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Predictive maintenance of systems subject to hard failure based on proportional hazards model

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ABSTRACT

The remaining useful lifetime (RUL) estimated from the in-situ degradation data has shown to be useful for online predictive maintenance. In the literature, the RUL is often estimated by assuming a soft-failure threshold for the degradation data. In practice, however, systems may not be subject to the degradation-induced soft failures. Instead, the systems are deemed to be fail when they cannot perform the intended function, and such failures are known as hard failures. Because there are no fixed thresholds for hard failures, the corresponding RUL estimation is not an easy task, which causes difficulties in finding the optimal maintenance schedule. In this study, a Weibull proportional hazards model is proposed to jointly model the degradation data and the failure time data. The degradation data are treated as the time-varying covariates so that the degradation does not directly lead to system failures, but increases the hazard rate of hard failures. A random-effects Wiener process is proposed to model the degradation data by considering the system heterogeneities. Based on the developed proportional hazards model, closed-form distribution of the RUL is derived upon each inspection and the optimal maintenance schedule is then obtained by minimizing the system maintenance cost. The proposed maintenance strategy is successfully applied to predictive maintenance of lead-acid batteries.

1. Introduction

An effective maintenance plan is essential to ensure high system reliability and availability. It can reduce the maintenance cost and increase production efficiency, as well as mitigate the failure risk that may incur numerous economic losses or even threats to lives. The simplest maintenance schedule might be the pure corrective maintenance, where maintenance is only performed upon unexpected failures. To achieve higher values of maintenance, it is usually necessary to determine a preventive maintenance schedule in the case of catastrophic failures. Classic preventive maintenance usually consists of two steps: (1) predicting the system failure time based on historical system lifetime data and (2) planning and conducting maintenance actions [20].

As an alternative to the lifetime data that are difficult to collect for many systems, continuously or periodically monitored degradation data can be utilized for better prognostics and health management of the system [12,31]. Degradation refers to the cumulative change of a subject's performance characteristic over time [15], and examples of degradation can be found in the capacity of batteries of hybrid-electric vehicles [1], the leveling defects of railway tracks [18], the tensile strength of carbon micro-composites [21], to name a few. Due to the advances in measurement technology and deployment of sensors, collection of degradation data becomes less costly, which can further facilitate maintenance decisions based on the in-situ degradation signals [9,14,25]. This is known as condition-based maintenance (CBM) or, more broadly, predictive maintenance, which has shown to be more effective than classic preventive maintenance strategies [30].

Most existing literature of CBM assumes that the system or product fails only when its degradation level exceeds a fixed threshold. Under this assumption, the system failure time can be deemed as the first hitting time of the degradation process to the fixed failure threshold [29]. Although it facilitates the mathematical modeling and analysis, the assumption may be inapplicable in many real applications. A motivating example is the failure behavior of automotive lead-acid batteries in [32], where the degradation and failure time data of 13 batteries are shown in Fig. 1. The dataset is from an accelerated battery aging test in SAE J2801 [23], where the 13 batteries are of the same type. The life test simulates high heat automotive service when the battery operates in a voltage regulated charging system. It subjects the battery to charge and discharge cycles comparable to those encountered

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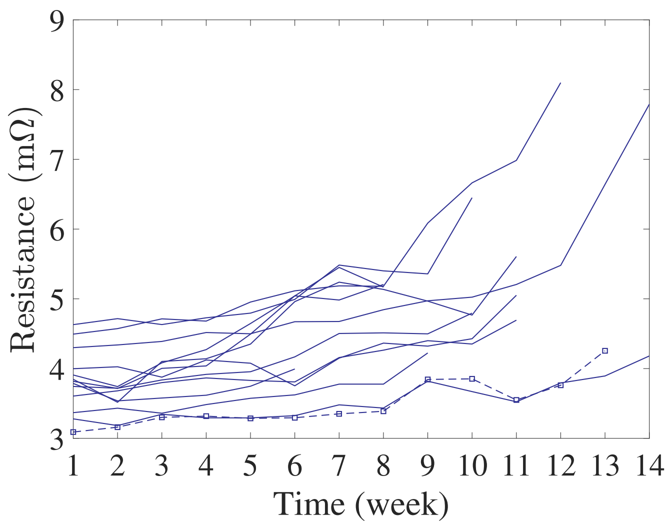

Fig. 1. Battery resistance and failure time data from an accelerated aging test.

in automotive service. Here, the degradation characteristic is the battery resistance, and the failure time data are based on the aging cycles, where a battery is deemed to fail when it fails to crank the engine. It is obvious that each battery fails at a different degradation level, and the assumption of the fixed failure threshold is not applicable here. This kind of instantaneous failure mode is called hard failure and it is usually characterized by a hazard rate function [17]. Some other examples of hard failures include high-voltage power transformers, bridge systems, and integrated digital communication systems [13,14,27], to name a few. In order to make an appropriate maintenance plan, it is necessary to model the failure behaviors of systems subject to hard failures.

In this study, a proportional hazards model is proposed to model the hard failure behaviors by integrating both the aging and degradation effects. The underlying idea is that although the degradation does not directly lead to system failure, it may be closely related to the system failure rate. For example, higher resistance often indicates higher failure probability of the batteries. With this in mind, we treat the degradation data as the covariates that affect the hazard rate of failure based on proportional hazards model. In fact, the similar treatment can also be found in the literature [e.g., [11,13,24,32]]. In these studies, the dynamics of degradation are usually captured by a general path model. Although it is mathematically convenient, the general path model can not reflect the time-varying volatility in the degradation data, which motivates us to propose a new joint model for degradation and failure data analysis.

