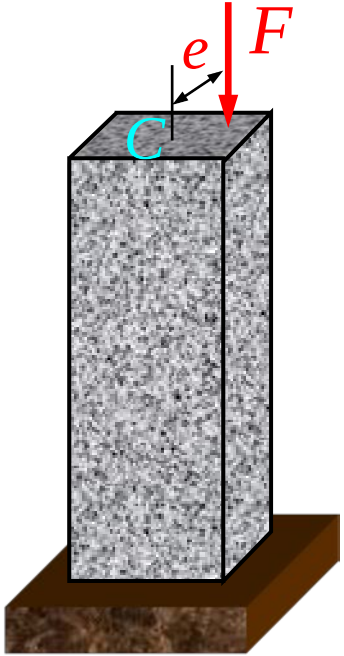
第八章　组合变形（2）

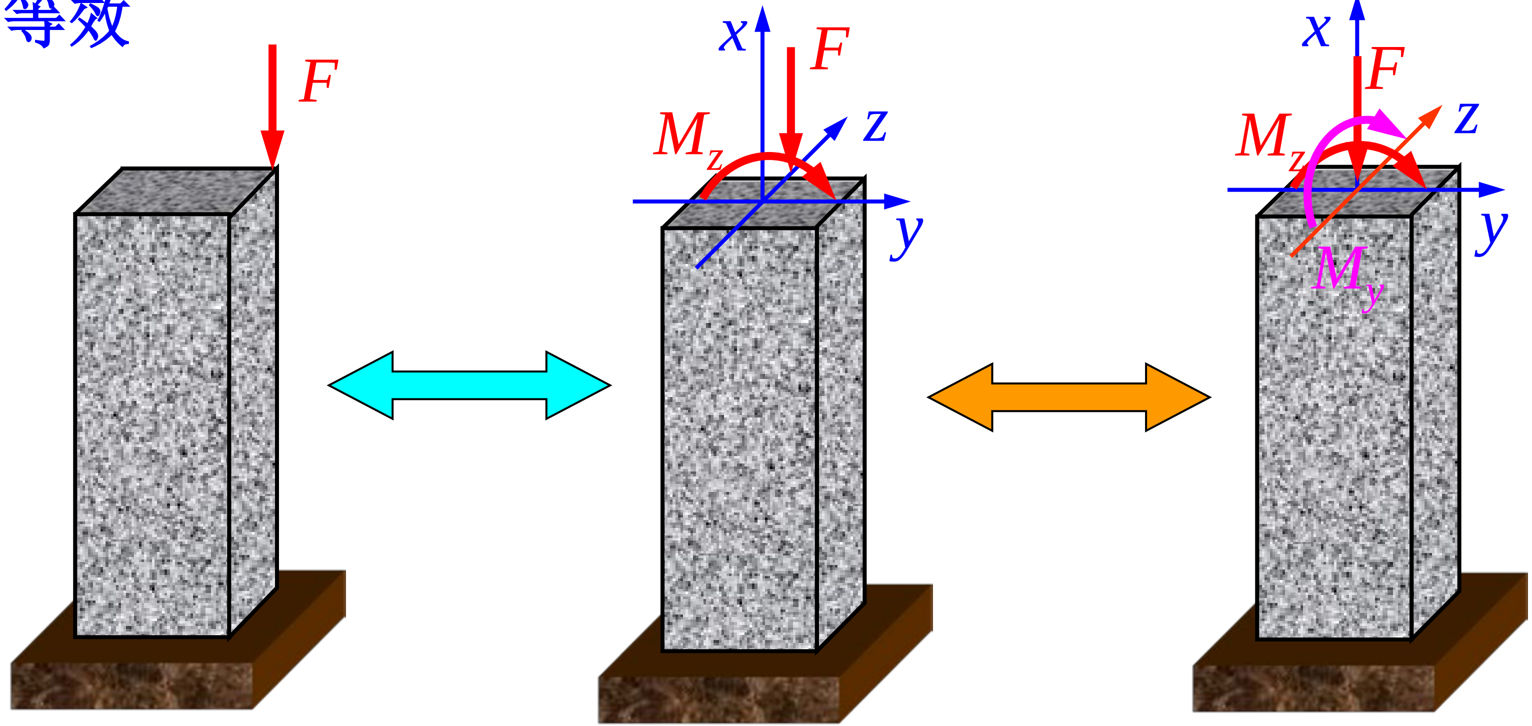
第 22 讲

§ 8.3　偏心压缩和截面核心

一、偏心压缩

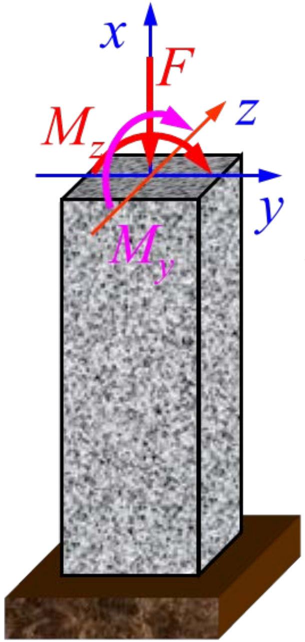


力系等效



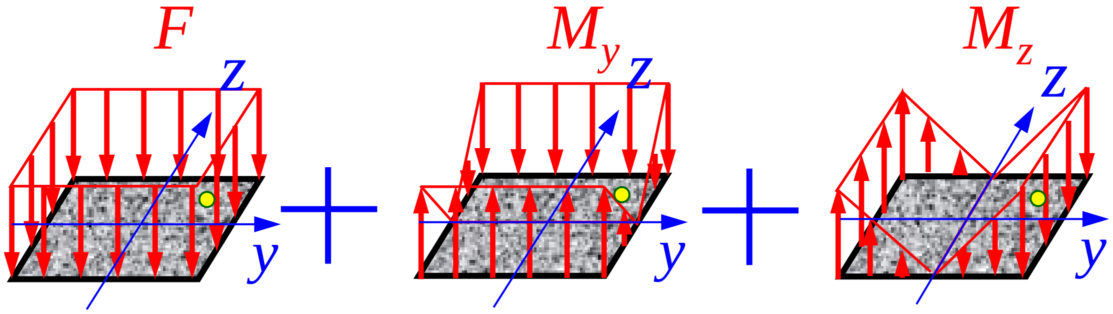
y和z轴为形心主惯性轴

（1）应力分析

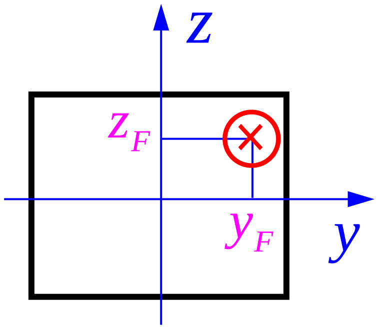


$$M_z = F \times y_F$$

$$M_y = F \times z_F$$



$$\sigma_{x,F} = -\frac{F}{A} \qquad \sigma_{x,M_y} = -\frac{M_y}{I_y} z \qquad \sigma_{x,M_z} = -\frac{M_z}{I_z} y$$

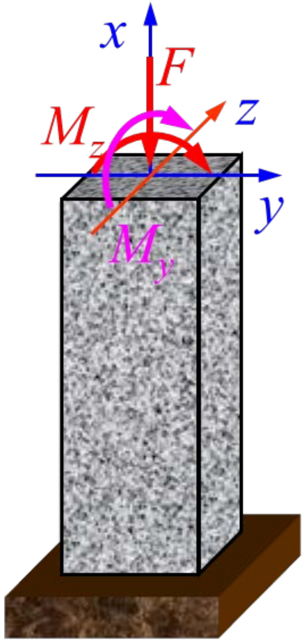


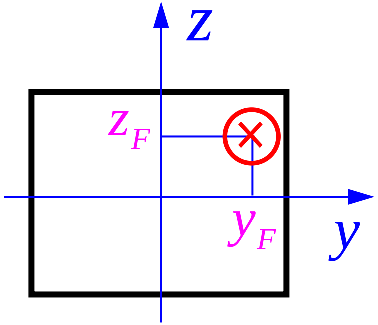
$$\sigma_x = -\left(\frac{F}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \right)$$

$$\sigma_x = -\left(\frac{F}{A} + \frac{F y_F \times y}{I_z} + \frac{F z_F \times z}{I_y} \right)$$

（2） 中性轴方程

$$\sigma_x = -\left(\frac{F}{A} + \frac{Fy_F \times y}{I_z} + \frac{Fz_F \times z}{I_y}\right)$$





$$\sigma_x = -\left(\frac{F}{A} + \frac{Fy_F \times y_0}{I_z} + \frac{Fz_F \times z_0}{I_y}\right) = 0$$

$$\frac{F}{A} + \frac{Fy_F y_0}{Ai_z^2} + \frac{Fz_F z_0}{Ai_y^2} = 0$$

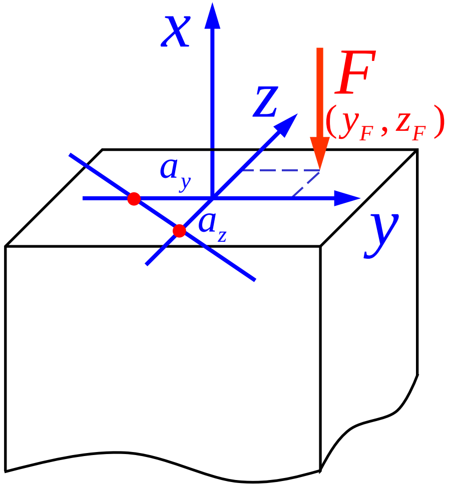
$$\frac{F}{A}(1 + \frac{y_F y_0}{i_z^2} + \frac{z_F z_0}{i_y^2}) = 0$$

$$1 + \frac{y_F y_0}{i_z^2} + \frac{z_F z_0}{i_y^2} = 0$$

中性轴方程

讨论：

$$1+\frac{y_F y_0}{i_z^2}+\frac{z_F z_0}{i_y^2}=0 \xrightarrow[\text{截距方程}]{\text{改写成}} \frac{y_0}{-\frac{i_z^2}{y_F}}+\frac{z_0}{-\frac{i_y^2}{z_F}}=1$$



