Problem 1 (40 points).

1) Generally speaking, a robot is a goal-oriented machine that can sense, ________ and act. (2 points)

2) When we say an object comprised by a set of points is **rigid**, we mean that these points should maintain a constant relative position with respect to ___________. (2 points)

   A. any static coordinate frame　　B. the object's coordinate frame

   C. the world coordinate frame　　D. None of them

3) When we talk about an object's **pose**, we mean its position and _____________. (2 points)

4) The following matrix **R** is a 2×2 rotation matrix. The missing element is __________. (2 points)

$$\mathbf{R}=\begin{bmatrix}\frac{\sqrt{3}}{2} & ?\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

5) Composition of 2D rotation matrices is not commutative. (True/False) __________. (2 points)

6) Given the following rotation matrix

$${}^{0}\mathrm{R}_{1}=\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix},$$

which of the following statement is wrong? ________ (2 points)

   A. Frame {1} has its y-axis in the z-direction of frame {0}

   B. Frame {1} has its z-axis in the negative y-direction of frame {0}

   C. Frame {0} has its y-axis in the negative z-direction of frame {1}

   D. None of them

7) Which of the following statement about Euler's rotation theorem and Euler angles is wrong? ________ (2 points)

   A. Any two independent orthonormal coordinate frames can be related by a sequence of rotations (not more than three) about coordinate axes

   B. No two successive rotations may be about the same axis

   C. The mapping from rotation matrix to Euler angles is unique

   D. None of them

8) List all sequences of Cardanian type for three-angle representation:

____

(2 points)

9) The expression of non-holonomic constraint for bicycle model is $\dot{y}\sin\theta - \dot{x}\cos\theta \neq 0$. (Ture/False) ___________ (2 points)

10) What does SLAM stand for? ____ (2 points)

11) Which of the following robotic tasks does not use reactive navigation? ___________ (2 points)

A. Heading towards a light

B. Following white line on the ground

C. Moving through a maze by using a map

D. Vacuuming a room by following a random path

12) Which of the following navigation method is most likely to find the shortest path from a point to a specified goal? _________ (2 points)

A. Bug2 B. Distance transform

C. PRM D. Voronoi roadmap method

13) Which of the following navigation method can support path planning for nonholonomic vehicle? _________ (2 points)

A. Bug2 B. Distance transform

C. RRT D. Voronoi roadmap method

14) Which of the following navigation method can support incremental replanning? _________ (2 points)

A. Bug2 B. D*

C. PRM D. RRT

15) Which of the following statement about Kalman Filter is wrong? _____________ (2 points)

A. Given the system inputs and sensor outputs, a Kalman filter estimates the unknown true state and how certain we are of the estimate

B. The Kalman filter is an optimal estimator for the case where the process or measurement noise are zero-mean Gaussian noise

C. Kalman filter has two steps: prediction and update

D. None of them

16) Which of the following statement about Kalman Filter localization is wrong? __________ (2 points)

A. Uncertainty in position grows without bound using dead-reckoning alone

B. All sensor covariance matrix W is diagonal

C. The innovation represents the difference between what the sensors measure and what the sensors are predicted to measure.

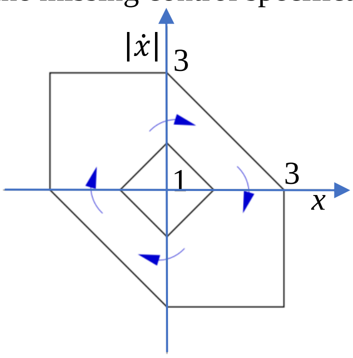
D. None of them

17) The joint structure of some robot arms can be described by a string with letters “R” and “P”. Please write down the string for the below manipulators.


