缩写及术语

Pose: position and orientation.
ASVs: Autonomous surface vehicles.
AUV: Autonomous underwater vehicle.
UAV: Unmanned aerial vehicle 无人机
AGVs: automated guided vehicles（fix）
Rigid: constituent points maintain a constant relative position w.r.t. {B}。刚性:构成点保持恒定的相对位置 w.r.t。{B}
Homogeneous trans: 齐次变换
Orthonormal rotat: 正交旋转矩阵 $R^{-1}=R^T$;特征值 1. **Rigid:** 相对物体坐标系
Quaternions: 四元数
Path: 空间路径 a locus in space that leads from an initial pose to a final pose.
Trajectory: path with specified timing. (smooth)
Cartesian motion: Another common requirement is a smooth path between two poses in SE(3) which involves change in position + orientation。
INS: Inertial navigation system, It measures its accelerations and angular velocities and integrates them over time to estimate velocity , orientation and position **PRM**: Probabilistic Roadmap method. **RRT**:Rapidly exploring Random Tree **Robot:** sense, plan and act. **Odemeter:** 里程计 a sensor that measures distance travelled, typically by measuring the angular rotation of the wheels.
Electronic campass: the direction of travel
Gyroscope or differential odometry: the change in heading can be measured.**Joint:**关节， **Link:** 连杆 **Mobile robots**: a class of robots that are able to move through the environment.**SCARA**: Selective Compliance Assembly Robot Arm(平面关节型机器人)**Gantry robot:** 桁式机器人 **Parallel-link manipulator:** 并联机器手 **ICR:** Instantaneous Center of Rotation(前后轮轴线交点)**Traversability:** how easy the teain is to drive over.**Dead reckoning:** 航位推算, estimation of location based on estimated speed, direction and time of travel.**Landmark:** a visible feature in environment whose location is known.**Kinematics(运动学)**: is the branch of mechanics that studies the motion of a body, or a system of bodies, without consideration given to its mass or the forces acting on it. **SLAM:**simultaneous localization and mapping. **CML:**concurrent mapping and localization
D-H: Denavit-Hartenberg Parameters

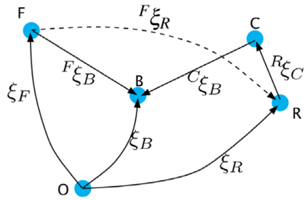
绪论

Field and service robotd 的两个挑战: 1.operate and move in a complex, cluttered and changing environment. **2.**operate safely in the presence of people.

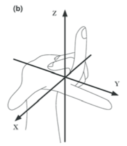
Fundation of robotics

Representation of Position and Orientation

P 位置的描述: 1.reference coordinate，2.P's position can be described by a coordinate vector.
$^A\xi_B$: A: reference coordinate frame.
ξ: frame being described
B: Pose of frame {B}. w.r.t frame{A}
P 位置的表示: $^Ap = {}^A\xi_B * {}^Bp$
坐标组合 composed compounded: $^A\xi_C = {}^A\xi_B \oplus {}^B\xi_C$
从左往右乘
$\xi_F \oplus {}^F\xi_B = \xi_R \oplus {}^R\xi_C \oplus {}^C\xi_B$
composition is not commutative
^VR_B describes how points are transformed from frameB to V
Det(R)=1 $R^T = R^{-1}$
齐次变换 ^AT_B = [^AR_B t; 0 0 1] t 是 A 下 B 原点坐标



2D(旋转): [cos θ , -sin θ ; sin θ , cos θ]
2D 旋转加平移: [cos θ , -sin θ ,x;
sin θ ,cos θ ,y;
0, 0, 1];
右手坐标系:



3D 旋转: x 轴 [1, 0, 0; 0, cos θ , -sin θ ; 0 sin θ , cos θ];
Y 轴 [cos θ , 0, sin θ ; 0, 1, 0; -sin θ , 0, cos θ];
Z 轴 [cos θ , -sin θ , 0; sin θ , cos θ , 0; 0, 0, 1];
欧拉式: XYZ, XZX, YXY, YZY, ZXZ, **ZYZ (aeronautics and mechanical dynamics,(Γ gamma=(Φphi, θ, ψpsi))**
欧拉旋转定理: Any two independent orthonormal coordinate frames can be related by a sequence of rotations (not more than three) about coordinate axes, where no two successive rotations may be about the same axis.
Euler's rotation theorem requires successive rotation about three axes such that no two successive rotations are about the same axis
卡尔丹: XYZ, XZY, YZX, YXZ, ZXY, ZYX
YPR 角: 1.Yaw—Z—偏航角(travel)—downward
ZYX(ypr)2.Pitch-Y-俯仰角(elevant,horizontal)—right
3.Roll—X—横滚(rotation)—forward
3D 旋转加平移: [R_{3*3}, t_{3*1}; 0_{1*3}, 1]，其中 T 为原点相对 original 的平移变化。
正交旋转矩阵:

$$^A\boldsymbol{p} = \begin{bmatrix} ^Ax \\ ^Ay \\ ^Az \end{bmatrix} = \begin{bmatrix} \hat{x}_A\cdot\hat{x}_B & \hat{x}_A\cdot\hat{y}_B & \hat{x}_A\cdot\hat{z}_B \\ \hat{y}_A\cdot\hat{x}_B & \hat{y}_A\cdot\hat{y}_B & \hat{y}_A\cdot\hat{z}_B \\ \hat{z}_A\cdot\hat{x}_B & \hat{z}_A\cdot\hat{y}_B & \hat{z}_A\cdot\hat{z}_B \end{bmatrix}\begin{bmatrix} ^Bx \\ ^By \\ ^Bz \end{bmatrix}$$