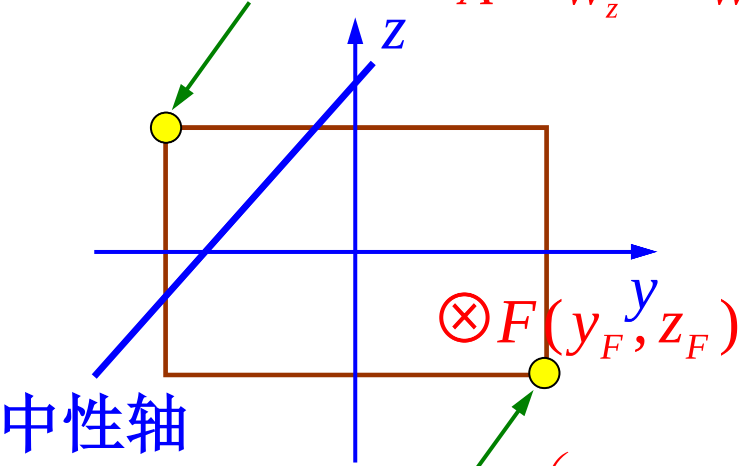
1. 力的作用点与中性轴永远在截面形心的两侧。 $a_y=-\frac{i_z^2}{y_F}, a_z=-\frac{i_y^2}{z_F}$

2. 力的作用点向截面形心方向移，即 $y_F\downarrow, z_F\downarrow$，则 $|a_y|\uparrow, |a_z|=\uparrow$，则中性轴向外移。

当力的作用点通过截面形心，有$y_F=0$、$z_F=0$，此时中性轴在无穷远处。

（3） 危险点（距中性轴最远的点）

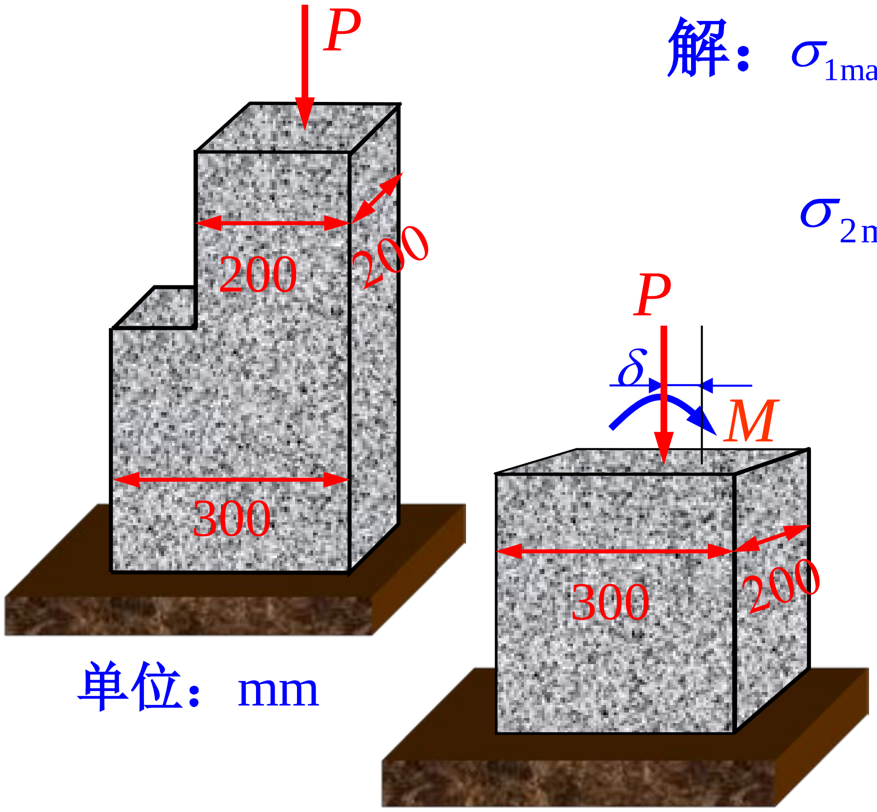
$$\sigma_{t\max} = -\frac{F}{A} + \frac{Fy_F}{W_z} + \frac{Fz_F}{W_y}$$



$$\sigma_{c\max} = -\left(\frac{F}{A} + \frac{Fy_F}{W_z} + \frac{Fz_F}{W_y}\right)$$

视频

例1 图示力P=350kN，求出两柱内的绝对值最大正应力



单位：mm

解：$\sigma_{1\max}=\dfrac{P}{A}=\dfrac{350\times10^3}{0.2\times0.2}=8.75\text{MPa}$ （压）

$$\sigma_{2\max}=\frac{P}{A_1}+\frac{M}{W_{z1}}$$

$$=\frac{350\times10^3}{0.2\times0.3}+\frac{350\times10^3\times0.05}{\frac{1}{6}\times0.2\times0.3^2}$$

