

# Introduction to Scientific Computing

## Statements

```
foo = x^2 + sin(5*y) / exp(67*z);
```

The semicolon at the end is optional.

## Built-in MATLAB functions

`sin(x)` , `cos(x)` , `exp(x)` , `log(x)` , `atan(x)` , `cosh(x)` , `sinh(x)` , `mean(x)` , `median(x)` ,etc.

## Array

```
x = [1,3,9,11,-10.2];
```

```
foo = x(3); % extracts third element in the array
```

```
x(2) = 27; % reassign element in the array
```

```
x = 1:15 % creates an array with the number 1 through 15
```

```
x = 4:2:28; % creates an array starting at 4 and going up by 2 until it gets to 28
```

```
x = -1.3:0.1:1.3; %creates an array starting at -1.3 and going up by 0.1 until it gets to 1.3
```

```
x = 1:2:6; % x = [1,3,5]
```

```
x = [1:2:6]; % x = [1,3,5]
```

# Operations on arrays

```
x = 0:0.1:10;
y = exp(5*x).*sin(x); % evaluate an expression on every element in the array x producing a new array
plot(x,y);

x = [1,2,3,4,5];
y = [7,8,9,10,11];
z = x + y; % element-wise addition adds corresponding elements
z = x.*y; % element-wise multiplication
z = x./y; % element-wise division
```

## Overflow

```
X = uint8(78); Y = uint8(190);
Z = X + Y;
% overflow - value will be clipped to 255
```

## Scientific notation

Note that numbers in scientific notation have the following components

- A sign - positive or negative
  - A mantissa(尾数) - ex. 2.13
  - An exponent - ex. 17
- e.g. +2.13e17

## Arrays and numerical types

When an array variable is created all of the numeric values in that array share the same numeric type. For example we can talk about an array of `uint16` s or an array of doubles.

```
x = single(0:0.1:100); % make an array of single precision (单精度) numbers
```

# A few useful MATLAB commands

- `whos`  
Lists all of the variables currently in your workspace and shows you their types
- `clear`  
Clears all of the variables in your workspace – you can also use this to clear specific variables
- `clc`  
Just clears the command window – has no effect on the workspace

## Function

```
function f = fact(n)
    f = prod(1:n);
end
```

```
function [output1,output2,output3] = myFunction(input1,input2,input3)
```

## Naming a Function

Must start with a letter from the alphabet

## Special Functions

```
function out1 = testFunction(in1)
% A test function
% this would report, in the help function, all of functionality of the function
% in1 = function input (units: not specified)
% out2 = function output (units : not specified)
```

(in the command window)

```

lookfor 'a test function'

% testFunction          - A test function

help testFunction

% A test
% this would report, in the help function, all of functionality of the function
% in1 = function input (units: not specified)
% out2 = function output (units : not specified)

```

## Search paths

- The working folder at startup can be changed using the `userpath` function

```
userpath('C:\ATRI')
```

- Paths can also be “permanently” added to the search path using the `addpath()` function
- Paths can be removed using the `rmpath()` function

## Commenting

- Highlight a selection or place your cursor on a line and press “ctrl+r” to comment a section or line of code, respectively
- “ctrl+t” uncomments the code

## Anonymous Function

```
function_name = @ (arguments) expression
```

```
FtoC = @ (F) 5*(F-32)./9
```

```
% examples
```

```
FA = @ (x) exp(x^2)/sqrt(x^2+5)
```

```
FA(2)
```

```
FA = @ (x) exp(x.^2)./sqrt(x.^2+5)
```

```
FA([1 0.5 2])
```

# Function Handles

```
f = @sin;  
m = fminbnd(f,0,2*pi)  
  
q = integral(@cubicPoly,0,1);
```

## Vectors(Arrays)

```
% Linspace and logspace  
x = linspace(1,5,5)  
x = logspace(1,5,5)  
  
x = [5, 9, 4, 1, 7, 3, 4, 8]  
% Accessing vector elements  
y = x(1)  
y = x(1:3)  
y = x(end)  
y = x(end-2)  
y = x([1,3,6]) % y = [5,4,3]  
  
% The indices start at 1, not 0.  
  
% Overwriting  
x(end) = 1;  
  
%Add ending indices  
x(end+1) = 8  
x(end+1:end+2) = [6,9]  
  
%Removing elements  
x(1) = []; % Remove the first element  
x(end-3,end) = [];
```

# Iteration

```
for jj = 1:20
    disp(jj)
end

a = 1;
while a < 10
    a = a + 1;
end

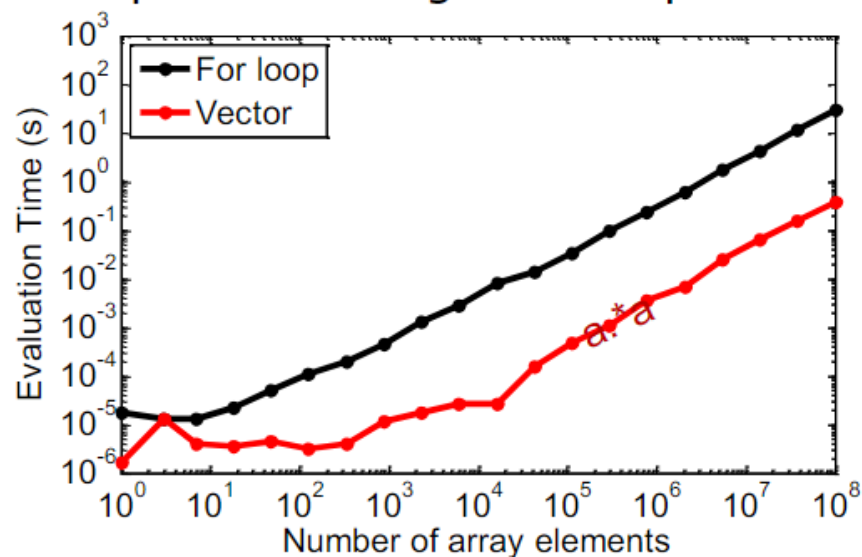
% Iterate through the array
x = [1,2,3]
for element = x
    disp(element);
end
```

## Array Operation vs Iteration

### Array operations increase computation speed

---

Time to Compute  $a*a$  using a for Loop vs. Vectorization



# The if statement

```
a = 1;
b = 2;
if a == 1;
    disp('a is equal to 1! Yeah');
end

elseif a == 1 && b == 2;
    disp('...')
end
```

## Logical Operators

Logical Operator(Scalars)	Logical Operator(Vectors)	Function Call
&&	&	and(a,b)
		or(a,b)
~	~	not(a)
none	none	xor(a,b)
Attention: ~= in matlab equals to != in other languages		

## Logical Vectors

```
a = [1 6 5] < 2
% a = [1,0,0]

x = [5 9 2 4 3];
v = logical([1 0 1 0 1]);
xp = x(v);
% xp = [5,2,3]
```

# Creating matrices

```
A = [1 2 3;4 5 6];
```

```
A = ones(2,5);
```

```
A = zeros(2,3);
```

```
A = eye(3); % Create a identity matrix
```

```
B = A' % Rows become columns and columns become rows.
```

```
% Expressions used to initialize arrays can include algebraic operati
```

```
a = [0 13*2];
```

```
b = [a(2) 13 a];
```

```
A = [1 5 9;2 4 8;3 6 7];
```

```
c = A[3,1]; % c == 3
```

```
c = A[4]; % Matrix indices may also be accessed using a single subscr
```

$$A = \begin{bmatrix} 1^1 & 5^4 & 9^7 \\ 2^2 & 4^5 & 8^8 \\ 3^3 & 6^6 & 7^9 \end{bmatrix}$$

# Subarrays

```
% Vectors
```

```
va =[1:5]
```

```
va(:) % all elements
```

```
va(m:n) % elements m through n
```

```
% Matrix
```

```
A = [1 2 3; 4 5 6; 7 8 9];
```

```
A(:,n) % all elements in column n
```

```
A(n,:) % all elements in row n
```

```
A(:,m:n) % elements in columns m through n
```

```
A(m:n,:) % elements in rows m through n
```

```
A(m:n,p:q)
```

```
b = [1 2 3 4; 5 6 7 8; 9 10 11 12];
```

```
b(2:2:end,2:end); % [6 7 8]
```



# Manipulating Matrices

Built-in function	description
<code>length(v)</code>	returns length of largest array dimension
<code>size(A)</code>	returns[m,n] where m and n are the size of A
<code>numel(A)</code>	returns the total number of elements in the array
<code>reshape(A,m,n)</code>	rearrange A to have m rows and n columns
<code>diag(v)</code>	creates square matrix with elements on the diagonal
<code>diag(A)</code>	creates vector from the diagonal elements

## Resizing a Matrix

```
A(:,2) = []; % deleting rows or columns from an array
A = [B C; D E]; % Arrays can be concatenated to form larger arrays.
```

## Strings

- An array of characters typed between single quotes ('),e.g. 'Hello World'
- char built-in function `My_string = char('string 1','string 2','string 3')`
- Work with strings `lower ; isspace ; isletter`

## Matrix Addition and Subtraction

1. Scalars may be added to any array, and the scalar value is added to all elements of the array
2. Arrays may be added and subtracted so long as they have the same dimensions

## Element-by-Element Operations

- Addition(+) and subtraction (-)
- Multiplication (.\* ) and division (./)
- Exponentiation (.^)
- Functions applied over arrays ( `sin , log ,...`)

- evaluating a function e.g.  $y = x^2 - 4x$

## Matrix Multiplication

- `a*B` Matrices may be multiplied by a scalar: Element-by-Element Multiplication
- `A.*B` Element-by-Element Multiplication
- `A*B` Matrix multiplication
- `cross(X,Y)` Cross product, special operation on two vectors.