To this end, this study uses a Wiener process to model the system degradation. As a stochastic process, the Wiener process is widely adopted in degradation modeling due to its ability to model the temporal variation [29]. Due to diverse usage and environmental differences, the degradation characteristics of different systems are often different. To account for the system-to-system heterogeneities, random effects are further incorporated into the degradation model. The proposed random-effects Wiener process is then used as a time-varying covariate for the hazard function of the hard failure. In addition, we choose the Weibull hazard function as the baseline hazard function, which is a popular choice in failure data analysis [3,8,19].

Based on the proposed Weibull proportional hazards model, we update the joint distribution of degradation parameters by using all the historical degradation data upon each inspection epoch. Afterwards, we derive the distribution of the system remaining useful life (RUL) using the Brownian bridge theory and then make use of the distribution to determine the optimal predictive maintenance schedule. Under the maintenance schedule, we can minimize the long-run average maintenance cost per cycle based on the degradation signals. The proposed maintenance policy is an online method in the sense that the optimal maintenance schedule is updated when new degradation signals are available. One related model may be found in [16], where a Wiener process was also used for degradation modeling. However, due to a lack of the closed-form expression of the RUL distribution, their method is computationally intensive. Moreover, system-to-system heterogeneities were not considered in that study.

The remainder of the study is organized as follows. Section 2 proposes the Weibull proportional hazards model to model hard failures. In addition, the predictive maintenance strategy is also proposed. Section 3 derives the system RUL distribution upon each decision epoch, which is used to obtain the optimal predictive maintenance schedule. Section 4 uses a case study to demonstrate the effectiveness of the proposed method. Section 5 concludes the study and discusses some possible future research.

2. Problem statement

2.1. Model development

Consider a system that is subject to hard failure. The hazard rate for the hard failure is assumed to be a function of the in-situ degradation level, which is given by

$$h(t, X(t)) = h_0(t)\varphi(X(t)). \tag{1}$$

The form of (1) is motivated by the well-known proportional hazards model [2], where $h_0(t)$ is the baseline hazard rate function and $\varphi(X(t))$ characterizes the effects of degradation on $h(t, X(t))$. The degradation is modeled by a drifted Wiener process as

$$X(t) = x_0 + \mu t + \sigma B(t), \tag{2}$$

where $x_0 > 0$ is the initial degradation level, $\mu > 0$ is the degradation rate, $\sigma > 0$ is the degradation volatility parameter, and $B(t)$ is a standard Brownian motion. Without loss of generality, we assume that the degradation process is generally increasing over time [7,13]. We adopt a linear form $\varphi(X(t)) = \beta X(t)$ in this study, where $\beta > 0$ is the corresponding coefficient. We also assume that the baseline hazard rate of the hard failure follows the Weibull model, i.e., $h_0(t) = mt^{m-1}/\eta^m$. The Weibull assumption has been widely used in reliability engineering as it has reasonable flexibility in fitting lifetime data [3,8,19]. In addition, the above two assumptions enable us to derive the explicit analytical form of the RUL distribution, which facilitates the online predictive maintenance in Sections 2.2 and 3.

In this study, we assume that the shape and the scale parameters m and η in the Weibull distribution, and the coefficient β in $\varphi(\cdot)$ fully characterize the population of interest. To focus on the optimal predictive maintenance schedule, these parameters are assumed to be known. In practice, these parameters can be estimated in advance based on the historical degradation and failure data of systems from the population. On the other hand, we assume the degradation rate and volatility parameters μ and σ are unknown and their values vary for different systems. This random-effects model has been commonly used in degradation literature to incorporate system-to-system heterogeneities [22,28]. To be specific, we assume that both the parameters follow a truncated normal distribution [26,28], i.e., $\mu \sim \mathcal{TN}(\omega, \kappa^{-2})$ and $\sigma \sim \mathcal{TN}(\delta, \gamma^{-2})$, where the probability density functions (PDF) of $\mathcal{TN}(\omega, \kappa^{-2})$ is given by

$$f(\mu|\omega, \kappa^{-2}) = \frac{\kappa\phi[\kappa(\mu - \omega)]}{1 - \Phi(-\kappa\omega)}, \quad \omega > 0, \quad \kappa > 0. \tag{3}$$

Here, $\phi(\cdot)$ and $\Phi(\cdot)$ are the standard normal PDF and the cumulative distribution function (CDF), respectively. The parameters ω, κ, δ, γ are assumed known or accurately estimated based on historical degradation data of systems from the same population. With the online inspection scheme in real applications, we can use the observed degradation data to update the joint distribution of μ and σ, and hence the optimal maintenance time.

2.2. Maintenance policy

In this study, we aim to develop a predictive maintenance model that determines the optimal maintenance time based on the observed degradation data. Consider an online inspection scheme where inspections will be conducted at predetermined time epochs $0 = t_0 < t_1 < t_2, \ldots$. In the literature, $t_0, t_1, t_2, \ldots$, are often assumed to be equally spaced. Here, we consider a more general setting that includes the periodic inspection as a special case. Upon each inspection, we first update the joint distributions of the degradation parameters μ and σ based on all the historical degradation data. The maintenance time interval is then obtained by minimizing the long-run average maintenance cost per cycle. In specific, let C_p and C_f be the costs of a planned replacement and a failure replacement, respectively. We follow the common assumption that $C_f > C_p$, since unexpected failures usually incur numerous downtime losses as well as an increased cost in materials and labor during maintenance. At the kth inspection epoch t_k, we update the maintenance time interval τ so that the maintenance will be conducted at time $t_k + \tau$. The long-run average maintenance cost per replacement cycle given τ can be represented as follows [4]:

$$C(\tau|\boldsymbol{X}_k) = \frac{C_p\bar{F}(\tau|\boldsymbol{X}_k) + C_f(1 - \bar{F}(\tau|\boldsymbol{X}_k))}{\int_0^\tau \bar{F}(z|\boldsymbol{X}_k)dz + t_k}, \tag{4}$$

