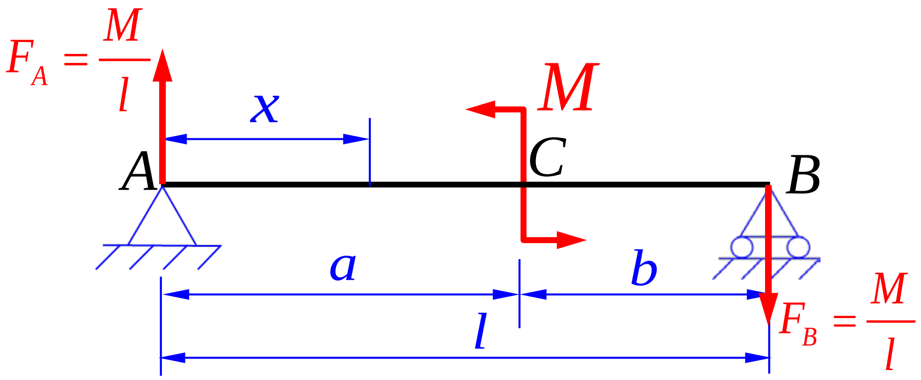
第四章　弯曲内力（二）

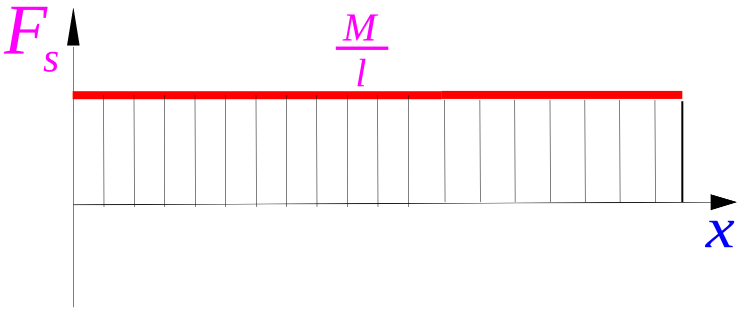
第12讲

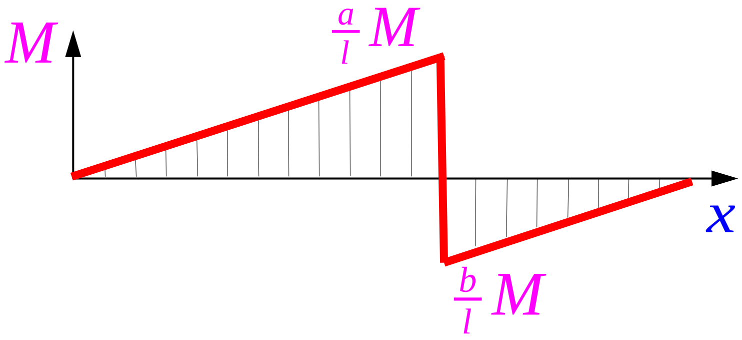
剪力图和弯矩图



$$F_s = F_s(x)$$

$$M = M(x)$$

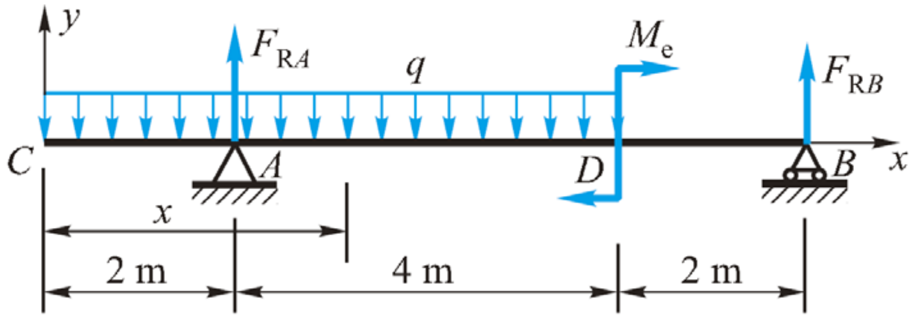




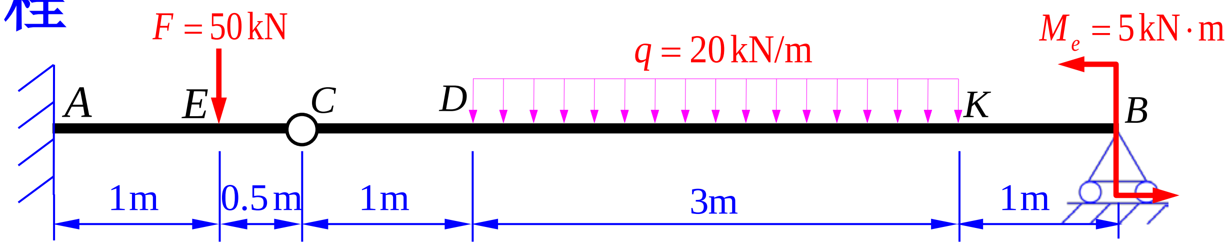
剪力图和弯矩图

应用：确定最大剪力和最大弯矩的位置和大小！

确定危险截面！



用剪力方程和弯矩方程作剪力图和弯矩图

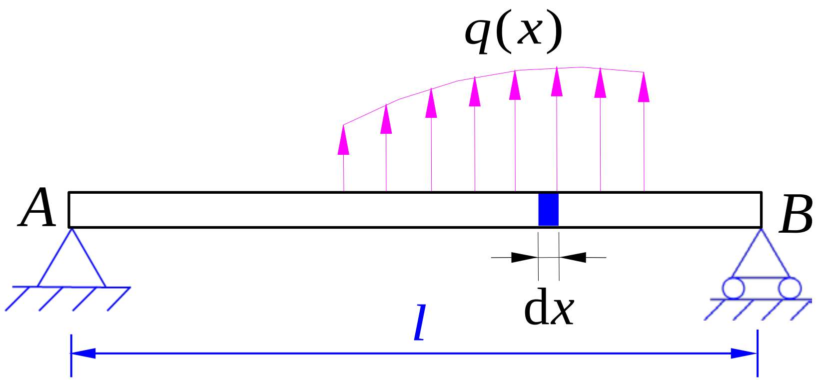


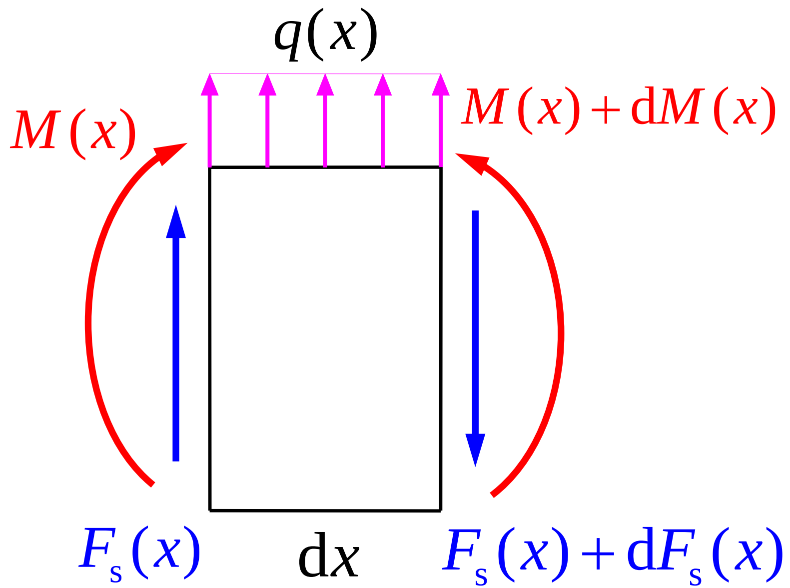
繁、烦

好的办法？

§ 4.5 载荷集度、剪力和弯矩间的关系

1. 载荷集度、剪力和弯矩间的微分关系

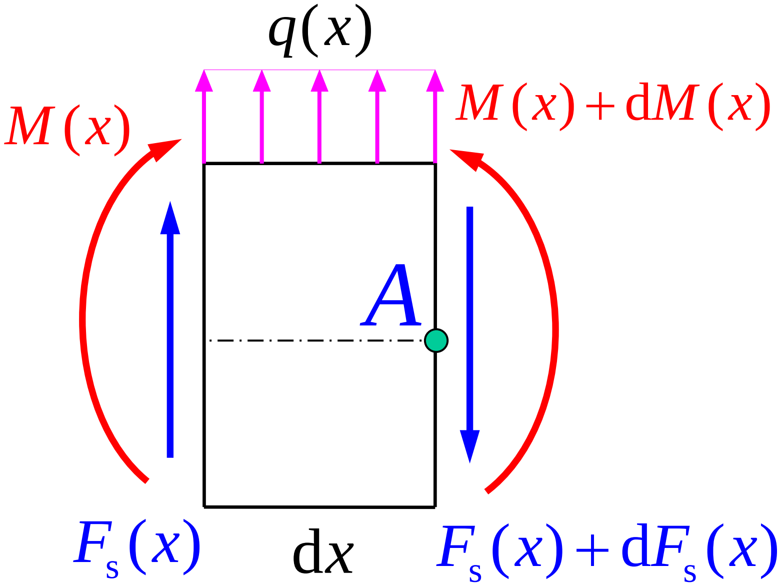




考察竖向力平衡：

$$F_s(x)+\mathrm{d}F_s(x)-F_s(x)-q(x)\mathrm{d}x=0$$

$$\Rightarrow \frac{\mathrm{d}\,F_s(x)}{\mathrm{d}\,x}=q(x)$$



考察对A点的力矩平衡：

$$M(x)+\mathrm{d}M(x)-M(x)$$