列: directions of frame {B}'s axes in terms of frame{A}
行: directions of frame {A}'s axes in terms of frame{B}
一个正交旋转矩阵总是有一个特征值为 1，和一对共轭 cos θ +i sin θ ，θ 就是旋转的角度。λ=1 的特征向量对应旋转向量 It implies that the corresponding eigenvector v is unchanged by the rotation. 变化如下:

$$R = I_{3\times3} + \sin\theta S(v) + (1-\cos\theta)(vv^T - I_{3\times3})$$

$$S(v) = \begin{pmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{pmatrix}$$

Time and motion

旋转速度矩阵:

$$^A\dot{R}_B = {}^AR_B[^B\omega]_\times [^AR_B]^T\, {}^AR_B = {}^AR_B[^B\omega]_\times \in \mathbb{R}^{3\times3}$$

$$^A\dot{R}_B = [^A\omega]_\times {}^AR_B \quad \text{Notice t}$$

$$\dot{\xi} \sim {}^A\dot{T}_B = \begin{bmatrix} ^A\dot{R}_B & ^A\dot{t}_B \\ 0_{1\times3} & 0 \end{bmatrix} \qquad \xi \sim {}^AT_B = \begin{bmatrix} ^AR_B & ^At_B \\ 0_{1\times3} & 1 \end{bmatrix}$$

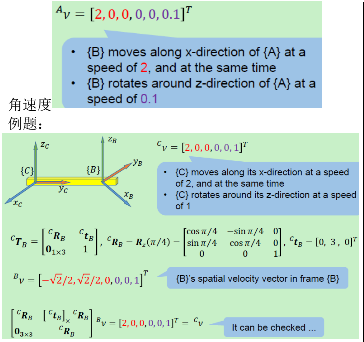
$$^A\dot{R}_B = [^A\omega]_\times {}^AR_B \qquad \dot{\xi} \sim {}^A\dot{T}_B = \begin{bmatrix} [^A\omega]_\times {}^AR_B & ^A\dot{t}_B \\ 0_{1\times3} & 0 \end{bmatrix}$$

空间速度变换:

$$^Av = \begin{bmatrix} ^AR_B & 0_{3\times3} \\ 0_{3\times3} & ^AR_B \end{bmatrix} {}^Bv$$

前三个是线速度，后三个为角速度

例题:

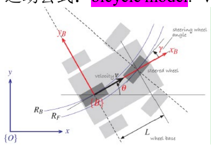


^0m = B[cosI; 0; sinI]' 其中 I 为 inclination angle
A horizontal projection of vector $\boldsymbol{m}$ points in the direction of magnetic north
Declination angle D is measured from true north clockwise to that projection
Inclination angle I is measured in a vertical plane downward from horizontal to $\boldsymbol{m}$

移动机器人

移动机器人

两种导航方式: reactive-system, plan-based
转弯半径: R_B = L/tan γ (γ 为 steering angle, L 车长，即前后轮距离) **angular velocity** $\dot{\theta}$ =V/R_B
运动公式: bicycle model. ^Bv = (v, 0)自身坐标系



Equations of motion

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v}{L}\tan\gamma$$

Fix steering wheel angle the car moves a circular arc. **Non-holonomic constraint:**
y 点 cos θ - x 点 sin θ 三 0;
Rate of change of heading: 也被称为 turn rate; heading rate; yaw rate
Moving to a point: （x*,y*）
速度控制器 V*=Kv ρ (ρ 为(x,y)和(x*,y*)距离)
转向角控制 γ=Kh(θ *⊖ θ)
θ * = arctan(y*-y)/(x*-x)
Following a Line: ax+by+c=0;

$$d = \frac{(a,b,c)\cdot(x,y,1)}{\sqrt{a^2+b^2}}$$

1.向线走 $\alpha_d = -K_d d,\ K_d > 0$

2. $\alpha_h = K_h(\theta^* \ominus \theta),\ K_h > 0 \qquad \theta^* = \tan^{-1}\frac{-a}{b}$

总: $\gamma = -K_d d + K_h(\theta^* \ominus \theta)$

Follow a path: (类似跟随动点，保持 d*距离)
1.pure pursuit: PI 控制器:

$$e = \sqrt{(x^*-x)^2 + (y^*-y)^2} - d^*$$
$$v^* = K_v e + K_i\int e\,dt$$

γ=Kh(θ *⊖ θ)

Moving to a pose
unicycle model (从 bicycle model 变化过来)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \qquad \gamma = \tan^{-1}\frac{\omega L}{v}$$

转换到极坐标系: （再由右边的控制器）

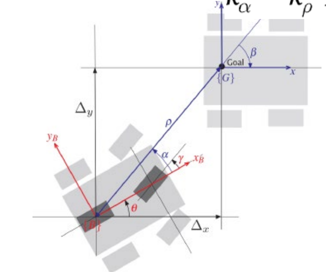
$$\rho = \sqrt{\Delta_x^2 + \Delta_y^2} \qquad \alpha = \tan^{-1}\frac{\Delta_y}{\Delta_x} - \theta$$
$$\beta = -\theta - \alpha$$

α ∈(-pi/2,pi/2]

$$\begin{pmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix} \qquad v = k_\rho \rho \qquad \gamma = k_\alpha\alpha + k_\beta\beta$$

β AB 连线世界系 x 轴夹角
α AB 连线与 A 系 x 轴角度
θ A 系 x 轴与世界系 x 轴的角度
当(ρ ， α ， β)走到 0 时 稳定条件:

$$\begin{pmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{pmatrix} = \begin{pmatrix} -k_\rho\cos\alpha \\ k_\rho\sin\alpha - k_\alpha\alpha - k_\beta\beta \\ -k_\rho\sin\alpha \end{pmatrix} \qquad k_\rho > 0 \quad k_\beta < 0 \quad k_\alpha - k_\rho > 0$$



$$v = k_\rho\rho$$
$$\gamma = k_\alpha\alpha + k_\beta\beta$$

差速车模型（W 为车宽）:

$$\dot{x} = v\cos\theta \qquad v = ½(v_R + v_L) \quad \text{average velocity}$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v_\Delta}{W} \qquad v_\Delta = v_R - v_L \quad \text{differential velocity}$$

