
James Clerk Maxwell
1831 – 1879

补充

Electromagnetic Fields and Waves

传输线例题

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课堂练习

1）短路传输线，当线长度小于四分之一波长的时候，相当于(　)

A．电阻　　B. 电感

C．电容　　D. 电阻、电感、电容 都有

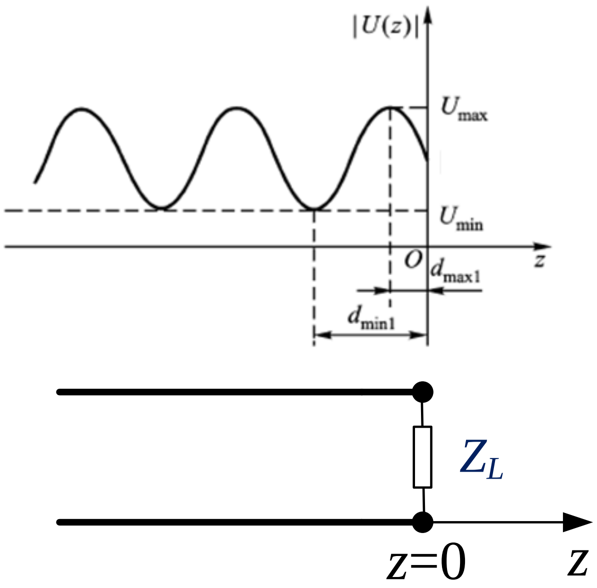
课堂练习

2）下图所示为传输线上电压的驻波分布，判别负载Z_L是什么性质的阻抗?

（ ）

A．纯电阻　　B. 电阻、电容都有

C．纯电抗　　D. 电阻、电感都有



3）特征阻抗为Z_0的均匀无耗传输线上传输行驻波，驻波系数为ρ，其电压波腹点处的阻抗为（   ），电流波腹点的阻抗为（   ）

A. Z_0 **B.** ρ/Z_0 **C.** ρZ_0 **D.** Z_0/ρ

4. 传输线特征阻抗为50Ω，电压为 $U(z)=10\mathrm{e}^{-\mathrm{j}kz}+5\mathrm{e}^{\mathrm{j}kz}$ **，则电流** $I(z)$ **为（ ）：**