$$-F_s(x)\mathrm{d}x-\tfrac{1}{2}q(x)\mathrm{d}x^2=0$$

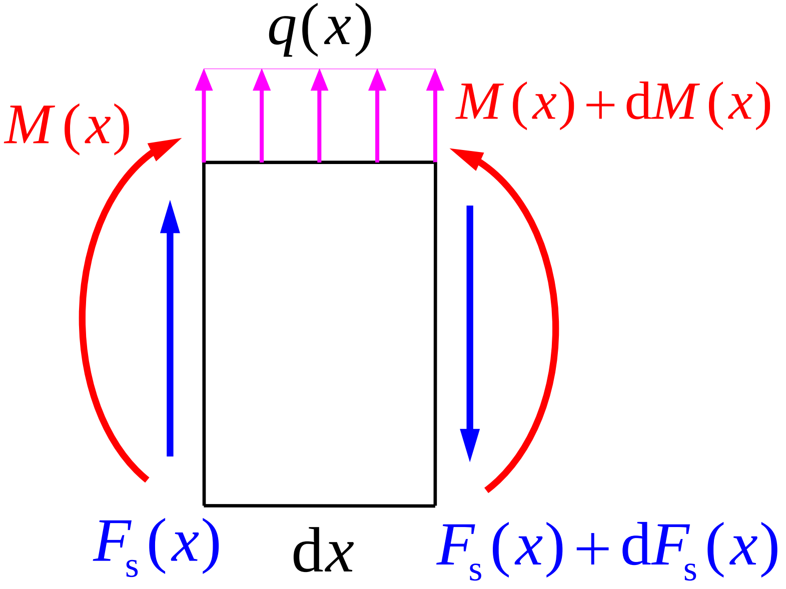
$$\Rightarrow \frac{\mathrm{d}\,M(x)}{\mathrm{d}\,x}=F_s(x)$$

弯矩、剪力和分布载荷集度间的微分关系

$$\frac{\mathrm{d}F_s(x)}{\mathrm{d}x}=q(x)$$

$$\frac{\mathrm{d}M(x)}{\mathrm{d}x}=F_s(x)$$

$$\frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2}=\frac{\mathrm{d}F_s(x)}{\mathrm{d}x}=q(x)$$

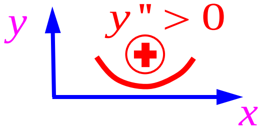
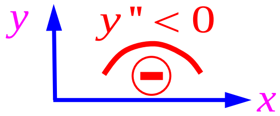


注意

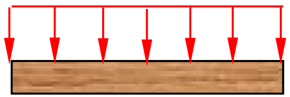
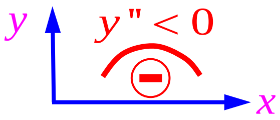
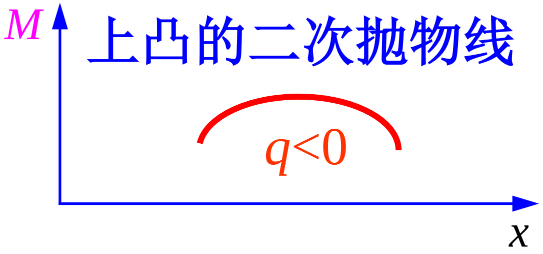
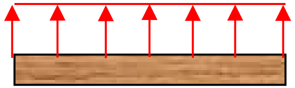
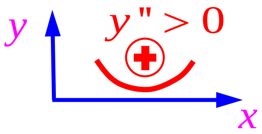
这里导出微分关系时，假设q的方向是向上的。