将 reference frame 前移 a 则有

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & -a\sin\theta \\ \sin\theta & a\cos\theta \end{pmatrix}\begin{pmatrix} v \\ \omega \end{pmatrix}$$

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{1}{a}\sin\theta & \frac{1}{a}\cos\theta \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

巡航 Navigation

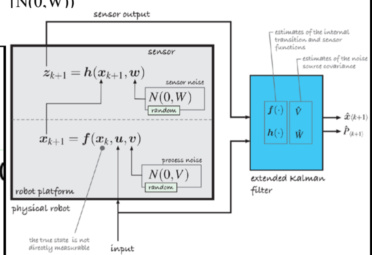
Reactive Navigation(机器人定义不用 plan)
eg1. Braitenberg Vehicles.
V = 2 – S_R – S_L
γ = k(S_R – S_L)
eg2. Simple Automata(bug2)
1. it moves along a straight line towards its goal
2.遇到障碍就绕（顺时针或逆时针）
3.直到到了离终点更近的点的线上继续沿线

Map-based Planning
Occupancy grid 0—free 1—occupied
eg1. D*算法(增量式重规划)最有用
不支持换 goal，因此产生了 roadmap 算法
从 goal 开始
eg2. Roadmap(支持更换起止点)
1. 泰森多边形路图（骨架）
2. 概率路图法 PRM（地图里撒点）
两部分: planning 建计划、query 找路
PRM 的优点是需要测试的点相对较少，以确定点和它们之间的路径是无障碍的，**距离阈值**
3.RRT: 将运动约束考虑到了，**快，避障直接**

Kalman Filter

$$\boldsymbol{x}\langle k+1\rangle = \boldsymbol{F}\boldsymbol{x}\langle k\rangle + \boldsymbol{G}\boldsymbol{u}\langle k\rangle + \boldsymbol{v}\langle k\rangle$$
$$\boldsymbol{z}\langle k\rangle = \boldsymbol{H}\boldsymbol{x}\langle k\rangle + \boldsymbol{w}\langle k\rangle$$

x: 状态向量 **z:** 输出向量 **F:** A describes dynamics of the system, i.e., how the states evolve with time. **G:** B describes how inputs are mapped to system states. **H:** describes how system states are mapped to observed outputs. **v:** process noise(Gaussian random variable N(0, V)) **w:** measurement noise(Gaussian random variable N(0,W))



1. How state $\boldsymbol{x}$ evolves as a function of the inputs state transition model $f(\cdot)$, and we know the inputs to the system $\boldsymbol{u}$
2. It is common to represent this uncertainty by an imaginary random number generator which is corrupting system state process noise
3. How the sensor output depends on state $\boldsymbol{x}$ sensor model $h(\cdot)$
4. Its uncertainty is also modeled by an imaginary random number generator sensor noise

$$g(x) = \frac{1}{\sqrt{\sigma^2 2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

1 维 δ 标准差
N 维， μ 为均值，p 为 covariance

$$g(x) = \frac{1}{\sqrt{\det(P)(2\pi)^n}} e^{-\frac{1}{2}(x-\mu)^T P^{-1}(x-\mu)}$$

第一步: 基于之前的状态和输入预测当前状态:

$$\hat{\boldsymbol{x}}\langle k+1|k\rangle = \boldsymbol{F}\hat{\boldsymbol{x}}\langle k\rangle + \boldsymbol{G}\boldsymbol{u}\langle k\rangle\ \hat{\boldsymbol{x}} \quad \text{estimate of the state}$$
$$\hat{\boldsymbol{P}}\langle k+1|k\rangle = \boldsymbol{F}\hat{\boldsymbol{P}}\langle k\rangle\boldsymbol{F}^T + \hat{\boldsymbol{V}} \quad \text{p:estimated covariance}$$

If we overestimate $\hat{V}$, our estimate of P will be larger than it really is giving a pessimistic estimate of our certainty in the state Conversely if we underestimate $\hat{V}$ the filter will be overconfident of its estimate

克服不确定性累计引入新的信息(z 实际传感 x 传感预测)

$$\boldsymbol{\nu}\langle k+1\rangle = \boldsymbol{z}\langle k+1\rangle - \boldsymbol{H}\hat{\boldsymbol{x}}\langle k+1|k\rangle$$

第二步: update step, maps innovation into a correction for prediction state
（K 为 Kalman gain）

$$\hat{\boldsymbol{x}}\langle k+1|k+1\rangle = \hat{\boldsymbol{x}}\langle k+1|k\rangle + \boldsymbol{K}\langle k+1\rangle\boldsymbol{\nu}\langle k+1\rangle$$
$$\hat{\boldsymbol{P}}\langle k+1|k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle - \boldsymbol{K}\langle k+1\rangle\boldsymbol{H}\hat{\boldsymbol{P}}\langle k+1|k\rangle$$

$$\boldsymbol{K}\langle k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle\boldsymbol{H}^T\underbrace{\left(\boldsymbol{H}\hat{\boldsymbol{P}}\langle k+1|k\rangle\boldsymbol{H}^T + \hat{\boldsymbol{W}}\right)^{-1}}_{S}$$

