

Introduction to Scientific Computing

Statements

```
foo = x^2 + sin(5*y) / exp(67*z);
```

The semicolon at the end is optional.

Built-in MATLAB functions

`sin(x)`, `cos(x)`, `exp(x)`, `log(x)`, `atan(x)`, `cosh(x)`, `sinh(x)`, `mean(x)`, `median(x)`, etc.

Array

```
x = [1,3,9,11,-10.2];
foo = x(3); % extracts third element in the array
x(2) = 27; % reassign element in the array

x = 1:15 % creates an array with the number 1 through 15
x = 4:2:28; % creates an array starting at 4 and going up by 2 until it gets to 28
x = -1.3:0.1:1.3; %creates an array starting at -1.3 and going up by 0.1 until it
gets to 1.3

x = 1:2:6; % x = [1,3,5]
x = [1:2:6]; % x = [1,3,5]
```

Operations on arrays

```
x = 0:0.1:10;
y = exp(5*x).*sin(x); % evaluate an expression on every element in the array x
producing a new array called y
plot(x,y);

x = [1,2,3,4,5];
y = [7,8,9,10,11];
z = x + y; % element-wise addition adds corresponding elements
z = x.*y; % element-wise multiplication
z = x./y; % element-wise division
```

Overflow

```
X = uint8(78); Y = uint8(190);
Z = X + Y;
```

```
% overflow - value will be clipped to 255
```

Scientific notation

Note that numbers in scientific notation have the following components

- A sign - positive or negative
- A mantissa(尾数) - ex. 2.13
- An exponent - ex. 17 e.g. +2.13e17

Arrays and numerical types

When an array variable is created all of the numeric values in that array share the same numeric type. For example we can talk about an array of `uint16s` or an array of doubles.

```
x = single(0:0.1:100);
```

 make an array of single precision (单精度) numbers

A few useful MATLAB commands

- `whos` Lists all of the variables currently in your workspace and shows you their types
- `clear` Clears all of the variables in your workspace – you can also use this to clear specific variables
- `clc` Just clears the command window – has no effect on the workspace

Function

```
function f = fact(n)
    f = prod(1:n);
end

function [output1,output2,output3] = myFunction(input1,input2,input3)
```

Naming a Function

Must start with a letter from the alphabet

Special Functions

```
function out1 = testFunction(in1)
% A test function
% this would report, in the help function, all of functionality of the function
% in1 = function input (units: not specified)
% out2 = function output (units : not specified)
```

(in the command window)

```

lookfor 'a test function'

% testFunction          - A test function

help testFunction

% A test
% this would report, in the help function, all of functionality of the function
% in1 = function input (units: not specified)
% out2 = function output (units : not specified)

```

Search paths

- The working folder at startup can be changed using the `userpath` function

```
userpath('C:\ATRI')
```

- Paths can also be “permanently” added to the search path using the `addpath()` function
- Paths can be removed using the `rmpath()` function

Commenting

- Highlight a selection or place your cursor on a line and press “ctrl+r” to comment a section or line of code, respectively
- “ctrl+t” uncomments the code

Anonymous Function

```

function_name = @ (arguments) expression

FtoC = @ (F) 5*(F-32)./9

% examples
FA = @ (x) exp(x^2)/sqrt(x^2+5)
FA(2)
FA = @ (x) exp(x.^2)./sqrt(x.^2+5)
FA([1 0.5 2])

```

Function Handles

```

f = @sin;
m = fminbnd(f,0,2*pi)

q = integral(@cubicPoly,0,1);

```

Vectors(Arrays)

```
% Linspace and logspace
x = linspace(1,5,5)
x = logspace(1,5,5)

x = [5, 9, 4, 1, 7, 3, 4, 8]
% Accessing vector elements
y = x(1)
y = x(1:3)
y = x(end)
y = x(end-2)
y = x([1,3,6]) % y = [5,4,3]

% The indices start at 1, not 0.

% Overwriting
x(end) = 1;

%Add ending indices
x(end+1) = 8
x(end+1:end+2) = [6,9]

%Removing elements
x(1) = []; % Remove the first element
x(end-3,end) = [];
```

Iteration

```
for jj = 1:20
    disp(jj)
end

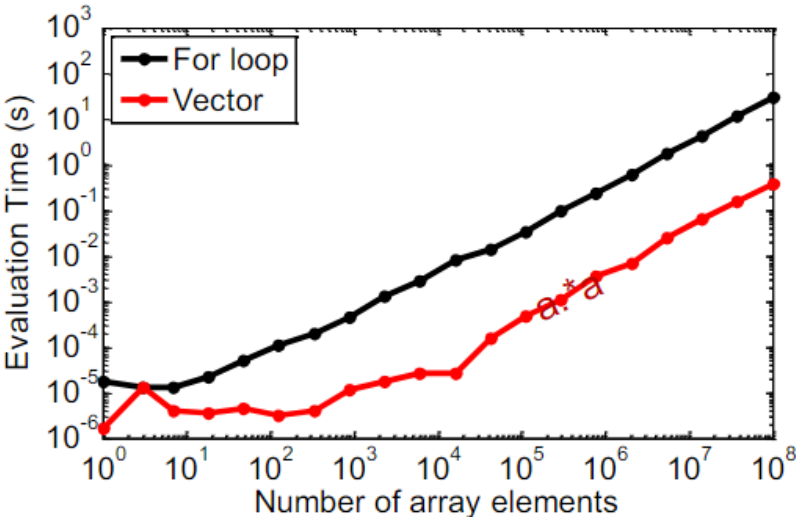
a = 1;
while a < 10
    a = a + 1;
end

% Iterate through the array
x = [1,2,3]
for element = x
    disp(element);
end
```

Array Operation vs Iteration

Array operations increase computation speed

Time to Compute $a*a$ using a for Loop vs. Vectorization



The `if` statement

```
a = 1;
b = 2;
if a == 1;
    disp('a is equal to 1! Yeah');
end

elseif a == 1 && b == 2;
    disp('...')
end
```

Logical Operators

Logical Operator(Scalars)	Logical Operator(Vectors)	Function Call
&&	&	and(a,b)
		or(a,b)
~	~	not(a)
none	none	xor(a,b)

Attention: `~=` in matlab equals to `!=` in other languages

Logical Vectors

```
a = [1 6 5] < 2
% a = [1,0,0]
```

```
x = [5 9 2 4 3];
v = logical([1 0 1 0 1]);
xp = x(v);
% xp = [5,2,3]
```

Creating matrices

```
A = [1 2 3;4 5 6];

A = ones(2,5);
A = zeros(2,3);
A = eye(3); % Create a identity matrix

B = A' % Rows become columns and columns become rows.

% Expressions used to initialize arrays can include
algebraic operations and (all or portions of) previously
defined arrays
a = [0 13*2];
b = [a(2) 13 a];

A = [1 5 9;2 4 8;3 6 7];
c = A[3,1]; % c == 3
c = A[4]; % Matrix indices may also be accessed using a
single subscript
```

$$A = \begin{bmatrix} 1^1 & 5^4 & 9^7 \\ 2^2 & 4^5 & 8^8 \\ 3^3 & 6^6 & 7^9 \end{bmatrix}$$

Subarrays

```
% Vectors
va =[1:5]
va(:) % all elements
va(m:n) % elements m through n

% Matrix
A = [1 2 3; 4 5 6; 7 8 9];
A(:,n) % all elements in column n
A(n,:) % all elements in row n
A(:,m:n) % elements in columns m through n
A(m:n,:) % elements in rows m through n
A(m:n,p:q)

b = [1 2 3 4; 5 6 7 8; 9 10 11 12];
b(2:2:end,2:end); % [6 7 8]
```