(SCARA robot, 3 DoF) ________________ (2 points)


(Gantry robot, 3 DoF) ________________ (2 points)

18) Given the following state space view of a reach control problem. Please write down the missing control specification. (4 points)



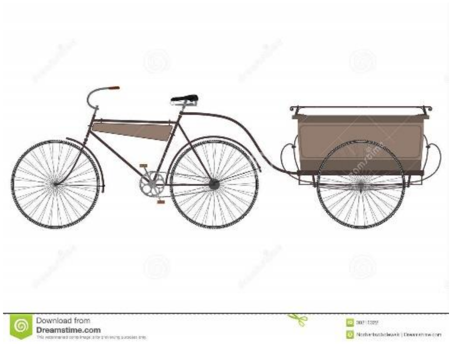
$|x| \leq 3, |\dot{x}| \leq 3$,

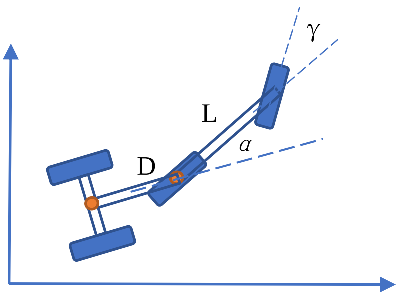
____

Problem 2 (15 points).

(a) Assuming a point $(x^*\langle t\rangle, y^*\langle t\rangle)$ is moving on a circle, design a controller (v, γ) for the bicycle model with configuration (x, y, θ) to follow the trajectory of $(x^*\langle t\rangle, y^*\langle t\rangle)$ while maintaining a distance d^* behind the pursuit point. (8 points)

(b) In the following figure, you can see a bicycle with a trailer (拖车).

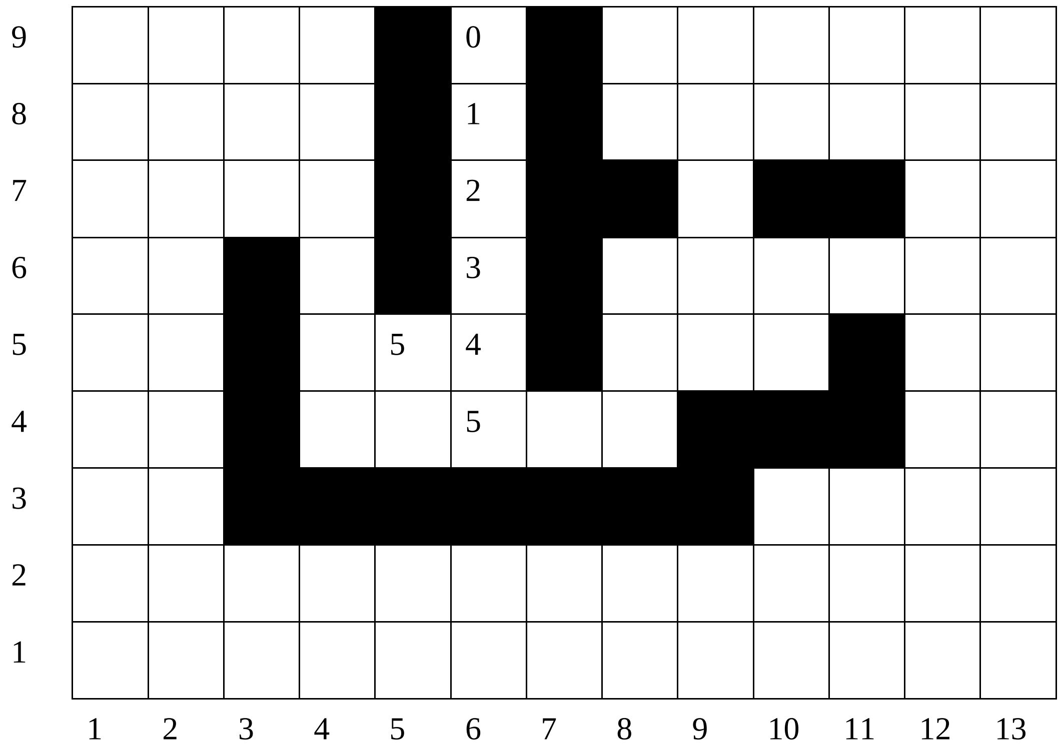




Now assuming that the bicycle is already moving with constant (v, γ), please drive the relationship between α and (γ, L, D), i.e., function $\alpha = f(\gamma, L, D)$. (7 points)

Problem 3 (15 points).