If $\hat{W} = 0$ and H^{-1} exists

$$K\langle k+1\rangle = \hat{P}\langle k+1|k\rangle H^T\left(H\hat{P}\langle k+1|k\rangle H^T\right)^{-1} = H^{-1}$$
$$\hat{x}\langle k+1|k+1\rangle = H^{-1}z\langle k+1\rangle$$

当 K = H^{-1} 有 $\hat{P}\langle k+1|k+1\rangle = 0$
3 个特点:
1. optimal(噪声为 0 均值 time invariant paramters 时最优)
2. recursive, 上一个输出为下一个的输入
3. asynchronous
对于 P 帽子矩阵，对角线上的元素为 variance 或 uncertainty in state xi，非对角线上元素 Pij 帽子为 correlations between states xi and xj

Extended Kalman Filter 非线性

$$\boldsymbol{x}\langle k+1\rangle = \boldsymbol{f}(\boldsymbol{x}\langle k\rangle, \boldsymbol{u}\langle k\rangle, \boldsymbol{v}\langle k\rangle)$$
$$\boldsymbol{z}\langle k\rangle = \boldsymbol{h}(\boldsymbol{x}\langle k\rangle, \boldsymbol{w}\langle k\rangle)$$

线性化

$$\boldsymbol{x}'\langle k+1\rangle \approx \boldsymbol{F}_x\boldsymbol{x}'\langle k\rangle + \boldsymbol{F}_u\boldsymbol{u}\langle k\rangle + \boldsymbol{F}_v\boldsymbol{v}\langle k\rangle$$
$$\boldsymbol{z}'\langle k\rangle \approx \boldsymbol{H}_x\boldsymbol{x}'\langle k\rangle + \boldsymbol{H}_w\boldsymbol{w}\langle k\rangle$$

$F_x = \partial f/\partial x \in \mathbb{R}^{n\times n}$, $F_u = \partial f/\partial u \in \mathbb{R}^{n\times m}$,
$F_v = \partial f/\partial v \in \mathbb{R}^{n\times n}$
$H_x = \partial h/\partial x \in \mathbb{R}^{p\times n}$ $H_w = \partial h/\partial w \in \mathbb{R}^{p\times p}$

$$\hat{\boldsymbol{x}}\langle k+1|k\rangle = \boldsymbol{f}(\hat{\boldsymbol{x}}\langle k\rangle, \boldsymbol{u}\langle k\rangle) \quad \text{prediction step}$$
$$\hat{\boldsymbol{P}}\langle k+1|k\rangle = \boldsymbol{F}_x\hat{\boldsymbol{P}}\langle k|k\rangle\boldsymbol{F}_x^T + \boldsymbol{F}_v\hat{\boldsymbol{V}}\langle k\rangle\boldsymbol{F}_v^T$$

$$\hat{\boldsymbol{x}}\langle k+1|k+1\rangle = \hat{\boldsymbol{x}}\langle k+1|k\rangle + \boldsymbol{K}\langle k+1\rangle\boldsymbol{\nu}\langle k+1\rangle \quad \text{update step}$$
$$\hat{\boldsymbol{P}}\langle k+1|k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle - \boldsymbol{K}\langle k+1\rangle\boldsymbol{H}_x\hat{\boldsymbol{P}}\langle k+1|k\rangle$$

$$\boldsymbol{\nu}\langle k+1\rangle = \boldsymbol{z}\langle k+1\rangle - \boldsymbol{h}(\hat{\boldsymbol{x}}\langle k+1|k\rangle) \quad \text{innovation}$$

Kalman gain

$$\boldsymbol{K}\langle k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle\boldsymbol{H}_x^T\left(\boldsymbol{H}_x\hat{\boldsymbol{P}}\langle k+1|k\rangle\boldsymbol{H}_x^T + \boldsymbol{H}_w\hat{\boldsymbol{W}}\boldsymbol{H}_w^T\right)^{-1}$$

不高斯
离散时间模型 for the evolution of configuration based on odometry δ <k>=(δ d, δ θ)位姿

$$\xi\langle k\rangle \sim \begin{pmatrix} \cos\theta\langle k\rangle & -\sin\theta\langle k\rangle & x\langle k\rangle \\ \sin\theta\langle k\rangle & \cos\theta\langle k\rangle & y\langle k\rangle \\ 0 & 0 & 1 \end{pmatrix}$$

$$\xi\langle k+1\rangle \sim \begin{pmatrix} \cos\theta\langle k\rangle & -\sin\theta\langle k\rangle & x\langle k\rangle \\ \sin\theta\langle k\rangle & \cos\theta\langle k\rangle & y\langle k\rangle \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & \delta_d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\delta_\theta & -\sin\delta_\theta & 0 \\ \sin\delta_\theta & \cos\delta_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} \cos(\theta\langle k\rangle + \delta_\theta) & -\sin(\theta\langle k\rangle + \delta_\theta) & x\langle k\rangle + \delta_d\cos\theta\langle k\rangle \\ \sin(\theta\langle k\rangle + \delta_\theta) & \cos(\theta\langle k\rangle + \delta_\theta) & y\langle k\rangle + \delta_d\sin\theta\langle k\rangle \\ 0 & 0 & 1 \end{pmatrix}$$

subject to both systematic and random error.
误差会累计。带误差的推导:（v 为误差）

$$\boldsymbol{x}\langle k+1\rangle = \boldsymbol{f}(\boldsymbol{x}\langle k\rangle, \delta\langle k\rangle, \boldsymbol{v}\langle k\rangle) = \begin{pmatrix} x\langle k\rangle + (\delta_d + v_d)\cos\theta\langle k\rangle \\ y\langle k\rangle + (\delta_d + v_d)\sin\theta\langle k\rangle \\ \theta\langle k\rangle + \delta_\theta + v_\theta \end{pmatrix}$$