$=5.83+5.83=11.66$ MPa （压）

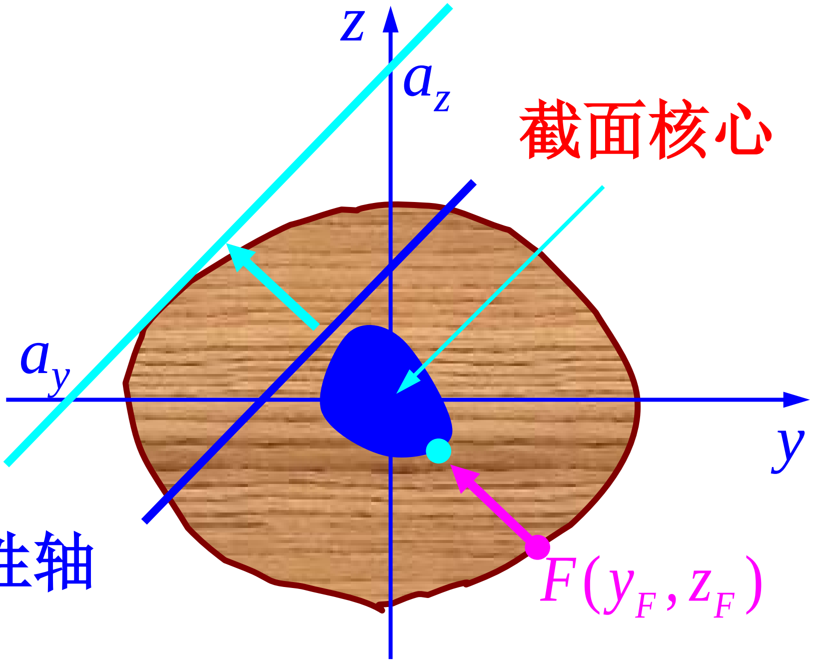
偏心压缩：

增大截面反而使应力增大！

（4）截面核心　当压力作用在此区域内时，横截面上无拉应力

$$\frac{y_0}{-\frac{i_z^2}{y_F}}+\frac{z_0}{-\frac{i_y^2}{z_F}}=1$$

$$\boxed{a_y=-\frac{i_z^2}{y_F}\qquad a_z=-\frac{i_y^2}{z_F}}$$



y和z轴为形心主惯性轴

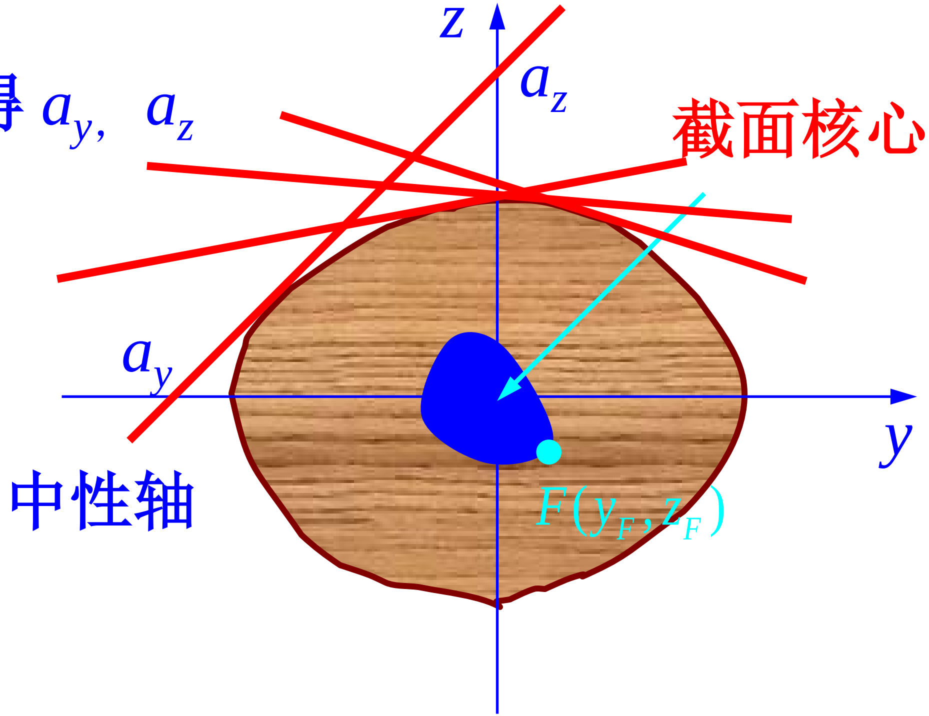
截面核心的确定：

找到截面上最远的中性轴，得 a_y，a_z

$$\frac{y_0}{-\frac{i_z^2}{y_F}} + \frac{z_0}{-\frac{i_y^2}{z_F}} = 1$$

$$-\frac{i_z^2}{y_F} = a_y \qquad -\frac{i_y^2}{z_F} = a_z$$

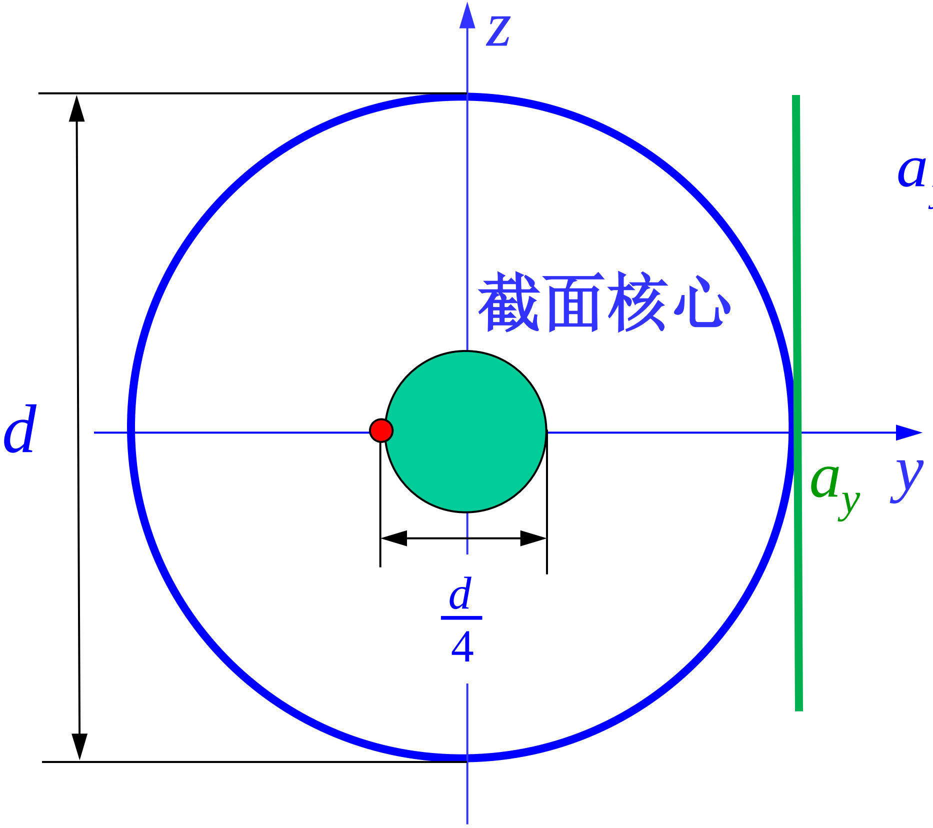
$$y_F = -\frac{i_z^2}{a_y}, \qquad z_F = -\frac{i_y^2}{a_z},$$



可求得对应该中性轴的力F 的一个作用点 (y_F, z_F)

几种截面的截面核心

1. 圆形截面



$$\frac{y_0}{a_y}+\frac{z_0}{a_z}=1$$

$$a_y=-\frac{i_z^2}{y_F} \qquad a_z=-\frac{i_y^2}{z_F}$$

$$y_F=-\frac{i_z^2}{a_y}$$

$$z_F=-\frac{i_y^2}{a_z}$$

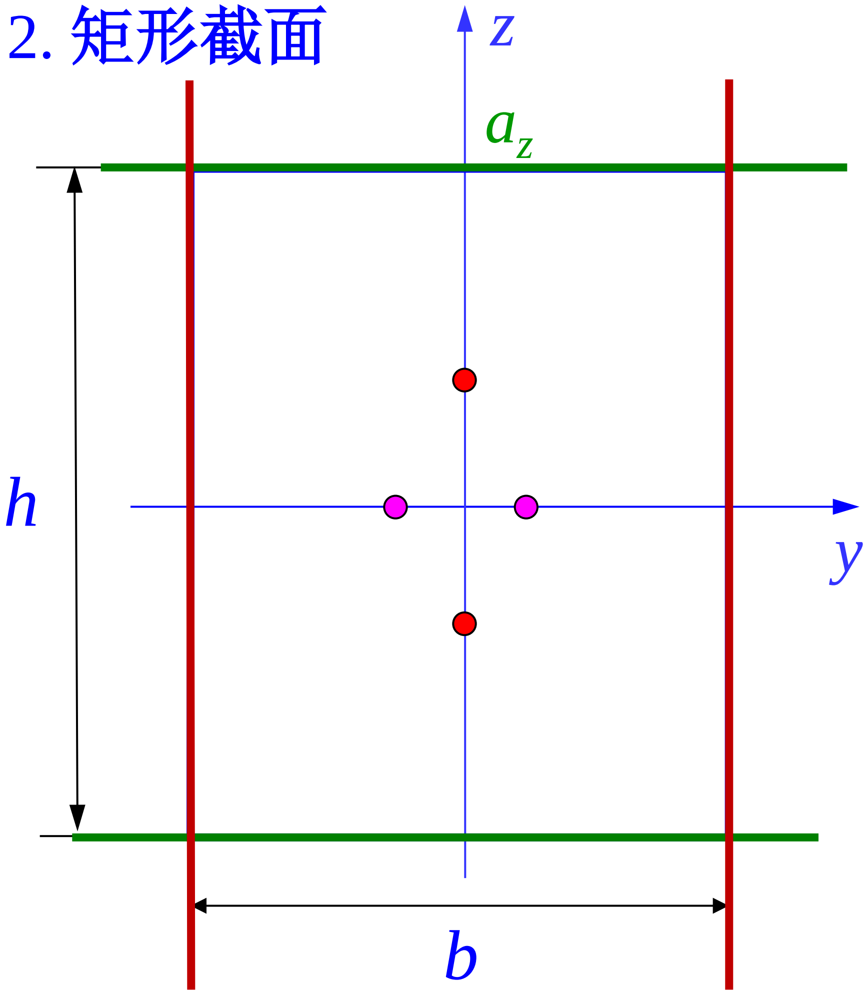
$$a_y=d/2 \qquad i_z^2=\frac{I_z}{A}=\frac{\frac{1}{64}\pi d^4}{\frac{1}{4}\pi d^2}=\frac{1}{16}d^2$$