在以后讨论和应用时，q的方向向上为正，向下为负。

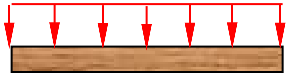
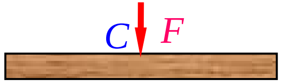
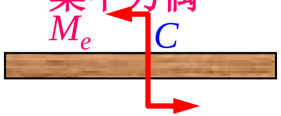
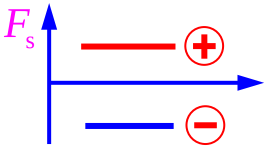
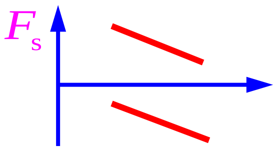
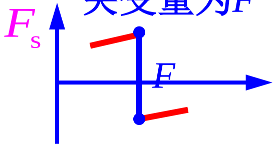
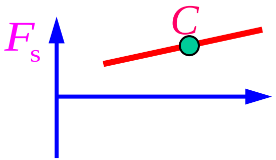
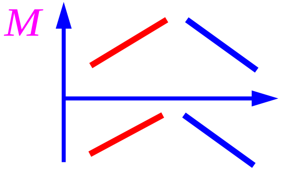
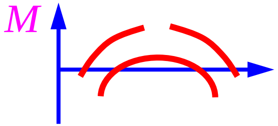
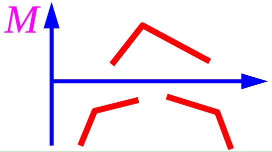
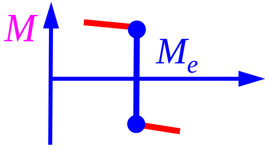
载荷集度、剪力和弯矩间微分关系的应用

坐标系	微分关系	物理意义
	$\frac{\mathrm{d}F_s(x)}{\mathrm{d}x}=q(x)$ $\frac{\mathrm{d}M(x)}{\mathrm{d}x}=F_s(x)$	$q \longleftrightarrow F_s$图上相应段的斜率 $F_s \longleftrightarrow M$图上相应段的斜率
	$\frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2}=q(x)$	二阶导数大于0，有极小值； 二阶导数小于0，有极大值；

q与弯矩图的特征

分布载荷q	q与极值间的关系	弯矩图的特征
 $q<0$(向下): 有极大值	$y''<0$: 有极大值 	上凸的二次抛物线 
 $q>0$(向上): 有极小值	$y''>0$: 有极小值 	下凸的二次抛物线 

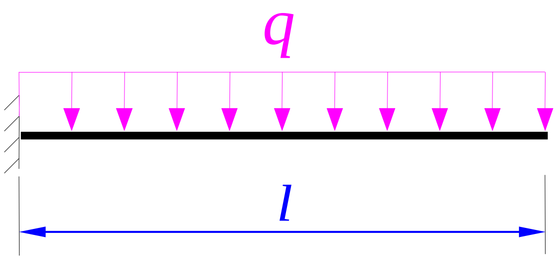
几种常见荷载下剪力图和弯矩图的特征

外力情况	无荷载 	向下的均布荷载 	集中力 	集中力偶 
F_s图特征	水平直线 	向下方倾斜的直线 	在C处有突变 突变量为F 	在C处无变化 
M图特征	斜直线 	上凸的二次抛物线 	F_s有突变，M不可导 M 图有尖角，但连续 	在C处有突变 突变量为M_e 

$$\frac{\mathrm{d}F_s(x)}{\mathrm{d}x}=q(x),\quad \frac{\mathrm{d}M(x)}{\mathrm{d}x}=F_s(x),\quad \frac{\mathrm{d}^2M(x)}{\mathrm{d}x^2}=q(x)$$