## Matrix division

### Inverse of a matrix

- `inv` function or raise to -1 power
- matrix must be square and invertible

### Left Division

Solve  $AX = B$  where  $X$  and  $B$  are column vectors.

$$X = A^{-1}B$$

`X=A\B`

### Right Division

Solve  $XC = D$  where  $X$  and  $D$  are column vectors.

$$X = DC^{-1}$$

`X=D/C`

## Rotating and flipping arrays

- `rot90` : Rotate counterclockwise by 90 degrees
- `flipplr` or `flipdim(A,1)` : Reverse the matrix by treating a row as a whole.
- `flipud` or `flipdim(A,2)` : Reverse the matrix by treating a column as a whole.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \left\{ \begin{array}{l} \xrightarrow{\text{rot90}(A)} \begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix} \\ \xrightarrow{\text{fliplr}(A) \text{ or } \text{flipdim}(A,1)} \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix} \\ \xrightarrow{\text{flipud}(A) \text{ or } \text{flipdim}(A,2)} \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \end{array} \right.$$

Actually, we use `flip` instead of `flipdim` now. And the grammar is absolutely the same.

The second parameter of `flip` (or `flipdim`) is the dimension that is manipulated. It can be set to 3 or more when the dimension of array is above 2.

## Built-in Matrices

- `zeros(n)` returns an n-by-n matrix with all the elements equal to zero
- `ones(n)` returns an n-by-n matrix with all the elements equal to one
- `magic(n)` returns an n-by-n matrix constructed from the integers 1 through n\*n with equal row and column sums. The order n must be a scalar greater than or equal to 3 in order to create a valid magic square.
- `eye(n)` return an n-by-n identity matrix

## Sparse arrays

Sparse matrices are defined by a list of elements containing non-zero elements while all other elements are assumed to be zero

```
% conventional way to create a matrix
```

```
A = zeros(1000,2000);
```

```
A(3,4) = 15;
```

```
A(100,1500) = 5;
```

```
A(1000,2000) = 9;
```

```
% create using a sparse matrix
```

```
A = sparse([3,100,1000],...
```

```
          [4,1500,2000],...
```

```
          [15,5,9],1000,2000);
```

%The original (full) matrix A could also be converted to a sparse matrix via the following

```
A = sparse(A);
```

Matrix type	Memory(MB)	Time to compute $A^2$ (ms)
Full	15.2	4
Sparse	0.015	0.04

## Summing array elements

Consider the following 3D array

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$$\text{sum}(A) = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 6 & 6 & 6 \end{bmatrix}$$

$$\text{sum}(\text{sum}(A)) = [6][12][18]$$

$$\text{sum}(\text{sum}(\text{sum}(A))) = [36]$$

$$\text{sum}(A, 3) = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 6 & 6 \end{bmatrix}$$

manipulate the matrix in the same way:

1. treat a row as a whole
2. treat a column as a whole
3. treat the 3rd dimension a whole

## Cumulative sums

- `cumsum()` sums cumulatively along a vector or some dimension of an array
- unless a dimension is specified, it works on the first non-singleton dimension

```
cumsum([1 2 3 4 5]);  
% [1 3 6 10 15]
```

```
cumsum([1 2 3; 4 5 6]);  
% ([1 2 3; 5 7 9])
```

## Product

```
x(:, :, 1) = [1 2 3; 4 5 6];  
x(:, :, 2) = [6 5 4; 3 2 1];  
y = prod(x)
```

```
% y(:, :, 1) == [4 10 18] and  
% y(:, :, 2) == [18 10 4]
```

## Cumulative product

`cumprod()` calculates the cumulative product of array elements along the first non-singleton dimension

```
x(:, :, 1) = [1; 2; 3];  
x(:, :, 2) = [2; 2; 2];  
y = cumprod(x)
```

```
% y(:, :, 1) = [1; 2; 6] and  
% y(:, :, 2) = [2; 4; 8]
```

# Input Commands

## input command

```
in1 = input('Enter data:');  
% Stores input value as a numerical value or array of values of type double  
  
in2 = input('Enter data:', 's');  
% Stores input value as a string (an array of characters)
```

# Output Commands

## disp command

```
disp(variable);  
disp('text to print')
```

## fprintf command

```
fprintf(format, data);
```

Formatted output placed on screen or saved to a file

# Manipulate the files

## save

% Save all variables in the workspace to a file

```

save('file_name');
save file_name;
% Writes file_name.mat file in binary format

% Save a few variables
save file_name var1 var2;

% Save in readable format
save -ascii file_name;
% Easy way to transfer data

```

## load

```

% Load the variables in a file
load('file_name');
load file_name;
% Reads file_name.mat in binary format and restores variables

load file_name var1 var2;
% Load a few variables
load file_name var1 var2;

% Load from a text(.txt) file
load file_name.txt;
var = load('file_name.txt');

% Options
% mat, -ascii

```

## Cell Arrays

```

c = {42,rand(5),'abcd'}

% c =
%      1x3 cell array
%      {[42]}    {5x5 double}    {'abcd'}

```

# A pleasing plot

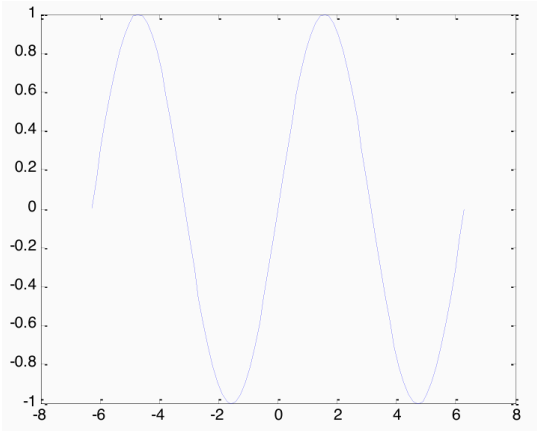
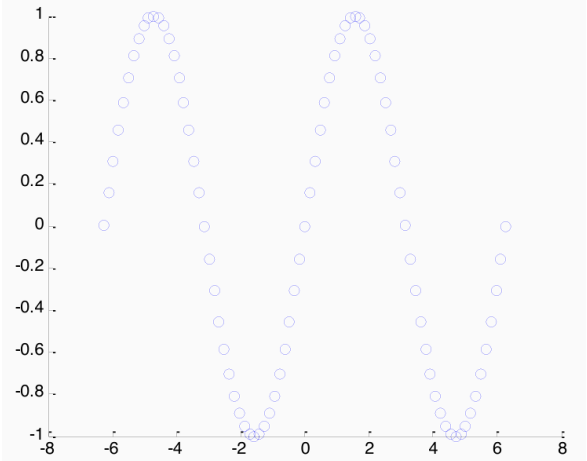
several objective measures of a good plot

- Labels with units
- Minimum amount of information to convey the objective (don't double label objects unless necessary, extraneous callouts should be avoided)
- Avoid overuse of eye candy
- Scatter plots for measured data and line plots for fits or closed form equations
- Sufficient resolution on domain to represent solution
- Line widths, data point sizes, and text large enough to read