where $\boldsymbol{X}_k \equiv [X(t_0), X(t_1), \ldots, X(t_k)]'$ denotes all the historical degradation data till epoch t_k and $\bar{F}(\tau|\boldsymbol{X}_k)$ is the corresponding survival function of the system RUL given $\boldsymbol{X}_k$. We can see that $C(\tau|\boldsymbol{X}_k)$ is directly related to the system RUL distribution $\bar{F}(\tau|\boldsymbol{X}_k)$. It means that we are essentially using the in-situ degradation data to make a better maintenance decision. However, it is challenging to obtain the value of $\bar{F}(\tau|\boldsymbol{X}_k)$, and we will derive its explicit analytical expression in Sections 3. Let τ^* be the minimizer of (4), i.e., the optimal updated maintenance time upon t_k, and C^* be the corresponding optimal maintenance cost rate. If $t_k + \tau^* \le t_{k+1}$, a preventive replacement is scheduled at $t_k + \tau^*$ so that the system returns to the as-good-as-new state. Otherwise, no action is taken until t_{k+1} and then the above procedure is repeated to update the maintenance time based on the degradation data from epochs t_0 to t_{k+1}, i.e., $\boldsymbol{X}_{k+1}$.

3. Remaining useful life distribution

3.1. RUL Distribution with known μ and σ

In order to derive the optimal predictive maintenance time τ^* to minimize $C(\tau|\boldsymbol{X}_k)$ in (4), we need to derive the RUL distribution $\bar{F}(\tau|\boldsymbol{X}_k)$ given the historical degradation data $\boldsymbol{X}_k$. This section first derives this survival function assuming the system-specific degradation parameters μ and σ are known. With the updated joint distribution of these parameters, the system RUL distribution and the optimal predictive maintenance time can then be obtained when μ and σ are unknown.

Without loss of generality, suppose that we are at the kth inspection epoch t_k and the historical degradation level is $\boldsymbol{X}_k = [x_0, x_1, \ldots, x_k]'$. Let the underlying remaining times to the hard failure be T_F. To derive $\bar{F}(\tau|\boldsymbol{X}_k)$, we first need to derive the distribution of $Y(t) \equiv \int_0^\tau h_0(s + t_k)\varphi(X(s + t_k|x_k))ds$. It follows from Hoel et al. [6, Page 134] that $Y(t)$ is normal distributed for all $\tau \ge 0$. In specific, following the construction procedure in Hoel et al. [6, Pages 134–135], we have

$$E\left\{\left[\int_0^\tau h_0(s)B(s)ds\right]^2\right\} = \int_0^\tau \left[H_0(\tau) - H_0(s)\right]^2 ds.$$

Therefore, the system RUL distribution only considering hard failure can be derived based on the moment generating function of a normal random variable. For notation convenience, let $A(t_k, x_k)$ denote the event that $X(t_k) = x_k$. After straightforward calculation, the system RUL distribution given μ and σ upon the kth inspection epoch can then be expressed as

$$\begin{aligned}\bar{F}(\tau|\mu, \sigma) &= \Pr(T_F > \tau|\boldsymbol{X}_k, \mu, \sigma) = \Pr(T_F > \tau|A(t_k, x_k), \mu, \sigma)\\ &= \mathrm{E}\left\{\exp\left[-\beta\int_{t_k}^{t_k+\tau} h_0(s)X(s)ds\right]\right\}\\ &= \exp\left[-\beta x_k H_0(t) - \beta\mu\int_{t_k}^{t_k+\tau} h_0(s)sds + \frac{\beta^2\sigma^2}{2}\int_{t_k}^{t_k+\tau}\left(H_0(t) - H_0(s)\right)^2 ds\right]\\ &= \exp\left[-\beta(x_k - \mu t_k)\left(\frac{(t_k+\tau)^m - t_k^m}{\eta^m}\right) - \frac{\beta\mu m}{m+1}\left(\frac{(t_k+\tau)^{m+1} - t_k^{m+1}}{\eta^m}\right)\right.\\ &\quad + \frac{\beta^2\sigma^2}{2\eta^{2m}}\left((t_k+\tau)^{2m}\tau - \frac{2(t_k+\tau)^m((t_k+\tau)^{m+1} - t_k^{m+1})}{m+1}\right.\\ &\quad \left.\left. + \frac{((t_k+\tau)^{2m+1} - t_k^{2m+1})}{2m+1}\right)\right],\end{aligned} \tag{5}$$

where $H_0(t) = \int_{t_k}^{t_k+\tau} h_0(s)ds$.

3.2. RUL Distribution with unknown μ and σ

The system RUL distribution in (5) can be derived for known μ and σ, whose values are unobservable in real applications. This section derives the RUL distribution when μ and σ are unknown. Note that we have assumed that μ and σ follow the truncated normal distributions in (3). By leveraging on the observed degradation data, we can update the joint distributions of μ and σ for a more cost-effective maintenance planning. Recall that the historical degradation data are $\boldsymbol{X}_k = [x_0, \ldots, x_k]'$ upon the kth inspection epoch t_k. Let $\Delta x_i = x_i - x_{i-1}$ and $\Delta t_i = t_i - t_{i-1}$, $i = 1, \ldots, k$, be the degradation increment and the time interval between the ith and the $(i + 1)$th inspections, respectively. For notation convenience, let $A(t_k, \tau, x_k, y)$ denote the event that $X(t_k) = x_k$ and $X(t_k + \tau) = y$.

