector momentum, then subtract, then find the magnitude.

1.P.117 Figure 1.56 shows a portion of the trajectory of a ball traveling through the air. Arrows indicate its momentum at several locations.
At various locations, the ball's momentum is:
$\vec{p}_B = \langle 3.03, 2.83, 0 \rangle$ kg · m/s
$\vec{p}_C = \langle 2.55, 0.97, 0 \rangle$ kg · m/s
$\vec{p}_D = \langle 2.24, -0.57, 0 \rangle$ kg · m/s
$\vec{p}_E = \langle 1.97, -1.93, 0 \rangle$ kg · m/s
$\vec{p}_F = \langle 1.68, -3.04, 0 \rangle$ kg · m/s
(a) Calculate the change in the ball's momentum between each pair of adjacent locations.
(b) On a copy of the diagram, draw arrows representing each $\Delta\vec{p}$ you calculated in part (a).
(c) Between which two locations is the magnitude of the change in momentum greatest?

Computational Problems

These problems are intended to introduce you to using a computer to model matter, interactions, and motion. You will build on these small calculations to build models of physical systems in later chapters.

Some parts of these problems can be done with almost any tool (spreadsheet, math package, etc.). Other parts are most easily done with a programming language. We recommend the free 3D programming environment VPython, obtainable at http://vpython.org. Your instructor will introduce you to an available computational tool and assign problems, or parts of problems, that can be addressed using the chosen tool.

1.P.118 **(a)** Write a program that makes an object move from left to right across the screen with velocity $\langle v_x, 0, 0\rangle$. Make v_x a variable, so you can change it later. Let the time interval for each step of the computation be a variable dt, so that the position x increases by an amount $v_x dt$ each time.
(b) Modify a copy of your program to make the object run into a wall and reverse its direction.
(c) Make a modification so that v_x is no longer a constant but changes smoothly with time. Is the change of the speed of the object clearly visible to an observer? Try to make one version in which the speed change is clearly noticeable, and another in which it is not noticeable.
(d) Corresponding to part (c), make a computer graph of x vs. t, where t is the time.
(e) Corresponding to part (c), make a computer graph of v_x vs. t, where t is the time.

1.P.119 **(a)** Write a program that makes an object move at an angle.
(b) Change the component of velocity of the object in the x direction but not in the y direction, or vice versa. What do you observe?
(c) Start the object moving at an angle and make it bounce off at an appropriate angle when it hits a wall.

1.P.120 Write a program that makes an object move smoothly from left to right across the screen with speed v, leaving a trail of dots on the screen at equal time intervals. If the dots are too close together, leave a dot every N steps, and adjust N to give a nice display.

1.P.121 This problem requires a 3D programming environment. Assume SI units, and give the objects reasonable and distinguishable attributes (such as sizes and colors). Run the program after each part, to make sure your display is correct.

- **(a)** Create a sphere representing a basketball, at location $\langle -5, 2, -3\rangle$ m.
- **(b)** Create an arrow to represent the position of the basketball, relative to the origin.
- **(c)** Create a second sphere to represent a volleyball, at location $\langle -3, -1, 3.5\rangle$ m.
- **(d)** Create an arrow to represent the position of the volleyball relative to the origin.
- **(e)** Using symbolic names (no numbers) for the positions and attributes of these balls, create an arrow to represent the position of the volleyball relative to the basketball.
- **(f)** Change the positions of the basketball and the volleyball to $\langle -4, 2, 5\rangle$ m and $\langle 3, 1, -2\rangle$ m, respectively. If you correctly used symbolic names in the previous part, the arrow representing the relative position vector should automatically adjust. Does it?
- **(g)** Add to your program instructions to print the position of each ball, and the position of the tip and the tail of the arrow (all 3D vectors).

1.P.122 Write a program to display the motion of a cart traveling on a track at constant velocity.

- **(a)** Create a box to represent a track 2 meters long, 10 cm wide, and 5 cm high. Orient the long axis of the track along the x axis.
- **(b)** Create a second box to represent a low-friction cart that is 10 cm long, 4 cm high, and 6 cm wide. Give this box a symbolic name so you can refer to it later in the program.
- **(c)** Position the cart so it is 1 cm above the track, and its left edge lines up with the left end of the track.
- **(d)** Create a vector variable to represent the velocity of the cart. Give the cart an initial velocity of $\langle 0.2, 0, 0\rangle$ m/s.
- **(e)** Using a time step of 0.01 seconds, write a loop to use the position update formula to animate the motion of the cart as it travels at constant velocity from one end of the track to the other.
- **(f)** Change your program to start the cart at the right end of the track and move to the left.
- **(g)** What would happen if the cart had a nonzero y velocity component? Try it.
- **(h)** Have your cart leave a trail behind it as it moves.