## 2D plot commands

### plot and scatter

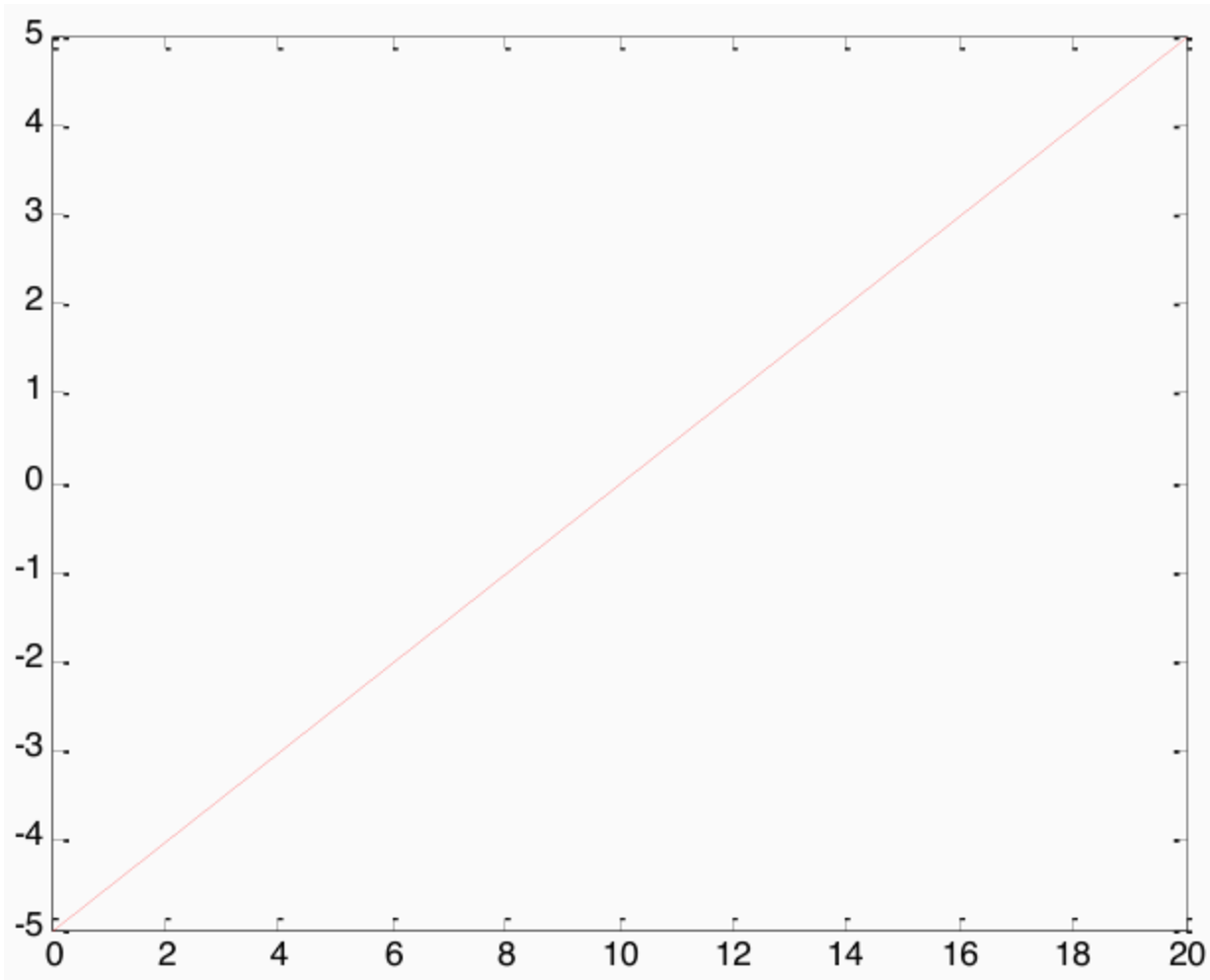
```
x = -2*pi:pi/20:2*pi;  
y = sin(x);
```

commands	plot()	scatter()
picture		

- `plot()` and `scatter` require vectors of equal length
- The default plot attributes are blue lines or circular blue data points, respectively
- These attributes can be modified by adjusting the line specifications of the plot `linespec`
- Straight lines may also be plotted by specifying the beginning and end points of the line

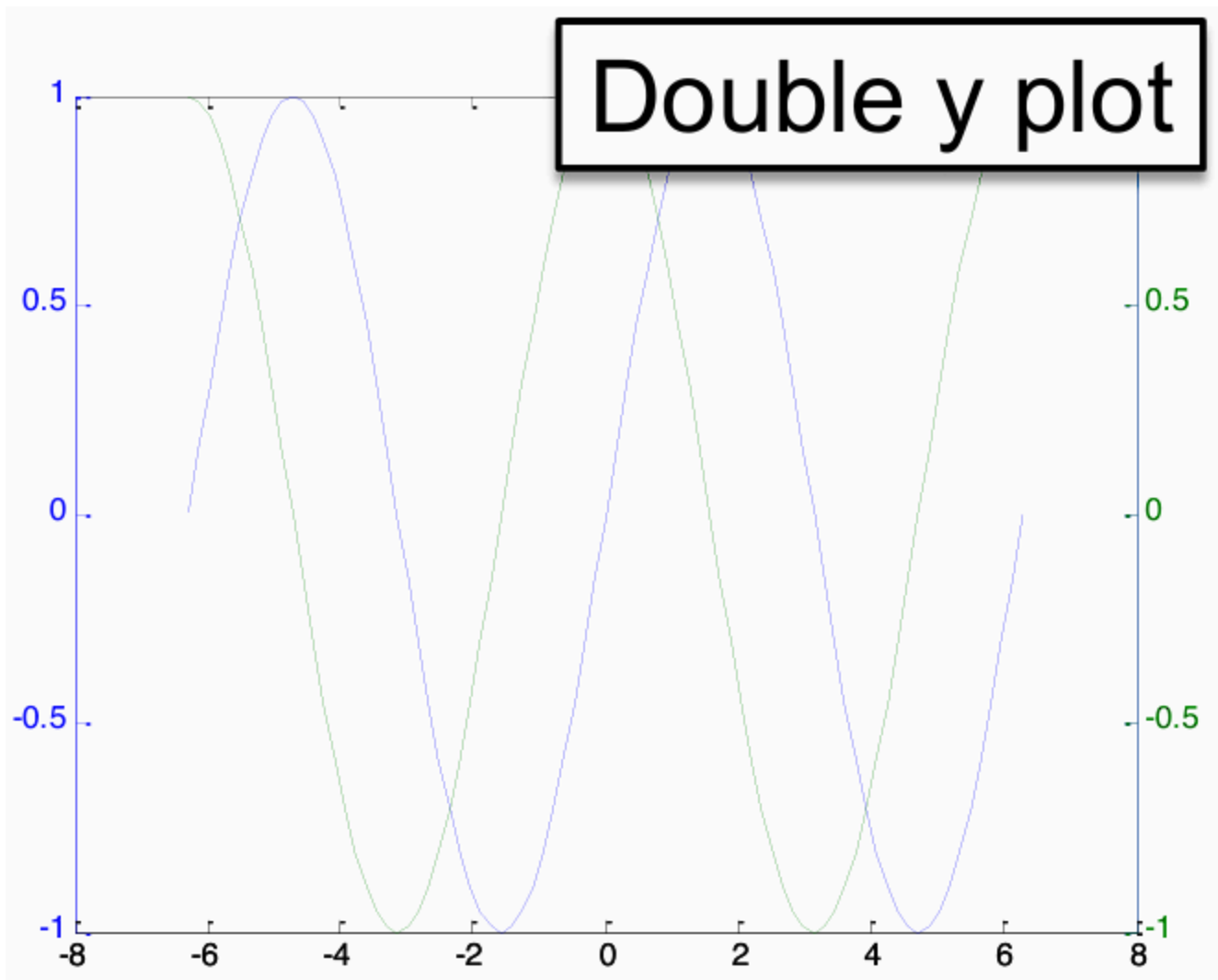


```
plot([0,20],[-5,5],'r-');
```



## Double y plot plotyy()

```
x = -2*pi:pi/20:2*pi;  
y1 = sin(x);  
y2 = cos(x);  
plotyy(x,y1,x,y2);
```



## Logarithmic plot

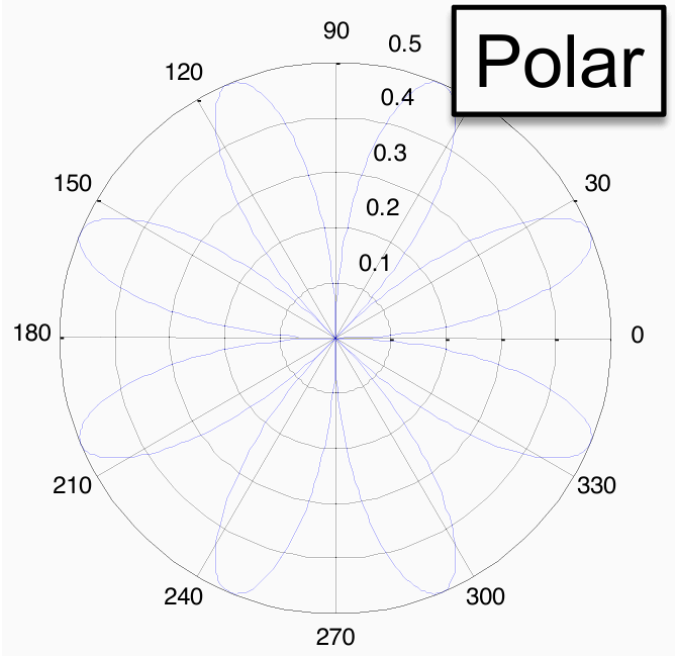
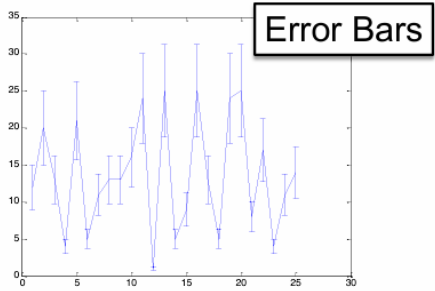
```
x = logspace(0,5,25);
y = x.^5;
loglog(x,y,'LineWidth',4)
```