Manipulating Matrices

Built-in function	description
-------------------	-------------

Built-in function	description
<code>length(v)</code>	returns length of largest array dimension
<code>size(A)</code>	returns[m,n] where m and n are the size of A
<code>numel(A)</code>	returns the total number of elements in the array
<code>reshape(A,m,n)</code>	rearrange A to have m rows and n columns
<code>diag(v)</code>	creates square matrix with elements on the diagonal
<code>diag(A)</code>	creates vector from the diagonal elements

Resizing a Matrix

```
A(:,2) = []; % deleting rows or columns from an array
A = [B C; D E;] % Arrays can be concatenated to form larger arrays.
```

Strings

- An array of characters typed between single quotes ('),e.g. `'Hello World'`
- char built-in function `My_string = char('string 1','string 2','string 3')`
- Work with strings `lower;isspace;isletter`

Matrix Addition and Subtraction

1. Scalars may be added to any array, and the scalar value is added to all elements of the array
2. Arrays may be added and subtracted so long as they have the same dimensions

Element-by-Element Operations

- Addition(+) and subtraction (-)
- Multiplication (.*) and division (./)
- Exponentiation (.^)
- Functions applied over arrays (`sin, log,...`)
- evaluating a function e.g. `$y=x^2-4x$`

Matrix Multiplication

- `a*B` Matrices may be multiplied by a scalar:Element-by-Element Multiplication
- `A.*B` Element-by-Element Multiplication
- `A*B` Matrix multiplication
- `cross(X,Y)` Cross product, special operation on two vectors.

Matrix division

Inverse of a matrix

- `inv` function or raise to -1 power

- matrix must be square and invertible

Left Division

Solve $AX=B$ where X and B are column vectors.

$$X=A^{-1}B$$

$$X=A \setminus B$$

Right Division

Solve $XC=D$ where X and D are column vectors.

$$X=DC^{-1}$$

$$X=D/C$$

Rotating and flipping arrays

- `rot90`: Rotate counterclockwise by 90 degrees
- `fliplr` or `flipdim(A,1)`: Reverse the matrix by treating a row as a whole.
- `flipud` or `flipdim(A,2)`: Reverse the matrix by treating a column as a whole. $A = \begin{bmatrix} 1&2&3 \\ 4&5&6 \end{bmatrix} \xrightarrow{\text{rot90}(A)} \begin{bmatrix} 3&6 \\ 2&5 \\ 1&4 \end{bmatrix} \xrightarrow{\text{fliplr}(A) \text{ or } \text{flipdim}(A,1)} \begin{bmatrix} 3&2&1 \\ 6&5&4 \end{bmatrix} \xrightarrow{\text{flipud}(A) \text{ or } \text{flipdim}(A,2)} \begin{bmatrix} 4&5&6 \\ 1&2&3 \end{bmatrix}$ Actually, we use `flip` instead of `flipdim` now. And the grammar is absolutely the same.
The second parameter of `flip`(or `flipdim`) is the dimension that is manipulated. It can be set to 3 or more when the dimension of array is above 2.

Built-in Matrices

- `zeros(n)` returns an n-by-n matrix with all the elements equal to zero
- `ones(n)` returns an n-by-n matrix with all the elements equal to one
- `magic(n)` returns an n-by-n matrix constructed from the integers 1 through n*n with equal row and column sums. The order n must be a scalar greater than or equal to 3 in order to create a valid magic square.
- `eye(n)` return an n-by-n identity matrix

Sparse arrays

Sparse matrices are defined by a list of elements containing non-zero elements while all other elements are assumed to be zero

```
% conventional way to create a matrix
A = zeros(1000,2000);
```



```
A(3,4) = 15;
A(100,1500) = 5;
A(1000,2000) = 9;

% create using a sparse matrix
A = sparse([3,100,1000],...
           [4,1500,2000],...
           [15,5,9],1000,2000);

%The original (full) matrix A could also be converted to a sparse matrix via the
following
A = sparse(A);
```

Matrix type	Memory(MB)	Time to compute A^2 (ms)
Full	15.2	4
Sparse	0.015	0.04

Summing array elements

Consider the following 3D array $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$

$\sum(A) = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$

$\sum(\sum(A)) = [6] [12] [18]$

$\sum(\sum(\sum(A))) = [36]$

$\sum(A,3) = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}$

manipulate the matrix in the same way:

- 1. treat a row as a whole
- 2. treat a column as a whole
- 3. treat the 3rd dimension a whole

Cumulative sums

- `cumsum()` sums cumulatively along a vector or some dimension of an array
- unless a dimension is specified, it works on the first non-singleton dimension

```
cumsum([1 2 3 4 5]);
% [1 3 6 10 15]

cumsum([1 2 3; 4 5 6]);
% ([1 2 3; 5 7 9])
```

Product

```
x(:, :, 1) = [1 2 3; 4 5 6];
x(:, :, 2) = [6 5 4; 3 2 1];
y = prod(x)
```

```
% y(:, :, 1) == [4 10 18] and  
% y(:, :, 2) == [18 10 4]
```

Cumulative product

`cumprod()` calculates the cumulative product of array elements along the first non-singleton dimension

```
x(:, :, 1) = [1; 2; 3];  
x(:, :, 2) = [2; 2; 2];  
y = cumprod(x)  
  
% y(:, :, 1) = [1; 2; 6] and  
% y(:, :, 2) = [2; 4; 8]
```

Input Commands

`input` command

```
in1 = input('Enter data:');  
% Stores input value as a numerical value or array of values of type double  
  
in2 = input('Enter data:', 's');  
% Stores input value as a string (an array of characters)
```

Output Commands

`disp` command

```
disp(variable);  
disp('text to print')
```

`fprintf` command

```
fprintf(format, data);
```

Formatted output placed on screen or saved to a file

Manipulate the files

`save`

% Save all variables in the workspace to a file

```
save('file_name');
save file_name;
% Writes file_name.mat file in binary format

% Save a few variables
save file_name var1 var2;

% Save in readable format
save -ascii file_name;
% Easy way to transfer data
```

load

```
% Load the variables in a file
load('file_name');
load file_name;
% Reads file_name.mat in binary format and restores variables

load file_name var1 var2;
% Load a few variables
load file_name var1 var2;

% Load from a text(.txt) file
load file_name.txt;
var = load('file_name.txt');

% Options
% mat, -ascii
```

Cell Arrays

```
c = {42,rand(5),'abcd'}

% c =
%      1x3 cell array
%      { [42] }    { 5x5 double }    { ['abcd'] }
```

A pleasing plot

several objective measures of a good plot

- Labels with units
- Minimum amount of information to convey the objective (don't double label objects unless necessary, extraneous callouts should be avoided)

- Avoid overuse of eye candy
- Scatter plots for measured data and line plots for fits or closed form equations
- Sufficient resolution on domain to represent solution
- Line widths, data point sizes, and text large enough to read

2D plot commands

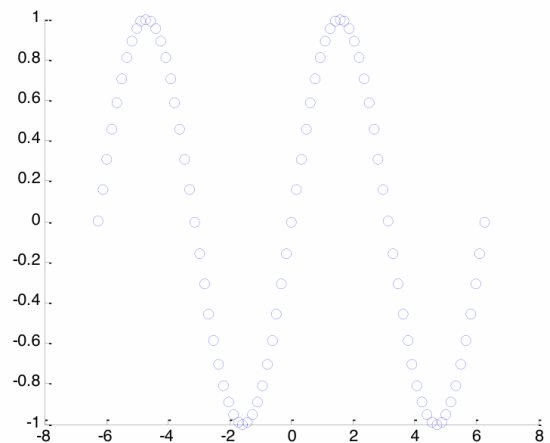
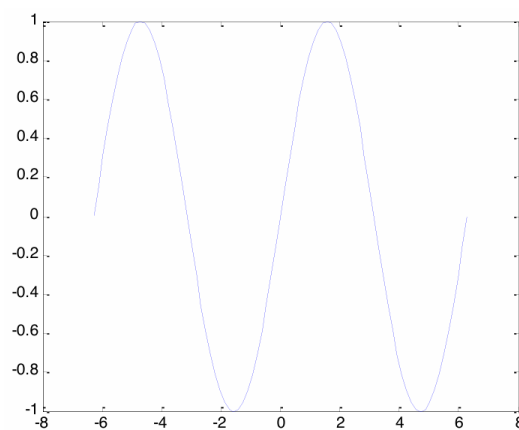
plot and scatter

```
x = -2*pi:pi/20:2*pi;
y = sin(x);
```

commands plot()