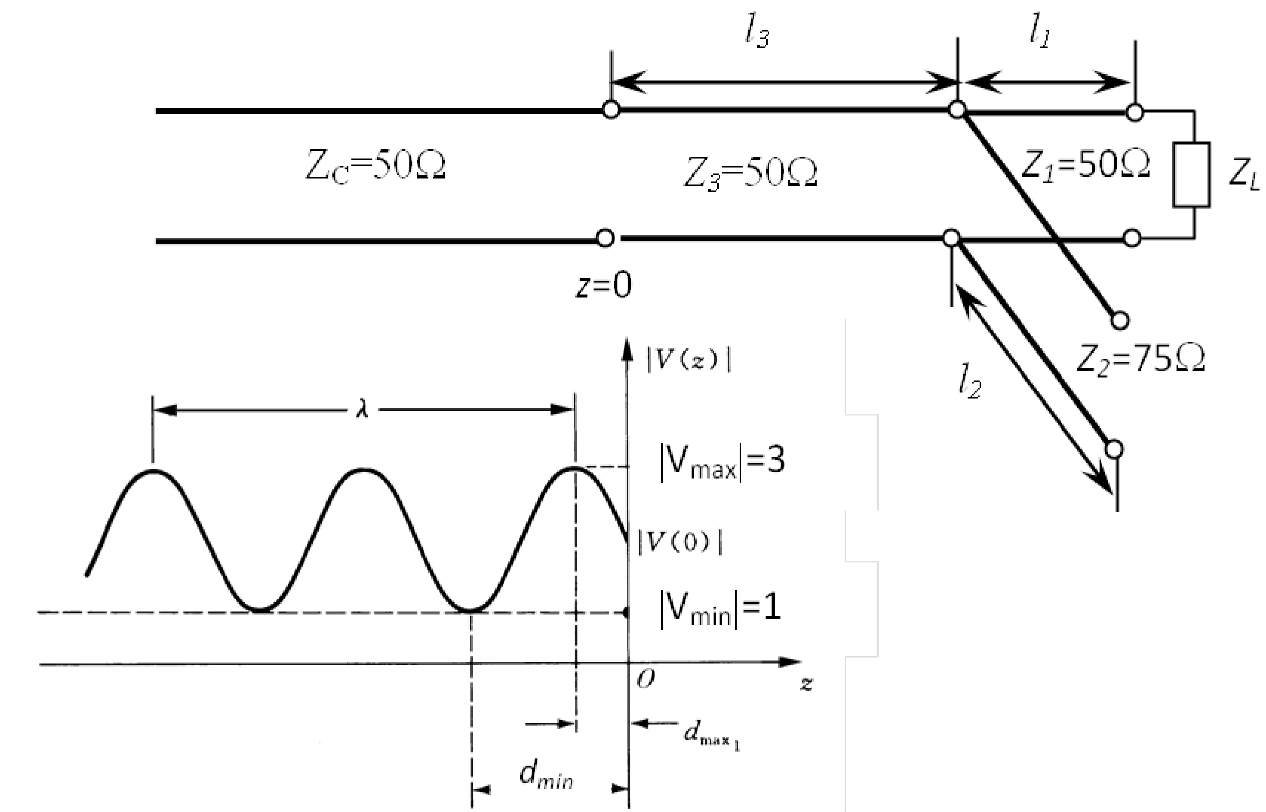
A. $0.2\mathrm{e}^{-\mathrm{j}kz}-0.1\mathrm{e}^{\mathrm{j}kz}$