By the conditional independence, the joint distribution of μ and σ given $\boldsymbol{X}_k$ is

$$\begin{aligned}f(\mu, \sigma|\boldsymbol{X}_k) = &\; Cf(\mu|\omega, \kappa^{-2})f(\sigma|\delta, \gamma^{-2})\\ &\times \prod_{i=1}^{k}\frac{1}{\sqrt{2\pi\Delta t_i\sigma^2}}\exp\left[-\frac{(\Delta x_i - \mu\Delta t_i)^2}{2\Delta t_i\sigma^2}\right]\\ &\Pr(T_F > \Delta t_i|A(t_{i-1}, \Delta t_i, x_{i-1}, x_i)),\end{aligned} \tag{6}$$

where C is a normalizing term independent of μ and σ. Meanwhile, $\Pr(T_F > \tau|A(t_k, \tau, x_k, y))$ denotes the probability that no hard failure occurs during time interval $[t_k, t_k + \tau]$ conditional on $A(t_k, \tau, x_k, y)$, which can be derived from the fact that $\{X(t|A(t_k, \tau, x_k, y)), t_k \le t \le t_k + \tau\}$ is a Brownian bridge. Based on [10], we have

$$E[X(t)] = \frac{y(t-t_k) + x_k(t_k+\tau-t)}{\tau}, \quad t_k \le t \le t_k+\tau,$$

Then, $Z = \int_{t_k}^{t_k+\tau} h_0(s)X(s)ds$ follows a Gaussian distribution, with mean

$$E[Z] = \int_{t_k}^{t_k+\tau} h_0(s)E[X(s)]ds = \int_{t_k}^{t_k+\tau} h_0(s)\frac{y(s-t_k) + x_k(t_k+\tau-s)}{\tau}ds,$$

and variance $\mathrm{Var}[Z] = E[Z^2] - E[Z]^2$, where

$$E[Z^2] = \int_{t_k}^{t_k+\tau} h_0(s)\int_{t_k}^{t_k+\tau} h_0(u)E[X(s)X(u)]duds,$$

$$E[X(s)X(u)] = \mathrm{Cov}[X(s), X(u)] + E[X(s)]E[X(u)],$$

and

$$\mathrm{Cov}[X(s), X(u)] = \frac{\sigma^2(t_k+\tau-\max\{s,u\})(\min\{s,u\}-t_k)}{\tau}.$$

The variance of Z can be then obtained based on the above values and it is given as

$$\mathrm{Var}(Z) = \int_{t_k}^{t_k+\tau}\int_{t_k}^{t_k+\tau} h_0(s)h_0(u)\left\{\frac{\sigma^2(t_k+\tau-\max\{s,u\})(\min\{s,u\}-t_k)}{\tau} + g(u)g(s)\right\}duds - E[Z]^2,$$

where $g(s) = [y(s-t_k) + x_k(t_k+\tau-s)]/\tau$. Based on this distribution, the closed-form of $\Pr(T_F > \tau|A(t_k, \tau, x_k, y))$ can be derived based on the moment generating function of the normal random variable, which is

$$\begin{aligned}\Pr(T_F > \tau|A(t_k,\tau,x_k,y)) = \exp\Bigg\{ & -\int_{t_k}^{t_k+\tau}\beta h_0(s)\frac{y(s-t_k)+x_k(t_k+\tau-s)}{\tau}ds \\ & + \frac{1}{2}\beta^2\int_{t_k}^{t_k+\tau}\int_{t_k}^{t_k+\tau} h_0(s)h_0(u) \\ & \Bigg[\frac{\sigma^2(t_k+\tau-\max\{s,u\})(\min\{s,u\}-t_k)}{\tau} \\ & + \frac{(y(u-t_k)+x_k(t_k+\tau-u))(y(s-t_k)+x_k(t_k+\tau-s))}{\tau^2}\Bigg]duds \\ & - \frac{1}{2}\left[\int_{t_k}^{t_k+\tau}\beta h_0(s)\frac{y(s-t_k)+x_k(t_k+\tau-s)}{\tau}ds\right]^2\Bigg\}.\end{aligned} \quad (7)$$

With this conditional joint distribution $f(\mu, \sigma|\boldsymbol{X}_k)$, we can now update the RUL distribution upon t_k by integrating μ and σ out, which is given by

$$\bar{F}(\tau|\boldsymbol{X}_k) = \int_0^\infty\int_0^\infty \bar{F}(\tau|\mu,\sigma)f(\mu,\sigma|\boldsymbol{X}_k)d\mu d\sigma, \quad \tau > 0. \quad (8)$$

3.3. The Metropolis-Hasting algorithm

Directly computing the value of $\bar{F}(\tau|\boldsymbol{X}_k)$ can be quite time-consuming. This is because we need to first compute the double integral in (7) to obtain $\bar{F}(\tau|\mu,\sigma)$, and then compute another double integral in (8). In view of this fact, we use a Monte Carlo simulation to approximate the value of $\bar{F}(\tau|\boldsymbol{X}_k)$ in this study. We first sample B groups of μ and σ from the PDF $f(\mu, \sigma|\boldsymbol{X}_k)$, where B is a sufficiently large number. For each group of μ and σ, we use the numerical integral to calculate (7), obtain $\bar{F}(\tau|\mu,\sigma)$ in (5), and take average of the B groups to obtain the value of (8). To generate the samples of μ and σ, we develop a Metropolis-Hasting algorithm, which is a well-known Markov chain Monte Carlo (MCMC) method [22], with detailed steps provided in the Appendix. The proposed algorithm enables us to obtain the variance and the kernel density distribution of $\bar{F}(\tau|\boldsymbol{X}_k)$ based on the generated samples (μ, σ) from $f(\mu, \sigma|\boldsymbol{X}_k)$. With $\bar{F}(\tau|\boldsymbol{X}_k)$ in (8), we can now obtain the value of the long-run average maintenance cost per cycle $C(\tau|\boldsymbol{X}_k)$ in (4) given any maintenance time τ. The optimal maintenance time τ^* can be efficiently obtained by a one-dimensional search in modern optimization software.