Estimate pose EKF(extended Kalman filter)

$$\hat{\boldsymbol{x}}\langle k+1\rangle = \hat{\boldsymbol{x}}\langle k\rangle + \boldsymbol{F}_x(\boldsymbol{x}\langle k\rangle - \hat{\boldsymbol{x}}\langle k\rangle) + \boldsymbol{F}_v\boldsymbol{v}\langle k\rangle$$

其中 θ v= θ <k>

$$\boldsymbol{F}_x = \left.\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}\right|_{v=0} = \begin{pmatrix} 1 & 0 & -\delta_d\sin\theta_v \\ 0 & 1 & \delta_d\cos\theta_v \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{F}_v = \left.\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{v}}\right|_{v=0} = \begin{pmatrix} \cos\theta_v & 0 \\ \sin\theta_v & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{\boldsymbol{x}}\langle k+1|k\rangle = \boldsymbol{f}(\hat{\boldsymbol{x}}\langle k\rangle, \delta\langle k\rangle, 0)$$
$$\hat{\boldsymbol{P}}\langle k+1|k\rangle = \boldsymbol{F}_x\langle k\rangle\hat{\boldsymbol{P}}\langle k|k\rangle\boldsymbol{F}_x\langle k\rangle^T + \boldsymbol{F}_v\langle k\rangle\hat{\boldsymbol{V}}\boldsymbol{F}_v\langle k\rangle^T$$

V:estimation of the covariance of the odometry noise **P:** covariance matrix, represents uncertainty in the estimated vehicle config
Localizing with map(同上式)
Sensor model: $\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{xv}, \boldsymbol{xi}, \boldsymbol{w})$ state,known,error
Observation function

$$\boldsymbol{z} = \begin{pmatrix} \sqrt{(y_i - y_v)^2 + (x_i - x_v)^2} \\ \tan^{-1}(y_i - y_v)/(x_i - x_v) - \theta_v \end{pmatrix} + \begin{pmatrix} w_r \\ w_\beta \end{pmatrix}$$

=(r, β)^T r:range, β :方位角 xi 第 i 个路标

$$\begin{pmatrix} w_r \\ w_\beta \end{pmatrix} \sim N(0, \boldsymbol{W}), \quad \boldsymbol{W} = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\beta^2 \end{pmatrix}$$

线性化得到：

$$\boldsymbol{z}\langle k\rangle = \hat{\boldsymbol{h}} + \boldsymbol{H}_x\left(\boldsymbol{x}\langle k\rangle - \hat{\boldsymbol{x}}\langle k\rangle\right) + \boldsymbol{H}_w \boldsymbol{w}\langle k\rangle$$

其中 $\hat{\boldsymbol{h}} = \boldsymbol{h}(\hat{x}\langle k\rangle, x_i, 0)$

$$\boldsymbol{H}_{x_i} = \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}_v}\right|_{w=0} = \begin{pmatrix} -\frac{x_i - x_v\langle k\rangle}{r} & -\frac{y_i - y_v\langle k\rangle}{r} & 0 \\ \frac{x_i - x_v\langle k\rangle}{r^2} & -\frac{y_i - y_v\langle k\rangle}{r^2} & -1 \end{pmatrix}$$

$$\boldsymbol{H}_w = \left.\frac{\partial \boldsymbol{h}}{\partial \boldsymbol{w}}\right|_{w=0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

W = 0 时，如下图所示

其中 W 为 Estimated covariance of sensor noise

$$\boldsymbol{S}\langle k+1\rangle = \boldsymbol{H}_x\langle k+1\rangle \hat{\boldsymbol{P}}\langle k+1|k\rangle \boldsymbol{H}_x\langle k+1\rangle^T + \boldsymbol{H}_w\langle k+1\rangle \hat{\boldsymbol{W}}\langle k+1\rangle \boldsymbol{H}_w\langle k+1\rangle^T$$

$$\Rightarrow \boldsymbol{S}\langle k+1\rangle = \boldsymbol{H}_x\langle k+1\rangle \hat{\boldsymbol{P}}\langle k+1|k\rangle \boldsymbol{H}_x\langle k+1\rangle^T$$

$$\boldsymbol{K}\langle k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle \boldsymbol{H}_x\langle k+1\rangle^T \boldsymbol{S}\langle k+1\rangle^{-1}$$

$$\Rightarrow \boldsymbol{K}\langle k+1\rangle = \boldsymbol{H}_x\langle k+1\rangle^{-1}$$

$$\hat{\boldsymbol{P}}\langle k+1|k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle - \boldsymbol{K}\langle k+1\rangle \boldsymbol{H}_x\langle k+1\rangle \hat{\boldsymbol{P}}\langle k+1|k\rangle$$

$$\Rightarrow \hat{\boldsymbol{P}}\langle k+1|k+1\rangle = 0$$

定义 uncertainty 为 根号下 det（p）

Creating a map

$\boldsymbol{x}_v=(x_v,y_v,\theta_v)$

遇到一个新的 landmark 时：

$$\hat{\boldsymbol{x}}\langle k+1|k\rangle = \hat{\boldsymbol{x}}\langle k|k\rangle \quad \hat{\boldsymbol{P}}\langle k+1|k\rangle = \hat{\boldsymbol{P}}\langle k|k\rangle$$