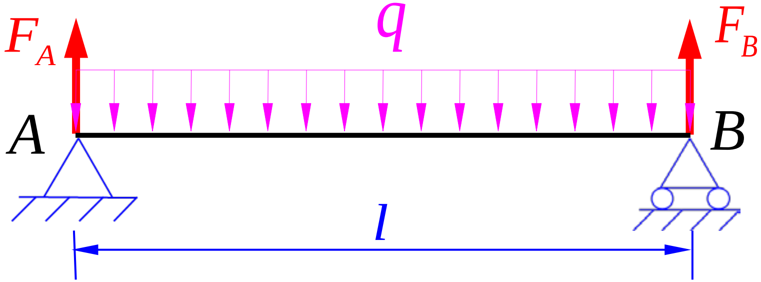
逆时针的力偶矩
向下跳跃

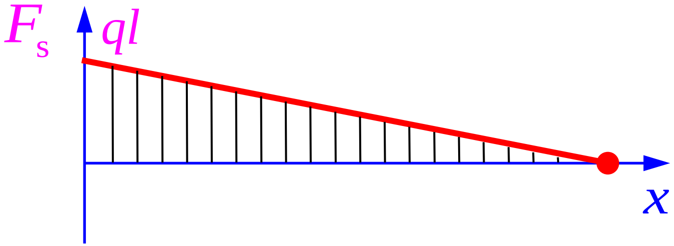
利用微分关系作剪力弯矩图（1）

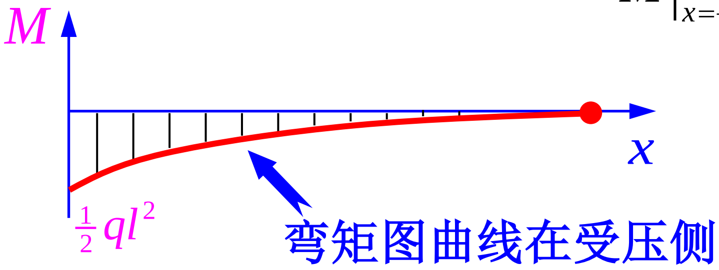


先求支座约束力

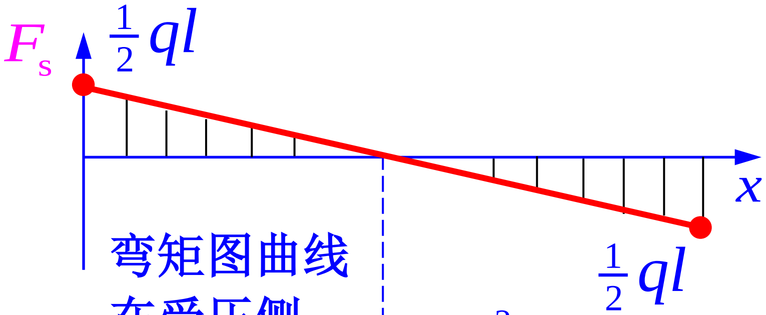
$$F_A = F_B = \frac{1}{2} ql$$

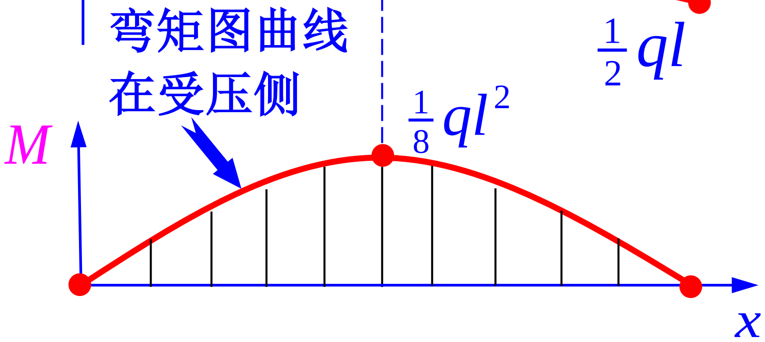




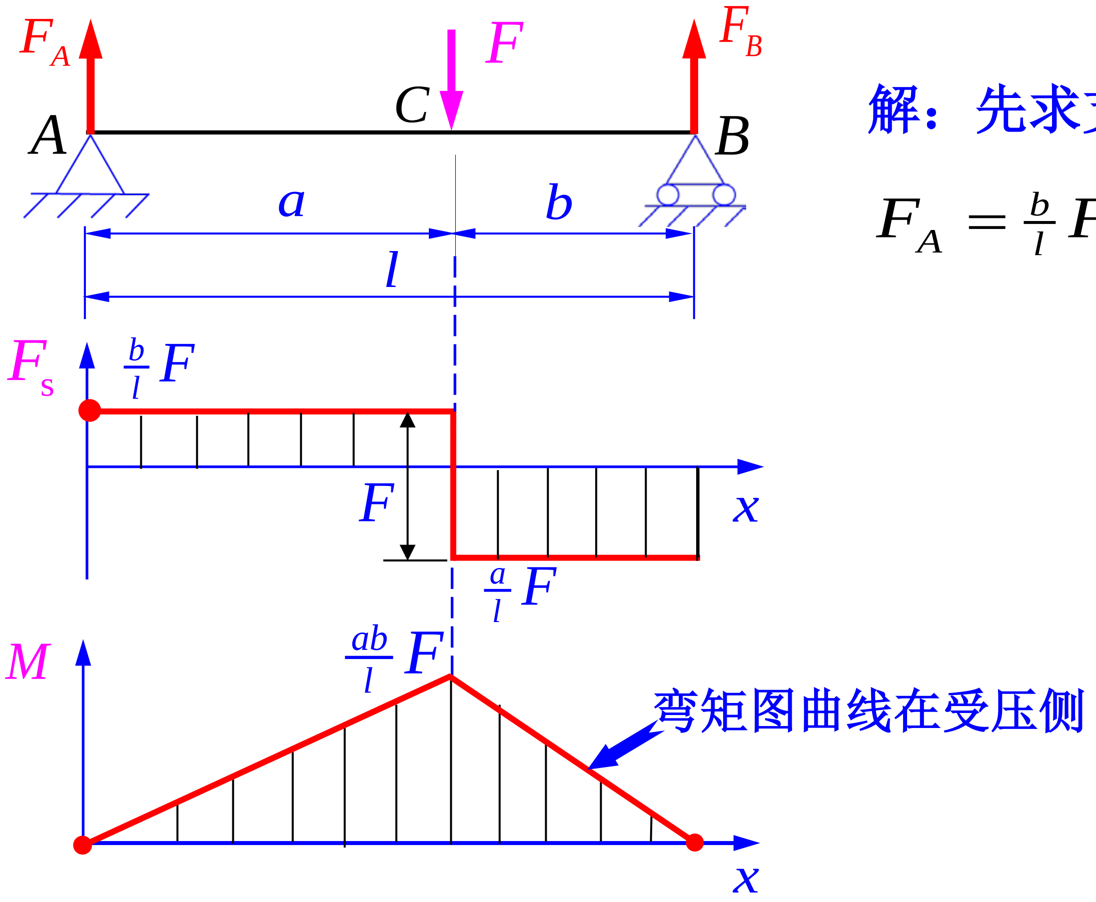


$$M\Big|_{x=\frac{l}{2}} = F_A \frac{l}{2} - \frac{1}{2} ql \times \frac{l}{4} = \frac{1}{8} ql^2$$





利用微分关系作剪力弯矩图（2）



解：先求支座约束力

$$F_A = \frac{b}{l}F, \quad F_B = \frac{a}{l}F$$

2. 载荷集度、剪力和弯矩间的积分关系

$$\frac{\mathrm{d}F_s(x)}{\mathrm{d}x}=q(x)$$

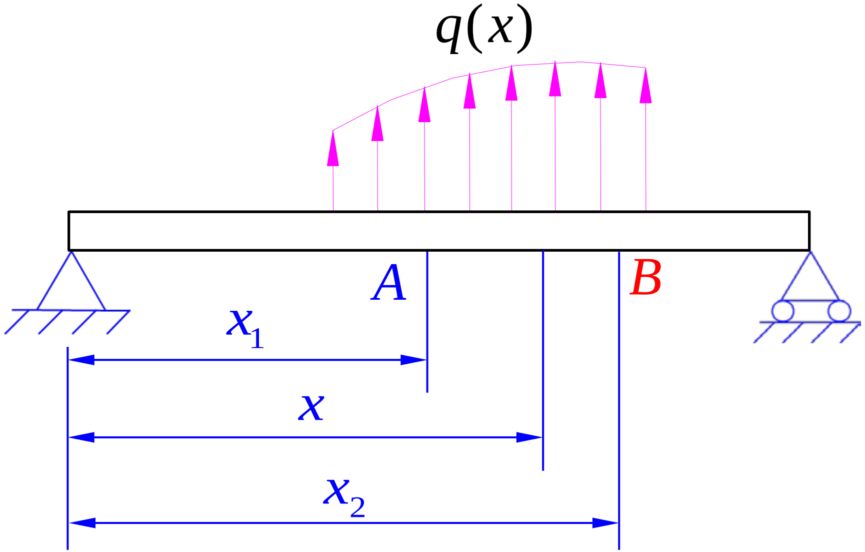
$$\mathrm{d}F_s(x)=q(x)\,\mathrm{d}x$$

$$\int_A^B \mathrm{d}F_s(x)=\int_{x_1}^{x_2} q(x)\,\mathrm{d}x$$

$$F_{sB}-F_{sA}=\int_{x_1}^{x_2} q(x)\,\mathrm{d}x$$

$$F_{sB}=F_{sA}+\int_{x_1}^{x_2} q(x)\,\mathrm{d}x$$

A、B两截面上的剪力差等于AB段梁上作用分布载荷的积分！



$$\frac{\mathrm{d}M(x)}{\mathrm{d}x}=F_{\mathrm{s}}(x)$$

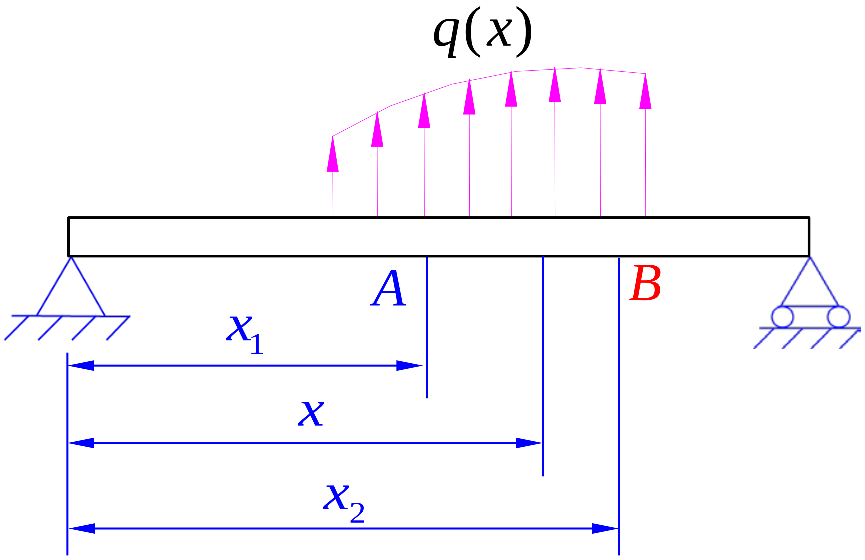
$$\mathrm{d}M(x)=F_{\mathrm{s}}(x)\,\mathrm{d}x$$

