电子电路基础

第八讲~磁耦合电路

课程纲要

- 6.1 磁耦合基本概念
  - 6.1.1 互感的概念及其计算
  - 6.1.2 具有互感的电感的伏安特性
  - 6.1.3 具有互感的电感元件的串并联
- 6.2 互感耦合电路分析
  - （运用电路定理，分析计算包含互感线圈的电路的参数）
- 6.3 变压器
  - 6.3.1 变压器的原理和结构
  - 6.3.2 包含变压器的电路的分析计算（只考虑理想变压器）

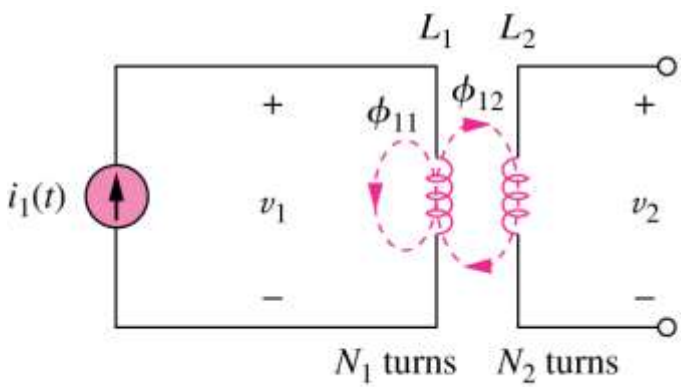
Magnetically Coupled Circuit Chapter 13

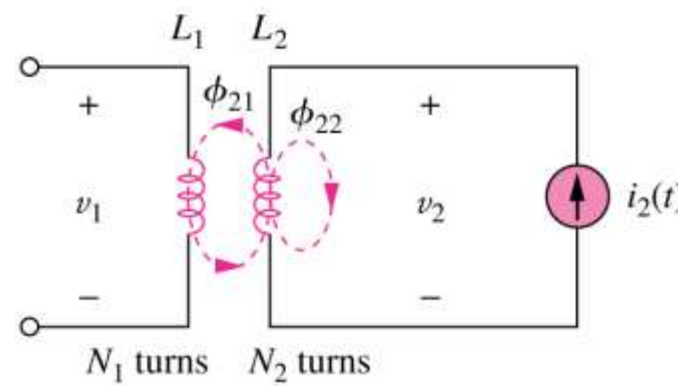
13.1 What is a transformer 变压器?

- It is an electrical device designed on the basis of the concept of magnetic coupling 基于磁耦合原理的电气元件
- It uses magnetically coupled coils to transfer energy from one circuit to another 利用磁耦合线圈在电路间传输能量
- It is the key circuit elements for stepping up or stepping down **ac** voltages or currents, impedance matching, isolation, etc. 主要应用：交流电压电流的升高或降低、阻抗匹配、隔离

13.2 Mutual Inductance 互感

- It is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H). **互感**：电感对临近电感产生感应电压降的能力；**自感**：相应的，电感自身的电感量就称为自感；