B. $0.2\mathrm{e}^{-\mathrm{j}kz}+0.1\mathrm{e}^{\mathrm{j}kz}$

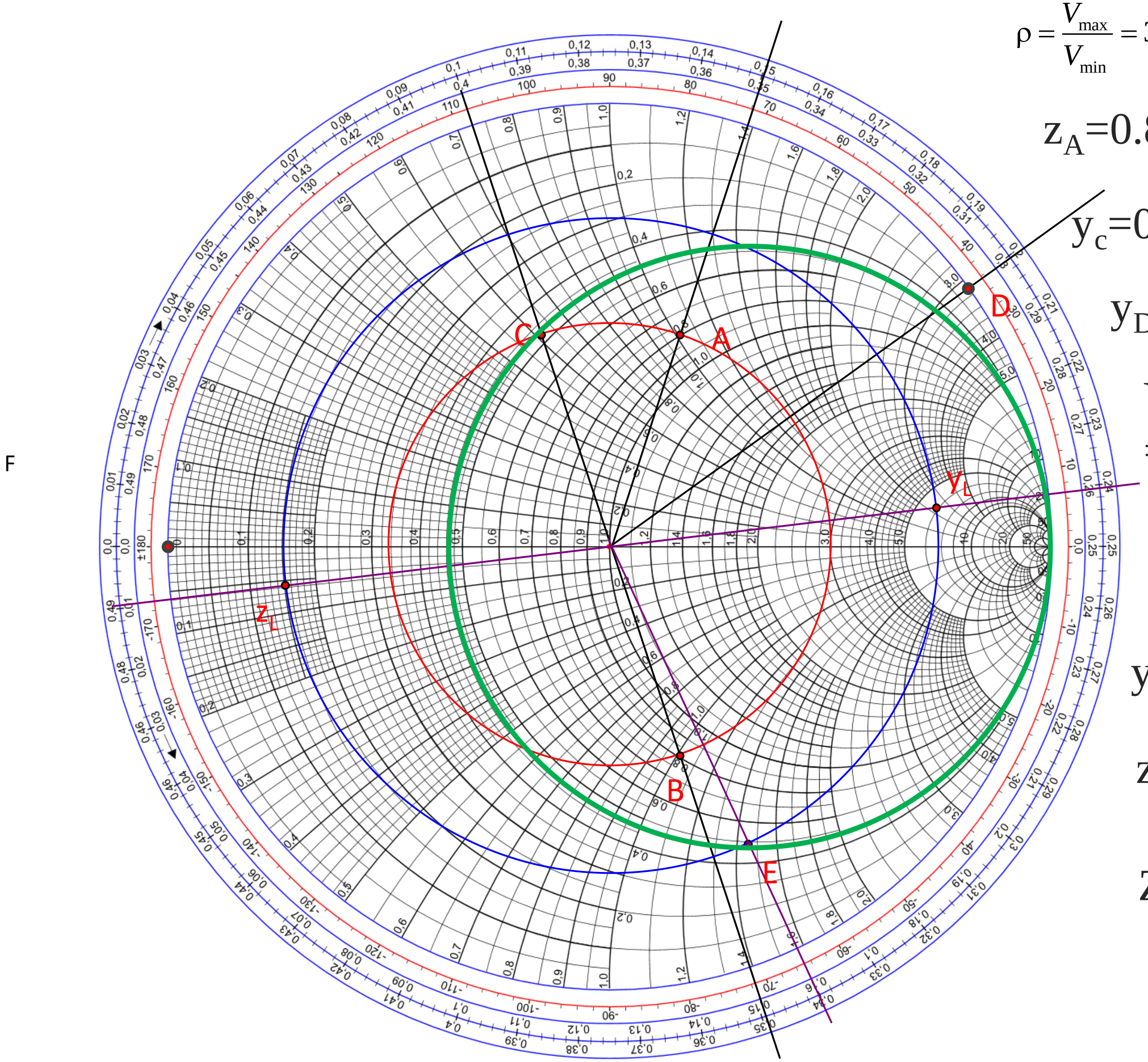
C. $0.1\mathrm{e}^{-\mathrm{j}kz}-0.2\mathrm{e}^{\mathrm{j}kz}$

D. $0.1\mathrm{e}^{-\mathrm{j}kz}+0.2\mathrm{e}^{\mathrm{j}kz}$

❖ 已知λ=10cm，l_1=1cm, l_2=2cm, l_3=3cm，在z=0输入端口左边传输线上的电压传播如下图所示，求终端负载Z_L=?



$d_{\min} = 0.35\lambda$



$$\rho = \frac{V_{\max}}{V_{\min}} = 3, d_{\min} = 0.35\lambda \Rightarrow |\Gamma| = 0.5, \varphi = 0.4\pi$$