$$\int_A^B \mathrm{d}M(x)=\int_{x_1}^{x_2}F_{\mathrm{s}}(x)\,\mathrm{d}x$$

$$M_B-M_A=\int_{x_1}^{x_2}F_{\mathrm{s}}(x)\,\mathrm{d}x$$

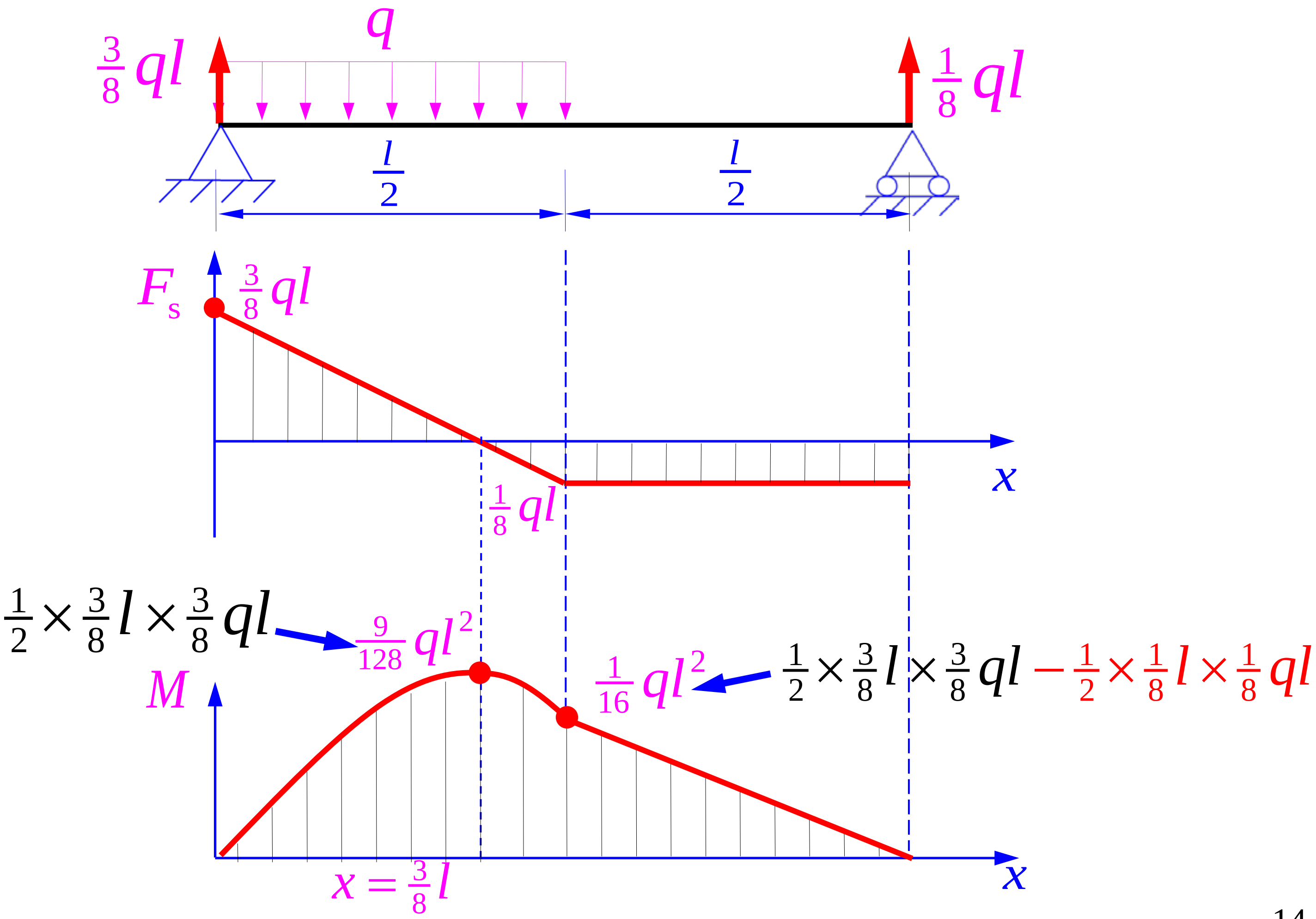
$$M_B=M_A+\int_{x_1}^{x_2}F_{\mathrm{s}}(x)\,\mathrm{d}x$$