$$y_F=-\frac{i_z^2}{a_y} \qquad y_F=-\frac{d^2/16}{d/2}=-\frac{d}{8}$$

$$a_z\to\infty$$

$$z_F=-\frac{i_y^2}{a_z} \qquad z_F=-\frac{d^2/16}{\infty}=0$$

2. 矩形截面



$$y_F = -\frac{i_z^2}{a_y}, \qquad z_F = -\frac{i_y^2}{a_z}$$

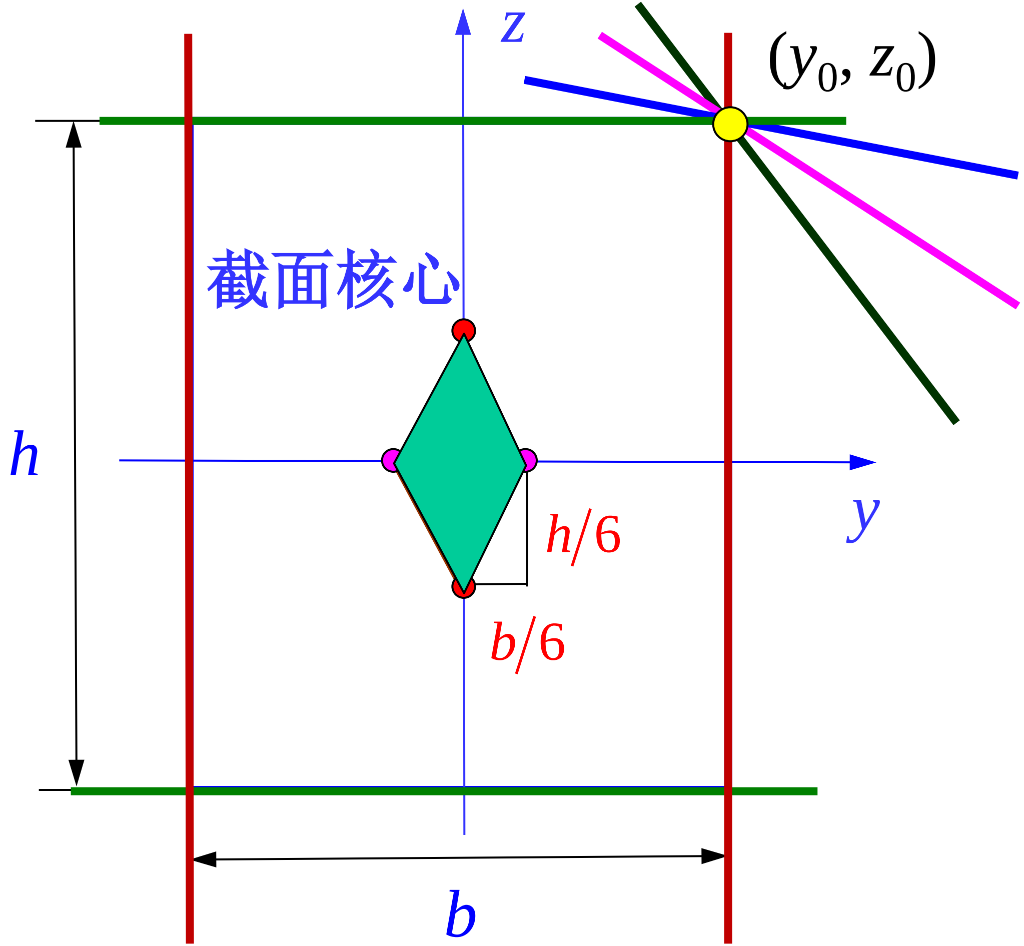
$$a_z = h/2 \qquad i_y^2 = \frac{I_y}{A} = \frac{\frac{1}{12}bh^3}{bh} = \frac{1}{12}h^2$$