只有变化的电流才能产生互感电压

$$v_2 = M_{21}\frac{di_1}{dt}$$

注意M的下标顺序

$$M_{12} = M_{21} = M$$

$$v_1 = M_{12}\frac{di_2}{dt}$$

The open-circuit mutual voltage across coil 2

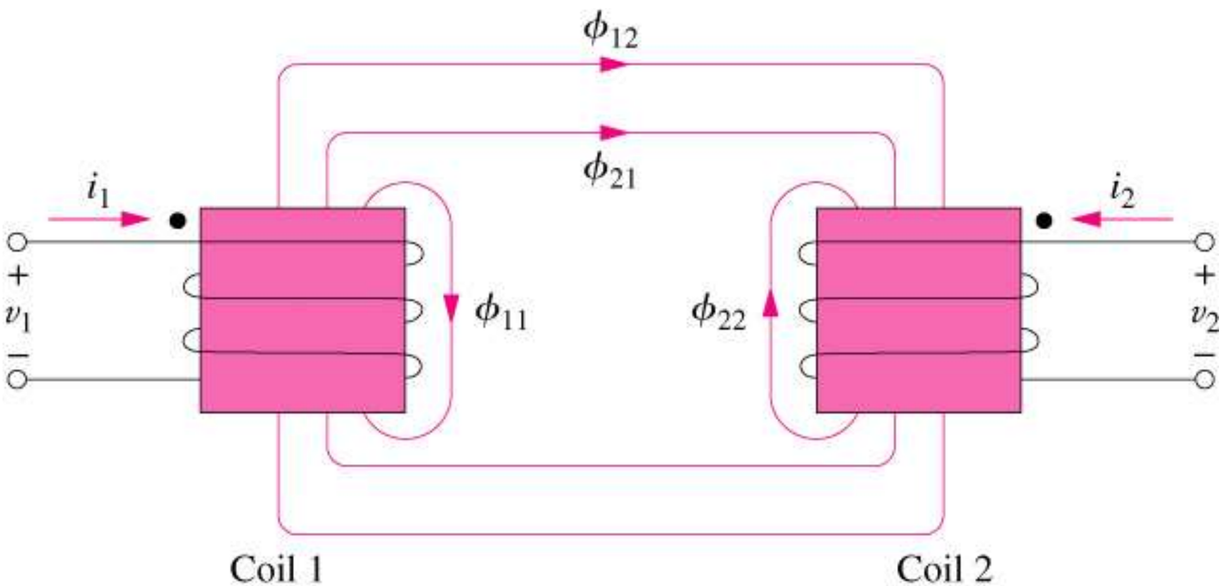
线圈1上的电流变化，在线圈2上所产生的开路电压

The open-circuit mutual voltage across coil 1

线圈2上的电流变化，在线圈1上所产生的开路电压

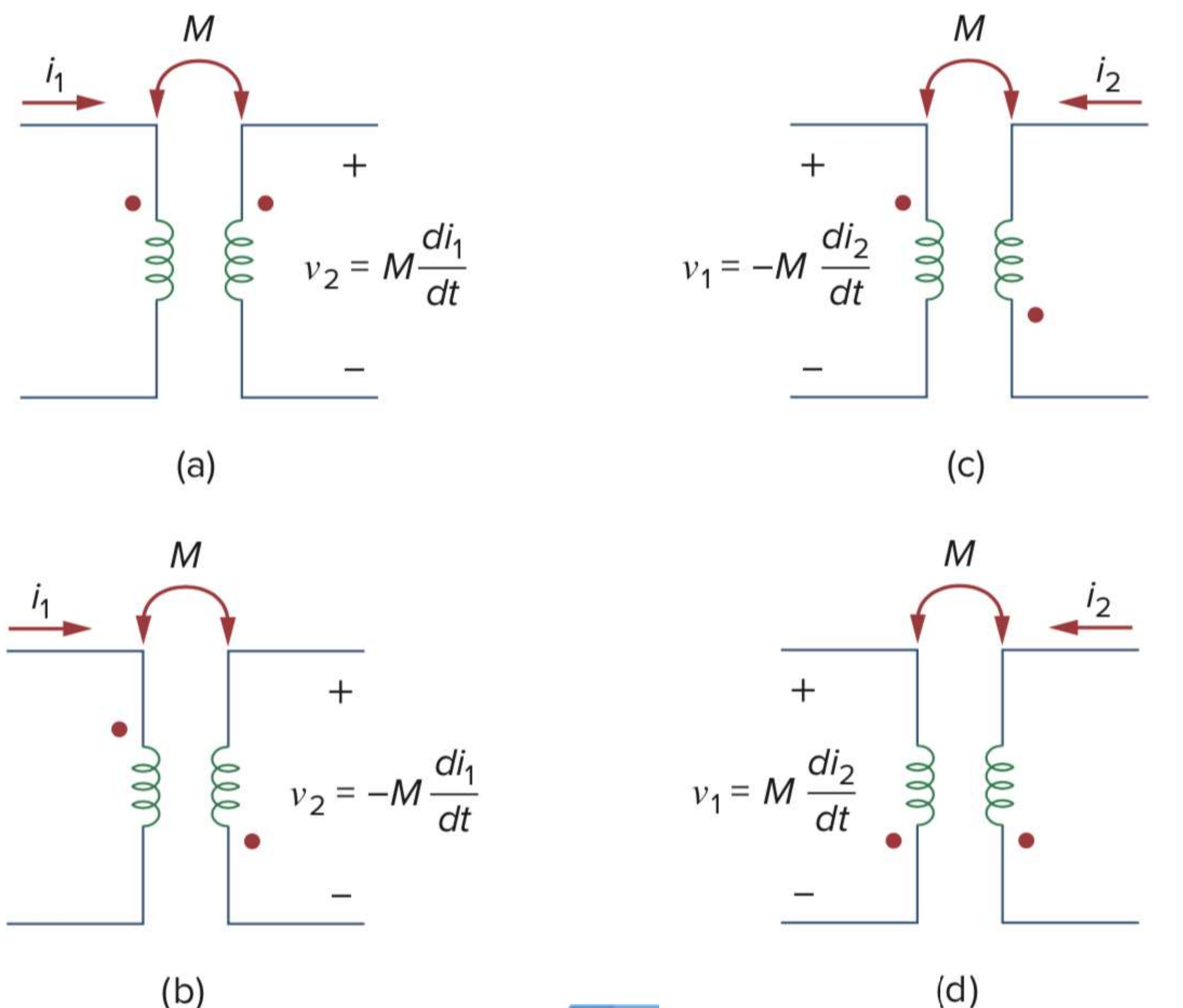
13.2 Mutual Inductance

- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.
- **同名端**（一对同标记端）：如果电流从某一线圈的**标记端流入**，那么其在另一线圈所产生的感应电压，**正极**在相应的**标记端**；