commands	semilogy()	semilogx()	loglog()
picture			

# polar() and Error Bars

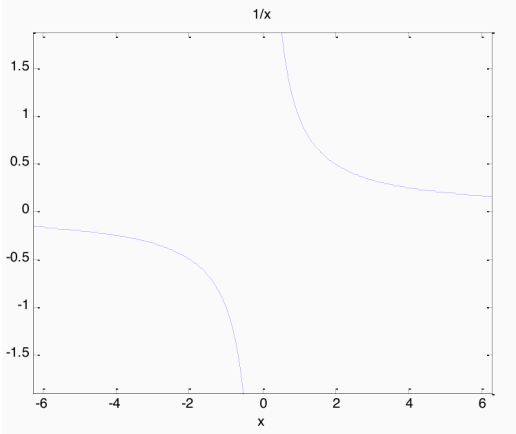
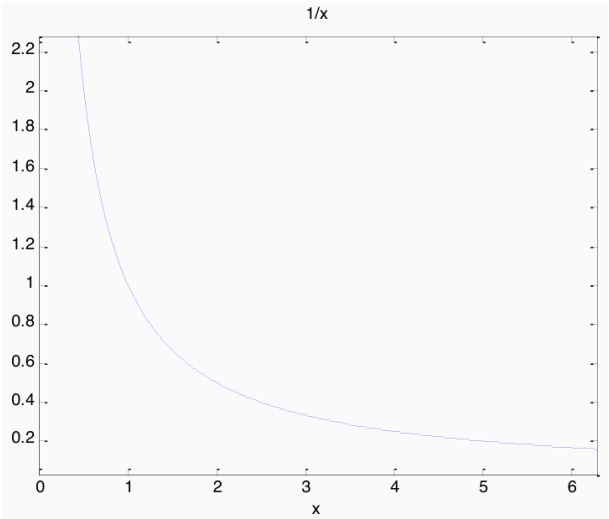
```
theta = 0:0.01:2*pi;  
rho = sin(2*theta).*cos(2*theta);  
polar(theta,rho)
```

```
x = 1:25;  
y = randi(25,1,length(x));  
y_err = 0.25*y;  
errorbar(x,y,y_err)
```

commands	polar()	errorbar()
picture		

## ezplot

ezplot produces simple plots without the need for discrete input of x/y vectors

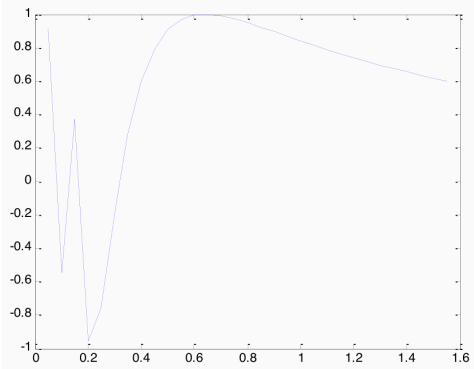
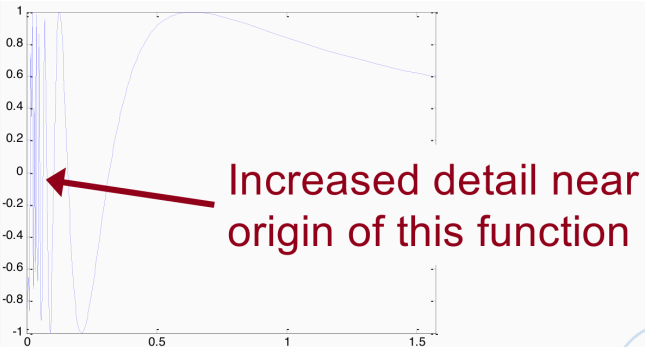
commands	<code>ezplot('1/x')</code>	<code>ezplot('1/x',[0 2*pi])</code>
picture	 A plot of the function 1/x. The x-axis ranges from -6 to 6 with major ticks every 2 units. The y-axis ranges from -1.5 to 1.5 with major ticks every 0.5 units. The plot shows two branches of the function, one in the second quadrant and one in the fourth quadrant, approaching the y-axis as x approaches 0 and the x-axis as  x  increases.	 A plot of the function 1/x. The x-axis ranges from 0 to 6 with major ticks every 1 unit. The y-axis ranges from 0.2 to 2.2 with major ticks every 0.2 units. The plot shows a single branch of the function in the first quadrant, starting at a high y-value near x=0 and decreasing as x increases towards 2*pi.
discription	The default domain is $-2\pi$ to $2\pi$	The domain may also be specified by a two element vector

## fplot

`fplot` intelligently selects x values based on changes to a function and may be used for rapidly changing functions

```
x = 0:.05:0.5*pi;
y = sin(1./x);
plot(x,y)
```

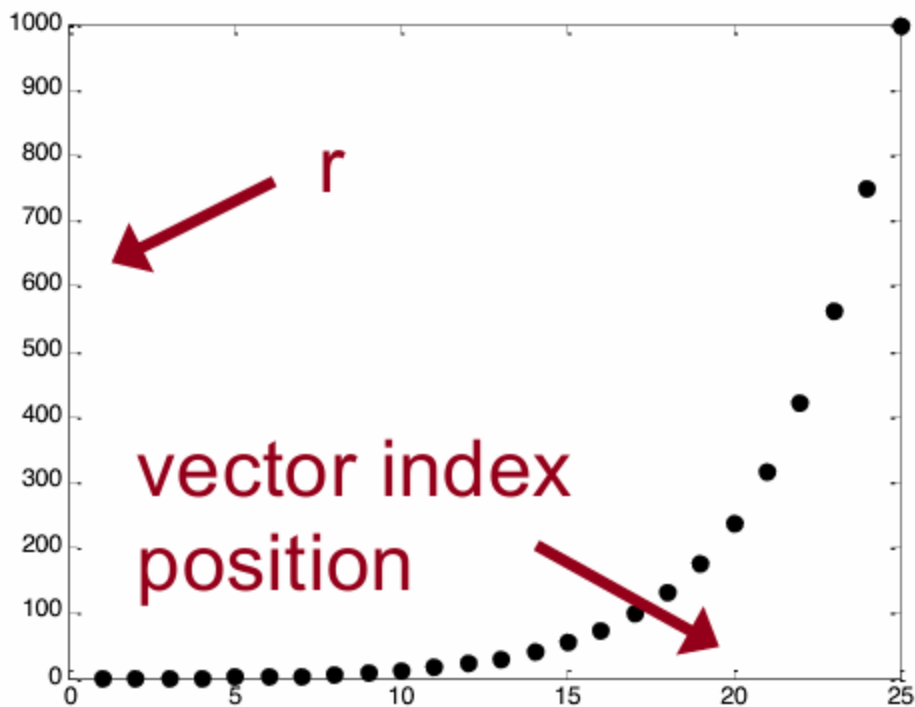
```
fplot('sin(1/x)',[0 0.5*pi]);
```

commands	<code>plot()</code>	<code>fplot()</code>
picture	 A plot of the function sin(1/x) using the <code>plot()</code> function. The x-axis ranges from 0 to 1.6 with major ticks every 0.2 units. The y-axis ranges from -1 to 1 with major ticks every 0.2 units. The plot shows the function oscillating rapidly near the origin and then settling down to a value of approximately 0.6 at x=1.6.	 A plot of the function sin(1/x) using the <code>fplot()</code> function. The x-axis ranges from 0 to 1.5 with major ticks every 0.5 units. The y-axis ranges from -1 to 1 with major ticks every 0.2 units. The plot shows the function oscillating rapidly near the origin and then settling down to a value of approximately 0.6 at x=1.5. A red arrow points to the origin with the text "Increased detail near origin of this function".

# Plotting a vector

Most 2D plot commands can be called with a single vector, which is often useful for debugging  
The x values are then assigned the index location

```
r = logspace(0,3,25)  
plot(r,'ko','MarkerFaceColor','k')
```



## Specifying plot attributes

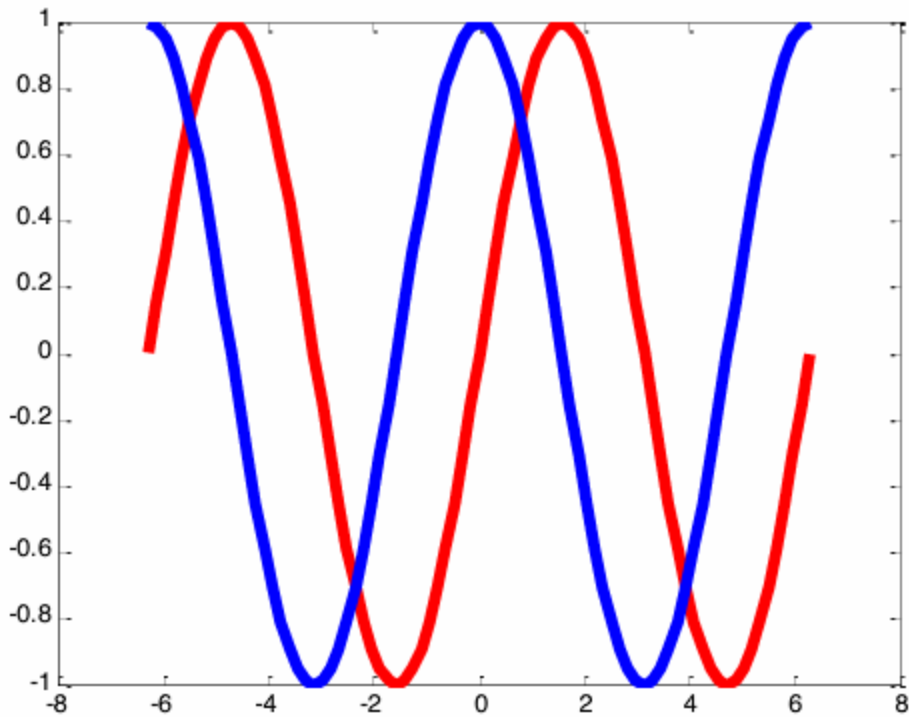
```
plot(x,y,'LineStyle','PropertyName',PropertyValue)
```