$z_A=0.8+j$ $z_B=0.8-j$

$y_c=0.48+0.61j$

$y_D = 3.07j$

$Y_{L1} = y_c \times Y_3 - 3.07j \times Y_2 = 0.0096-0.0287j$

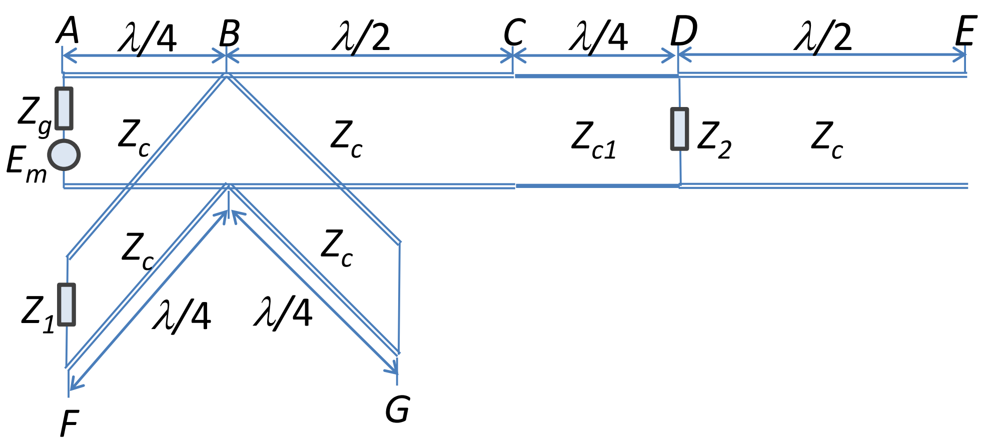
$y_{L1} = 0.48-1.435j$ E点

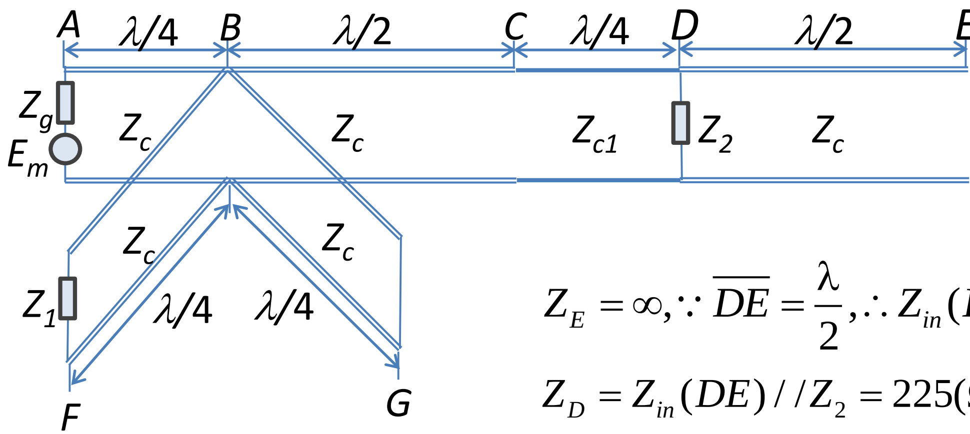
$y_L = 5.7615 + 2.3006j$

$z_L=0.15-0.06j$

$Z_L=7.5-3j$

由若干段传输线和负载组成的电路如下图所示，λ为波长。已知，$Z_g = Z_c = Z_1 = 100\Omega$，$Z_{c1} = 150\Omega$，$Z_2 = 225\Omega$，$|E_m| = 50\text{mV}$。试分析$AB$、$BC$、$CD$、$DE$、$BF$、$BG$各段传输线的工作状态，并计算各段传输线的始段、末端电压放电流振幅，画图示意沿各段传输线电压主、电流振幅的相对分布。





$$Z_E=\infty,\because \overline{DE}=\frac{\lambda}{2},\therefore Z_{in}(DE)=\infty,$$

DE段纯驻波

$$Z_D=Z_{in}(DE)//Z_2=225(\Omega)$$

CD段行驻波

$$\because \overline{CD}=\frac{\lambda}{4},\therefore Z_{in}(CD)=\frac{Z_{c1}^2}{Z_D}=100(\Omega)$$