1.P.123 Write a program to display the motion of a hockey puck sliding at constant velocity on ice.

- **(a)** Represent the ice surface by a box that is large in the x and z dimensions and thin in the y dimension. Represent the puck by a small cylinder, positioned just above the ice surface. (If you do not have a 3D programming environment, represent the puck by a circle rather than a cylinder.) Give the puck a symbolic name so you can refer to it later in the program.
- **(b)** Position the puck at one corner of the ice surface.
- **(c)** Create a vector variable to represent the velocity of the puck. Assign velocity components so the puck will slide diagonally across the ice.
- **(d)** Using a time step of 0.001 second, write a loop to use the position update formula to animate the motion of the puck as it slides from one corner of the ice surface to another.

ANSWERS TO EXERCISES

1.X.1 a, d
1.X.2 a, b, c, e
1.X.3 a, b, c
1.X.4 a, d
1.X.5 d, e
1.X.6 three
1.X.7 one
1.X.8 5.10 m

1.X.9 no
1.X.10 d
1.X.11 no
1.X.12 $\langle -0.09, 4.2, -78.0\rangle$
1.X.13 $\langle 1, -1.5, 2.5\rangle$
1.X.14 $\langle 6, -9, 15\rangle$ m/s
1.X.15 They are the same.
1.X.16 $\langle 0, 1, 0\rangle$
1.X.17 $\langle 0.873, 0.436, -0.218\rangle$
1.X.18 7
1.X.19 no
1.X.20 361, 335, 577, 696, no
1.X.21 $\langle 0, 0, 0\rangle$, 0, 7.07×10^3, 7.07×10^3, 1.41×10^4
1.X.22 $\langle 450, -300, -200\rangle$, $\langle 150, 300, -200\rangle$, $\langle -150, -300, 200\rangle$
1.X.23 a, b, e
1.X.24 (a) $\langle 9, 2, 20\rangle$ m, (b) 120 s
1.X.25 $\langle -1, 0, -1\rangle$ m
1.X.26 $\langle -0.940, 0.342, 0\rangle$
1.X.27 $\langle -0.643, 0.766, 0\rangle$
1.X.28 2.67×10^{-3} m/s
1.X.29 (a) $\langle 8, -10, 12\rangle$ m/s, (b) 17.55 m/s, (c)$\langle 0.456, -0.570, 0.684\rangle$
1.X.30 (a) $\langle 25, -55, -30\rangle$ m/s, (b) 67.45 m/s, (c) $\langle 0.371, -0.815, -0.445\rangle$
1.X.31 $\langle 2.1 \times 10^5, 1.4 \times 10^5, -2.8|5 \times 10\rangle$ m
1.X.32 0.5 s
1.X.33 Average velocity is the displacement (change in position) divided by the total time; instantaneous velocity is the limit of the average velocity as the time interval gets very small.
1.X.34 1a, 2h, 3g, 4f, 5e
1.X.35 5 m/s/s, about half as big
1.X.36 $\langle 5, 8t, 2 - 18t^2\rangle$, $\langle 0, 8, -16t\rangle$, $\langle 5, 0, 2\rangle$, $\langle 0, 0, -16\rangle$
1.X.37 650 kg · m/s
1.X.38 2.415×10^{-22} kg · m/s
1.X.39 *b*
1.X.40 $\langle 0.4, 0, 10\rangle$ m
1.X.41 $\langle 111.31, 0, 41.54\rangle$ m
1.X.42 (a) The diagram should show the ball bouncing back to the left with almost its initial speed, (b) $\langle -5.586, 0, 0\rangle$ kg · m/s, (c) -0.114 kg · m/s
1.X.43 (a) $\langle -1.6 \times 10^{28}, 0, 1.6 \times 10^{28}\rangle$ kg · m/s; (b) and (c) if you make a careful diagram, with the two arrows tail to tail, you'll see that the arrow representing $\Delta\vec{p}$ from C to D points up and to the right on the page (toward the Sun).
1.X.44 (a) $\langle 7, 0, 0\rangle$, (b) $\langle 0, 0, 2\rangle$
1.X.45 a, b