# 2025秋冬机器人技术与实践 实验五

## 1. 逆运动学解析解

利用 ${}^1T_5 = {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 = {}^0T_1^{-1} \cdot T \cdot {}^5T_6^{-1}$

其中, $c_1 = \cos(\theta_1), c_{23} = \cos(\theta_2 + \theta_3)$

$$
{}^1T_5 = {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 \cdot {}^4T_5 = \begin{bmatrix} -s_5c_{234} & s_{234} & c_5c_{234} & X \\ -s_5s_{234} & -c_{234} & c_5s_{234} & X \\ c_5 & 0 & s_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_1^{-1} \cdot T \cdot {}^5T_6^{-1} = \begin{bmatrix} (n_xc_1 + n_ys_1)c_6 - (o_xc_1 + o_ys_1)s_6 & s_6(n_xc_1 + n_ys_1) + c_6(o_xc_1 + o_ys_1) & X & X \\ -n_zc_6 + o_zs_6 & -n_zs_6 - o_zc_6 & X & X \\ c_6(n_yc_1 - n_xs_1) + s_6(o_xs_1 - o_yc_1) & s_6(n_yc_1 - n_xs_1) - c_6(o_xs_1 - o_yc_1) & -a_zs_1 + a_yc_1 & d_6(a_zs_1 - a_yc_1) - p_zs_1 + p_yc_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

可得 $d_6(a_zs_1 - a_yc_1) - p_zs_1 + p_yc_1 = d_4$ , 得到:

$$
(p_y - d_6a_y)c_1 - (p_z - d_6a_z)s_1 = d_4
$$

令 $m_1 = p_y - d_6a_y, n_1 = p_z - d_6a_z$ , 解得:

$$
\boxed{\theta_1 = \arctan 2(m_1, n_1) - \arctan 2(d_4, \pm\sqrt{m_1^2 + n_1^2 - d_4^2})}
$$

可得 $a_yc_1 - a_zs_1 = s_5$

得到:

$$
\boxed{\theta_5 = \arcsin(a_yc_1 - a_zs_1) \ \text{ or } \ \pi - \arcsin(a_yc_1 - a_zs_1)}
$$

$$
c_6(n_yc_1 - n_xs_1) + s_6(o_xs_1 - o_yc_1) = c_5
$$

$$
s_6(n_yc_1 - n_xs_1) - c_6(o_xs_1 - o_yc_1) = 0
$$

令 $m_6 = n_yc_1 - n_xs_1, n_6 = o_yc_1 - o_xs_1$ , 有:

$$
m_6c_6 - n_6s_6 = c_5
$$

$$
m_6s_6 + n_6c_6 = 0
$$

可得 $m_6^2 + n_6^2 = c_5^2$ , 得到:

$$
\boxed{\theta_6 = \arctan 2(m_6, n_6) - \arctan 2(c_5, \pm\sqrt{m_6^2 + n_6^2 - c_5^2}) = \arctan 2(m_6, n_6) - \arctan 2(c_5, 0)}
$$

$$
{}^1T_4 = {}^1T_2 \cdot {}^2T_3 \cdot {}^3T_4 = {}^0T_1^{-1} \cdot T \cdot {}^5T_6^{-1} \cdot {}^4T_5^{-1}
$$

$$
{}^1T_4 = \begin{bmatrix} c_{234} & s_{234} & 0 & a_3s_{23} + a_2s_2 \\ s_{234} & -c_{234} & 0 & -a_3c_{23} - a_2c_2 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_1^{-1} \cdot T \cdot {}^5T_6^{-1} \cdot {}^4T_5^{-1} = \begin{bmatrix} X & X & X & -d_5(s_6(n_xc_1 + n_ys_1) + c_6(o_xc_1 + o_ys_1)) - d_6(a_zc_1 + a_ys_1) + p_zc_1 + p_ys_1 \\ X & X & X & d_1 + a_zd_6 + d_5(o_zc_6 + n_zs_6) - p_z \\ X & X & X & X \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

又有:

$$
m_3 = -d_5(s_6(n_xc_1 + n_ys_1) + c_6(o_xc_1 + o_ys_1)) - d_6(a_zc_1 + a_ys_1) + p_zc_1 + p_ys_1 = a_3s_{23} + a_2s_2
$$

$$
n_3 = -d_1 - a_zd_6 - d_5(o_zc_6 + n_zs_6) + p_z = a_3c_{23} + a_2c_2
$$

可得, $m_3^2 + n_3^2 = a_3^2 + a_2^2 + 2a_2a_3(c_{23}c_2 + s_{23}s_2) = a_3^2 + a_2^2 + 2a_2a_3c_3$ , 得到:

$$
\boxed{\theta_3 = \pm \arccos\left(\frac{m_3^2 + n_3^2 - a_2^2 - a_3^2}{2a_2a_3}\right)}
$$

$$
\boxed{\theta_2 = \arctan 2(m_2m_3 - n_2n_3, n_2m_3 + m_2n_3)}
$$

$$
m_4 = s_{234} = (n_xc_1 + n_ys_1)s_6 + (o_xc_1 + o_ys_1)c_6
$$

$$
n_4 = c_{234} = n_zs_6 + o_zc_6
$$

得到:

$$
\boxed{\theta_4 = \arctan 2(m_4, n_4) - \theta_2 - \theta_3}
$$

# 2. 计算末端位姿参数对应的5组关节角

## IK逆运动学求解器

```
姿态 1 的解为:
[[ 1.04691599  0.5432342   0.53145444 -0.55151199  0.52390861  0.69854057]
 [ 1.04691599  1.05168993 -0.53145444  0.00294117  0.52390861  0.69854057]]
姿态 2 的解为:
[[ 1.57152579  0.46051711  0.6608339  -0.07451221  0.52363516  0.00132214]
 [ 1.57152579  1.09236785 -0.6608339   0.61530485  0.52363516  0.00132214]]
姿态 3 的解为:
[[ 0.63810904  0.78999757  1.34326212 -1.31694921 -0.01049602  0.01012841]]
姿态 4 的解为:
[[-0.06591846  0.82735639  0.82452922 -1.08013164  0.05459678 -0.03619458]]
姿态 5 的解为:
[[-0.7356519   1.10250941  1.05320703 -0.87536185  0.07485627 -0.01280543]]
```

## 解析解

姿态 1

[1.0469159881245618, 1.051315179254894, -0.5486274344503987, 0.02048989088368326, 0.4476705426936607, 0.6985405717445783]

[1.0469159881245618, 0.5264675685912314, 0.5486274344503987, -0.5519183494334516, 0.4476705426936607, 0.6985405717445783]

姿态 2

[1.5715257919759826, 1.060122555018591, -0.6303737057933186, 0.6170899442006842, 0.3521271666862505, 0.0013221366719161028

[1.5715257919759826, 0.45730114686717105, 0.6303737057933186, -0.04083605923453304, 0.3521271666862505, 0.001322136671916

姿态 3

[0.638109037379833, 0.7899963428791525, 1.3432633260992533, -1.3169491925111272, -0.01023672833574483, 0.01012840925262248

姿态 4

[-0.06591845703403737, 0.8273564118138991, 0.8245292269940321, -1.080131665407864, 0.05459672952874233, -0.03619458095722

姿态 5

[-0.7356518980607447, 1.102416736502623, 1.053276432244526, -0.8753385735468382, 0.07175868323524985, -0.0128054296748054

# 仿真验证

## 姿态一

### IK逆运动学求解器

x: +0.11698 y: +0.33417 z: +0.49866
a: -115.765 b: -3.329 g: -125.473
x: +0.11693 y: +0.33427 z: +0.49852
a: -115.803 b: -3.303 g: -125.494

### 解析解

x: +0.12197 y: +0.33128 z: +0.49866
a: -116.856 b: +0.899 g: -125.496
x: +0.12192 y: +0.33138 z: +0.49852
a: -116.894 b: +0.927 g: -125.516

## 姿态二

### IK逆运动学求解器

x: -0.06608 y: +0.33903 z: +0.44367
a: -150.095 b: -30.008 g: -179.997
x: -0.06611 y: +0.33912 z: +0.44353
a: -150.11 b: -29.954 g: -179.994

### 解析解

x: -0.05282 y: +0.33904 z: +0.44368
a: -150.081 b: -20.183 g: -179.991
x: -0.05284 y: +0.33912 z: +0.44355
a: -150.095 b: -20.131 g: -179.988

## 姿态三

### IK逆运动学求解器

```
x: +0.30004   y: +0.24992   z: +0.25964
a: -151.346   b: +33.759    g: -134.622
```

### 解析解

```
x: +0.30003   y: +0.24994   z: +0.25964
a: -151.333   b: +33.749    g: -134.629
```

## 姿态四

### IK逆运动学求解器

```
x: +0.42009   y: 0.000    z: +0.35956
a: +179.904   b: +57.199   g: -89.975
```

### 解析解

```
x: +0.42009   y: 0.000    z: +0.35956
a: +179.904   b: +57.199   g: -89.975
```

## 姿态五

### IK逆运动学求解器

```
x: +0.31979   y: -0.250   z: +0.15954
a: +171.943   b: +15.075   g: -48.148
```

### 解析解

```
x: +0.31962   y: -0.25019   z: +0.15954
a: +171.808   b: +14.955    g: -48.113
```