4. Case study

This section applies the proposed predictive maintenance policy to the automotive lead-acid batteries in Fig. 1. This dataset is from an accelerated battery aging test in SAE J2801 [23]. The resistance level is treated as the degradation indicator and the degradation level is observed weekly, i.e., $t_k = k$. As seen, there are no soft-failure thresholds for the resistance level and the battery is considered as failed when it fails to crank the engine. In this section, we first use the battery of the dash line marked with squares in Fig. 1 to demonstrate the proposed policy. Then, the proposed policy is applied to each battery respectively, and the average cost rate of all the batteries is calculated. For comparison purposes, the proposed maintenance policy is further compared with two traditional predictive maintenance policies, i.e., time-based maintenance and degradation-limit conditional maintenance. Finally, sensitivity analysis of the proposed maintenance policy is conducted. Throughout this case study, the model parameters are set as $\kappa^{-2} = 0.1$, $\omega = 0.1$ (mΩ/week), $\gamma^{-2} = 0.1$, and $\delta = 0.08$ (mΩ/week). The baseline hazard rate function is captured by a Weibull distribution as $h_0(t) = 4t^3/14.5^4$ and let $\beta = 0.45$. The values of these parameters can be obtained based on the historic degradation/failure time data of batteries of the same population. The cost of preventive replacement and corrective maintenance are set to be $C_\mathrm{p} = 1$ and $C_\mathrm{f} = 3$, respectively.

4.1. Optimal predictive maintenance policy

Upon each inspection epoch, the joint distribution of the unit-specific degradation parameters μ and σ are updated based on the observed degradation signals. The Metropolis-Hasting algorithm in the Appendix is adopted to draw the samples of μ and σ. We implement the MCMC algorithm in MATLAB, where 5000 samples are generated for the burn-in period and the subsequent 10,000 samples are used for obtaining the joint distribution. The convergence of every MCMC replication is monitored based on the Gelman-Rubin ratio [5]. Once the parameter

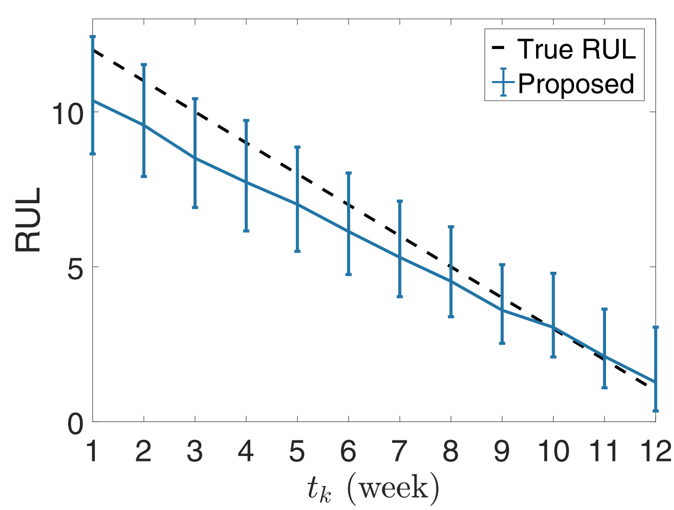

Fig. 2. The true and the estimated RUL of the battery at all prediction epochs. The band shows the corresponding 95% credible interval.

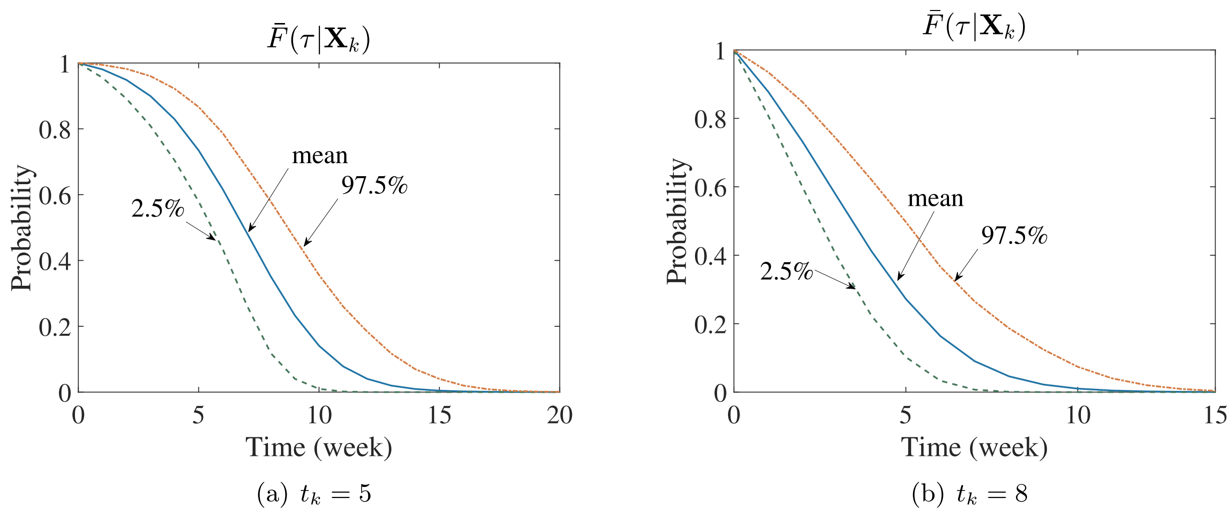

Fig. 3. RUL distribution of the unit at selected time points.