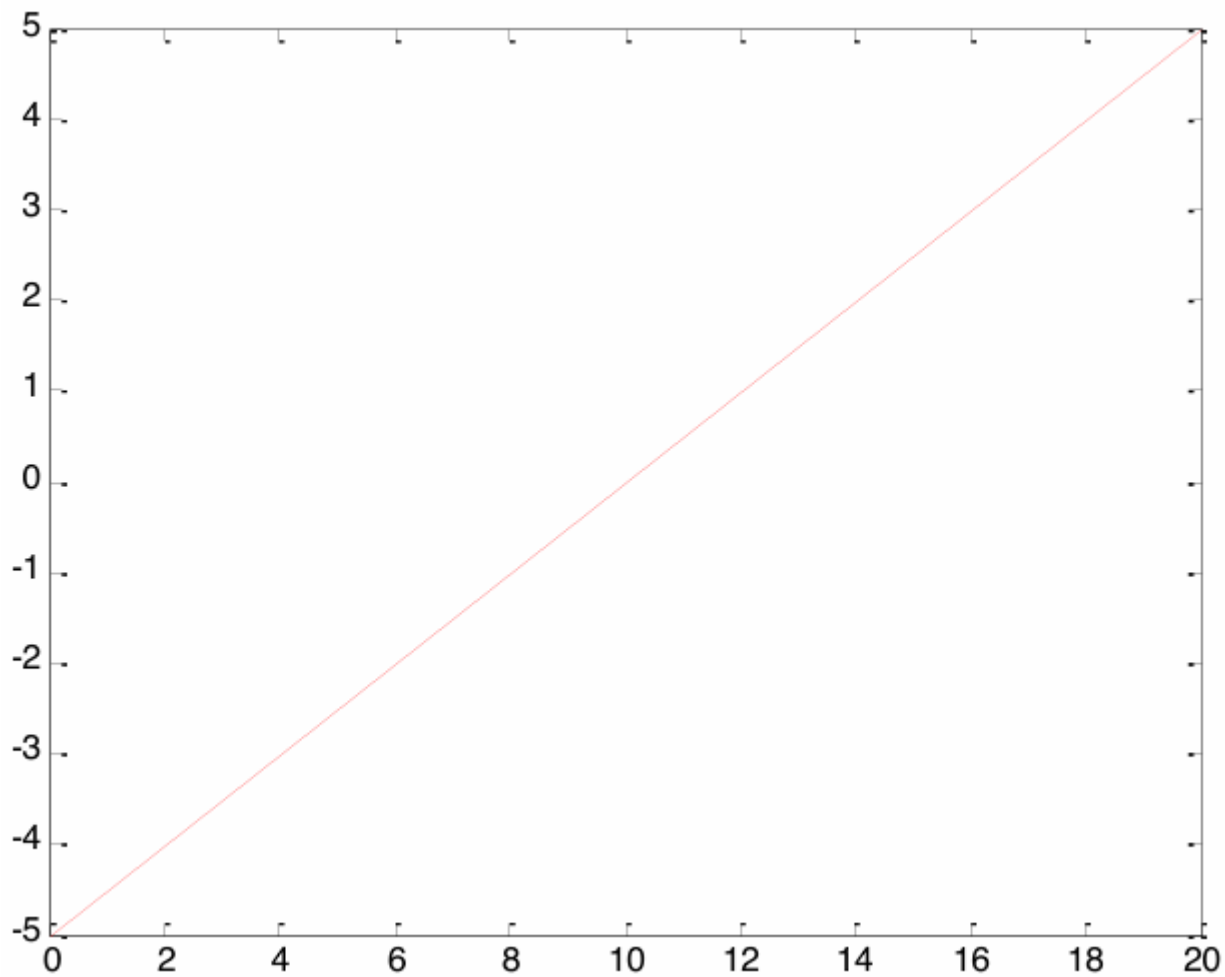
scatter()

picture



- `plot()` and `scatter` require vectors of equal length
- The default plot attributes are blue lines or circular blue data points, respectively
- These attributes can be modified by adjusting the line specifications of the plot `linespec`
- Straight lines may also be plotted by specifying the beginning and end points of the line

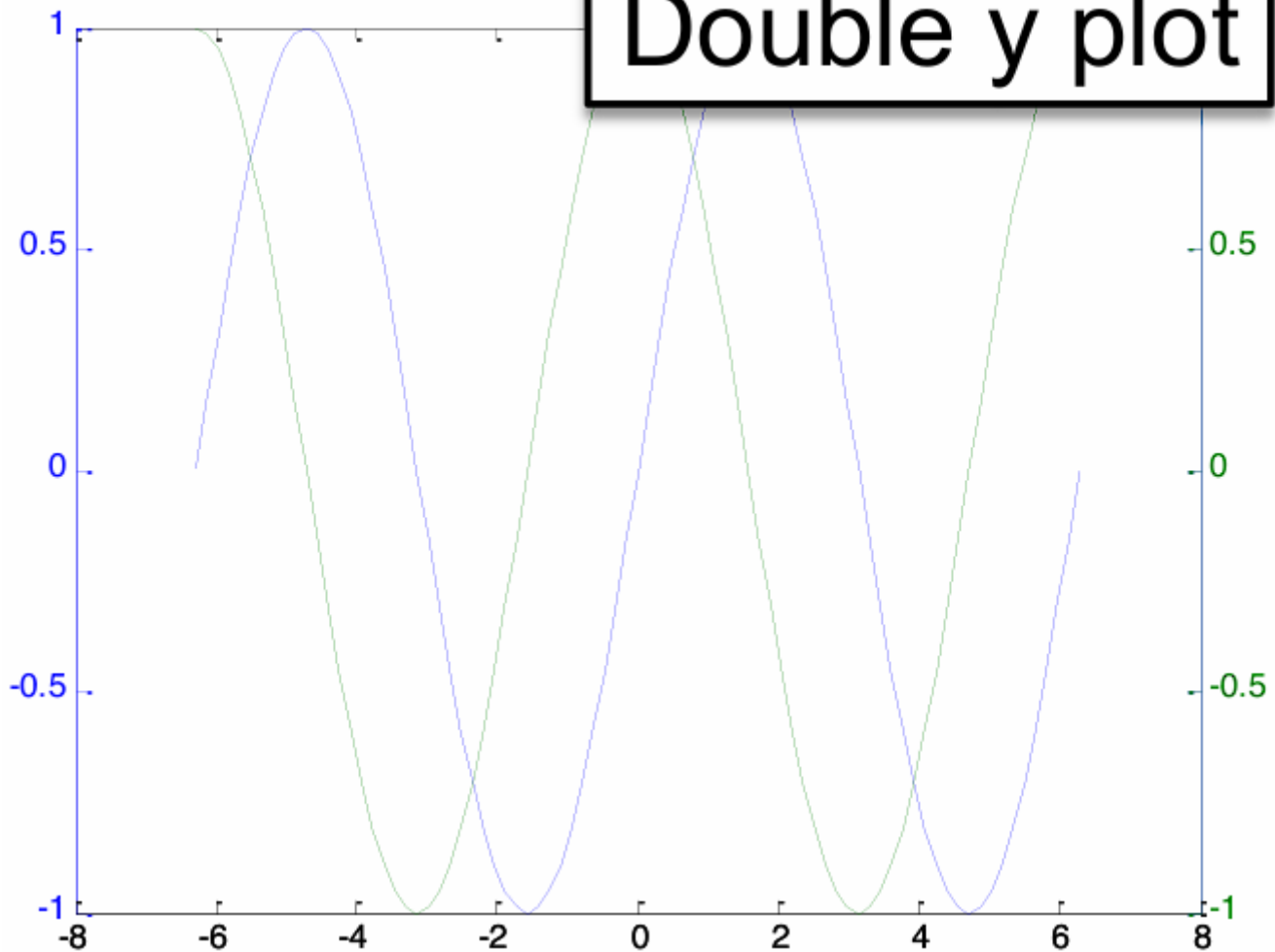
```
plot([0,20],[ -5,5], 'r-');
```



Double y plot `plotyy()`

```
x = -2*pi:pi/20:2*pi;  
y1 = sin(x);  
y2 = cos(x);  
plotyy(x,y1,x,y2);
```