$$z_F = -\frac{i_y^2}{a_z} \qquad z_F = -\frac{h^2/12}{h/2} = -\frac{h}{6}$$

$$a_y \to \infty$$

$$y_F = -\frac{i_z^2}{a_y} \qquad y_F = -\frac{b^2/12}{\infty} = 0$$

$$y_F = -\frac{b}{6}, \quad z_F = 0$$



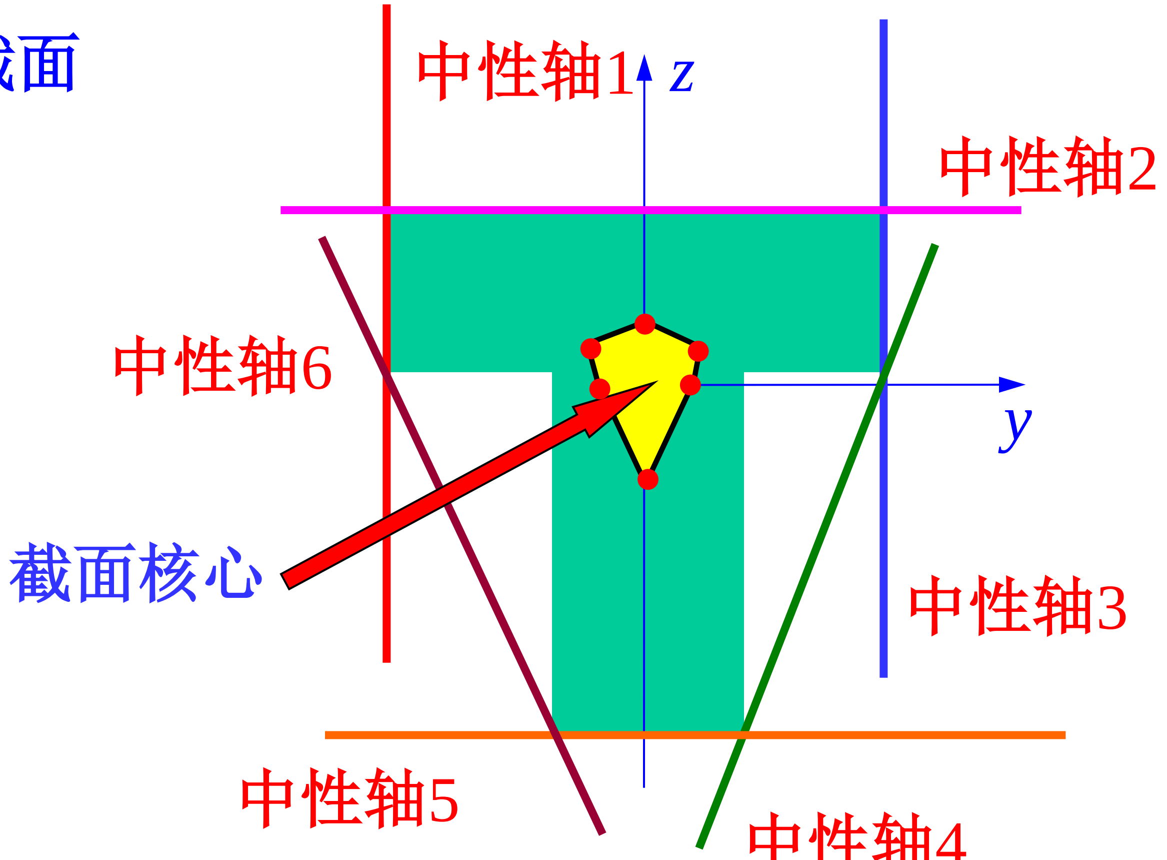
利用中性轴方程：

$$1+\frac{y_F y_0}{i_z^2}+\frac{z_F z_0}{i_y^2}=0$$

可知，通过同一点(y_0, z_0)的中性轴方程，其对应力的作用点(y_F, z_F)满足直线方程：

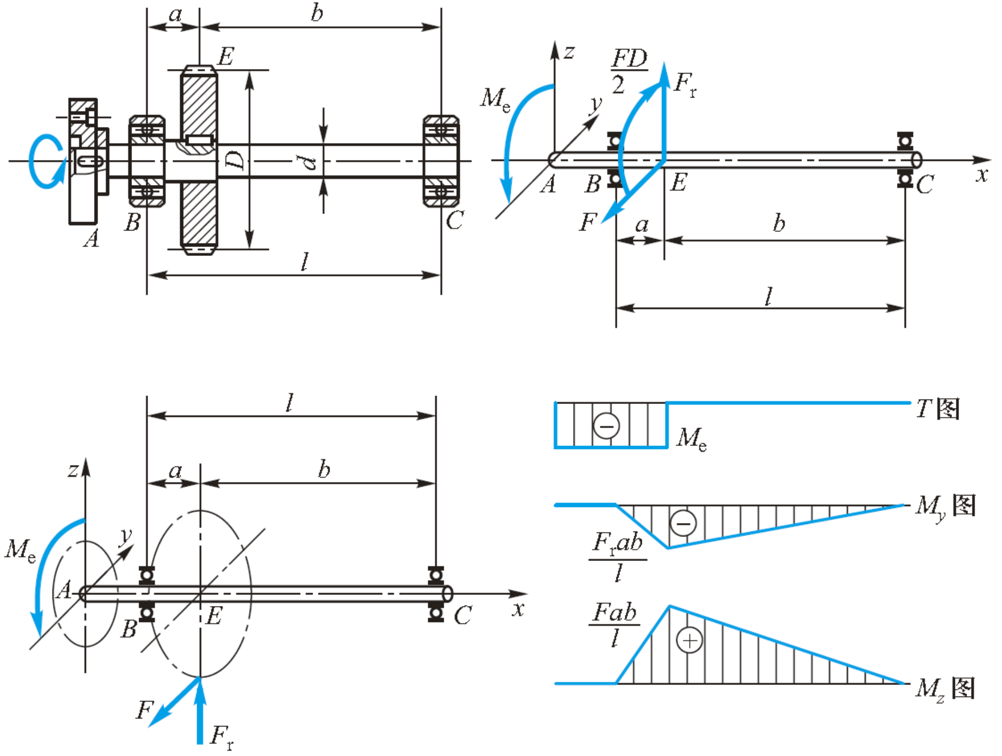
连接四点，即得截面核心的包络线。

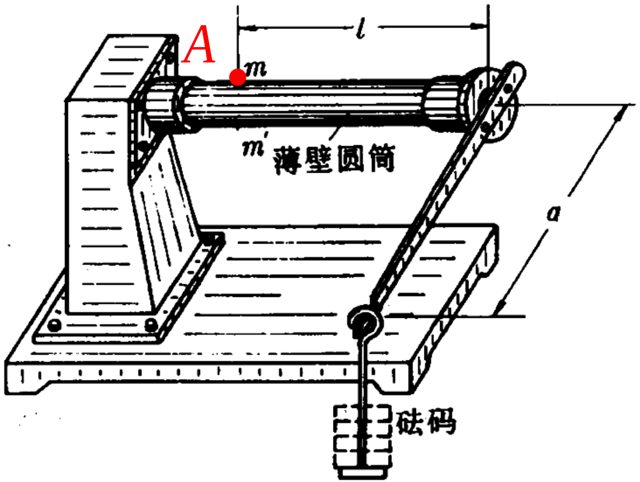
3. T形截面

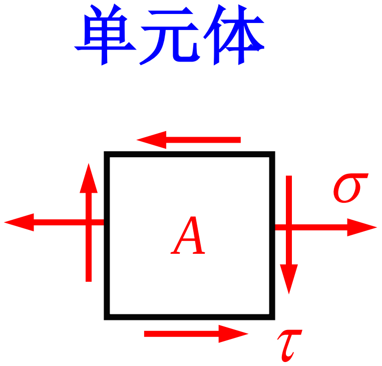


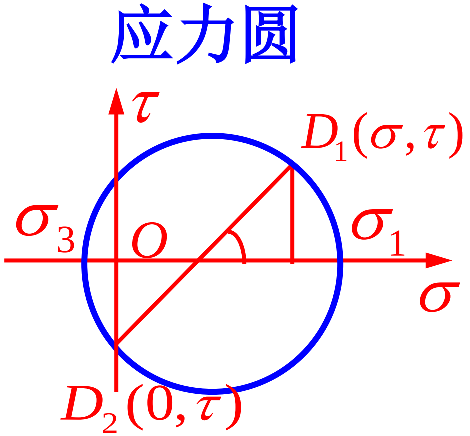
§ 8.4 扭转与弯曲的组合

图示传动轴的*BE*段的变形，是扭转与弯曲的组合。









强度条件（表面点 $\sigma_2=0$ ）：

$$\sigma_1=\sigma/2+\sqrt{(\sigma/2)^2+\tau^2}$$

$$\sigma_{r3}=\sigma_1-\sigma_3=\sqrt{\sigma^2+4\tau^2}\le[\sigma]$$

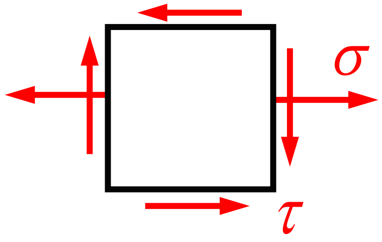
$$\sigma_3=\sigma/2-\sqrt{(\sigma/2)^2+\tau^2}$$

$$\sigma_{r4}=\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}=\sqrt{\sigma^2+3\tau^2}\le[\sigma]$$

圆轴受扭转与弯曲的组合作用：

$$\sigma_{r3}=\sqrt{\sigma^2+4\tau^2}=\sqrt{\left(\frac{M}{W}\right)^2+4\left(\frac{T}{W_p}\right)^2}=\frac{\sqrt{M^2+T^2}}{W}$$

$$\sigma_{r4}=\sqrt{\sigma^2+3\tau^2}=\sqrt{\left(\frac{M}{W}\right)^2+3\left(\frac{T}{W_p}\right)^2}=\frac{\sqrt{M^2+0.75T^2}}{W}$$



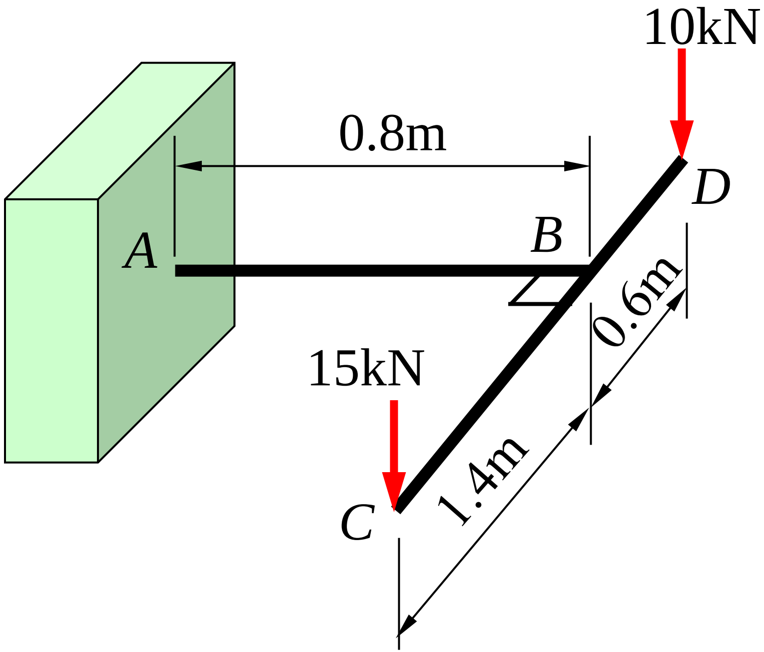
对于直径为d 的实心圆形截面 $W_p=2W$

$$W_P=\frac{1}{16}\pi d^3 \qquad W=\frac{1}{32}\pi d^3$$

对于外直径为D，内直径为d 的圆环形截面 $W_p=2W$

$$W_P=\frac{1}{16}\pi D^3(1-\alpha^4) \qquad W=\frac{1}{32}\pi D^3(1-\alpha^4) \qquad \alpha=\frac{d}{D}$$

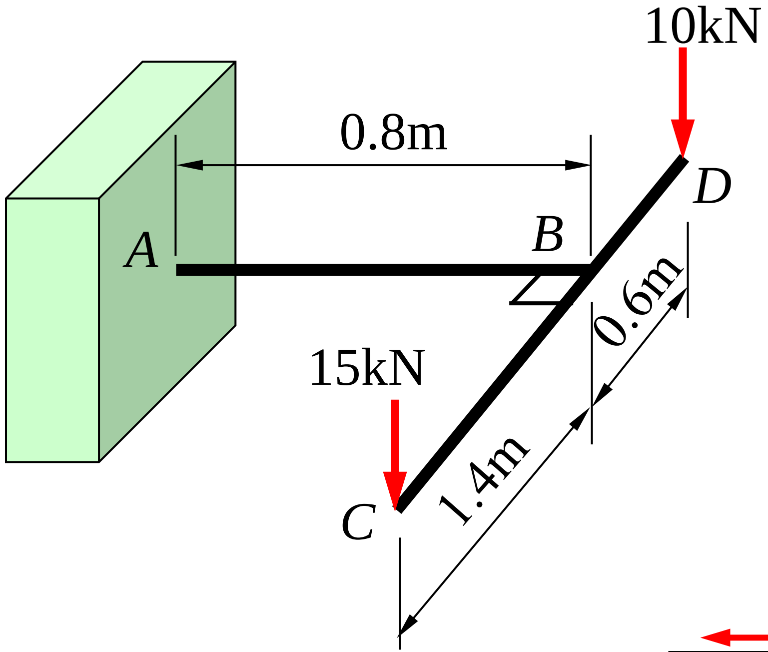
例2 空心圆杆*AB*和*CD*杆焊接成整体结构，受力如图。*AB*杆的外径 D=140mm，内、外径之比$\alpha = d/D$=0.8，材料的许用应力$[\sigma]$=160MPa。试用第三强度理论校核*AB*杆的强度。



分析：

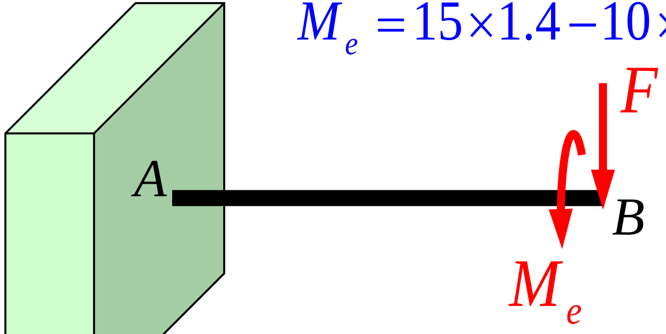
*AB*杆的变形特征

——扭弯组合



$$F = 25\text{kN}$$

$$M_e = 15\times1.4-10\times0.6=15\text{kN}\cdot\text{m}$$



$$T = M_e = 15\text{kN}\cdot\text{m}$$

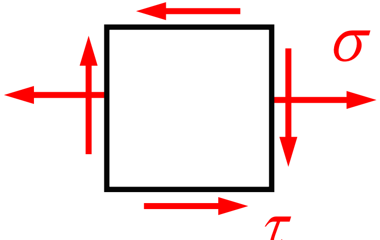
$$M_{\max} = 25\times0.8 = 20\text{kN}\cdot\text{m}$$

解：扭弯组合变形

危险截面和危险点？

危险截面：固定端 A

危险点：上表面点



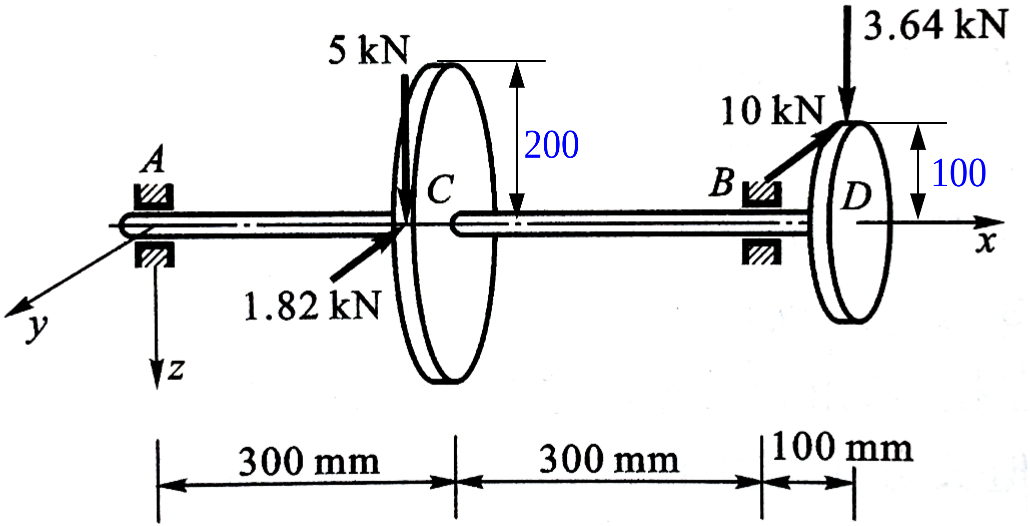
单元体

$$\sigma_{r3} = \frac{\sqrt{M_{\max}^2 + T^2}}{W} = 157.2\text{MPa}$$

$$< [\sigma] = 160\text{MPa}$$

AB杆的强度满足要求！

例3 图示钢制实心圆轴，齿轮C上作用有铅垂切向力5kN，径向力1.82kN；齿轮D上作用有水平切向力10kN，径向力3.64kN。齿轮C的直径$d_1=400$mm，齿轮D的直径$d_2=200$mm。若轴的直径 $d=60$mm，$[\sigma]=100$MPa，试按第四强度理论校核轴的强度。