Line specifiers include line style, line color and marker.(e.g. ro- )

Property names include LineWidth , MarkerSize , MarkerEdgeColor , MarkerFaceColor

# Overlaid plots

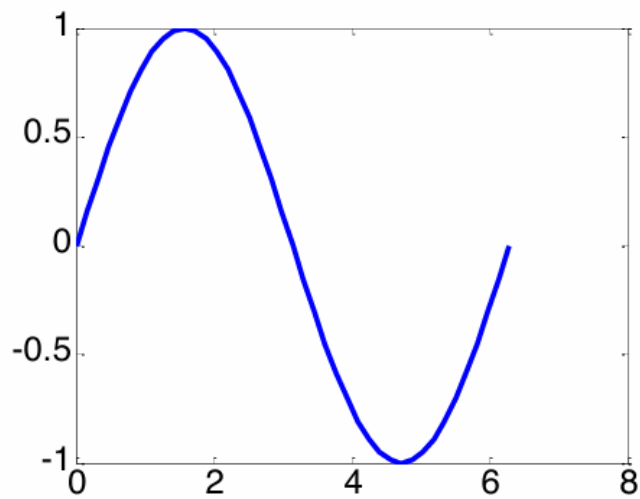
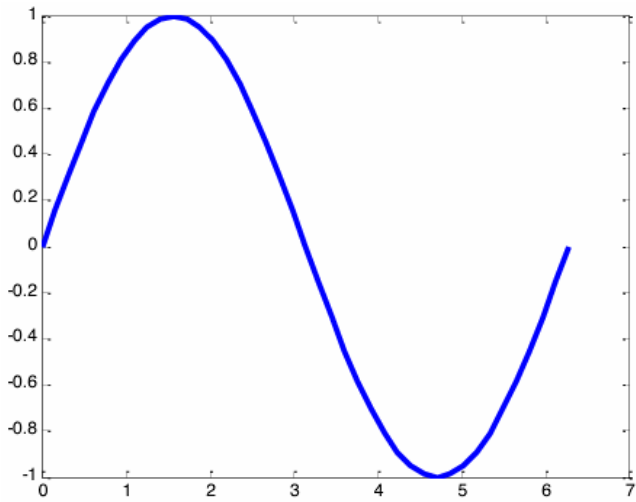
```
x = -2*pi:pi/20:2*pi;  
y1 = sin(x);  
y2 = cos(x)  
  
% first method  
plot(x,y1,'r-',x,y2,'b-', 'LineWidth',5)  
  
% second method  
plot(x,y1,'r-', 'LineWidth',5)  
hold on  
plot(x,y2,'b-', 'LineWidth',5)  
hold off
```



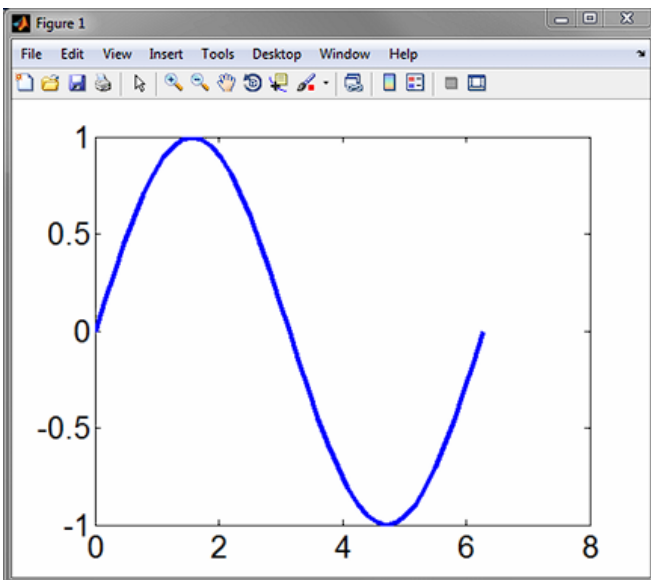
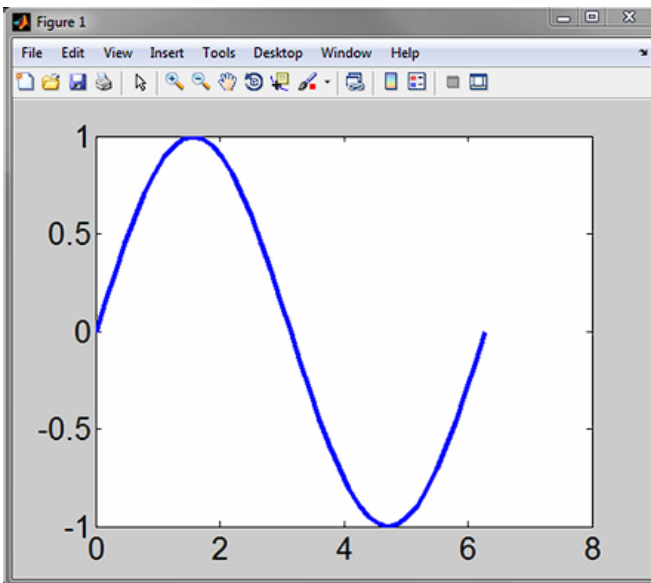
- Plot color may be specified for each vector, but line specifications such as `LineWidth` invoked within the plot function will be applied to all data series
- `hold on` is invoked after the first plot command and indicates that more data will be attributed to the existing axes;
- `hold off` indicates that additional plotting will overwrite the axes

# Setting axes or figure properties

```
set(gca,'FontSize',20)
```



```
set(gcf,'Color','w')
```



## Plotting multiple figures

The second plot command executed (without specifying hold on) will overwrite the first plot. You need to specify a new figure establish a new parent figure window

```
x = -2*pi:pi/20:2*pi;  
y1 = sin(x);  
y2 = cos(x);  
plot(x,y1,'k-','LineWidth',5)  
figure  
plot(x,y2,'k-','LineWidth',5)
```

Figures numbers may also be specified with a figure number, ex. `figure(3)`



Plots may then be closed using the close command

`close` : closes the last figure

`close(n)` : closes figure number n

`close all` : closes all figures

## Multiple axes on a figure

Multiple axes may be assigned to a single figure using the `subplot` command or by specifying the position of multiple axes

`subplot(rows,columns,position)`

- rows: rows of axes
- columns: columns of axes
- position: position of current axes (counted along rows)

```
x = 0:pi/20:2*pi;  
y1 = sin(x-0);  
y2 = sin(x-0.5);  
y3 = sin(x-1);  
y4 = sin(x-1.5);  
y5 = sin(x-2);  
y6 = sin(x-2.5);
```

```
subplot(2,3,1)  
plot(x,y1)  
axis tight
```

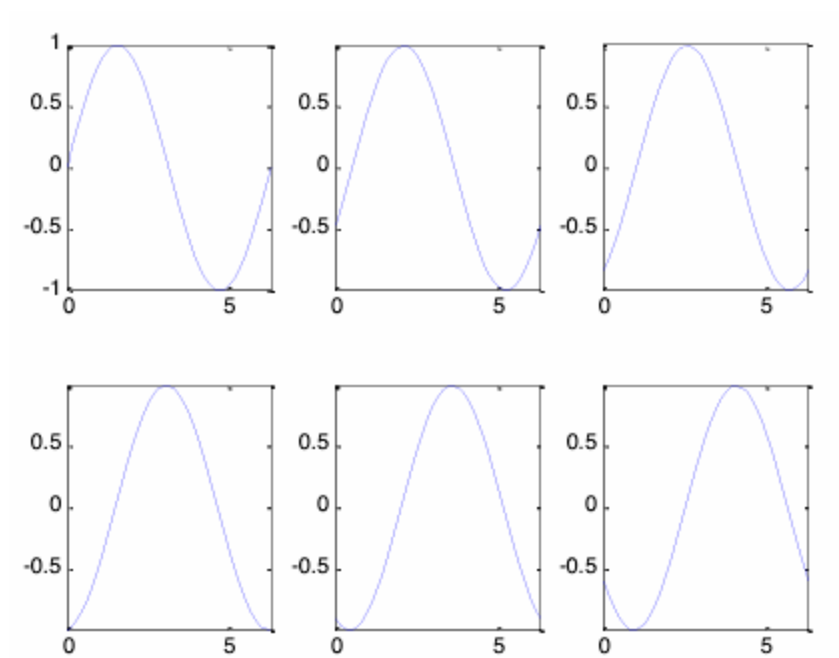
```
subplot(2,3,2)  
plot(x,y2)  
axis tight
```

```
subplot(2,3,3)  
plot(x,y3)  
axis tight
```

```
subplot(2,3,4)  
plot(x,y4)  
axis tight
```

```
subplot(2,3,5)  
plot(x,y5)  
axis tight
```

```
subplot(2,3,6)  
plot(x,y6)  
axis tight
```



```
axes('Position',[x,y,w,h])
```