Double y plot



Logarithmic plot

```
x = logspace(0,5,25);
y = x.^5;
loglog(x,y,'LineWidth',4)
```

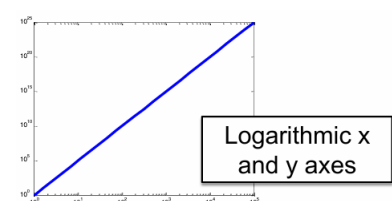
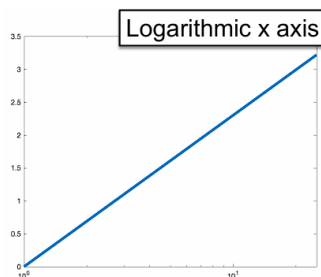
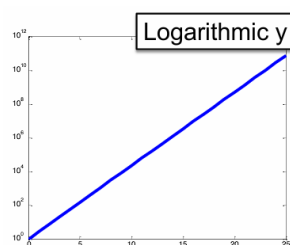
commands

`semilogy()`

`semilogx()`

`loglog()`

picture



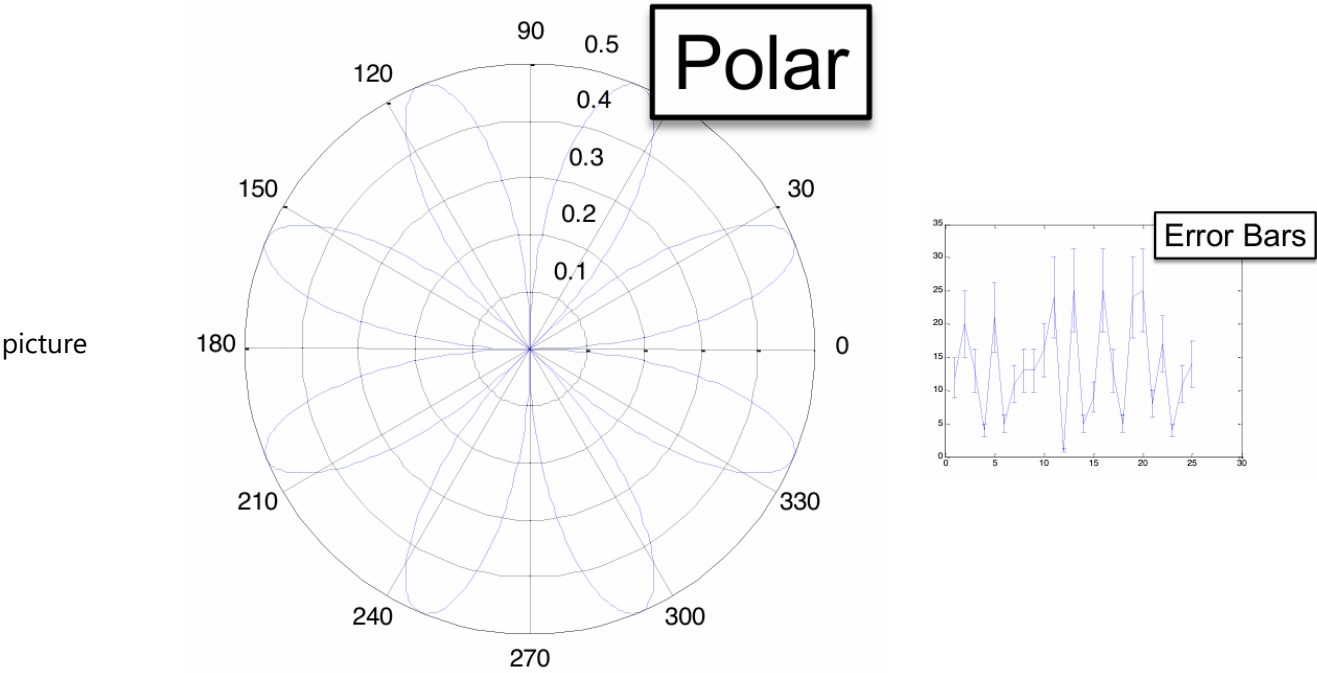
`polar()` and Error Bars

```
theta = 0:0.01:2*pi;
rho = sin(2*theta).*cos(2*theta);
```

```
polar(theta,rho)

x = 1:25; y = randi(25,1,length(x));
y_err = 0.25*y;
errorbar(x,y,y_err)
```

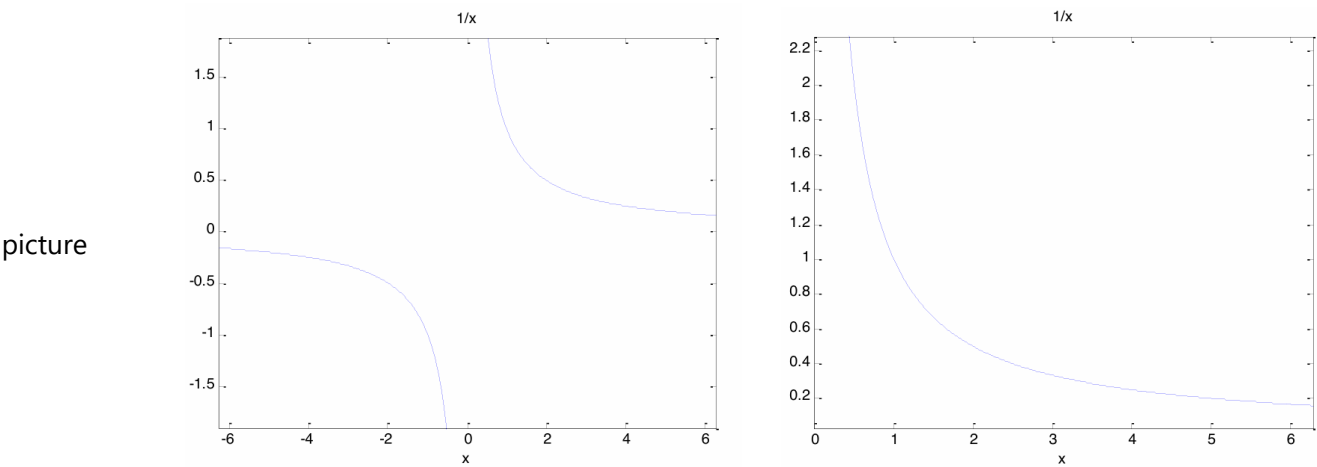
commands `polar()` `errorbar()`



ezplot

ezplot produces simple plots without the need for discrete input of x/y vectors

commands `ezplot('1/x')` `ezplot('1/x',[0 2*pi])`



discription The default domain is -2π to 2π

The domain may also be specified by a two element vector

fplot

`fplot` intelligently selects x values based on changes to a function and may be used for rapidly changing functions