(a) In the following grid map, fill in each cell its distance to the goal point (6,9), and draw the shortest paths from initial points (8,1) and (10,2) to the goal point, respectively. Manhattan distance $d = |\Delta_x| + |\Delta_y|$ is used. Moving diagonally is not allowed here. (7 points)



(b) In (a), we use the Manhattan distance as the travelling cost. Consider the following cost map, the number represents the cost to travel through each cell instead of the Manhattan distance cost. Moving diagonally is not allowed here. Please draw the shortest paths from initial points (10,2) and (13,9) to the goal point (6,9), respectively. (8 points)

The minimum cost from (10,2) to (6,9) is ________

The minimum cost form (13,9) to (6,9) is ________

9	1	1	1	1	∞	1	∞	1	1	1	1	1	1
8	1	1	1	1	∞	3	∞	1	2	1	1	1	1
7	1	1	1	2	∞	3	∞	∞	3	∞	∞	1	1
6	1	1	∞	3	∞	3	∞	1	2	2	3	2	1
5	1	1	∞	2	2	2	∞	2	1	1	∞	1	1
4	1	1	∞	2	2	2	3	3	∞	∞	∞	1	1
3	1	1	∞	∞	∞	∞	∞	∞	∞	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10	11	12	13

Problem 4 (15 points).

(a) When using Kalman filter to estimate the pose of a bicycle-modeled robot, we need a function $f(\cdot)$ that describes how the vehicle's configuration changes from time step $\langle k \rangle$ to time step $\langle k+1 \rangle$, which is

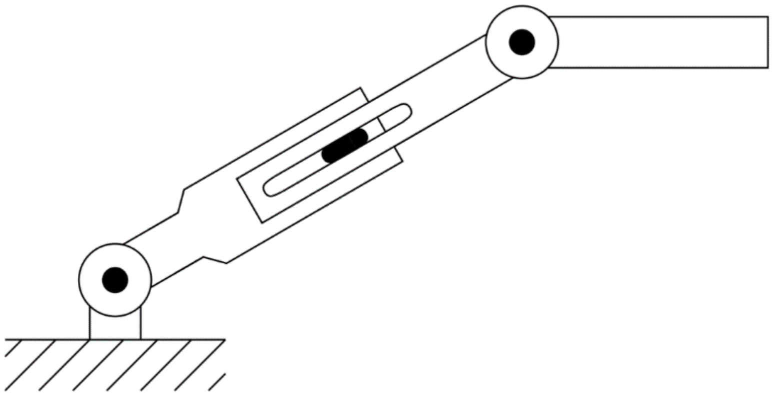
$$\mathbf{x}\langle k+1 \rangle = f\left(\mathbf{x}\langle k \rangle, \boldsymbol{\delta}\langle k \rangle, \mathbf{v}\langle k \rangle\right)$$

where $\mathbf{x}\langle k+1 \rangle = \left(x\langle k+1 \rangle, y\langle k+1 \rangle, \theta\langle k+1 \rangle\right)$ is the robot's configuration at $\langle k+1 \rangle$, $\mathbf{x}\langle k \rangle = \left(x\langle k \rangle, y\langle k \rangle, \theta\langle k \rangle\right)$ is the robot's configuration at $\langle k \rangle$, $\boldsymbol{\delta}\langle k \rangle = \left(\delta_d\langle k \rangle, \delta_\theta\langle k \rangle\right)$ is the odometry measurement, $\mathbf{v}\langle k \rangle = \left(v_d, v_\theta\right)$ is the random measurement noise. Please derive the expression for $f(\cdot)$. (7 points)

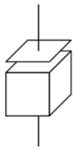
(b) The vehicle's motion obtained in (a) is nonlinear. To apply the extended Kalman filter, we need a local linearization of $f(\cdot)$. Please write down the expression of this linearization and derive the expression for Jacobians $F_{\mathbf{x}}$ and $F_{\mathbf{v}}$. (8 points)

Problem 5 (15 points).

(a) Consider the following three-link planar manipulator, derive its forward kinematic matrix ${}^{0}\xi_{E}$. Define the variables by yourself and label them in the following figure. (8 points)



(b) Given the position of end-effector (d_x, d_y, d_z), please solve the inverse kinematics (θ_1, d_2, d_3) for the following cylindrical manipulator. (7 points)



The left element represents a prismatic joint.