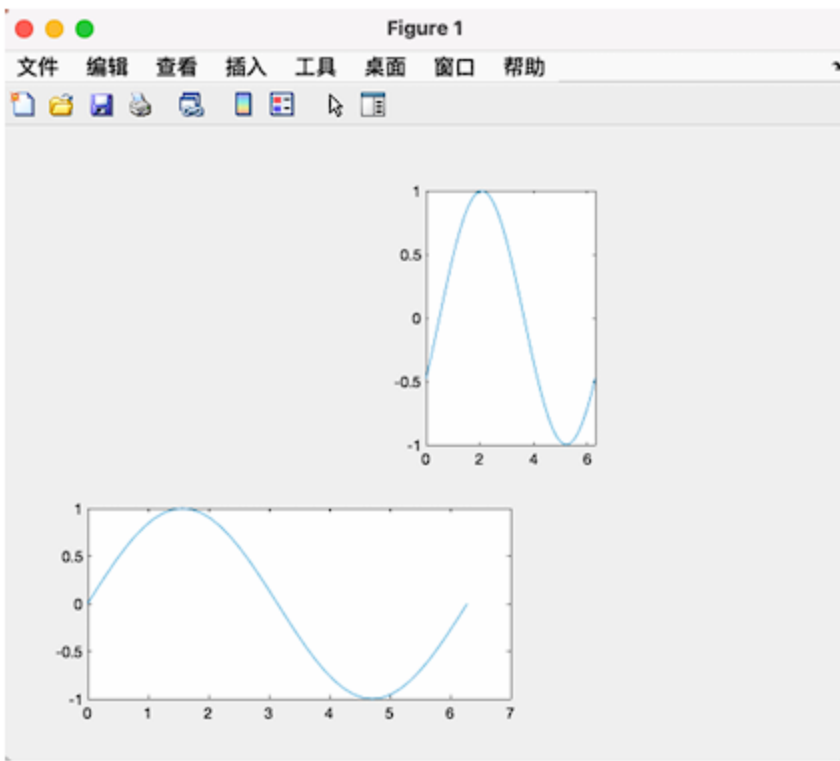
- x: lower left hand corner x coordinate
- y: lower left hand corner y coordinate
- w: width of axes
- h: height of axes

```
axes('Position',[.1 .1 .5 .3])
```

```
plot(x,y1)
```

```
axes('Position',[.5 .5 .2 .4])
```

```
plot(x,y3)
```



4th Week, 2024 Summer

Lec 3

## Specifying the domain and range

A specific domain and range may be specified using `axis` or `xlim/ylim`

`axis([xmin xmax ymin ymax])` OR

```
xlim([xmin xmax])
```

```
ylim([ymin ymax])
```

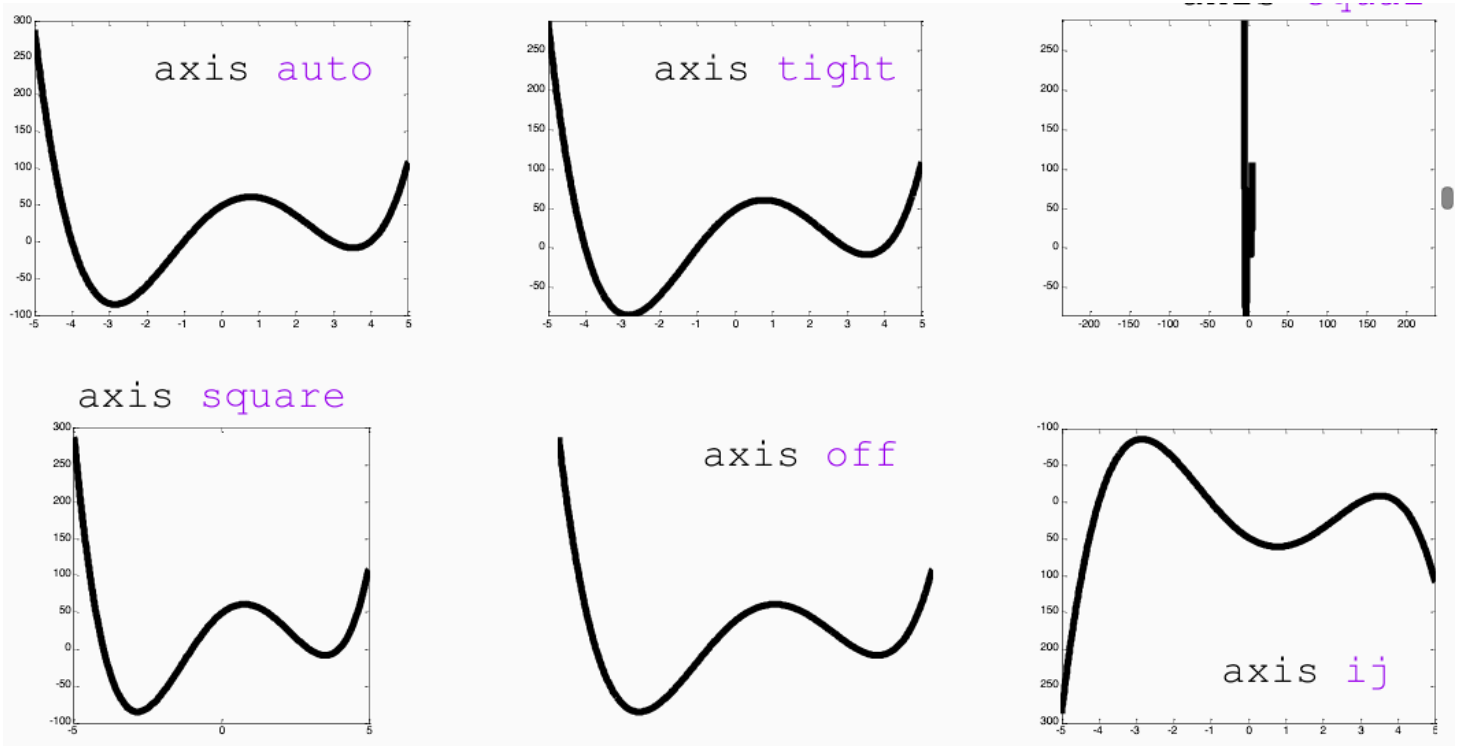
e.g.

```
plot(x,y1,'LineWidth',5)
```

```
axis([0 1.5*pi 0 1.5])
```

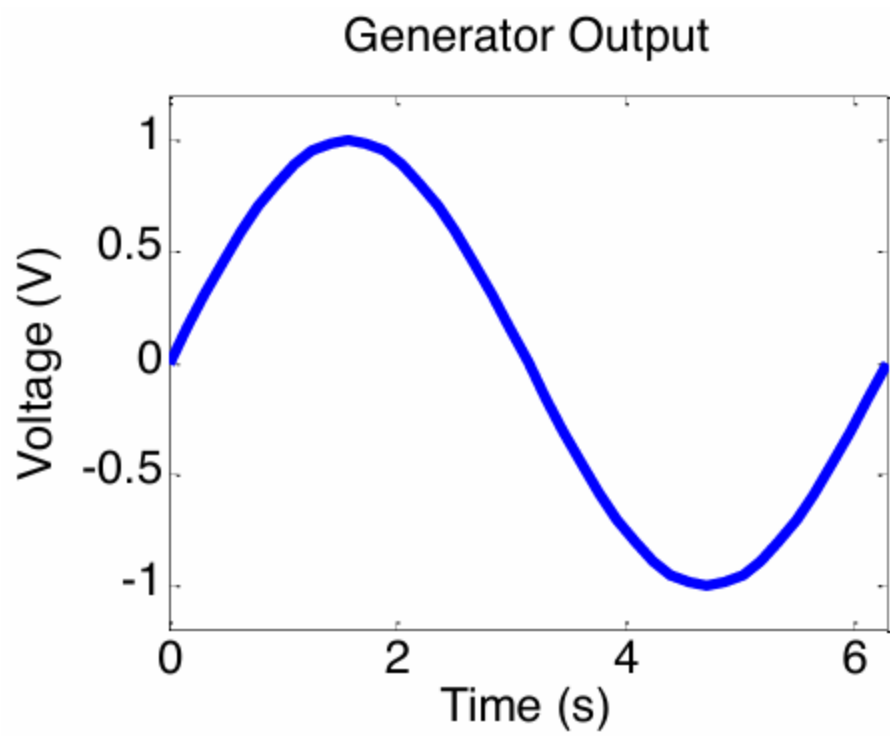
```
set(gca,'FontSize',20)
```

The `axis` command offers many predefined axis scaling/sizing operations.