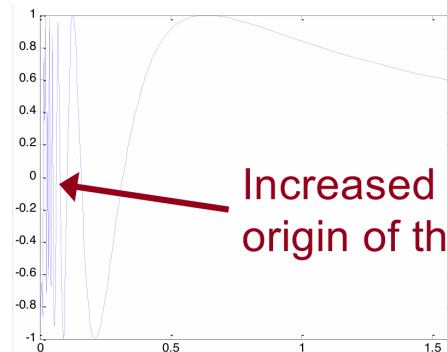
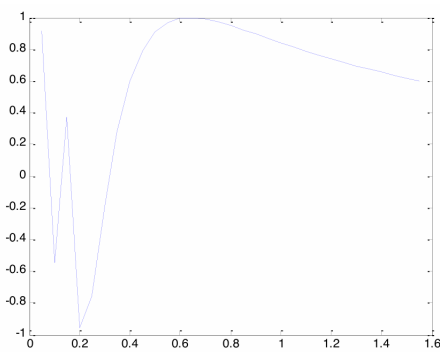
```
x = 0:.05:0.5*pi;
y = sin(1./x);
plot(x,y)

fplot('sin(1/x)',[0 0.5*pi]);
```

commands `plot()`

`fplot()`

picture

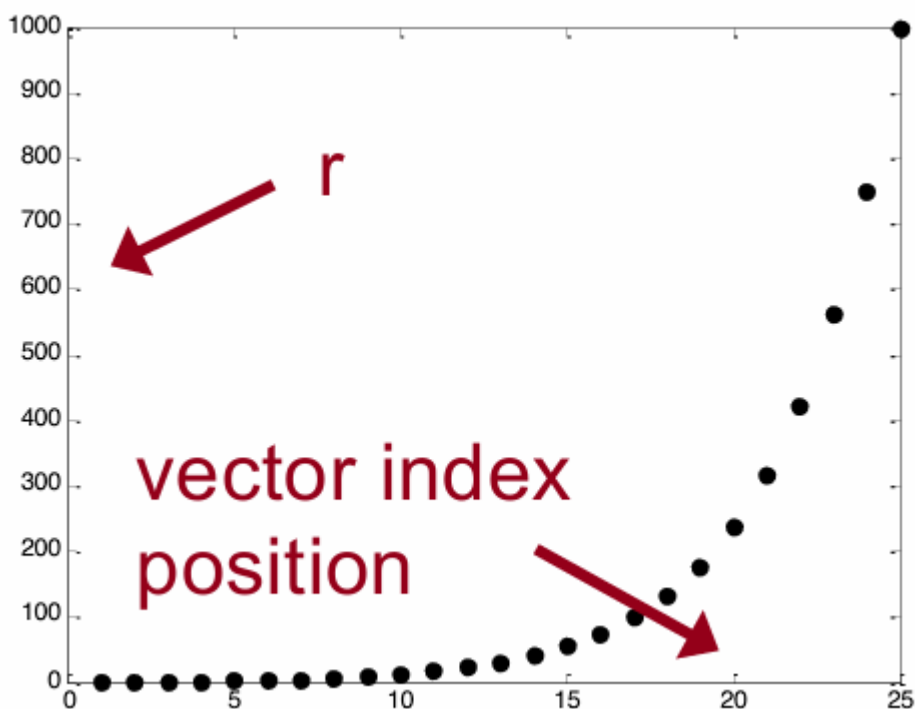


Increased detail near origin of this function

Plotting a vector

Most 2D plot commands can be called with a single vector, which is often useful for debugging
The x values are then assigned the index location

```
r = logspace(0,3,25)
plot(r,'ko','MarkerFaceColor','k')
```



Specifying plot attributes

```
plot(x,y,'LineSpecifiers','PropertyName',PropertyValue)
```

Line specifiers include line style, line color and marker.(e.g. `ro-`)

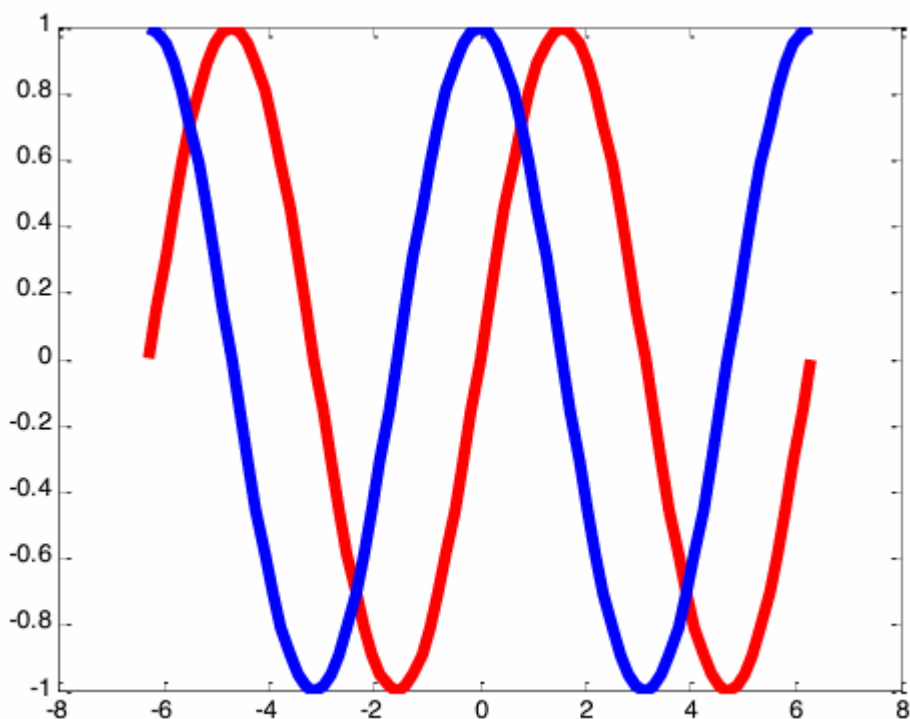
Property names include `LineWidth`, `MarkerSize`, `MarkerEdgeColor`, `MarkerFaceColor`

Overlaid plots

```
x = -2*pi:pi/20:2*pi;
y1 = sin(x);
y2 = cos(x)

% first method
plot(x,y1,'r-',x,y2,'b-', 'LineWidth',5)

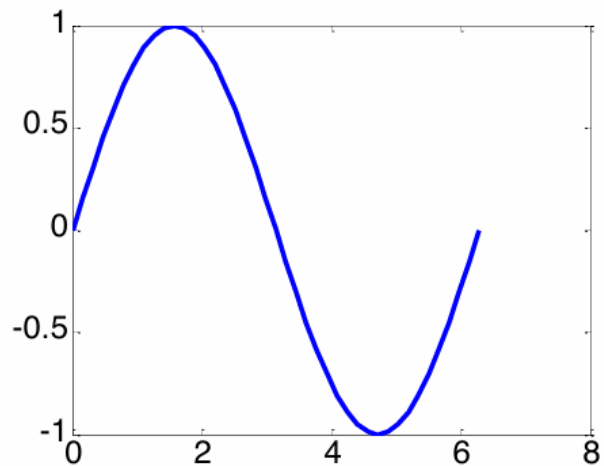
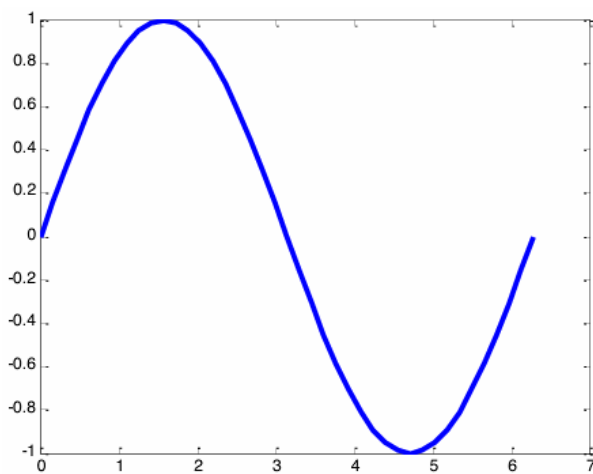
% second method
plot(x,y1,'r-', 'LineWidth',5)
hold on
plot(x,y2,'b-', 'LineWidth',5)
hold off
```