Xi = g（‘）

$$\boldsymbol{x}\langle k\rangle^* = \boldsymbol{y}(\boldsymbol{x}\langle k\rangle, \boldsymbol{z}\langle k\rangle, \boldsymbol{x}_v\langle k\rangle) = \begin{bmatrix} \boldsymbol{x}\langle k\rangle \\ \boldsymbol{g}(\boldsymbol{x}_v\langle k\rangle, \boldsymbol{z}\langle k\rangle) \end{bmatrix} \qquad \boldsymbol{g}(\boldsymbol{x}_v, \boldsymbol{z}) = \begin{pmatrix} x_v + r_z\cos(\theta_v + \theta_z) \\ y_v + r_z\sin(\theta_v + \theta_z) \end{pmatrix}$$

y 将观测估计与已知的匹配

$$\hat{\boldsymbol{P}}\langle k|k\rangle^* = \boldsymbol{Y}_z \begin{pmatrix} \hat{\boldsymbol{P}}\langle k|k\rangle & 0 \\ 0 & \hat{\boldsymbol{W}} \end{pmatrix} \boldsymbol{Y}_z^T$$

$$\boldsymbol{Y}_z = \begin{bmatrix} \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{x}} & \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{z}} \\ \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}} & \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 2} \\ \boldsymbol{0}_{2\times n} & \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} \end{bmatrix}$$

$$\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}} = \begin{bmatrix} \cos(\theta_v + \theta_z) & -r_z\sin(\theta_v + \theta_z) \\ \sin(\theta_v + \theta_z) & r_z\cos(\theta_v + \theta_z) \end{bmatrix} := \boldsymbol{G}_z$$

最终可得（***Hxi*** is at the location corresponding to the state ***xi***）

$$\hat{\boldsymbol{P}}\langle k|k\rangle^* = \begin{bmatrix} \hat{\boldsymbol{P}}\langle k|k\rangle & \boldsymbol{0}_{n\times 2} \\ \boldsymbol{0}_{2\times n} & \boldsymbol{G}_z \hat{\boldsymbol{W}} \boldsymbol{G}_z^T \end{bmatrix}$$

考虑 observation function 有

$$\hat{\boldsymbol{P}}\langle k+1|k+1\rangle = \hat{\boldsymbol{P}}\langle k+1|k\rangle - \boldsymbol{K}\langle k+1\rangle \boldsymbol{H}_x\langle k+1\rangle \hat{\boldsymbol{P}}\langle k+1|k\rangle$$

$$\boldsymbol{H}_{x_i} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}_i} = \begin{pmatrix} \frac{x_i - x_v}{r} & \frac{y_i - y_v}{r} \\ -\frac{x_i - x_v}{r^2} & \frac{y_i - y_v}{r^2} \end{pmatrix}$$

其中

臂型机器人(R 旋转 P 平移)

$$\xi_E(\boldsymbol{q}) = \mathscr{R}_z(q_1) \oplus \mathscr{T}_x(a_1)$$

先旋转后平移（对 3D，ZYZ）

R:revolute θ 可变 d 固定

P：prismatic dj 可变， θ 固定

DH:4 个参数，2 个(joint space or coinfig space)

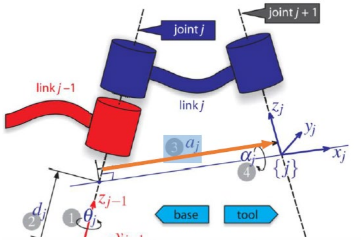
Joint angle θj Xj-1 Xj 关于 Zj-1 夹角

Link offset dj j-1 的原点到 xj 沿 Zj-1 的距离

Link length aj Zj-1 和 Zj 沿 Xj 的距离，平行则为 Zj-1×Zj

Link twist αj 从 Zj-1 到 Zj 沿 Xj 的角度

Joint type σj R 或 P



Notice that $\|v\|^2 = v^T v \Rightarrow \|v\| = +\sqrt{v^T v}$. Therefore,

$$\|Rv\| = +\sqrt{(Rv)^T Rv} = \sqrt{v^T R^T R v} = \sqrt{v^T v} = \|v\|$$

$${}^{j-1}\boldsymbol{A}_j = \begin{pmatrix} \cos\theta_j & -\sin\theta_j\cos\alpha_j & \sin\theta_j\sin\alpha_j & a_j\cos\theta_j \\ \sin\theta_j & \cos\theta_j\cos\alpha_j & -\cos\theta_j\sin\alpha_j & a_j\sin\theta_j \\ 0 & \sin\alpha_j & \cos\alpha_j & d_j \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

.For all-revolute robot, joint space ==joint angles

.joint coordinates == pose of the manipulator

逆运动学：两种方法

1.A closed form or analytic solution can be determined using geometric or algebraic approaches。存在问题：关节数越多，计算越困难；对一些机臂没有闭环解

2.An iterative numerical solution can be used。

.A necessary condition for a closed form solution of a 6 axis robot is a spherical wrist mechanism。

数值算法(numeric)缺点在于慢，优点在于 it has the great advantage of being able to work with manipulators at singularities and manipulators with less than six or more than six joints。

Trajectory:

1.joint-space motion

2.cartesian motion

关节速度与末端执行器速度：（雅可比）

$${}^0\nu = {}^0J(q)\dot{q}$$

行代表 Cartesian degree of freedom

列代表一个关节造成末端速度的关系

$${}^A\nu = \begin{pmatrix} {}^A\boldsymbol{R}_B & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & {}^A\boldsymbol{R}_B \end{pmatrix} {}^B\nu = {}^A\boldsymbol{J}_B\left({}^A\xi_B\right) {}^B\nu$$

$${}^E\nu = {}^E\boldsymbol{J}_0\left({}^E\xi_0\right) {}^0\boldsymbol{J}(q)\dot{q} = \begin{pmatrix} {}^E\boldsymbol{R}_0 & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & {}^E\boldsymbol{R}_0 \end{pmatrix} {}^0\boldsymbol{J}(q)\dot{q} = {}^E\boldsymbol{J}(q)\dot{q}$$

$${}^A\nu = \begin{pmatrix} {}^A\boldsymbol{R}_B & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{3\times 3} & {}^A\boldsymbol{R}_B \end{pmatrix} {}^B\nu = {}^A\boldsymbol{J}_B\left({}^A\xi_B\right) {}^B\nu$$