A、B两截面上的弯矩差等于AB段梁上作用剪力的积分！



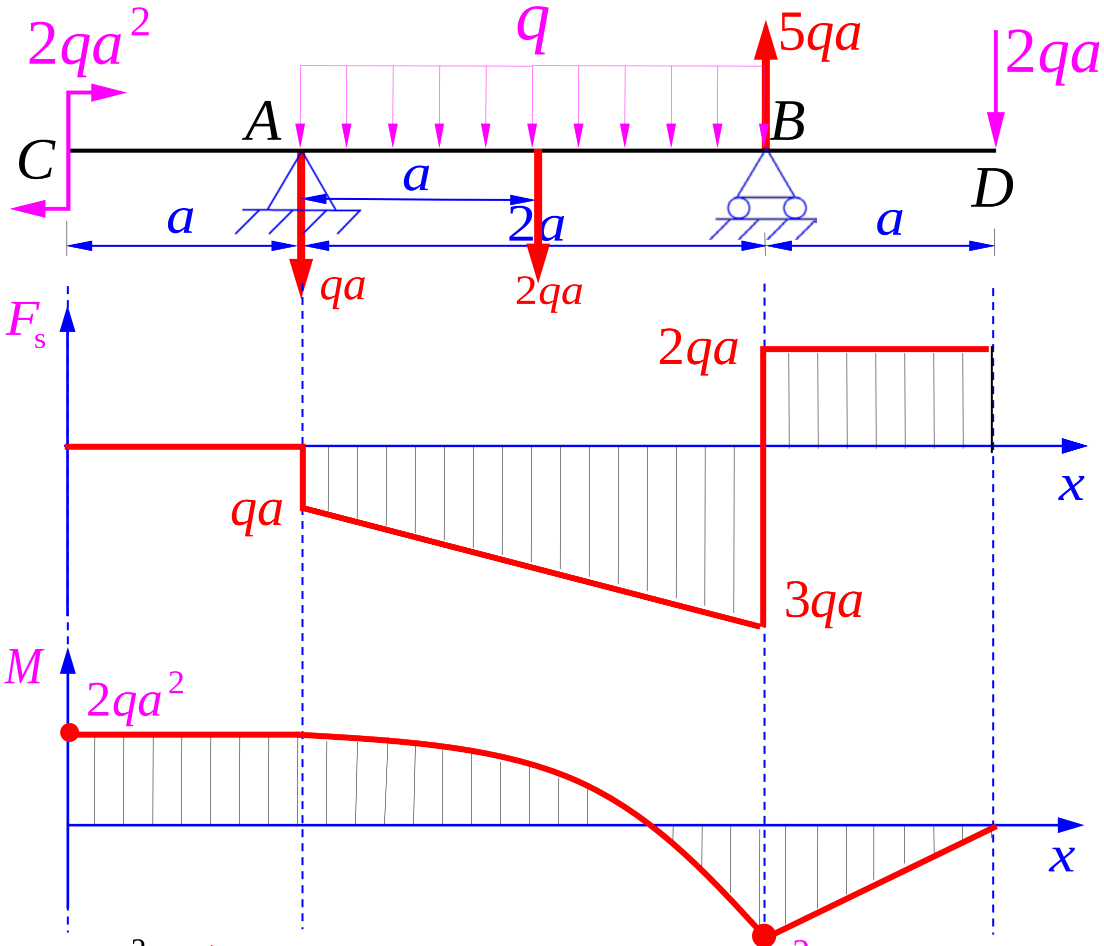
就是剪力图上相应段上与水平轴包围的面积！

作内力图(1)



q
$\frac{3}{8}ql$
$\frac{1}{8}ql$
$\frac{l}{2}$
$\frac{l}{2}$
F_s
$\frac{3}{8}ql$
x
$\frac{1}{8}ql$
$\frac{1}{2}\times\frac{3}{8}l\times\frac{3}{8}ql$
$\frac{9}{128}ql^2$
$\frac{1}{16}ql^2$
$\frac{1}{2}\times\frac{3}{8}l\times\frac{3}{8}ql-\frac{1}{2}\times\frac{1}{8}l\times\frac{1}{8}ql$
M
$x=\frac{3}{8}l$
x

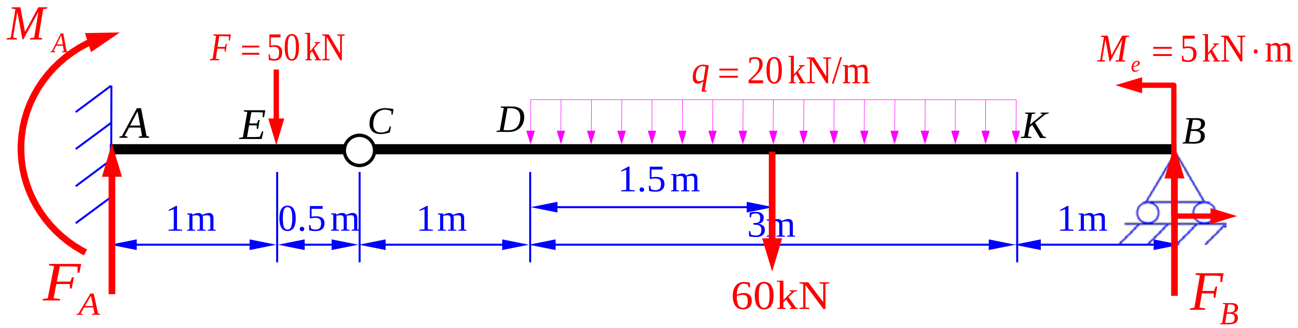
作内力图(2)



$$2qa^2 - \frac{1}{2}(qa + 3qa) \times 2a \longrightarrow 2qa^2$$

有中间铰的梁

作内力图(3)



1. 取CB梁：$\sum M_C=0$

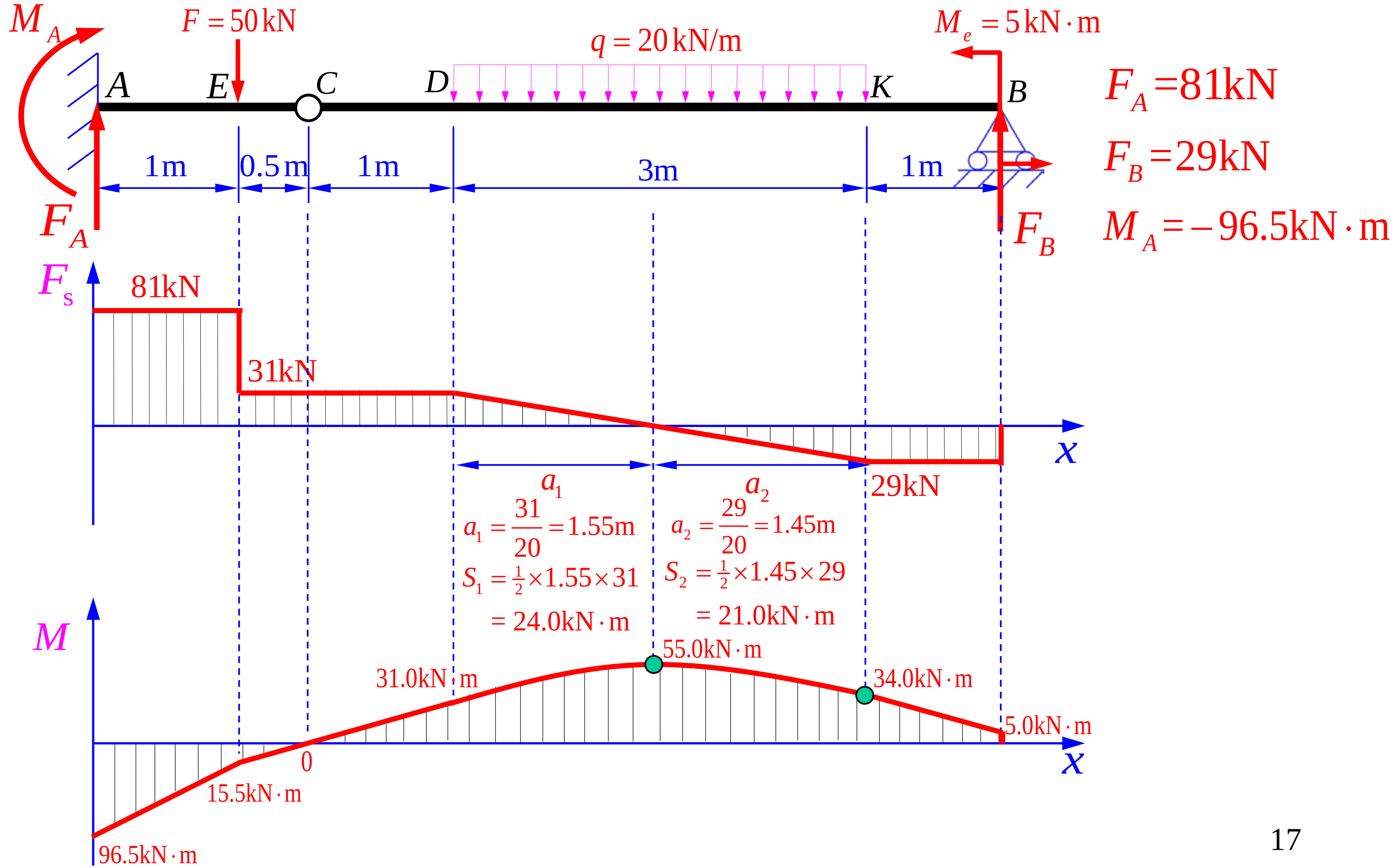
$$F_B\times5+5-60\times2.5=0\Rightarrow F_B=29\,\text{kN}$$