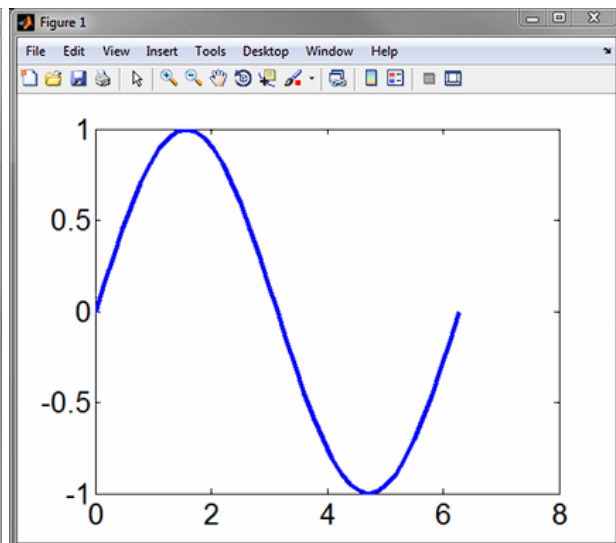
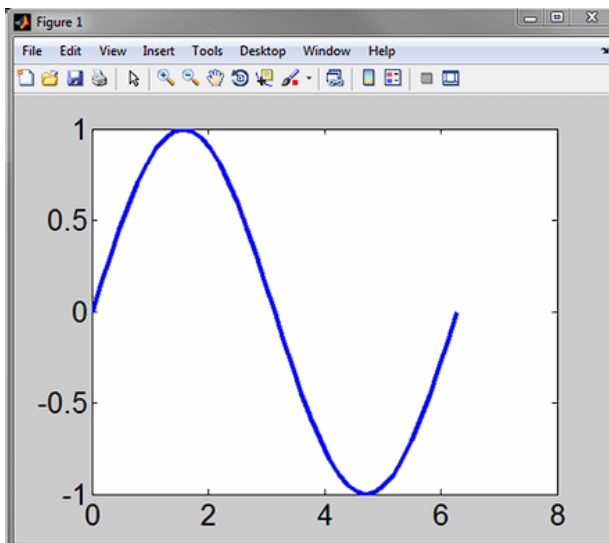
- Plot color may be specified for each vector, but line specifications such as `LineWidth` invoked within the plot function will be applied to all data series
- `hold on` is invoked after the first plot command and indicates that more data will be attributed to the existing axes;
- `hold off` indicates that additional plotting will overwrite the axes

Setting axes or figure properties

```
set(gca,'FontSize',20)
```



```
set(gcf,'Color','w')
```



Plotting multiple figures

The second plot command executed (without specifying hold on) will overwrite the first plot. You need to specify a new figure establish a new parent figure window

```
x = -2*pi:pi/20:2*pi;  
y1 = sin(x);  
y2 = cos(x);  
plot(x,y1,'k-','LineWidth',5)  
figure  
plot(x,y2,'k-','LineWidth',5)
```

Figures numbers may also be specified with a figure number, ex. `figure(3)`

Plots may then be closed using the close command

`close`: closes the last figure

`close(n)`: closes figure number n

`close all`: closes all figures

Multiple axes on a figure

Multiple axes may be assigned to a single figure using the `subplot` command or by specifying the `position` of multiple axes

`subplot(rows,columns,position)`

- rows: rows of axes
- columns: columns of axes
- position: position of current axes (counted along rows)

```
x = 0:pi/20:2*pi;  
y1 = sin(x-0);  
y2 = sin(x-0.5);  
y3 = sin(x-1);  
y4 = sin(x-1.5);  
y5 = sin(x-2);  
y6 = sin(x-2.5);
```

```
subplot(2,3,1)  
plot(x,y1)  
axis tight
```

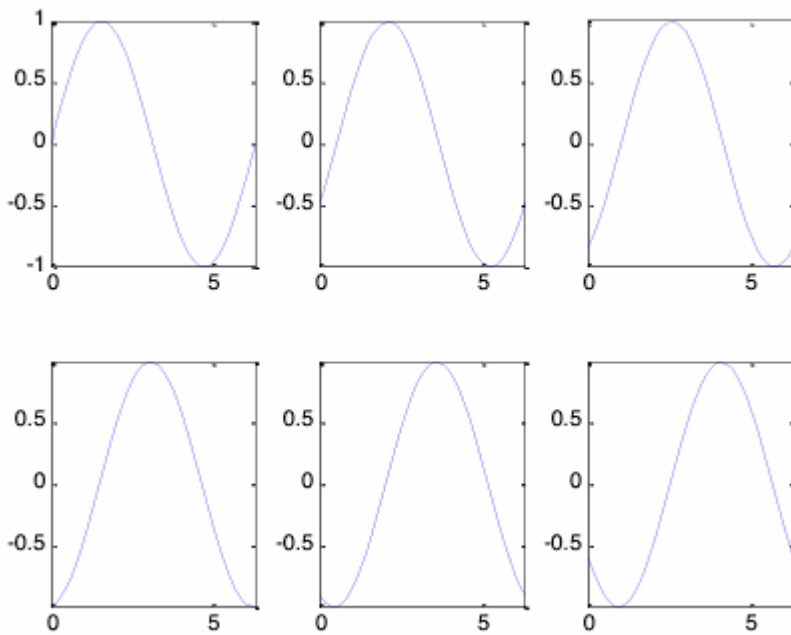
```
subplot(2,3,2)  
plot(x,y2)  
axis tight
```

```
subplot(2,3,3)  
plot(x,y3)  
axis tight
```

```
subplot(2,3,4)  
plot(x,y4)  
axis tight
```

```
subplot(2,3,5)  
plot(x,y5)  
axis tight
```

```
subplot(2,3,6)  
plot(x,y6)  
axis tight
```

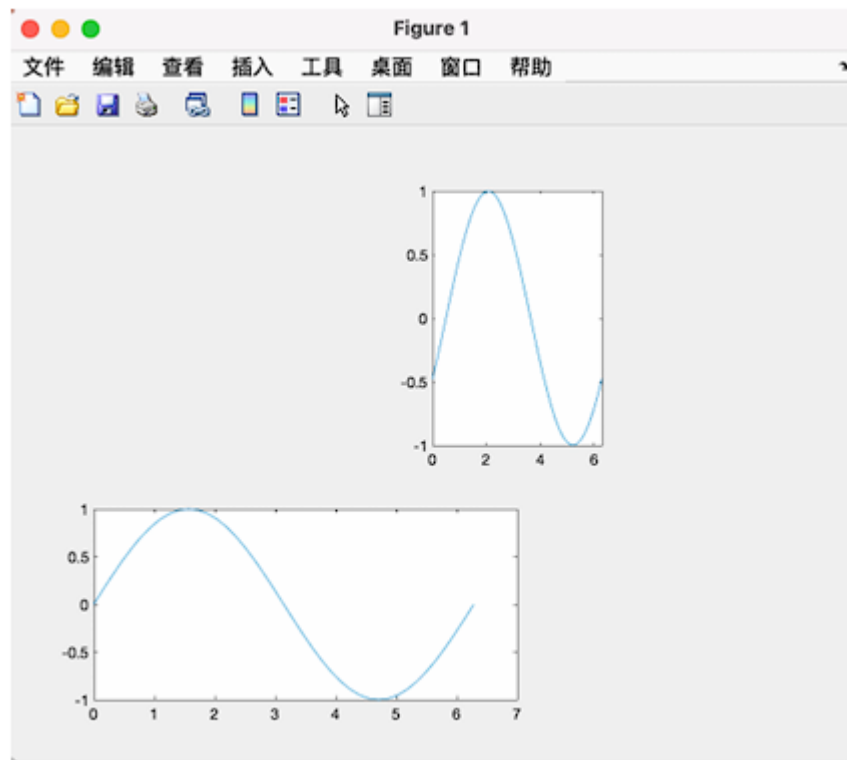


```
axes('Position',[x,y,w,h])
```

- x: lower left hand corner x coordinate
- y: lower left hand corner y coordinate
- w: width of axes
- h: height of axes

```
axes('Position',[.1 .1 .5 .3])  
plot(x,y1)
```

```
axes('Position',[.5 .5 .2 .4])  
plot(x,y3)
```