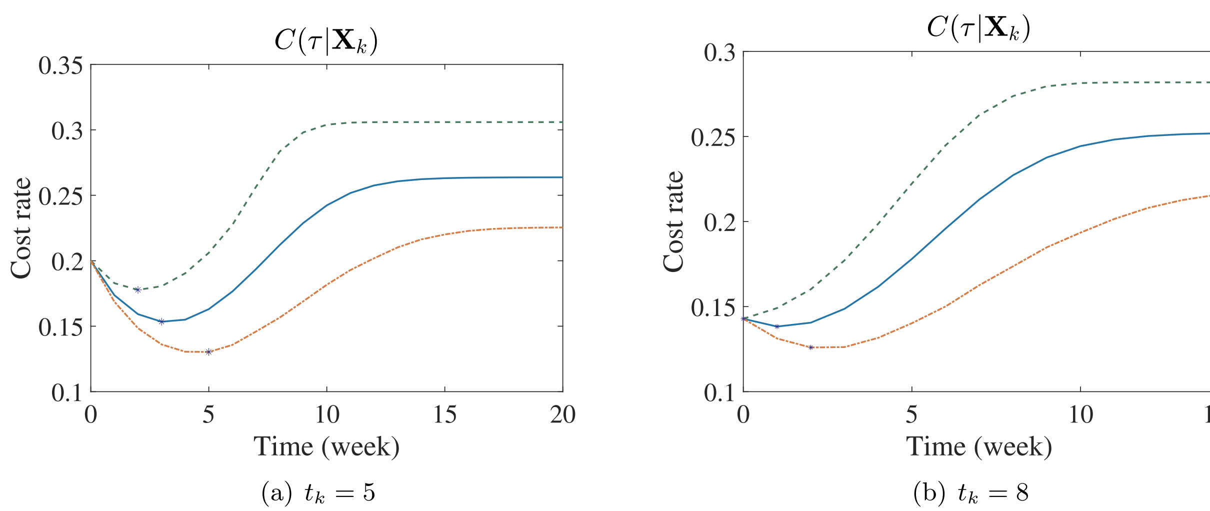

Fig. 4. Conditional cost rate of the unit at selected time points. The upper, lower, and middle curves correspond to the case when the 2.5th, the 97.5th point-wise percentile, and the mean of $\bar{F}(\tau)$ is used, respectively.

Table 1
Optimal maintenance time and corresponding cost rate at selected time points.

	$\bar{F}(\tau\|\mathbf{X}_k)$, $t_k = 5$				$\bar{F}(\tau\|\mathbf{X}_k)$, $t_k = 8$		
	Mean	2.5%	97.5%		Mean	2.5%	97.5%
(τ^*, C^*)	(3,0.15)	(2,0.18)	(5,0.13)	(τ^*, C^*)	(1,0.13)	(0,0.14)	(2,0.12)

Table 2
Summary of the different model parameters in the sensitivity analysis .

	C_f/C_p	ω	κ^{-2}	δ	γ^{-2}
Base	3	0.1	0.1	0.08	0.1
Low	1.5	0.05	0.05	0.04	0.05
High	6	0.2	0.2	0.16	0.2

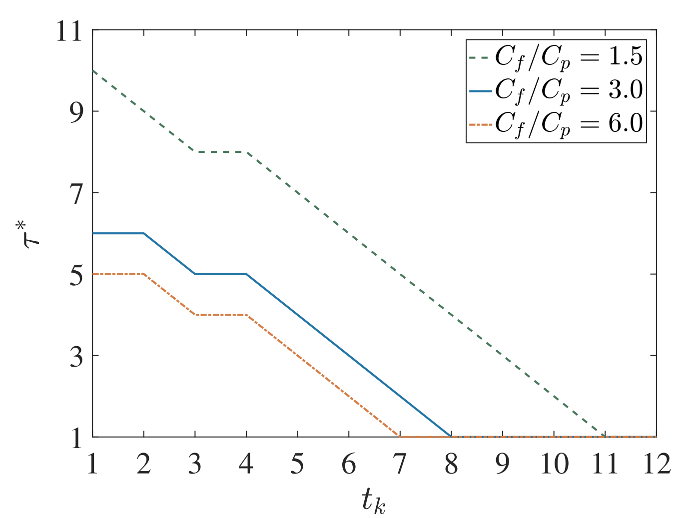

Fig. 5. Sensitivity of maintenance policy with respect to cost ratio C_f/C_p.

samples are available, they can be used to predict the RUL upon each inspection epoch. Fig. 2 shows the predicted RULs as well as the true RUL for the selected battery at $t_k = 1, \cdots, 12$. As seen, the proposed method generally predicts the RULs well as the 95% confidences intervals contains all the true RULs. In addition, the prediction accuracy improves with the amount of degradation data.