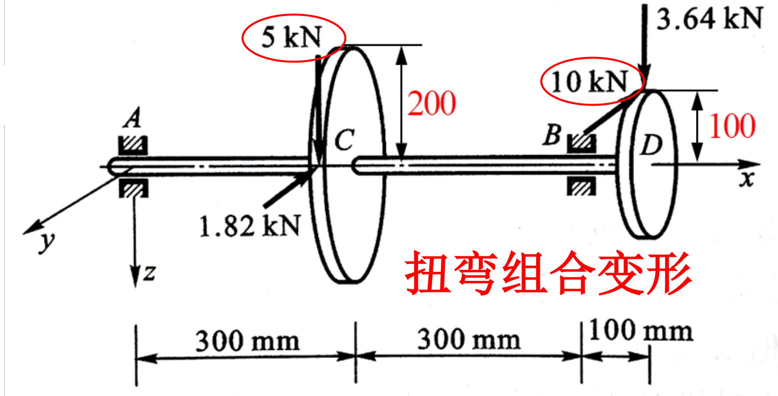


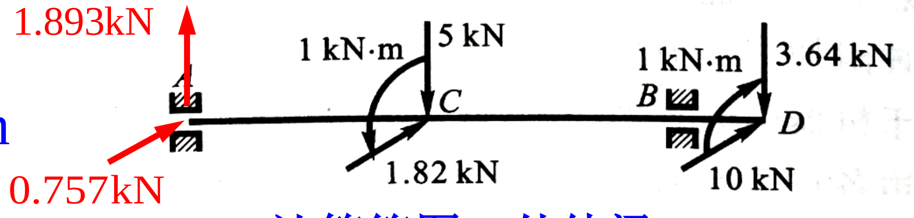
分析：

扭弯组合变形

解：受力分析

CD段：$T = -10\text{kN} \times 0.1\text{m} = -1\text{kN} \cdot \text{m}$

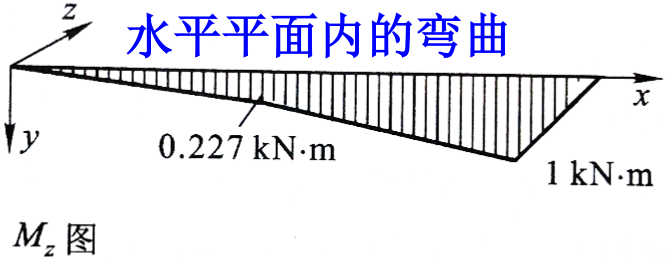




计算简图：外伸梁

叠加原理：水平平面内的弯曲+竖直平面内的弯曲+扭转

水平平面内的弯曲

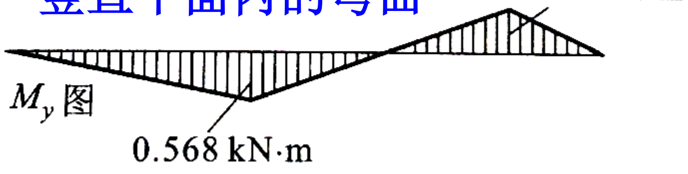


危险截面和危险点？

$$M_B = \sqrt{M_{yB}^2 + M_{zB}^2} = \sqrt{0.364^2 + 1^2} = 1.064\ \text{kN} \cdot \text{m}$$

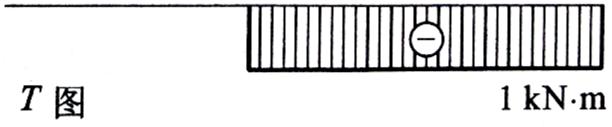
$$M_C = \sqrt{M_{yC}^2 + M_{zC}^2} = \sqrt{0.227^2 + 0.568^2} = 0.612\text{kN} \cdot \text{m}$$

竖直平面内的弯曲



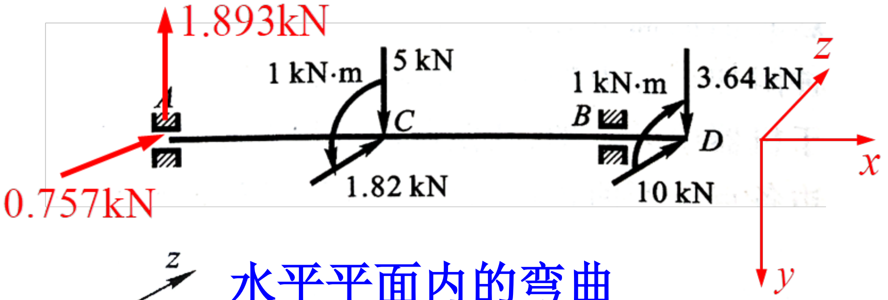
支座B处为危险截面！

$$M_{\max} = M_B = 1.064\ \text{kN.m}$$

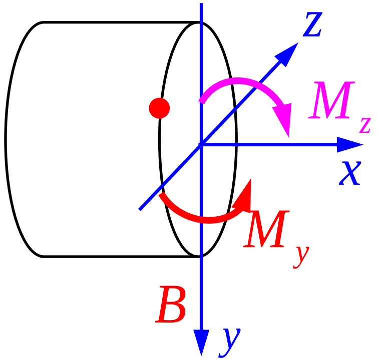


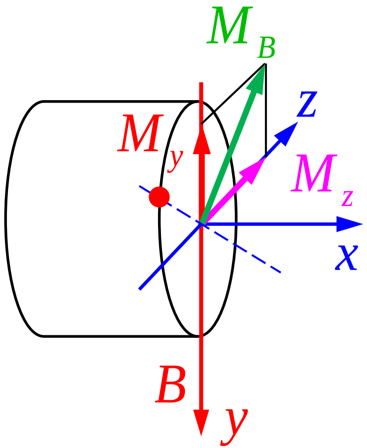
$$M_{\max} = M_B = 1.064\ \text{kN.m}$$

$$T_B = 1\ \text{kN.m}$$

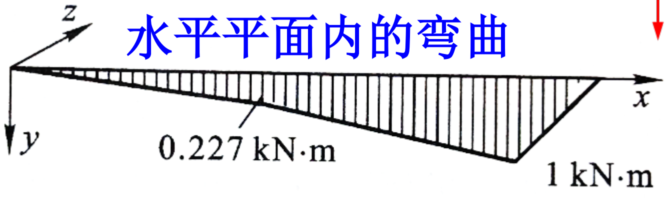


危险点：



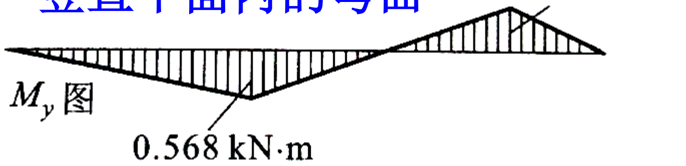


水平平面内的弯曲



M_z 图

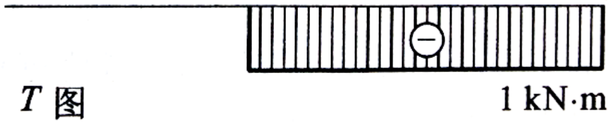
竖直平面内的弯曲



M_y 图

$$\sigma_{r4} = \frac{\sqrt{M_B^2 + 0.75T_B^2}}{W} = 64.7\text{MPa} < [\sigma] = 100\text{MPa}$$