2. 取整体梁：$\sum F_y=0$

$$F_A+F_B-50-60=0\Rightarrow F_A=81\,\text{kN}$$

3. 取CA梁：$\sum M_C=0$

$$M_A+F_A\times1.5-50\times0.5=0\Rightarrow M_A=-96.5\,\text{kN}\cdot\text{m}$$



M_A
F = 50kN
q = 20kN/m
M_e = 5kN·m
A
E
C
D
K
B
1m
0.5m
1m
3m
1m
F_A
F_B
F_A=81kN
F_B=29kN
M_A=−96.5kN·m
F_s
81kN
31kN
29kN
x
a_1
a_2
a_1 = 31/20 = 1.55m
a_2 = 29/20 = 1.45m
S_1 = ½×1.55×31
= 24.0kN·m
S_2 = ½×1.45×29
= 21.0kN·m
M
55.0kN·m
31.0kN·m
34.0kN·m
5.0kN·m
0
15.5kN·m
96.5kN·m
x

§ 4.6 平面刚架和曲杆的内力图

作平面刚架和曲杆的内力图时，习惯上按下列约定：

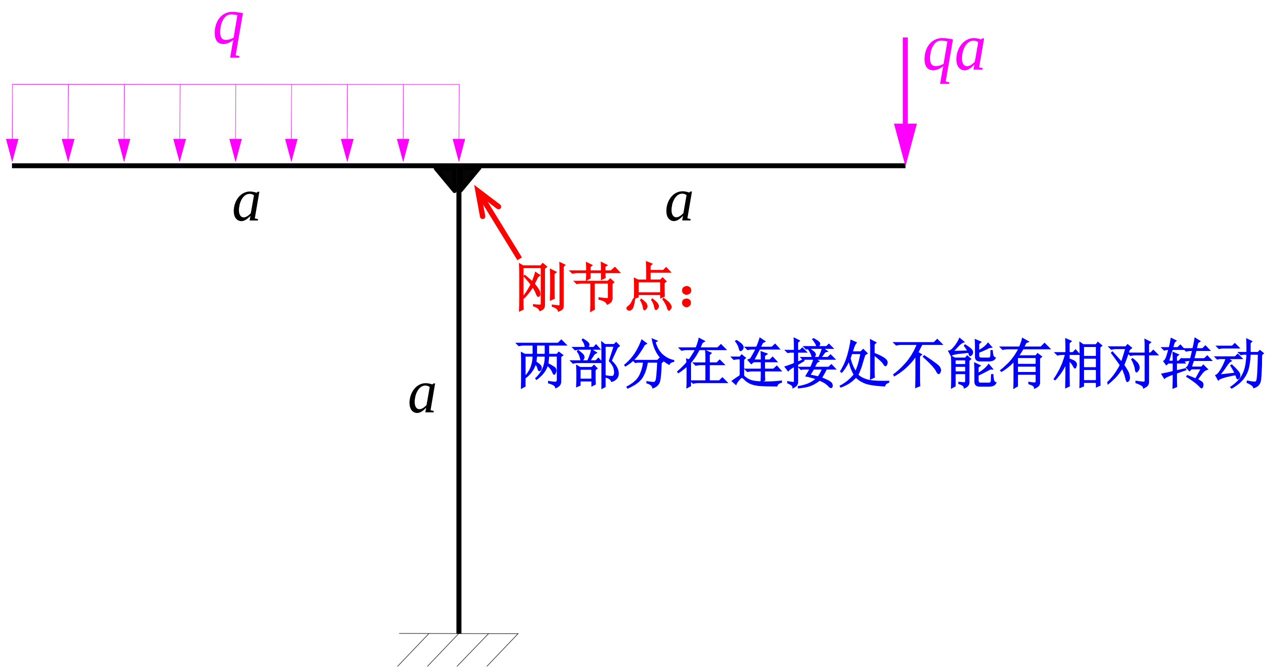
轴力图：引起拉伸变形的轴力为正

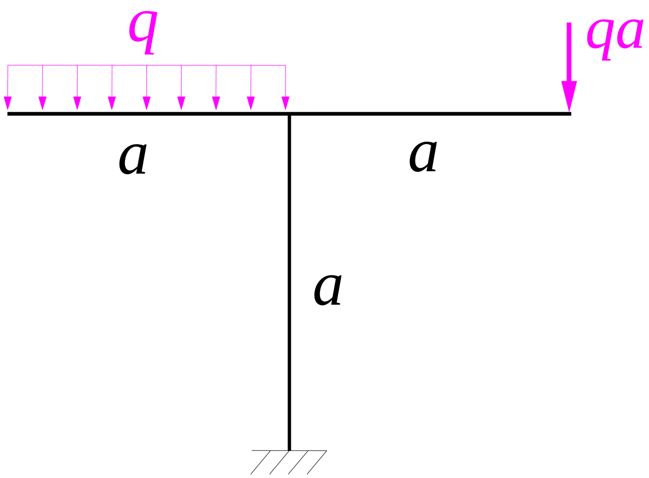
剪力图：对考虑一段杆件内任一点取矩，若力矩为顺时针方向，剪力为正

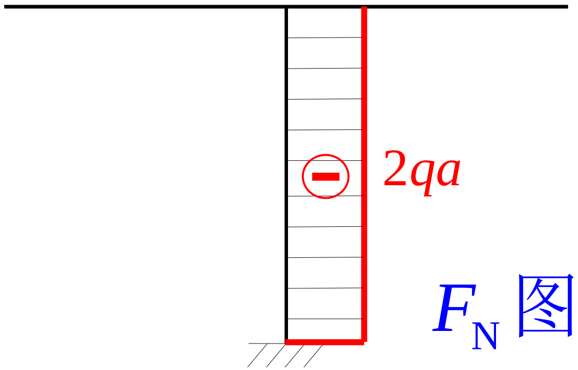
弯矩图（刚架）：约定把弯矩图画在杆件弯曲变形凹入的一侧（即受压的一侧）

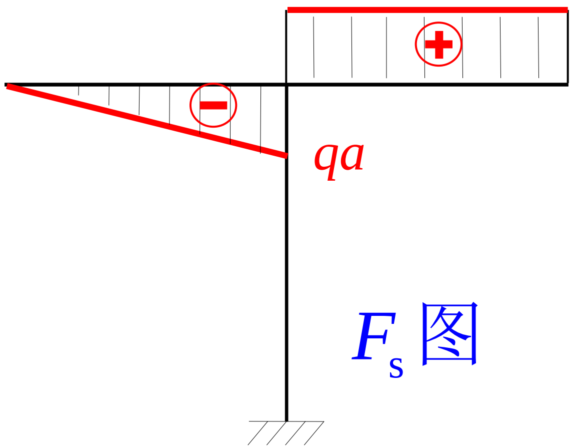
弯矩图（曲杆）：使轴线曲率增加的弯矩为正（受压的一侧），M 画在轴线的法线方向！