$$Z_C=Z_{in}(CD)=100(\Omega)$$

BC段行波

$$\overline{BC}=\frac{\lambda}{2},\therefore Z_{in}(BC)=Z_{in}(CD)=100(\Omega)$$

$$\because Z_F=Z_1=Z_0,Z_{in}(BF)=Z_0=100(\Omega)$$

BF段行波

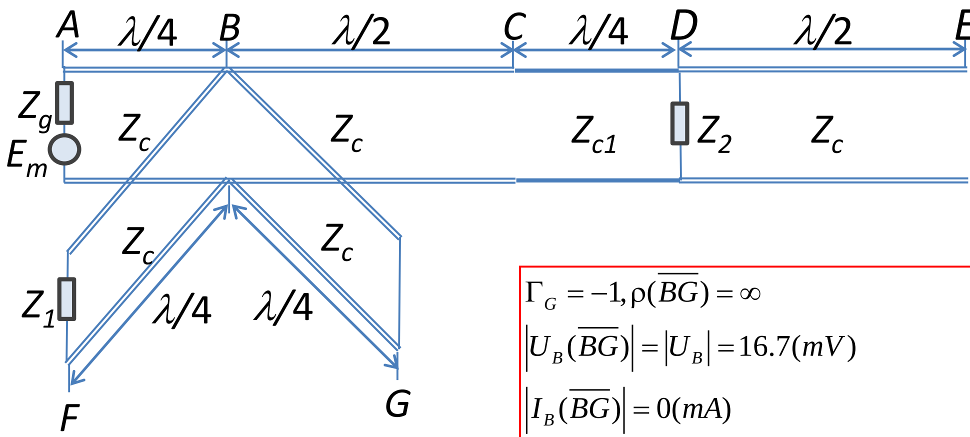
$$\because \overline{BG}=\frac{\lambda}{4},Z_G=0,Z_{in}(BG)=\infty$$

BG段纯驻波

$$Z_B=Z_{in}(BC)//Z_{in}(BF)//Z_{in}(BG)=50(\Omega)$$

AB段行驻波

$$\because \overline{AB}=\frac{\lambda}{4},Z_A=\frac{Z_0^2}{Z_B}=200(\Omega)$$



$$Z_A = 200(\Omega); Z_B = 50(\Omega)$$
$$Z_C = 100(\Omega); Z_D = 225(\Omega)$$
$$Z_E \to \infty$$

$$|U_A| = \frac{Z_A}{Z_A + Z_g}|E_m| = 33.4(mV)$$
$$|I_A| = \frac{|U_A|}{Z_A} = 0.167(mA)$$
$$\Gamma_B = \frac{Z_B - Z_0}{Z_B + Z_0} = -\frac{1}{3}, \rho(\overline{AB}) = 2$$
$$|U_B| = \frac{|U_A|}{\rho(\overline{AB})} = 16.7(mV)$$
$$|I_B| = \rho(\overline{AB})|I_A| = 0.334(mA)$$

$$\Gamma_G = -1, \rho(\overline{BG}) = \infty$$
$$|U_B(\overline{BG})| = |U_B| = 16.7(mV)$$
$$|I_B(\overline{BG})| = 0(mA)$$
$$|U_i(\overline{BG})| = \frac{|U_B|}{2} = 8.35(mV)$$
$$|I_i(\overline{BG})| = \frac{|U_i(\overline{BG})|}{Z_0} = 0.0835(mA)$$
$$|U_G(\overline{BG})| = 0(mV)$$
$$|I_G(\overline{BG})| = (1-\Gamma_G)|I_i(\overline{BG})| = 0.167(mA)$$

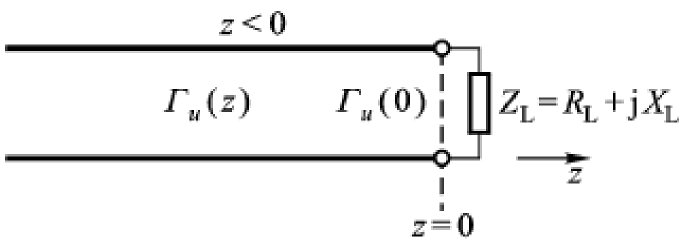
$$\Gamma_D = \frac{Z_D - Z_{01}}{Z_D + Z_{01}} = 0.2, \rho(\overline{CD}) = 1.5$$
$$|U_C(\overline{CD})| = |U_C| = 16.7(mV)$$
$$|I_C(\overline{CD})| = \left|\frac{U_C(\overline{CD})}{Z_C}\right| = 0.167(mA)$$
$$|U_D(\overline{CD})| = |U_C(\overline{CD})| \times \rho(\overline{CD}) = 25(mV)$$
$$|I_D(\overline{CD})| = \frac{|U_D(\overline{CD})|}{Z_D} = \frac{|I_C(\overline{CD})|}{\rho(\overline{CD})} = 0.111(mA)$$