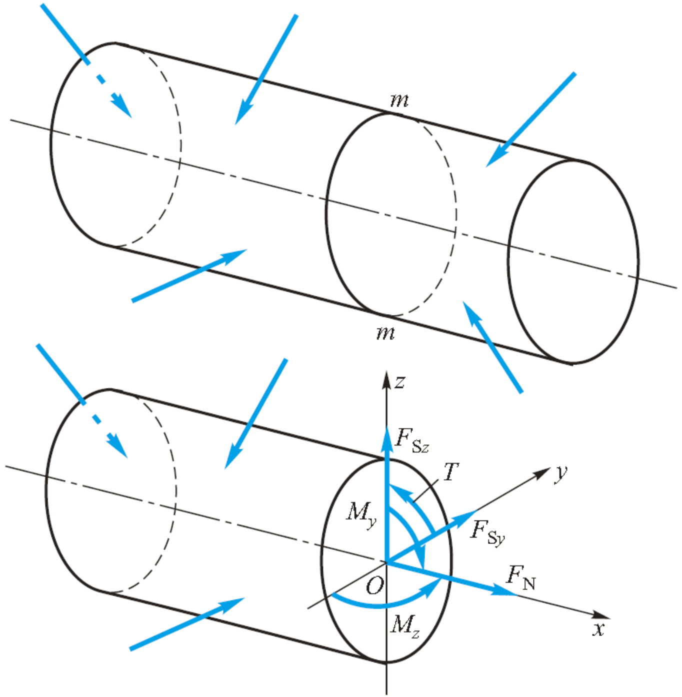
满足强度要求！



T 图

*§8.5 组合变形的普遍情况

任意载荷作用下的等直杆，研究杆件任意截面m-m上的应力时，取杆件的轴线为x轴，截面的形心主惯性轴为y轴和z轴。



计算出截面m-m上的内力或内力矩分量

$$F_N, F_{Sy}, F_{Sz}, T, M_y, M_z$$

轴力F_N对应着拉伸（压缩）变形

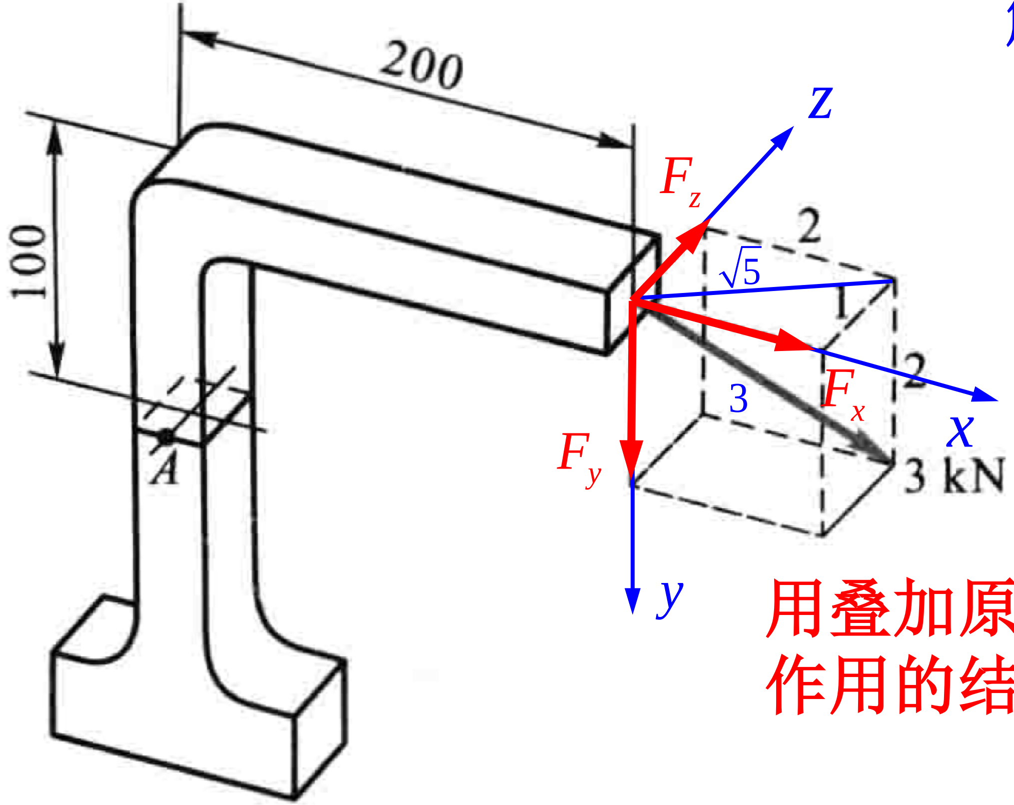
剪力F_{Sy}和F_{Sz}对应着剪切变形

扭矩T对应着扭转变形

弯矩M_y和M_z对应着弯曲变形

叠加上述内力和内力矩分量所对应的应力，即为组合变形的应力。

例4　图示折杆，横截面为边长12mm的正方形。用单元体表示A点的应力状态，并确定主应力。



解：建立坐标系如图

将力分解到 x, y, z 三个方向

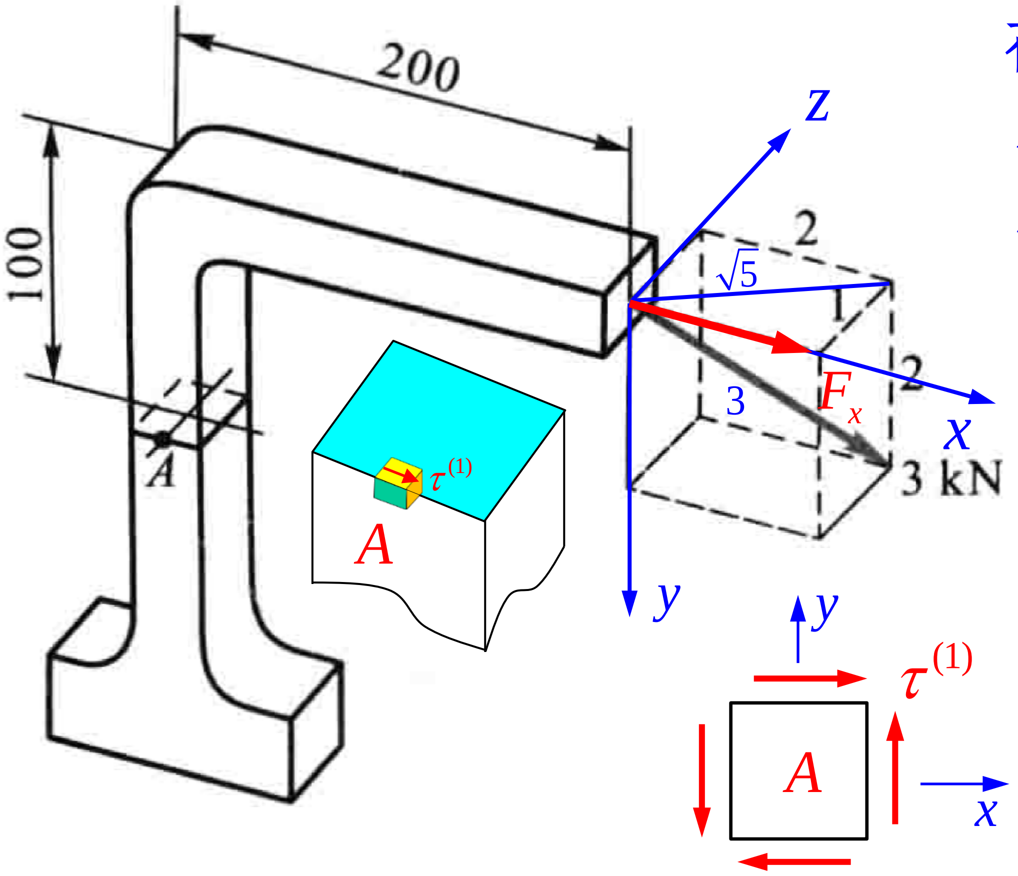
$$F_x = \frac{2}{3}F = 2\,\mathrm{kN}$$

$$F_y = \frac{2}{3}F = 2\,\mathrm{kN}$$

$$F_z = \frac{1}{3}F = 1\,\mathrm{kN}$$

用叠加原理，先计算 F_x, F_y和F_z单独作用的结果，然后叠加！

$F_x = 2\,\text{kN}$， $F_y = 2\,\text{kN}$， $F_z = 1\,\text{kN}$



1. 考虑 F_x 作用

在A点所在的截面上，内力有

$$F_{sx} = F_x = 2\,\text{kN}$$

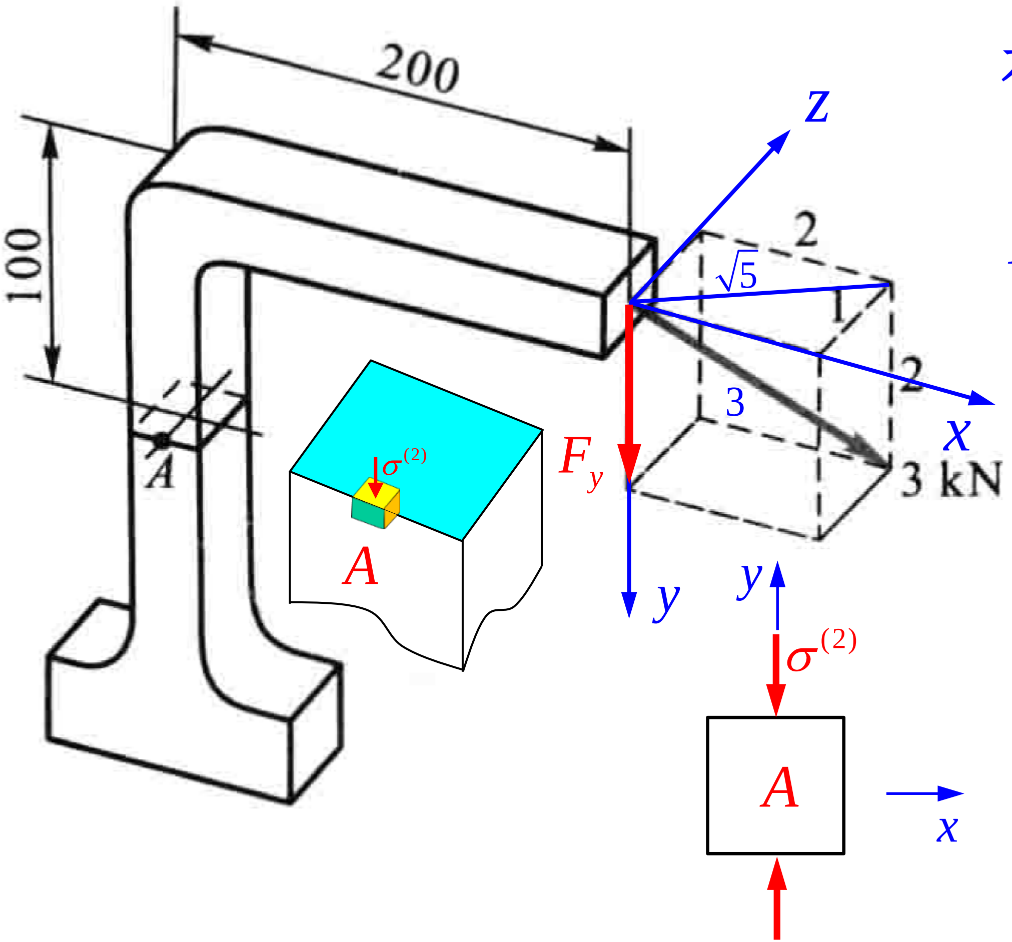
$$M_z = F_x \times 0.1 = 2 \times 0.1 = 0.2\text{kN} \cdot \text{m}$$