对于一个 rpy 旋转矩阵

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} s\theta_p\dot{\theta}_r + \dot{\theta}_y \\ -c\theta_p s\theta_y\dot{\theta}_r + c\theta_y\dot{\theta}_p \\ c\theta_p c\theta_y\dot{\theta}_r + s\theta_y\dot{\theta}_p \end{pmatrix} \qquad \omega = \begin{pmatrix} s\theta_p & 0 & 1 \\ -c\theta_p s\theta_y & c\theta_y & 0 \\ c\theta_p c\theta_y & s\theta_y & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta}_r \\ \dot{\theta}_p \\ \dot{\theta}_y \end{pmatrix}$$

ω =A(Γ)$\dot{\Gamma}$ This matrix ***A*** is itself a Jacobian that maps XYZ roll pitch yaw angle rates to angular velocity

Analytical jacobian $J_a(q) = \begin{pmatrix} I_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & A^{-1}(\Gamma) \end{pmatrix} J(q)$

机械臂速度

雅可比满秩存在的问题(逆运动学)：some cartesian end-effector velocities requires very high joint rates-at the singularity those rates will go to infinity 但旋转不存在问题

$$m = \sqrt{\det(JJ^T)}$$

The shape of the ellipsoid describes how well conditioned the manipulator is for making certain motions Manipulability is a succinct scalar measure that describes how spherical the ellipsoid is, for instance the ratio of the smallest to the largest radius。

逆运动学

$$\dot{q} = J(q)^{-1}\nu \qquad \nu^T\left(J(q)J(q)^T\right)^{-1}\nu = 1$$

例题：

1.旋转长度不变：

Notice that $\|V\|^2=V^TV \Rightarrow \|V\| = +\text{sqrt}(V^TV)$,所以 $\|RV\| = +\text{sqrt}((RV)^T(RV)) = \text{sqrt}(V^TR^TRV) = \text{sqrt}(V^TV) = \|V\|$

2.Express the incremental rotation ${}^B R_\Delta$ as an exponential series and verify $\boldsymbol{R}_B\langle t+\delta_t\rangle \approx \boldsymbol{R}_B\langle t\rangle + \delta_t \boldsymbol{R}_B\langle t\rangle[\boldsymbol{\omega}]_\times$.

$$\boldsymbol{R}_B(t+\delta_t) = \boldsymbol{R}_B(t)\,{}^B\boldsymbol{R}_\Delta$$

$${}^B\boldsymbol{R}_\Delta = e^{[{}^B\boldsymbol{\omega}]_\times \delta_t} = \sum_{k=0}^{\infty} \frac{\left([{}^B\boldsymbol{\omega}]_\times \delta_t\right)^k}{k!} \approx \boldsymbol{I} + [{}^B\boldsymbol{\omega}]_\times \delta_t$$

3. Suppose that $\boldsymbol{R}(t)=\boldsymbol{R}x(\theta(t))$. Compute $d\boldsymbol{R}(t)/dt$ directly using the chain rule. And show $d\boldsymbol{R}(t)/dt=\omega\times\boldsymbol{R}(t)$

$dR_x(\theta(t))/dt=(dR_x(\theta)/d\theta)*(d\theta(t)/dt)=[0\ \ 0\ \ 0\ ;\ 0\ \ -\sin(\theta(t))\omega_x\ \ -\cos(\theta(t))\omega_x\ ;\ 0\ \ \cos(\theta(t))\omega_x\ \ -\sin(\theta(t))\omega_x]$

$S(\omega)R_x(\theta(t))=[0\ 0\ 0;\ 0, -\omega_x\sin(\theta(t)), -\omega_x\cos(\theta(t));\ 0, \omega_x\cos(\theta(t)) -\omega_x\sin(\theta(t))]$

4. Write down Kalman filter for a robot moving on a one dimensional straight line for the following tasks:

1) Dead Reckoning

We need to estimate the coordinate of robot x_v

$x_v\langle k+1\rangle=x_v\langle k\rangle+\delta_d+v_d$ 其中 v_d is a zero-mean Gaussian process with variance σ_d^2

a) Kalman filter prediction equations

$x_v^{帽子}\langle k+1|k\rangle = x_v^{帽子}\langle k|k\rangle+\delta d$

$P^{帽子}\langle k+1|k\rangle = P^{帽子}\langle k|k\rangle+\sigma_d^2$

b) Kalman filter update equations

$x_v^{帽子}\langle k+1|k+1\rangle=x_v^{帽子}\langle k+1|k\rangle$

$P^{帽子}\langle k+1|k+1\rangle=P^{帽子}\langle k+1|k\rangle$

2) Localizing with a map

Sensor model:

$z\langle k\rangle=x_i-x_v\langle k\rangle+w_r$ (Here we assume that X_i is always larger than x_v) w_r is a zero-mean Gaussian process with variance σ_r^2

a) Prediction step

$x_v^{帽子}\langle k+1|k\rangle = x_v^{帽子}\langle k|k\rangle+\delta d$

$P^{帽子}\langle k+1|k\rangle = P^{帽子}\langle k|k\rangle+\sigma_d^2$

b) Update step

$v=z\langle k+1\rangle-(x_i - x_v^{帽子}\langle k+1|k\rangle)$

$K= P^{帽子}\langle k+1|k\rangle(P^{帽子}\langle k+1|k\rangle+ \sigma_r^2)^{-1}$

$x^{帽子}\langle k+1|k+1\rangle = x^{帽子}\langle k+1|k\rangle+ K v$

$P^{帽子}\langle k+1|k+1\rangle = P^{帽子}\langle k+1|k\rangle - K P^{帽子}\langle k+1|k\rangle$