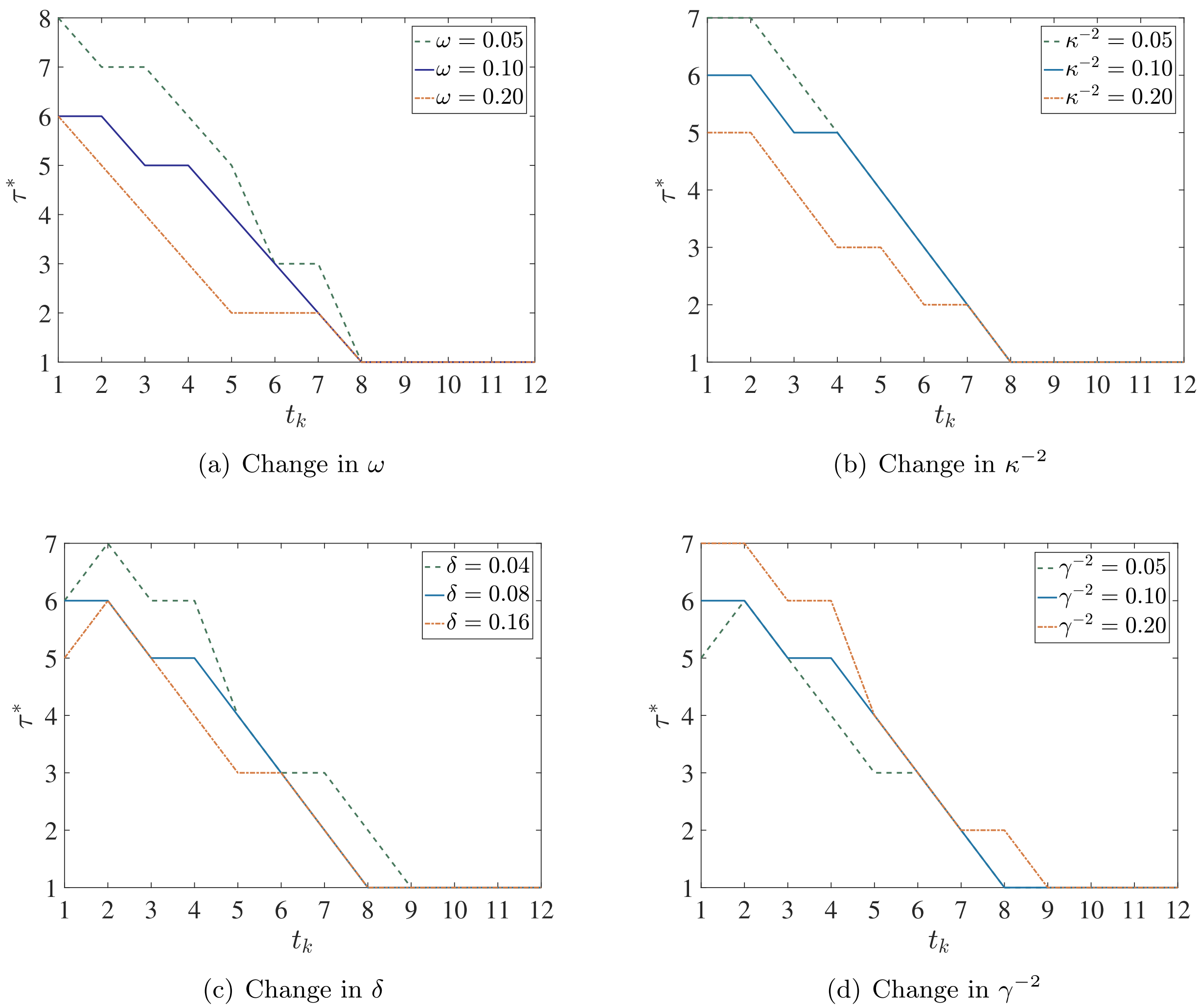

Fig. 6. Sensitivity of maintenance policy with respect to random-effects parameters.

Based on the predicted RULs, we can then obtain the optimal maintenance schedule by minimizing the long-run average maintenance cost upon each inspection epoch. For illustration, we present the results at the inspection epochs $t_k = 5$ and 8, respectively. From 10,000 Monte Carlo samples, we can compute the mean and the 2.5th/97.5th point-wise percentile of the unit RUL. The results are shown in Fig. 3. Under the mean, the 2.5th, and the 97.5th point-wise percentile of the unit RUL, we can compute the corresponding maintenance cost as a function of the maintenance time τ as shown in Fig. 4. Note that we only consider the situation where $\tau = 1, 2, \ldots$ so that the maintenance action can only be taken on a weekly basis. A one dimensional search efficiently finds the optimal maintenance time τ^* at each inspection epoch for each case of $\bar{F}(\tau)$, where the optimal (τ^*, C^*) are also shown in Table 1.

It can be seen that the proposed maintenance strategy determines the optimal maintenance time dynamically upon each inspection based on all the available degradation signals. At $t_k = 8$, the point-wise credible interval of $\bar{F}(\tau)$ becomes narrower compared the one at $t_k = 5$, due to the increased amount of the unit information as t_k increases. It is worth pointing out that $\tau^* = 1$ at $t_k = 8$ based on the mean of RUL, indicating the prediction process should be stopped and an preventive replacement should be implemented at the next week.

4.2. Comparison with other maintenance policies

The proposed predictive maintenance policy is then applied to other batteries. Here, all the results are obtained based on the mean of the RUL distribution. The average cost rate of all the batteries is obtained as 0.14. Other two policies, i.e., time-based maintenance and degradation-limit maintenance policies are adopted for comparison. Following the tradition in the literature, the heterogeneities are assumed negligible under both polices. In other words, the degradation parameters are the same for all the batteries and they are obtained based on the historical degradation data in advance.

Under the time-based maintenance policy, a preventive replacement is implemented at t_{k+1} when the optimal maintenance interval T_{T} satisfies $t_k \le T_{\mathrm{T}} < t_{k+1}$, or a corrective replacement is implemented when a failure occurs before that. The optimal maintenance interval T_{T} can be obtained by minimizing

$$C(T_{\mathrm{T}}) = \frac{C_{\mathrm{p}}\bar{F}(T_{\mathrm{T}}|A(0, x_0)) + C_{\mathrm{f}}(1 - \bar{F}(T_{\mathrm{T}}|A(0, x_0)))}{\int_0^{T_{\mathrm{T}}} \bar{F}(z|A(0, x_0))dz},$$

where $\bar{F}(T_{\mathrm{T}}|A(0, x_0)) = \Pr(T_{\mathrm{F}} > T_{\mathrm{T}}|A(0, x_0))$ is given by (5). The average cost rate of all the batteries can be obtained as 0.20. As a result, it is increased by 42.9%(=(0.20-0.14)/0.14) compared with the proposed policy. Under the degradation-limit maintenance policy, a preventive replacement is implemented at t_k when the measured degradation level x_k first exceeds the threshold X_c, or a corrective replacement is implemented when a failure occurs before that. The optimal degradation threshold X_c can be obtained by minimizing

$$C(X_c) = \int_0^\infty \frac{C_p \Pr(T_F > t_c | A(0, t_c, x_0, X_c)) + C_f(1 - \Pr(T_F > t_c | A(0, t_c, x_0, X_c)))}{\int_0^{t_c} \Pr(T_F > t_c' | A(0, t_c', x_0, X_c)) dt_c'} f_{X_c|x_0}(t_c|0) dt_c,$$

where $\Pr(T_F > t_c | A(0, t_c, x_0, X_c))$ is given by (7). Meanwhile, $f_{X_c|x_0}(t_c|0)$ is the PDF of the first reach time of the degradation level to X_c, which is

$$f_{X_c|x_0}(t_c|0) = \frac{X_c - x_0}{\sqrt{2\pi\sigma^2 t_c^3}} \exp\left[-\frac{(X_c - x_0 - \mu t_c)^2}{2\sigma^2 t_c}\right].$$

The average cost rate of all the batteries can be obtained as 0.17. It is increased by 21.4%(=(0.17-0.14)/0.14) compared with the proposed policy. All these results validate the effectiveness of our proposed predictive maintenance policy.