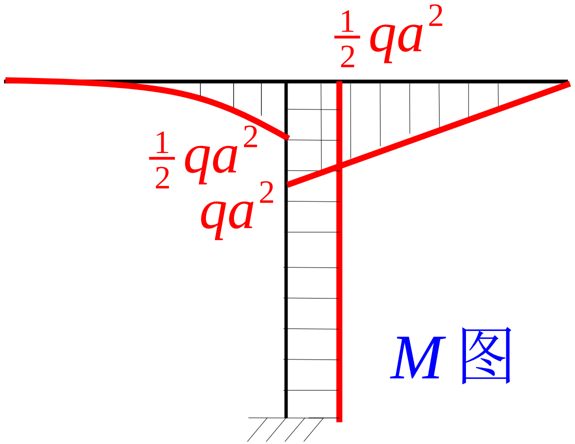
例题　作图示刚架的轴力图、剪力图和弯矩图。



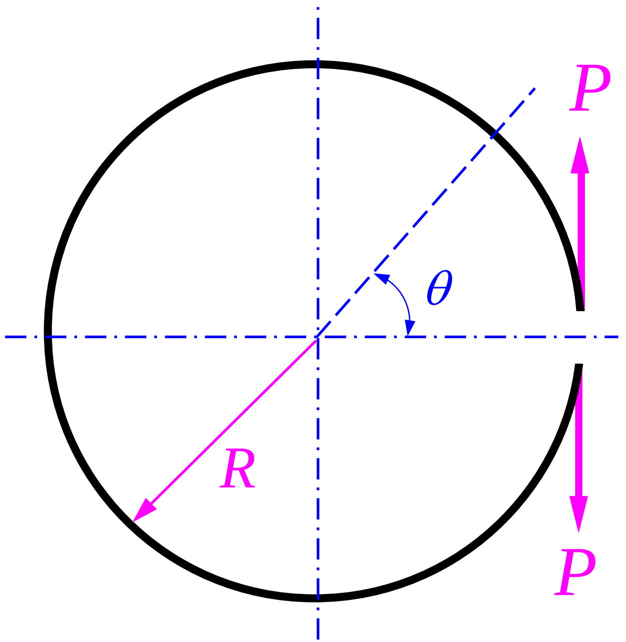

q
qa
a
a
a


2qa
F_N 图


qa
F_s 图


1/2 qa^2
1/2 qa^2
qa^2
M 图

例题 图示曲杆，其轴线为圆形，写出其轴力、剪力方程和弯矩方程，并作弯矩图。



解：轴力方程

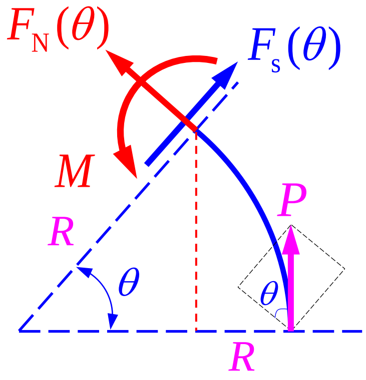
剪力方程

弯矩方程

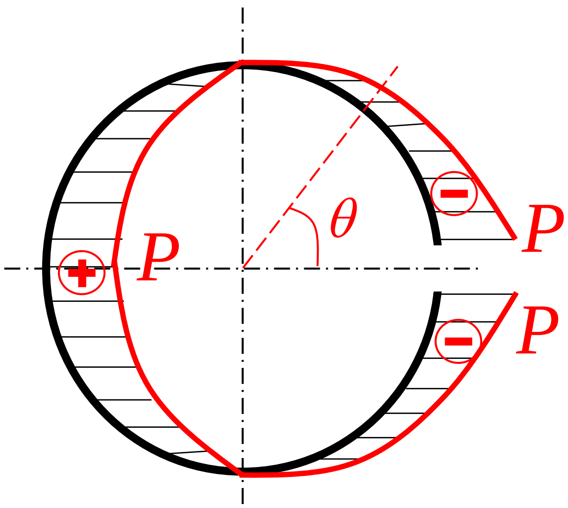
$$F_{\mathrm{N}}(\theta) = -P \cdot \cos\theta$$

$$F_{\mathrm{s}}(\theta) = -P \cdot \sin\theta$$

$$M(\theta) = -P(R - R\cos\theta)$$
$$= -PR(1 - \cos\theta)$$

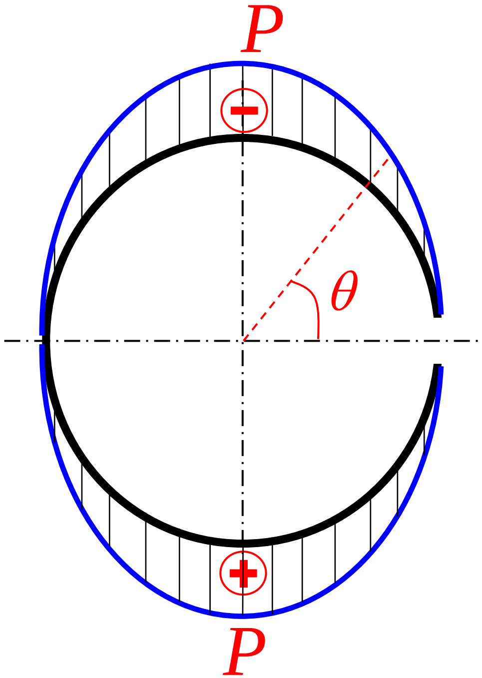


$F_N(\theta) = -P \cdot \cos\theta$



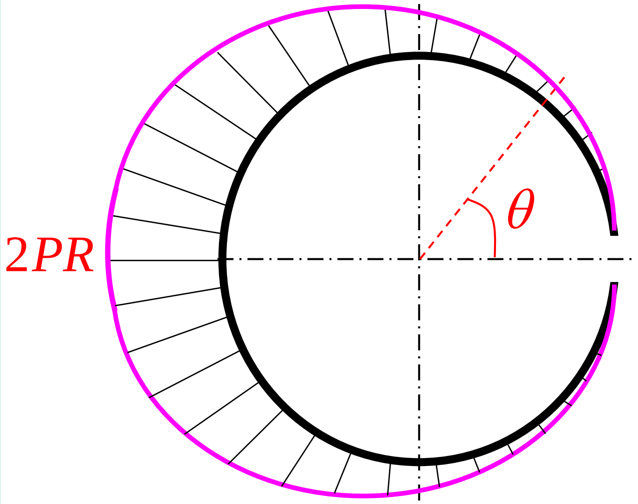
F_N 图

$F_s(\theta) = -P \cdot \sin\theta$



F_s 图

$M(\theta) = -PR(1-\cos\theta)$



M 图

谢谢大家

作业

P143：4.6(a)

P144-145：4.8

P147-148：4.18

对应第6版的题号 P135-136：4.6(a)、4.8；P141：4.18

下次课讲 第五章 弯曲应力