3) Creating a map (with perfect localization)

We need to estimate the coordinates of the landmarks $x=(x_1,x_2,\cdots,x_M)^T$

a). Prediction step

$x^{帽子}\langle k+1|k\rangle = x^{帽子}\langle k|k\rangle$

$P^{帽子}\langle k+1|k\rangle = P^{帽子}\langle k|k\rangle$

b). Update step

b1). When an old landmark i is observed

$Hx=0\cdots1\cdots0,\ Hw=1$

$S=H_x P^{帽子}\langle k+1|k\rangle H_x^T+H_w w^{帽子} H_w^T$

$= P_{ii}^{帽子}\langle k+1|k\rangle+ \sigma_r^2$

$K= P^{帽子}\langle k+1|k\rangle H_x^T S\langle k+1\rangle^{-1}$

$= (1/s)*[0\ldots P_{ii}^{帽子}\langle k+1|k\rangle\ldots 0]^T$

Here we will $P_{ii}^{帽子}\langle k+1|k\rangle$ to represent the element at row i and column i.

$x^{帽子}\langle k+1|k+1\rangle = x^{帽子}\langle k+1|k\rangle+ K v$

$P^{帽子}\langle k+1|k+1\rangle=P^{帽子}\langle k+1|k\rangle-KH_x P^{帽子}\langle k+1|k\rangle$

b2). When new landmark is observed

$x^{帽子}\langle k+1|k+1\rangle = [x^{帽子}\langle k+1|k\rangle;\ x_v\langle k+1|k\rangle+z]$

$P^{帽子}\langle k+1|k+1\rangle=[P^{帽子}\langle k+1|k\rangle, 0\ ;\ 0\ , \sigma_r^2]$

4) Localization and mapping

We need to estimate the coordinates of the vehicle and the landmarks $x=(x_1,x_2,\cdots,x_M)^T$

a). Prediction equations

$x^{帽子}\langle k+1|k\rangle=[x^{帽子}\langle k|k\rangle+ \sigma_d;\ x_{1:M}^{帽子}\langle k|k\rangle]$

$P^{帽子}\langle k+1|k\rangle= P^{帽子}\langle k|k\rangle+[\ \sigma_d^2\ldots0;\ldots.;0\ldots.0]$

b). Update step

b1). When an old landmark i is observed

$H_x=(1,0,\cdots,1,\cdots,0),\quad H_w=1$

$S=H_x P^{帽子}\langle k+1|k\rangle H_x^T+H_w w^{帽子} H_w^T$

$= (P_{vv}^{帽子}+ P_{vi}^{帽子}+ P_{iv}^{帽子}+ P_{ii}^{帽子})+ \sigma_r^2$

$K\langle k+1\rangle= P^{帽子}\langle k+1|k\rangle H_x^T S\langle k+1\rangle^{-1}$

$= (1/s)*[\ P_{vv}^{帽子}+ P_{vi}^{帽子};\ P_{1v}^{帽子}+ P_{1i}^{帽子};\ldots..;\ P_{Mv}^{帽子}+ P_{Mi}^{帽子}]$

$x^{帽子}\langle k+1|k+1\rangle = x^{帽子}\langle k+1|k\rangle+ K v$

$P^{帽子}\langle k+1|k+1\rangle=P^{帽子}\langle k+1|k\rangle-K\ H_x\ P^{帽子}\langle k+1|k\rangle$

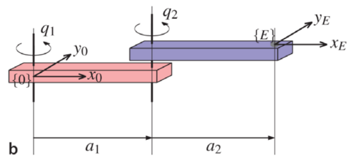
b2). When new landmark is observed

$x^{帽子}\langle k+1|k+1\rangle = [x^{帽子}\langle k+1|k\rangle;\ x_v\langle k+1|k\rangle+z]$

$P^{帽子}\langle k+1|k+1\rangle=Y_Z[P^{帽子}\langle k+1|k\rangle,0;\ 0,\ \sigma_r^2]\ Y_Z^T$

Yz = 主对角线为 1，左下角一个元素为 1。

5. Derive the forward kinematics for the 2-link robot. What is end-effector position (x, y) given link lengths (a1, a2) and joint angles (q1, q2)?



$$x = a_1\cos\theta_1 + a_2\cos(\theta_1 + \theta_2)$$
$$y = a_1\sin\theta_1 + a_2\sin(\theta_1 + \theta_2)$$

2. Derive the inverse kinematics for the 2-link robot. What are the joint angles (q1, q2) given the end-effector coordinates (x, y)?

$x^2=a_1^2\cos^2\theta_1+2a_1a_2\cos\theta_1\cos(\theta_1+\theta_2)+a_2^2\cos^2(\theta_1+\theta_2)$

$y^2=a_1^2\sin^2\theta_1+2a_1a_2\sin\theta_1\sin(\theta_1+\theta_2)+a_2^2\sin^2(\theta_1+\theta_2)$

$\theta_2=\cos^{-1}[(x^2+y^2-a_1^2-a_2^2)/(2a_1a_2)]$

$\cos\theta_2=(x^2+y^2-a_1^2-a_2^2)/(2a_1a_2):= c_2$

$\sin\theta_2=\pm\text{sqrt}(1-c_2^2):=s2$

$x=a_1\cos\theta_1+a_2\cos(\theta_1+\theta_2)$

$=a_1c_1+a_2c_1c_2-a_2s_1s_2$

$y = a_1\sin\theta_1+a_2\sin(\theta_1+\theta_2)$

$=a_1s_1+a_2s_1c_2+a_2c_1s_2$

$\cos\theta_1 = (a_1+a_2s_2(x+y))/((a_1+a_2c_2)*(a_1+a_2s_2+a_2^2s_2^2))$