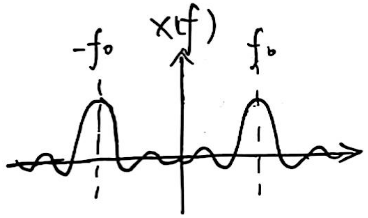
(2)

$$\cos 2\pi f_0 t = \frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$$

$$\Rightarrow \quad x(t) = \frac{1}{2}\left[w(t)e^{j2\pi f_0 t} + w(t)e^{-j2\pi f_0 t} \right]$$

$$\Rightarrow \quad X(f) = \frac{1}{2}\left[W(f-f_0) + W(f+f_0) \right]$$

$$= \frac{ET}{2}\left\{ \mathrm{sinc}[\pi(f-f_0)T] + \mathrm{sinc}[\pi(f+f_0)T] \right\}$$



(3)

$$x(t) \Leftrightarrow X(f)$$

$$x(t+t_0) \Leftrightarrow X(f)e^{j2\pi f t_0}$$

$$x(-at+t_0) \Leftrightarrow \frac{1}{a}X\left(-\frac{f}{a}\right)e^{j2\pi\left(-\frac{f}{a}\right)t_0}$$

即 $\frac{1}{a}X\left(-\frac{f}{a}\right)e^{-j2\pi\frac{f}{a}t_0}$

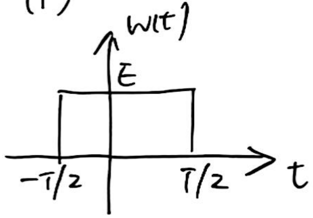
(4)

$$(-j2\pi t)^2 x(t) \Leftrightarrow \frac{d^2}{df^2}X(f)$$

$$-4\pi^2 t^2 x(t) \Leftrightarrow \frac{d^2}{df^2}X(f)$$

$$t^2 x(t) \Leftrightarrow -\frac{1}{4\pi^2}\frac{d^2}{df^2}X(f)$$

(1)



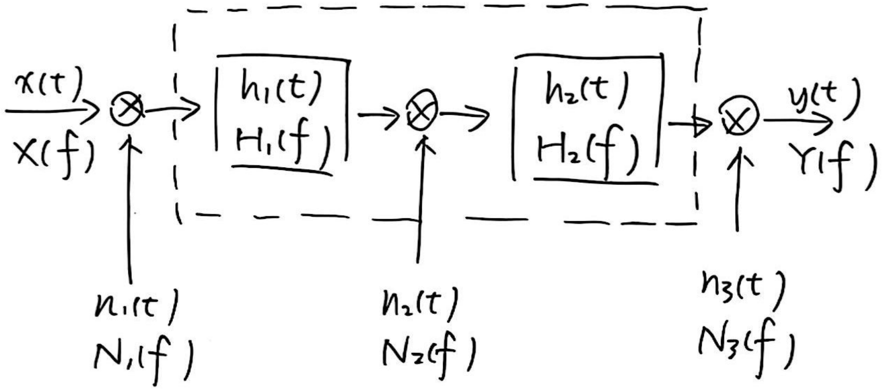
$$W(f) = \int_{-\infty}^{+\infty} w(t)e^{-j2\pi f t}\,dt$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} E e^{-j2\pi f t}\,dt$$

$$= E \cdot \frac{1}{-j2\pi f} e^{-j2\pi f}\Big|_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \frac{E}{\pi f}\sin(\pi f T) = ET\frac{\sin(\pi f T)}{\pi f T} = ET\,\mathrm{sinc}(\pi f T)$$

2



$$\underbrace{y(t)}_{\text{总输出}} = \underbrace{x'(t)}_{\text{无噪声影响下 } x(t) \text{ 的输出}} + n_1'(t) + n_2'(t) + \underbrace{n_3'(t)}_{\text{噪声 } n_3(t) \text{ 的输出}}$$

与 $x(t)$ 互相关函数

$$\Rightarrow R_{xy}(\tau) = R_{xx'}(\tau) + \underbrace{R_{xn_1'}(\tau) + R_{xn_2'}(\tau) + R_{xn_3'}(\tau)}$$

因输入与噪声独立无关，后三项均为0

$$\Downarrow$$

$$R_{xy}(\tau) = R_{xx'}(\tau)$$

由此可得 $y(t)$ 中 $x(t)$ 的成分

$$\Downarrow$$

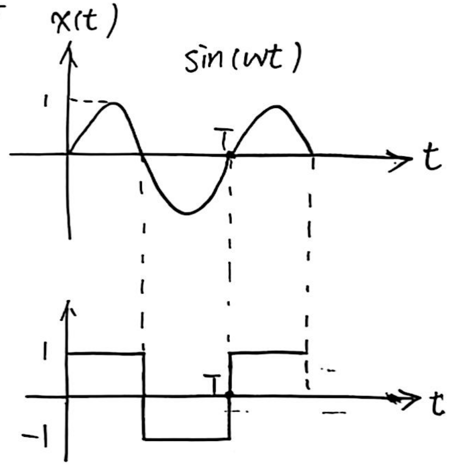
$$S_{xy}(f) = S_{xx'}(f) = H(f)S_x(f)$$

$$\Downarrow$$

$$H(f) = \frac{S_{xy}(f)}{S_x(f)}$$

这与无噪声时的结论一致
也就实现了滤噪

3



互相关函数

$$R_{xy}(\tau) = \frac{1}{T}\int_0^T x(t)y(t+\tau)\,dt$$

$$= \frac{1}{T}\int_0^T x(t-\tau)y(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} x(t-\tau)\,dt - \int_{\frac{T}{2}}^{T} x(t-\tau)\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^{\frac{T}{2}} \sin(wt-w\tau)\,dt - \int_{\frac{T}{2}}^{T} \sin(wt-w\tau)\,dt\right]$$

$$= \frac{1}{wT}\left[\int_0^{\frac{T}{2}} \sin(wt-w\tau)\,d(wt) - \int_{\frac{T}{2}}^{T} \sin(wt-w\tau)\,d(wt)\right]$$

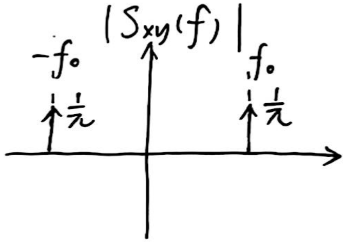
$$= \frac{1}{2\pi}\left[\int_0^{\pi} \sin(x-w\tau)\,dx - \int_{\pi}^{2\pi} \sin(x-w\tau)\,dx\right]$$

$$= \frac{1}{2\pi}\cdot 4\cos(w\tau) = \frac{2}{\pi}\cos(w\tau)$$

因 $\cos(w\tau) = \frac{1}{2}(e^{jw\tau} + e^{-jw\tau})$

令 $2\pi f_0 = w$，即 $f_0 = w/2\pi$，则 $\cancel{\cos(w\tau)} = R_{xy}(\tau) = \frac{1}{\pi}[e^{j2\pi f_0\tau} + e^{-j2\pi f_0\tau}]$

互功率谱 $S_{xy}(f) = \frac{1}{\pi}[\delta(f-f_0) + \delta(f+f_0)]$



$\boxed{4}$ (2)