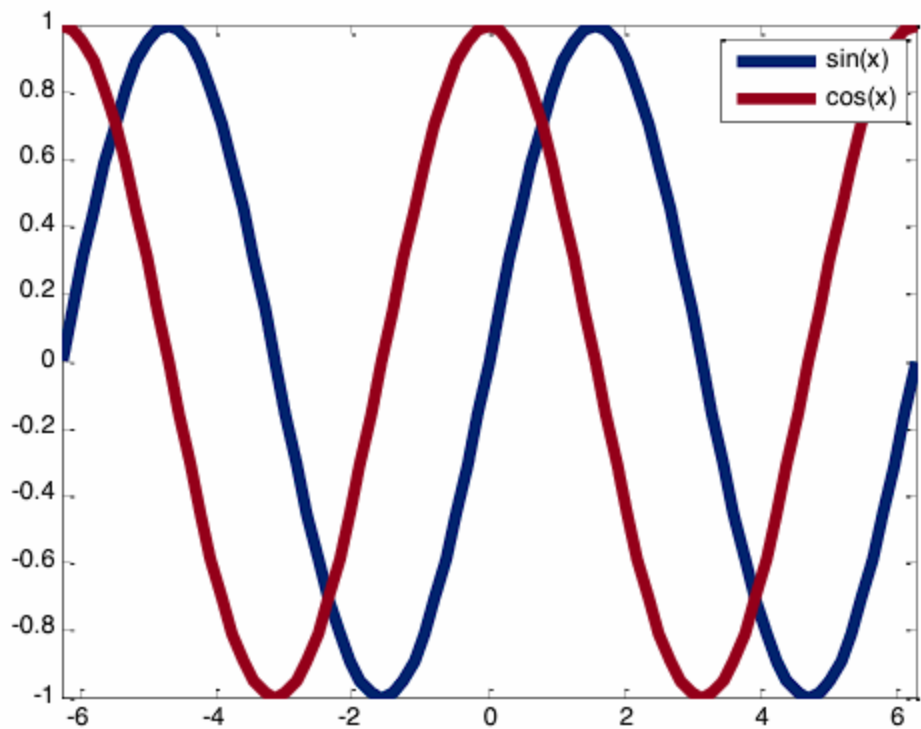
## Labeling and titling axes

```
plot(x,y1,'LineWidth',5)
set(gca,'FontSize',20)
xlabel('Time (s)')
ylabel('Voltage (V)')
title('Generator Output')
axis([0 2*pi -1.2 1.2])
```



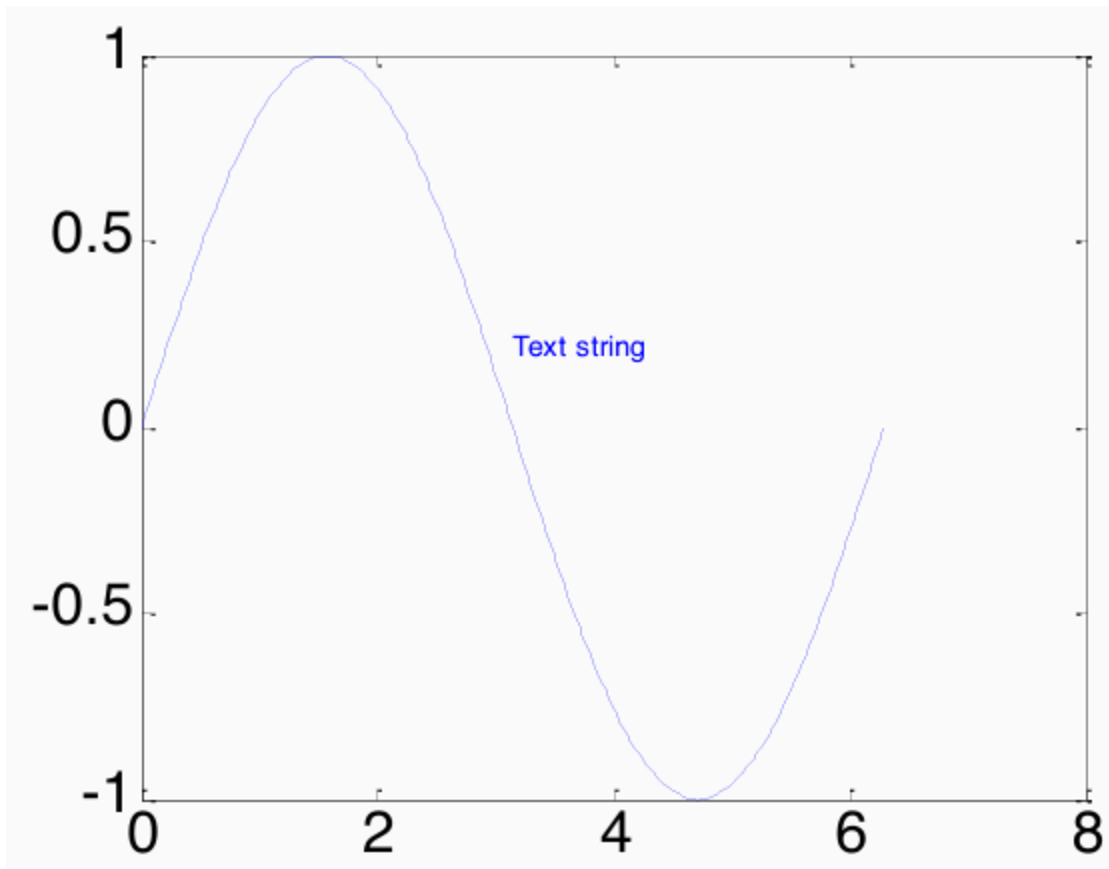
## Adding a legend

```
legend('sin(x)', 'cos(x)', 'Location', 'NorthEast')
```



## Placing text on a plot

```
text(pi,0.2,'Text string','Color','b')
```



## 3D plot

Three steps

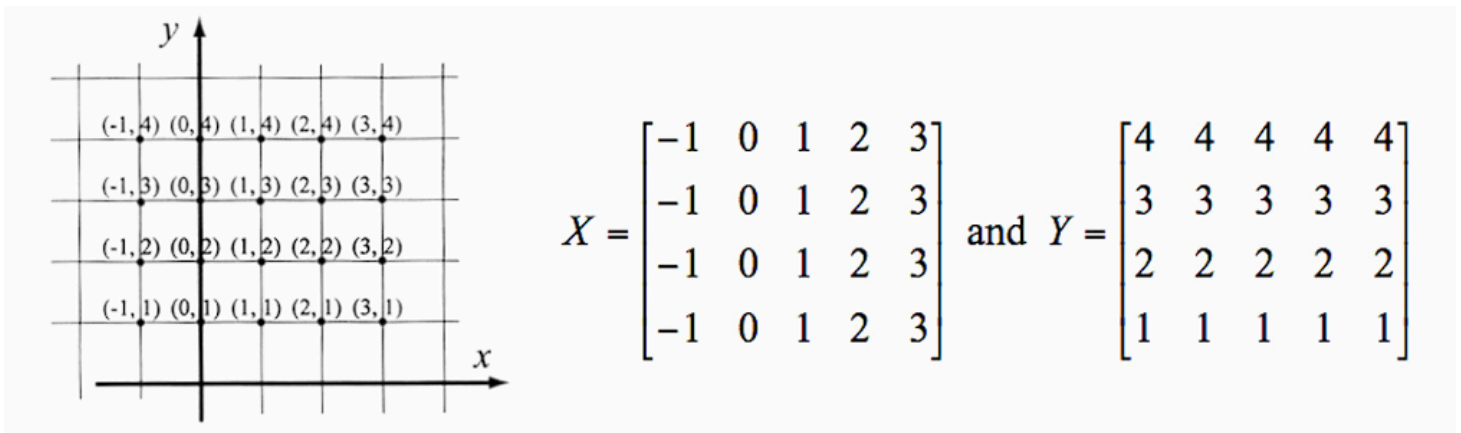
1. Make a grid in the x-y plane
2. Find the value of z for each point of the grid
3. Draw the mesh or surface plot

```
plot3(x, y, z, 'line specifiers','PropertyName', PropertyValue)
```

## meshgrid

```
[X, Y] = meshgrid (x, y)
```

```
x = linspace(-1, 3);
y = linspace(4, 1, -1);
[X, Y] = meshgrid(x, y);
```



- $X, Y$ : matrices of  $x$  and  $y$  coordinates, respectively
- $x, y$ : vectors that define the domain of the grid