$$\Gamma_F = 0, \rho(\overline{BF}) = 1$$
$$|U_B(\overline{BF})| = |U_i(\overline{BF})| = |U_B| = |U_F(\overline{BF})| = 16.7(mV)$$
$$|I_B(\overline{BF})| = |I_i(\overline{BF})| = |I_F(\overline{BF})| = 0.167(mA)$$

$$|U_B(\overline{BC})| = |U_i(\overline{BC})| = |U_B| = 16.7(mV)$$
$$|I_B(\overline{BC})| = |I_i(\overline{BC})| = \left|\frac{U_i(\overline{BC})}{Z_0}\right| = 0.167(mA)$$
$$|U_C(\overline{BC})| = |U_i(\overline{BC})| = 16.7(mV)$$
$$|I_C(\overline{BC})| = |I_i(\overline{BC})| = 0.167(mA)$$

$$\Gamma_E = 1, \rho(\overline{BG}) = \infty$$
$$|U_D(\overline{DE})| = |U_D| = 25(mV)$$
$$|I_D(\overline{DE})| = \frac{|U_D(\overline{DE})|}{\infty} = 0(mA)$$
$$|U_E(\overline{DE})| = |U_D(\overline{DE})| = 25(mV)$$
$$|I_E(\overline{DE})| = |I_D(\overline{DE})| = 0(mA)$$

传输线上传输的功率



❖ 传输线上传输的功率可按下式计算

$$P(z)=\frac{1}{2}\mathrm{Re}\left[U(z)\cdot I^*(z)\right]$$

❖ $U(z)$、$I(z)$由入射波、反射波两项构成

$$P(z)=\frac{1}{2}\mathrm{Re}\left[U^{\mathrm{i}}(1+\varGamma_u(z))\cdot\frac{U^{\mathrm{i}*}}{Z_{\mathrm{c}}^*}(1-\varGamma_u^*(z))\right]$$

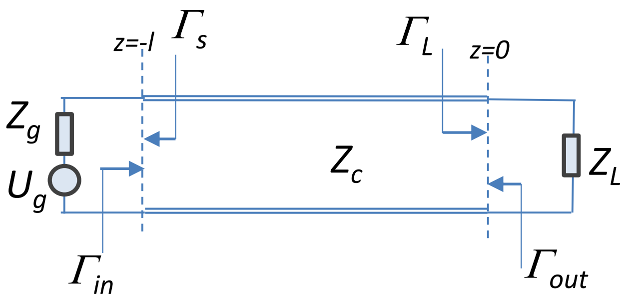
$$=\frac{1}{2}\mathrm{Re}\left[\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}^*}-\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}^*}|\varGamma_u(z)|^2+\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}^*}\left(\varGamma_u(z)-\varGamma_u^*(z)\right)\right]$$

❖ 对于无损耗传输线，Z_c是实数，则上式第三项等于零。 $|\varGamma_u|$为常数

❖ 所以$P(z)=P$，不随位置而变

$$P=\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}-\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}|\varGamma_u|^2=P^{\mathrm{i}}-P^{\mathrm{r}}\qquad P^{\mathrm{i}}=\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}$$