Specifying the domain and range

A specific domain and range may be specified using `axis` or `xlim/ylim`

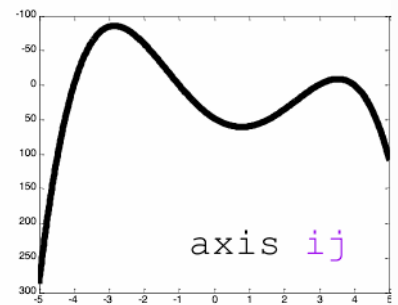
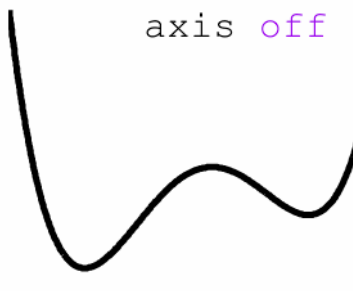
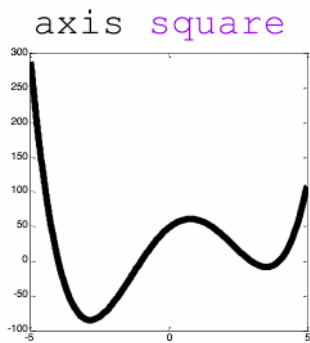
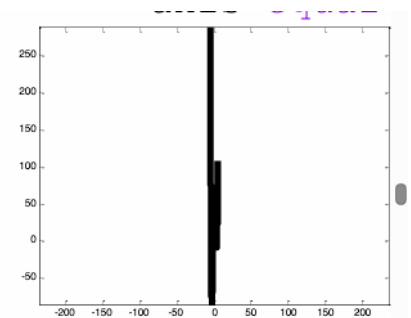
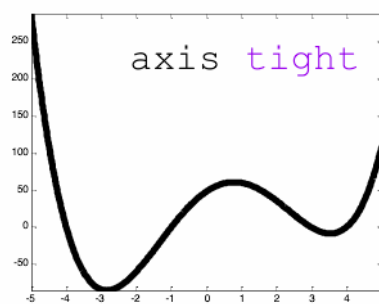
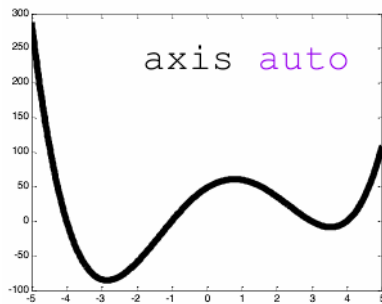
`axis([xmin xmax ymin ymax])` or

```
xlim([xmin xmax])  
ylim([ymin ymax])
```

e.g.

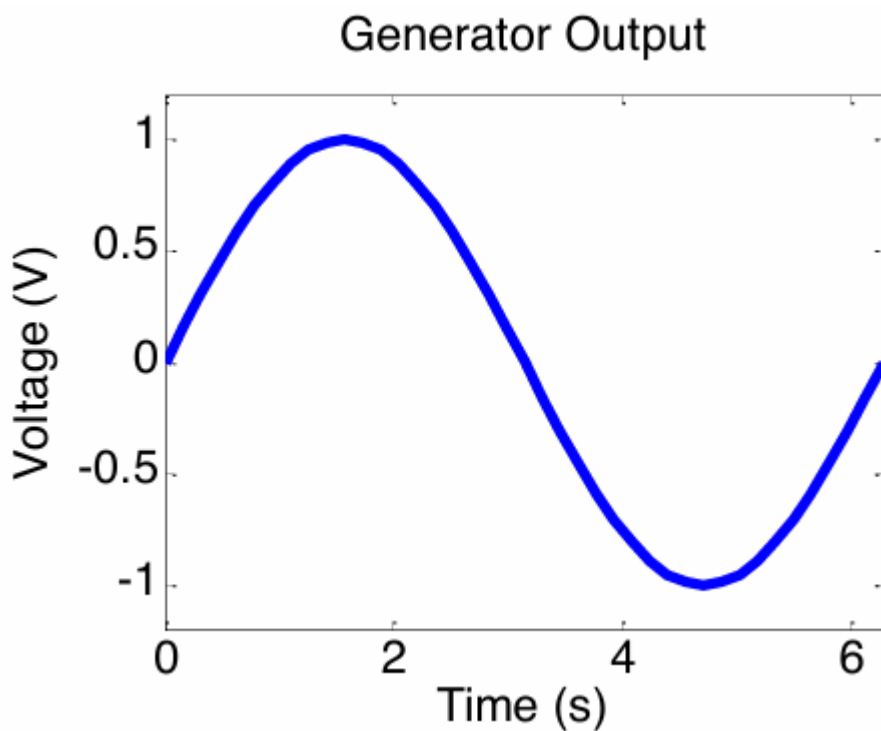
```
plot(x,y1,'LineWidth',5)  
axis([0 1.5*pi 0 1.5])  
set(gca,'FontSize',20)
```

The axis command offers many predefined axis scaling/sizing operations.



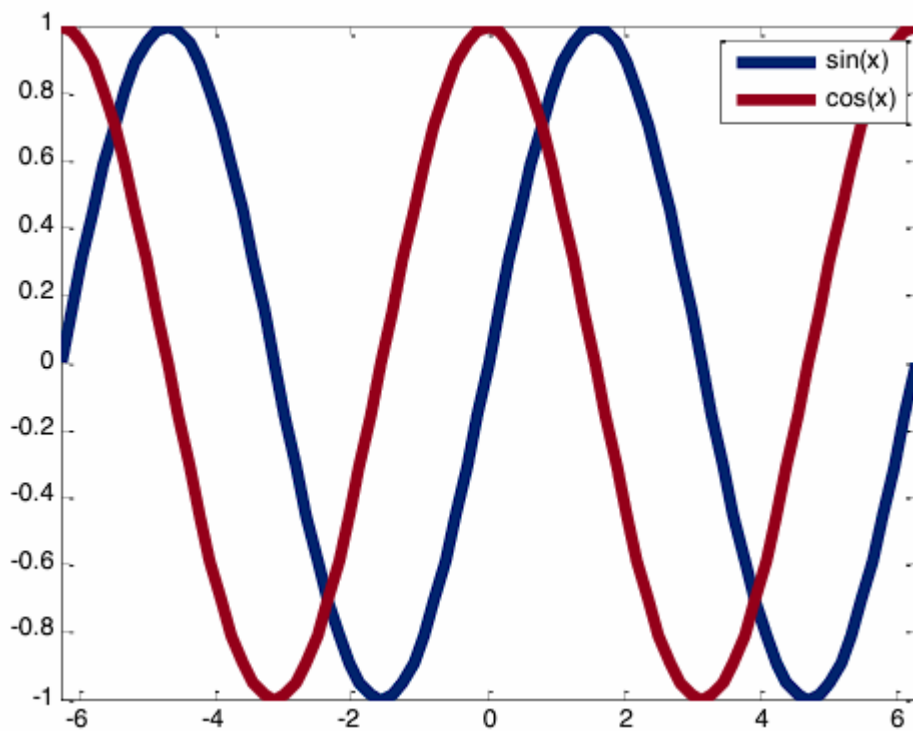
Labeling and titling axes

```
plot(x,y1,'LineWidth',5)
set(gca,'FontSize',20)
xlabel('Time (s)')
ylabel('Voltage (V)')
title('Generator Output')
axis([0 2*pi -1.2 1.2])
```