## Mesh and surface plots

- `mesh(X,Y,Z)` lines connecting the points
- `surf(X,Y,Z)` areas colored

## Other 3D Plots

- Sphere
- Cylinder
- 3D bar
- 3D stem
- 3D scatter
- 3D pie

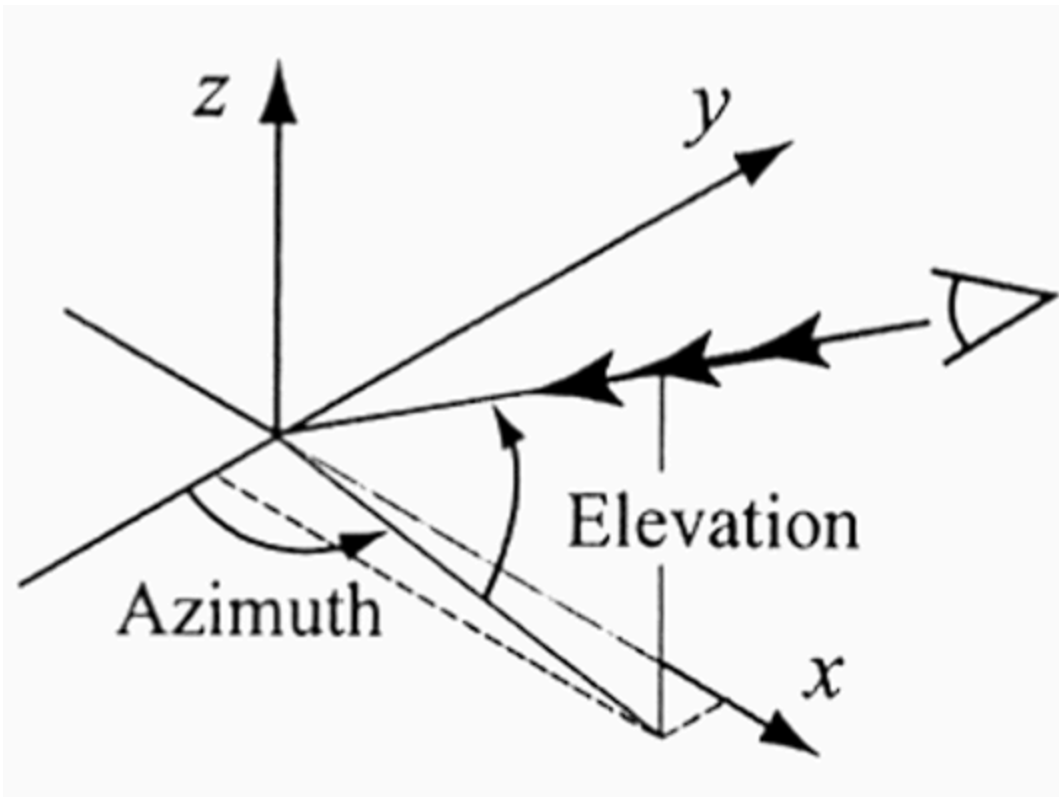
## Controlling the View in 3D

`view(az, el)` or `view([az, el])`

`az` : azimuth relative to negative  $y$ -axis (default is  $-37.5^\circ$ )

`el` : elevation from  $x$ - $y$  plane (default is  $30^\circ$ )





## Graphics handles

Figures, axes, and plots can all be assigned handles

These handles look a lot like “variables” and serve as a pointer to the object

```
myHandle1 = plot(x,y);
myFig = figure;
h = axes;
```

### set

```
set(<handle>,<property>,<propertyValue>)
```

e.g.

```
p11 = plot(x,y);
set(p11,'LineWidth',2)
```

### get

```
get(<handle>,<property>)
```

e.g.

```
fig1 = figure
pl1 = plot(x,y);
get(fig1,'Position')
```

## ODE

say we have an equation of the form

$$\ddot{y} + p\ddot{y} + q\dot{y} + ry = 0$$

We start by assigning generalized variables (say  $x$ ) to derivatives of  $y$  of increasing order as such

$$x_1 = y, x_2 = \dot{y}, x_3 = \ddot{y}$$

Next, we rearrange the ODE to get the variable of highest order on the left side of the equation and divide out any coefficients on this variable

$$\ddot{y} = -p\ddot{y} - q\dot{y} - ry$$

Next, we rewrite the derivatives of the generalized variables with respect to each other and the ODE

$$\dot{x}_1 = x_2, \dot{x}_2 = x_3, \dot{x}_3 = -px_3 - qx_2 - rx_1$$

```

function xout = thirdOrderODE(t,x)
% time and initial condition serve as inputs
p = 2;
q = 1.5;
r = 0.1;

dx1 = x(2);
dx2 = x(3);
dx3 = -p*x(3)-q*x(2)-r*x(1);

xout = [dx1; dx2; dx3];
% the derivatives of the generalized variables are returned as a column vector
end

[t,x] = ode45(@thirdOrderODE,[0 10],[10 1 2]);
% [0 10] indicates the start and end time of the solution
% [10 1 2] indicates initial conditions. x(1) = 10, x(2) = 1, etc.

```

## Polynimials

MATLAB represents polynomials as a vector

```

p = [8 5] % 8x+5
p = [5 0 0 6 -7 0] % 5x^5+6x^2-7x

```

Value of a polynomial

```

v = polyval(p,x)
% p is vector of coefficients
% x is the point to evaluate, it can be scalar, vector or matrix

```

## Polynomial Functions

- Roots of a polynomial  
`r = roots(p)`
- Polynomial from its roots  
`p = poly(r)`
- Polynomial multiplication(convolution)  
`c = conv(a,b)`

- Polynomial division (deconvolution)  
 $[q, r] = \text{deconv}(a, b)$
- Derivatives of polynomials  
 $pd = \text{polyder}(p)$

## Curve Fitting

### The Method of Least Square

Minimize sum of squares of residuals at all data points

$p = \text{polyfit}(x, y, n)$

$x, y$  : vectors of data coordinates

$n$  : degree of polynomial

$p$  : vector of coefficients for polynomial fit

### Fitting with other than Polynomials

Fitting Methods	relationship	linear form	code
Power	$y = bx^m$	$\ln y = m \ln x + \ln b$	$p = \text{polyfit}(\log(x), \log(y), 1)$
Exponential	$y = be^{mx}$ $y = b10^{mx}$	$\ln y = mx + \ln b$ $\log y = mx + \log b$	$p = \text{polyfit}(x, \log(y), 1)$ $p = \text{polyfit}(x, \log_{10}(y), 1)$
Logarithmic	$y = m \ln x + b$ $y = m \log x + b$		$p = \text{polyfit}(\log(x), y, 1)$ $p = \text{polyfit}(\log_{10}(x), y, 1)$
Reciprocal	$\frac{1}{mx + b}$	$\frac{1}{y} = mx + b$	$p = \text{polyfit}(x, 1./y, 1)$

## Interpolation

$yi = \text{interp1}(x, y, xi, 'method')$

$x, y$  : vectors of data coordinates

$xi$  : horizontal coordinate of interpolation point

$method$  : method to use for interpolation

## methods

nearest value of data point nearest

linear linear spline

spline cubic spline interpolation

pchip piecewise cubic Hermite interpolation

## Probability and Statistics

### The basics

avg = mean(x)

med = median(x)

### Histograms

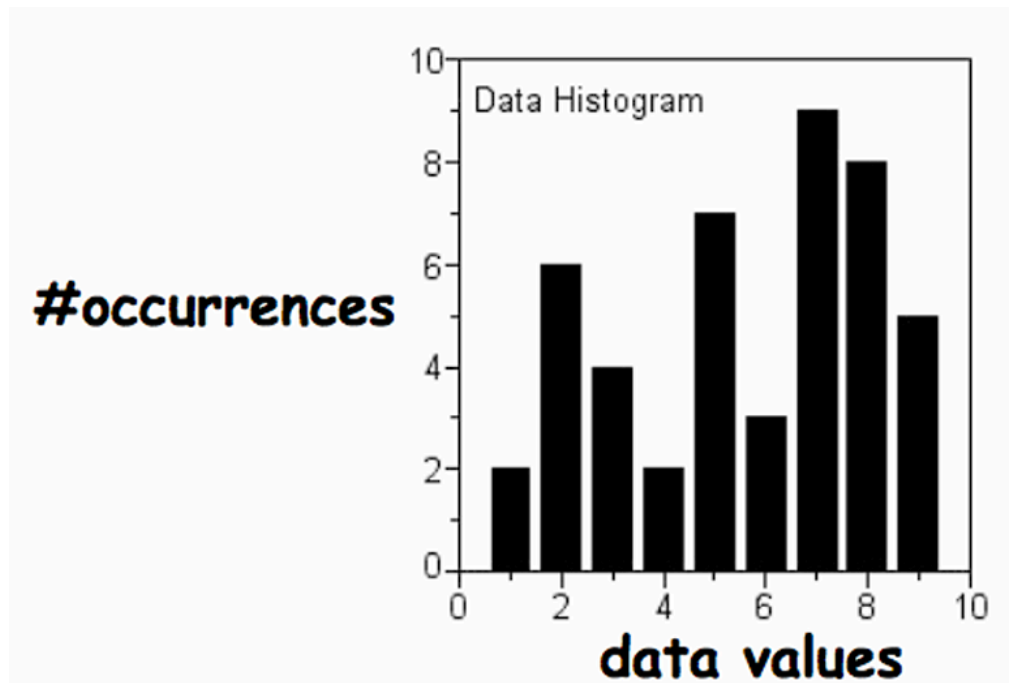
Histograms plot the frequency of occurrence of the data values not the data themselves

hist(y) Data divided into 10 bins

hist(y,n) Data divided into n (scalar) bins

hist(y,x) Vector x specifies bin centers

n = hist(y) Vector of number of points in each bin



### Error Function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

\*高斯函数是正态分布的密度函数

# Random Numbers

`v=rand` random value in  $[0,1]$

`rand(n)` an  $n \times n$  matrix with random values

`rand(r,c)` an  $r \times c$  matrix with random values

`X = rand(s,___)` `s`: randstreams

# Root Finding

`x = fzero(fun,x_0)`

`fun`: function to be solved

`x0`: initial guess at solution

`x`: solution

Function can be specified as

- a mathematical expression as a string with no predefined variables
- function handle
- name of an anonymous function

# sort

## 1D

`[y,I] = sort(x)`

`x` = input

`y` = sorted array

`I` = permutation index

## 2D

`y = sort(x,dim,mode)`

`dim`: selects a dimension along which to sort

`mode`: 'ascend' , 'descend'

# Vocabulary

- Scalars 标量
- portions 部分
- extraneous 无关的

- eye candy 视觉上具吸引力，但没什么实质内容的东西
- resolution 分辨率
- domain 范围，领域
- consecutive 连续的
- convolution 卷积
- interpolate 插值
- combinatorial 组合的