$$\sigma^{(1)} = 0$$

$$\tau^{(1)} = \frac{3}{2}\frac{F_{sx}}{A} = \frac{3}{2} \times \frac{2\times10^3}{(12\times10^{-3})^2}$$

$$= 20.83\times10^6\,\text{Pa} = 20.83\text{MPa}$$

$F_x = 2\,\text{kN}，\quad F_y = 2\,\text{kN}，\quad F_z = 1\,\text{kN}$



2. 考虑 F_y 的作用

在A点所在的截面上，内力有

$$F_N = F_y = -2\,\text{kN}$$

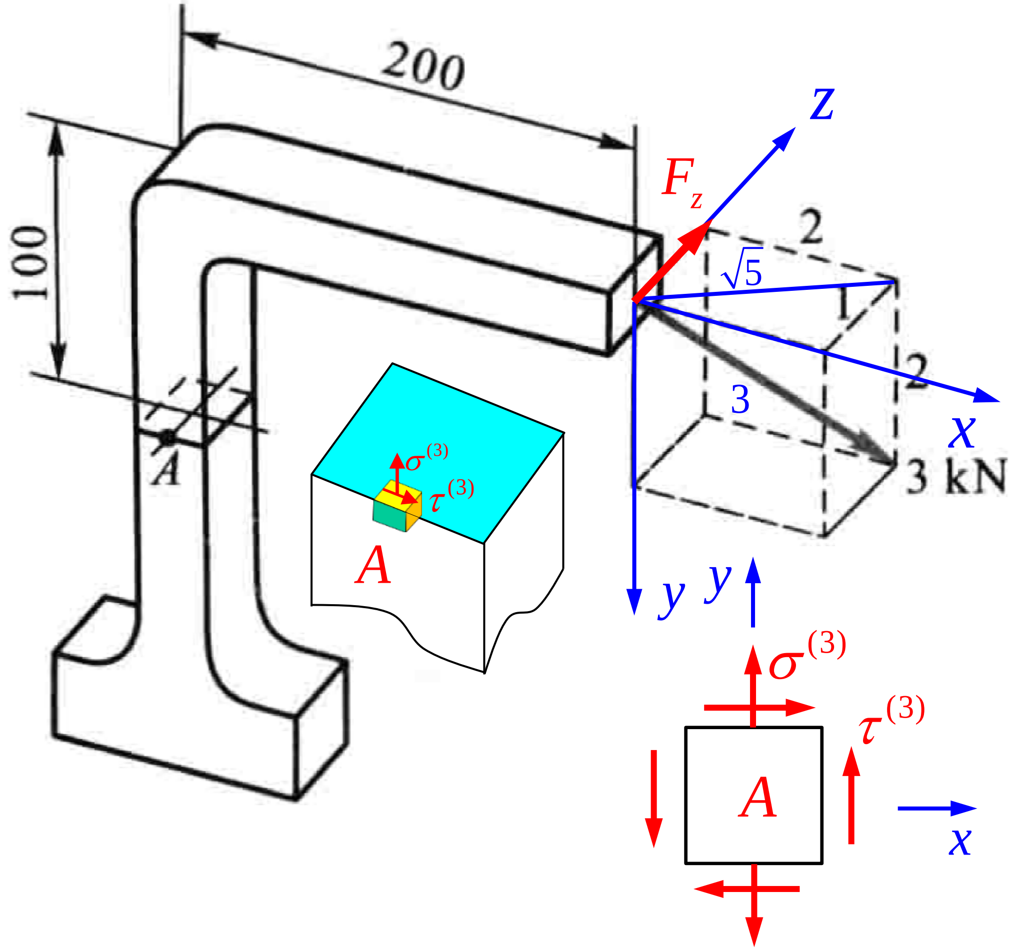
$$M_z = F_y \times 0.2 = 2\times 0.2 = 0.4\text{kN}\cdot\text{m}$$

$$\sigma^{(2)} = \frac{F_N}{A} = -\frac{2\times 10^3}{(12\times 10^{-3})^2}$$

$$= -13.89\times 10^6\,\text{Pa} = -13.89\text{MPa}$$

$$\tau^{(2)} = 0$$

$F_x = 2\text{kN}$，$F_y = 2\text{kN}$，$F_z = 1\text{kN}$



3. 考虑 F_z 的作用

在A点所在的截面上，内力有

$$F_{sz} = F_z = 1\text{kN}$$

$$T = F_z \times 0.2 = 1 \times 0.2 = 0.2\text{kN}\cdot\text{m}$$

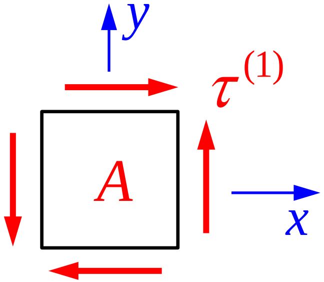
$$M_x = F_z \times 0.1 = 1 \times 0.1 = 0.1\text{kN}\cdot\text{m}$$

$$\sigma^{(3)} = \frac{M_x}{W} = \frac{0.1\times 10^3}{\frac{1}{6}\times(12\times 10^{-3})^3} = 347.22\text{MPa}$$

$$\tau^{(3)} = \frac{T}{W_t} = \frac{T}{\alpha h b^2} = \frac{T}{0.208 h b^2}$$

$$= \frac{0.2\times 10^3}{0.208\times(12\times 10^{-3})^3} = 556.45\text{MPa}$$

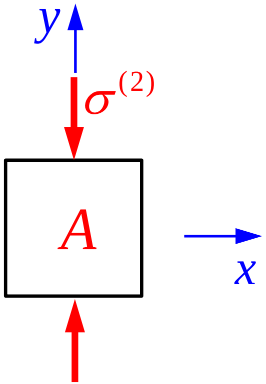
1. 考虑 F_x 的作用



$$\sigma^{(1)} = 0$$

$$\tau^{(1)} = 20.83\text{MPa}$$

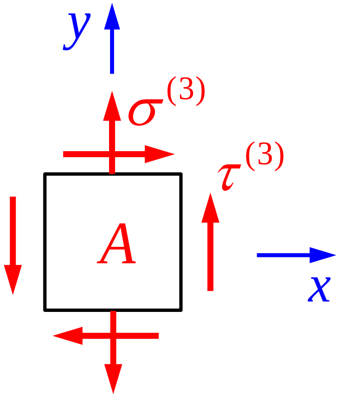
2. 考虑 F_y 的作用



$$\sigma^{(2)} = -13.89\text{MPa}$$

$$\tau^{(2)} = 0$$

3. 考虑 F_z 的作用



$$\sigma^{(3)} = 347.22\text{MPa}$$

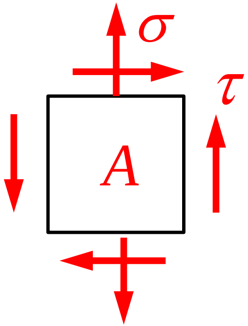
$$\tau^{(3)} = 556.45\text{MPa}$$

$$\sigma = \sigma^{(1)} + \sigma^{(2)} + \sigma^{(3)} = 0 - 13.89 + 347.22 = 333.33\text{MPa}$$

$$\tau = \tau^{(1)} + \tau^{(2)} + \tau^{(3)} = 20.83 + 0 + 556.45 = 577.28\text{MPa}$$

$$\sigma_2 = 0$$

$$\sigma_{1,3} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{333.33}{2} \pm \sqrt{\left(\frac{333.33}{2}\right)^2 + 577.28^2} = \begin{cases} 767.5\text{MPa} \\ -434.2\text{MPa} \end{cases}$$



谢 谢 大 家

作业

P305: 8.10(b)

P306: 8.14

P309: 8.19

对应第6版的题号 P298-300: 8.10(b)、 8.14、8.19

下次课讲　第九章 压杆稳定