4.3. Sensitivity analysis

In this subsection, we conduct the sensitivity analysis of the parameters on the predictive maintenance policy. Based on the model studied in Section 4.1, we vary the parameter values and obtain the corresponding optimal maintenance time. Table 2 summarizes the parameter values used in the sensitivity analysis. The 'Base' row contains the parameters used in Section 4.1, and the 'Low' and 'High' rows list the alternative values used in the sensitivity study. Again, all the results are obtained based on the mean of the RUL distribution.

We first investigate the sensitivity of the maintenance policy on the cost ratio C_f/C_p. Fig. 5 shows the optimal maintenance time over time interval [1, 12] when C_f/C_p changes. We can see that the policy becomes more conservative in terms of a shorter maintenance time when C_f/C_p increases. On the other hand, it becomes less attractive to implement preventive replacement for the unit before its failure when C_f/C_p decreases.

We next examine the influence of model parameters ω, κ^{-2}, δ and γ^{-2}, which characterize the degradation heterogeneity, on the maintenance policy. The results are shown in Fig. 6. Recall that $\mu \sim \mathcal{TN}(\omega, \kappa^{-2})$ and $\sigma \sim \mathcal{TN}(\delta, \gamma^{-2})$. A larger ω implies faster degradation, leading to a shorter maintenance time at early ages. This is because only limited degradation signals are available during this period, and the decision relies heavily on the initial distributions. When more degradation data are available, we may estimate the true degradation rate μ accurately. Consequently, the influence of ω on the optimal maintenance time is not significant as t_k increases. Meanwhile, a larger κ^{-2} indicates a more significant unit heterogeneity. It is reasonable to observe from Fig. 6(b) that, the maintenance time decreases more smoothly over time as κ^{-2} decreases. Similar results for δ and γ^{-2} can also be seen from Figs. 6(c) and (d), respectively. It is worth mentioning that the optimal maintenance time τ^* is nearly the same for different levels of model parameters when the degradation data are sufficient (i.e., t_k is near 10). This implies the merit of the online adjustment of maintenance plan in the sense that the initial estimate error of μ and σ will not affect the optimal maintenance time significantly.

5. Conclusion

This study successfully modeled the failure behavior of systems that are subject to aging and degradation. The degradation of system was modeled by a random-effects Wiener process and the in-situ degradation levels were used to update the conditional joint distribution of degradation parameters. Meanwhile, the hazard rate of the hard failure is a function of both the time and the in-situ system degradation level, where a higher degradation level implies a higher instantaneous probability of the hard failure. We have developed a predictive maintenance model that determines the optimal maintenance time for the system based on the historical degradation data. To minimize the long-run average maintenance cost per cycle, we derived the explicit RUL distribution based on the Brownian bridge theory. The optimal predictive maintenance time was updated upon each inspection epoch. The predictive maintenance strategy was successfully applied to determine the maintenance time of lead-acid batteries.

There are a number of extensions and topics for future research. In this study, MCMC is adopted to sample parameters from the joint distribution. The decision process is efficient due to the derived closed-form RUL distribution, and the proposed maintenance policy is capable for real applications. In future studies, an extended Kalman filter or particle filter can be adopted for some specific scenarios to achieve more efficient computation. In addition, only hard failures are considered in this study. In some applications, systems may be subject to both hard failures and degradation-induced soft failures. It is of interest to investigate maintenance optimization for such systems. Moreover, a linear form is adopted for the link function in this study. Other forms such as the exponential form can be explored based on the degradation and failure time data in future studies.

Declaration of Competing Interest

None.

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Appendix

The detailed steps of the MCMC are provided in Algorithm 1. As introduced in Section 3.3, we used 5000 samples for burn-in and the subsequent 10,000 samples for generating the sample, so that $T = 5000 + 10,000 = 15,000$. Meanwhile, the normal distribution is adopted as the proposal distribution in the Metropolis-Hasting algorithm.

Input: Initial distributions of μ and σ and historical degradation data $\boldsymbol{X}_k$ upon the kth inspection.
Output: Samples generated from the conditional joint distribuion $f(\mu, \sigma|\boldsymbol{X}_k)$.
begin
  Set $t = 1$;
  Generate an initial value $\boldsymbol{u} = (\mu, \sigma)$, and set $\boldsymbol{\theta}^{(t)} = \boldsymbol{u}$;
  while $t \leq T$ **do**
    1. $t = t + 1$;
    2. Generate a candidate $\boldsymbol{\theta}^{\star}$ from a proposal distribution $q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t-1)})$;
    3. Evaluate the acceptance probability $\alpha = \min\left\{1, \frac{f(\boldsymbol{\theta}^{\star})}{f(\boldsymbol{\theta}^{(t-1)})} \frac{q(\boldsymbol{\theta}^{(t-1)}|\boldsymbol{\theta}^{\star})}{q(\boldsymbol{\theta}^{\star}|\boldsymbol{\theta}^{(t-1)})}\right\}$;
    4. Generate a random variable u from the Uniform(0,1) distribution;
    if $u \leq \alpha$ **then**
      Accept the candidate and set $\boldsymbol{\theta}^{(t)} = \boldsymbol{\theta}^{\star}$;
    end
    if $u > \alpha$ **then**
      Set $\boldsymbol{\theta}^{(t)} = \boldsymbol{\theta}^{(t-1)}$;
    end
  end
end

Algorithm 1. The MCMC algorithm to generate the samples from the joint distribution.

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