❖ 传输线上任一点功率等于入射波功率与反射波功率之差，而且 $\frac{P^{\mathrm{r}}}{P^{\mathrm{i}}}=|\varGamma_u|^2$



$$P=\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}-\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}|\Gamma_u|^2=P^{\mathrm{i}}-P^{\mathrm{r}}$$

$$P=\frac{1}{2}\frac{|U^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}(1-|\Gamma_{in}|^2)$$

$$U^{\mathrm{i}}=\frac{U_{in}}{1+\Gamma_{in}}=\frac{U_g}{1+\Gamma_{in}}\left(\frac{Z_{in}}{Z_{in}+Z_g}\right)$$

$$Z_{in}=Z_c\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\qquad Z_g=Z_c\frac{1+\Gamma_s}{1-\Gamma_s}$$

$$U^{\mathrm{i}}=\frac{U_g}{2}\frac{(1-\Gamma_s)}{(1-\Gamma_s\Gamma_{in})}$$

$$P=\frac{1}{8}\frac{\left|U_g\right|^2}{Z_{\mathrm{c}}}\frac{\left|1-\Gamma_s\right|^2}{\left|1-\Gamma_s\Gamma_{in}\right|^2}(1-|\Gamma_{in}|^2)$$

$$\boxed{P=\frac{1}{8}\frac{\left|U_g\right|^2}{Z_{\mathrm{c}}}\frac{\left|1-\Gamma_s\right|^2}{\left|1-\Gamma_s\Gamma_L e^{-j2kl}\right|^2}(1-|\Gamma_L e^{-j2kl}|^2)}$$

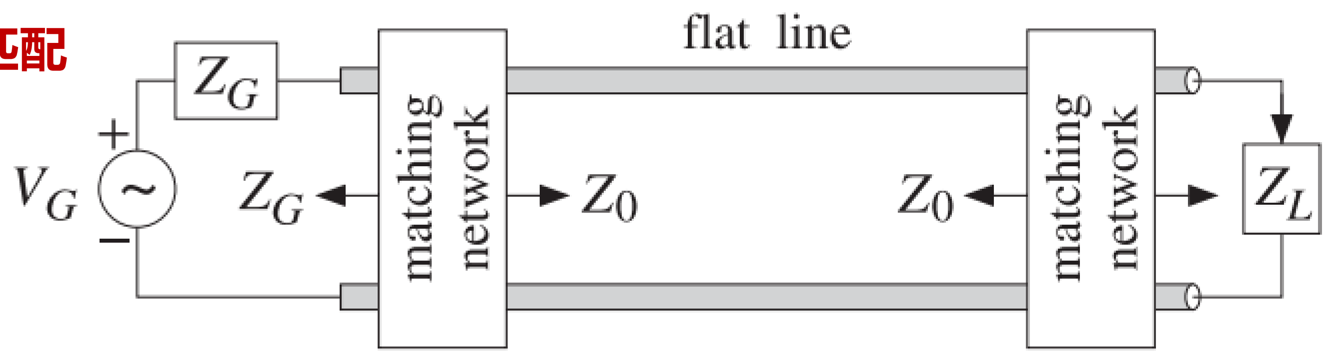
$$\boxed{\Gamma_{in}=\Gamma_L e^{-j2kl}}$$

$$\boxed{P_L=\frac{1}{2}\frac{|U_L^{\mathrm{i}}|^2}{Z_{\mathrm{c}}}(1-|\Gamma_L|^2)\qquad \left|U_L^{\mathrm{i}}\right|=\left|U_{in}^{\mathrm{i}}\right|e^{-\alpha l}\qquad P=\frac{1}{8}\frac{\left|U_g\right|^2}{Z_{\mathrm{c}}}\frac{\left|1-\Gamma_s\right|^2}{\left|1-\Gamma_s\Gamma_{in}\right|^2}e^{-2\alpha l}(1-|\Gamma_L|^2)}$$