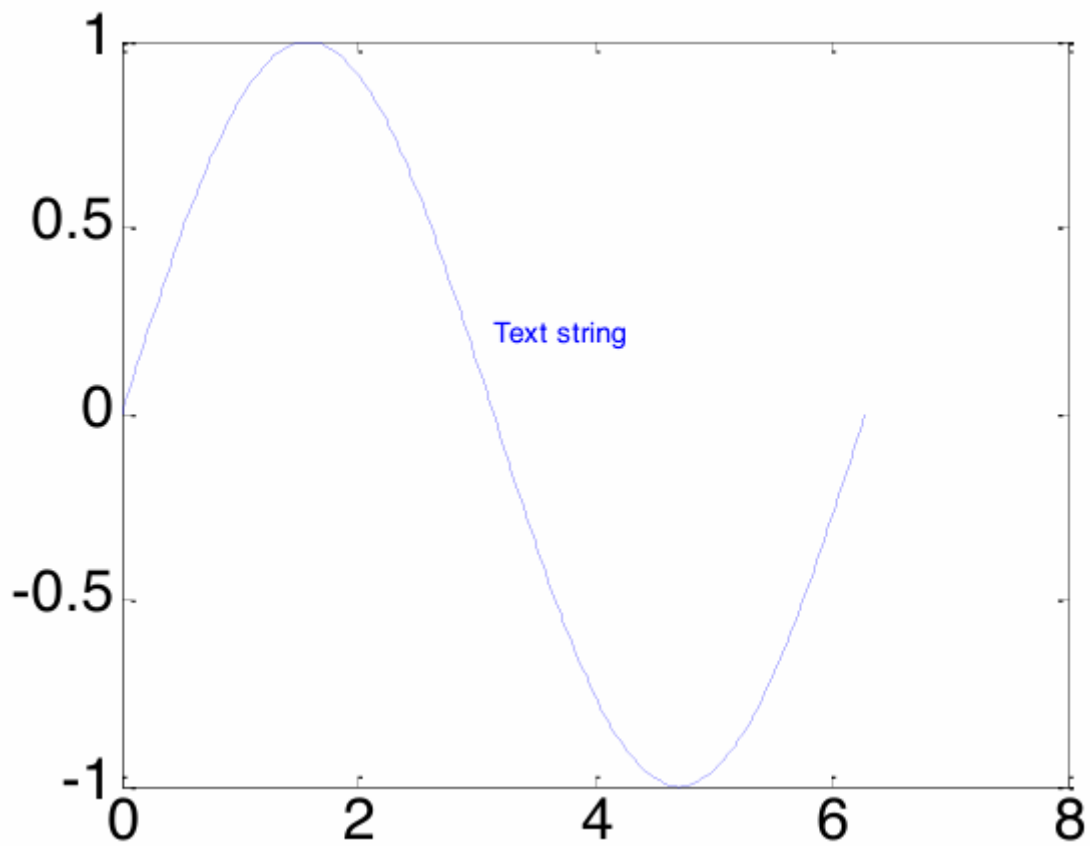
Adding a legend

```
legend('sin(x)', 'cos(x)', 'Location', 'NorthEast')
```



Placing text on a plot

```
text(pi, 0.2, 'Text string', 'Color', 'b')
```



3D plot

Three steps

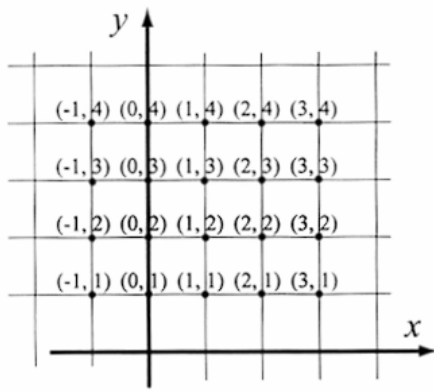
1. Make a grid in the x-y plane
2. Find the value of z for each point of the grid
3. Draw the mesh or surface plot

```
plot3(x, y, z, 'line specifiers','PropertyName', PropertyValue)
```

meshgrid

```
[X, Y] = meshgrid (x, y)
```

```
x = linspace(-1, 3);  
y = linspace(4, 1, -1);  
[X, Y] = meshgrid(x, y);
```

$$X = \begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \\ -1 & 0 & 1 & 2 & 3 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- X, Y : matrices of x and y coordinates, respectively
- x, y : vectors that define the domain of the grid

Mesh and surface plots

- `mesh(X,Y,Z)` lines connecting the points
- `surf(X,Y,Z)` areas colored

Other 3D Plots

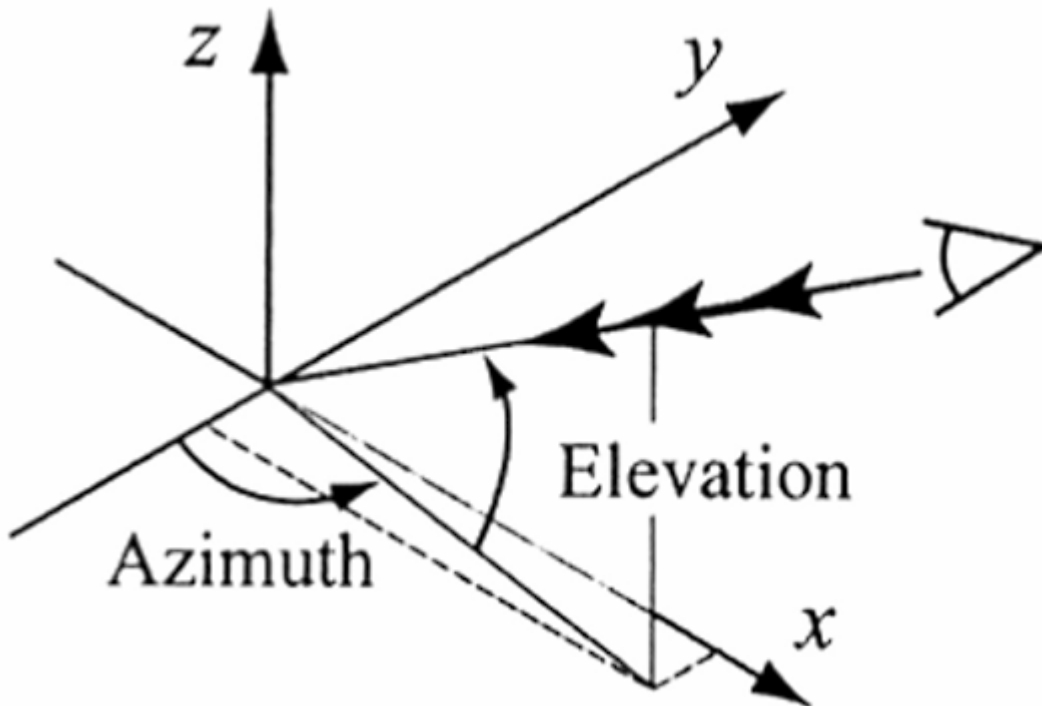
- Sphere
- Cylinder
- 3D bar
- 3D stem
- 3D scatter
- 3D pie

Controlling the View in 3D

`view(az, el)` or `view([az, el])`

`az`: azimuth relative to negative y -axis (default is -37.5°)

el: elevation from x-y plane (default is 30°)



Graphics handles

Figures, axes, and plots can all be assigned handles

These handles look a lot like “variables” and serve as a pointer to the object

```
myHandle1 = plot(x,y);
myFig = figure;
h = axes;
```

set

`set(<handle>, <property>, <propertyValue>)`

e.g.

```
p11 = plot(x,y);
set(p11, 'LineWidth', 2)
```

get

`get(<handle>, <property>)`

e.g.

```
fig1 = figure
pl1 = plot(x,y);
get(fig1, 'Position')
```

Vocabulary

- Scalars 标量
- portions 部分
- extraneous 无关的
- eye candy 视觉上具吸引力，但没什么实质内容的东西
- resolution 分辨率
- domain 范围，领域
- consecutive 连续的