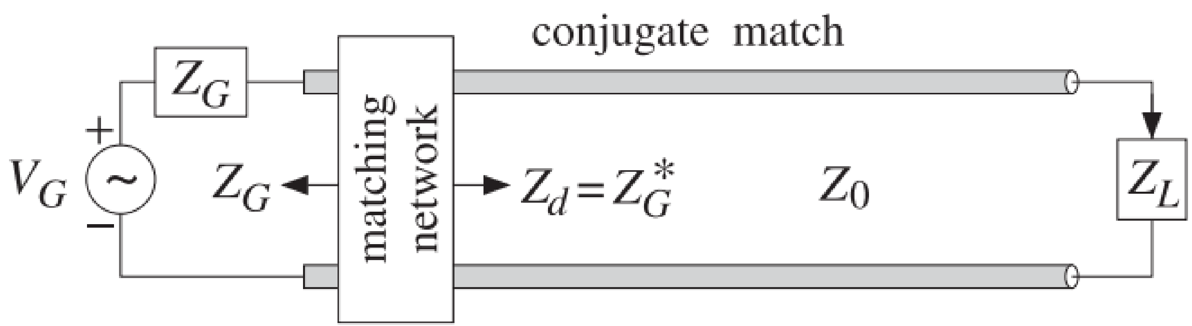
匹配网络

- 广义上说阻抗匹配并不仅限于实现无反射传输而在源和负载之间进行阻抗变换，实际的匹配网络除了减少功率损耗外，还有其他功能，减少噪声干扰，提供功率容量，提供频率响应的线性度等。
- 匹配网络的作用就是实现阻抗变换，将给定的阻抗值变换为更合适的阻抗值。
- 用集中参数电路元件时，电抗元件与复数阻抗串联在复数阻抗对应点的等电阻圆上移动，并联则在对应点的等电导圆移动

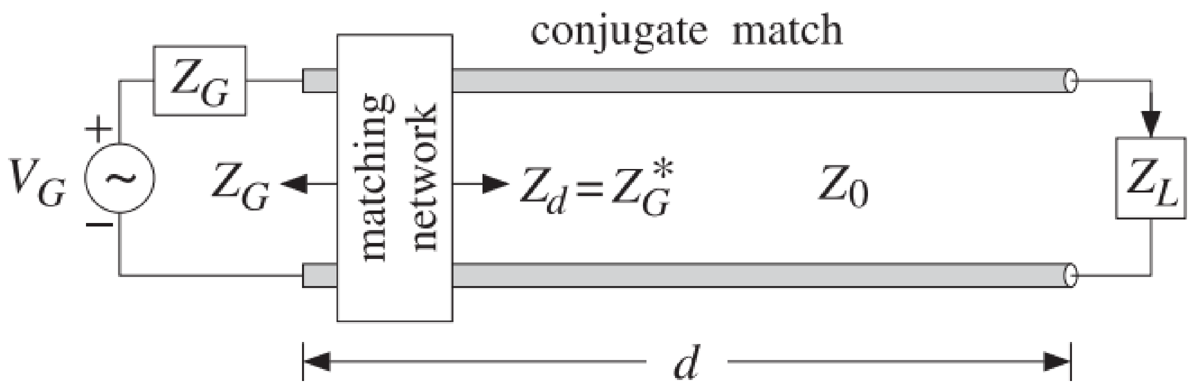
阻抗匹配



In the first, referred to as a *flat line*, both the generator and the load are matched so that effectively, $Z_G = Z_L = Z_0$. There are no reflected waves and the generator (which is typically designed to operate into Z_0) transmits maximum power to the load, as compared to the case when $Z_G = Z_0$ but $Z_L \neq Z_0$.



In the second case, the load is connected to the line without a matching circuit and the generator is conjugate-matched to the input impedance of the line, that is, $Z_d = Z_G^*$. As we mentioned above, the line remains conjugate matched everywhere along its length, and therefore, the matching network can be inserted at any convenient point, not necessarily at the line input.



$$\boxed{\Gamma_d = \Gamma_L e^{-2j\beta d} = \Gamma_G^*} \Leftrightarrow \boxed{Z_d = Z_G^*} \quad \text{(conjugate match)}$$

$$\boxed{\Gamma_l = \Gamma_L e^{-2j\beta l} = \Gamma_G^* e^{2j\beta(d-l)}} \quad \text{(conjugate match)}$$

The End.

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