为何需要同名端：因为线圈可以顺时针绕，也可以逆时针绕

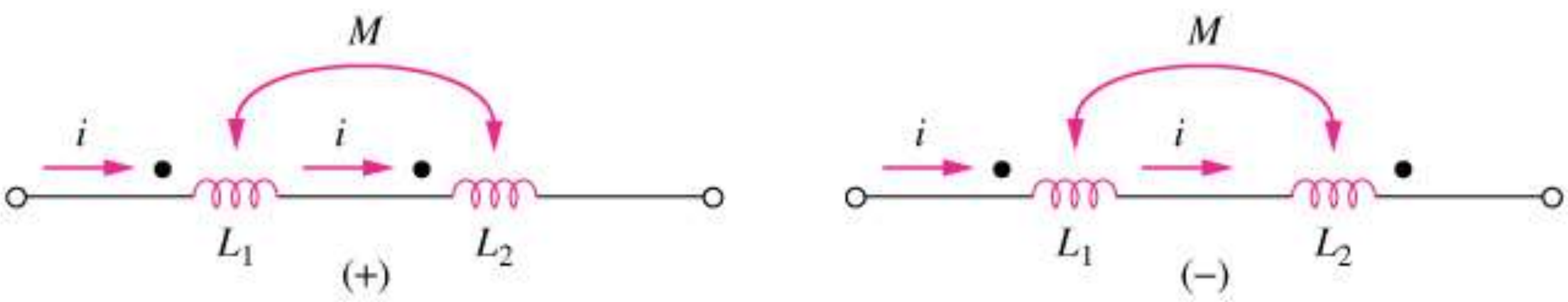
Illustration of the **dot convention**.



注意：未标红点的一对端点，也可以看作是一对同名端

13.2 Mutual Inductance

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage; (a) series-aiding connection, (b) series-opposing connection.



(a) (b)

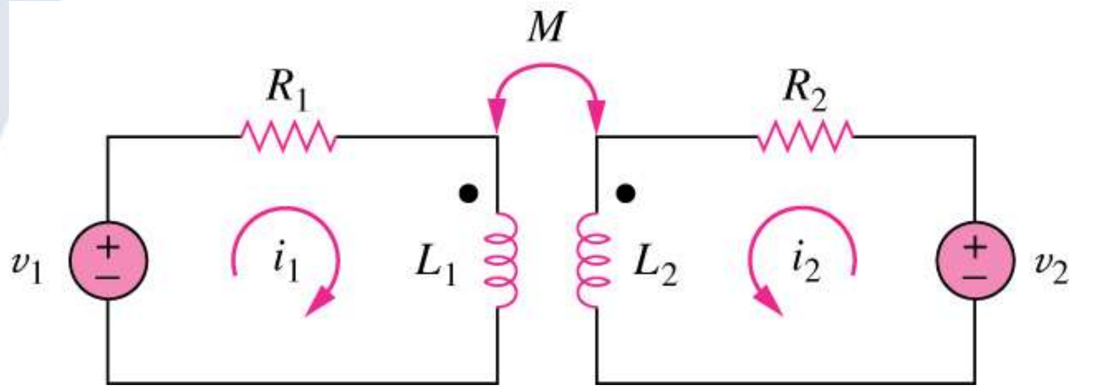
$L = L_1 + L_2 + 2M$ (Series-aiding connection)

$L = L_1 + L_2 - 2M$ (Series-opposing connection)

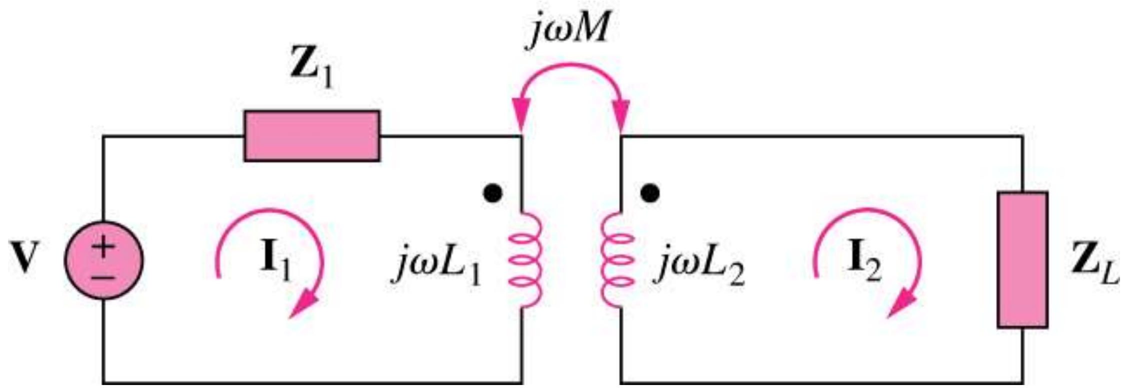
同向串联：电流都从标记端流入，感应电压与自身电压相加 → 电感加强

反向串联：电流从某一标记端流入，从另一标记端流出，感应电压与自身电压相减 → 电感减弱

13.2 Mutual Inductance



Time-domain analysis of a circuit containing coupled coils.



Frequency-domain analysis of a circuit containing coupled coils

等效电路：
电流从标记端**流入**→ 在另一侧相应的标记端**同向串联**感应电压源（标红点是一对同名端，未标红点的也是一对同名端）

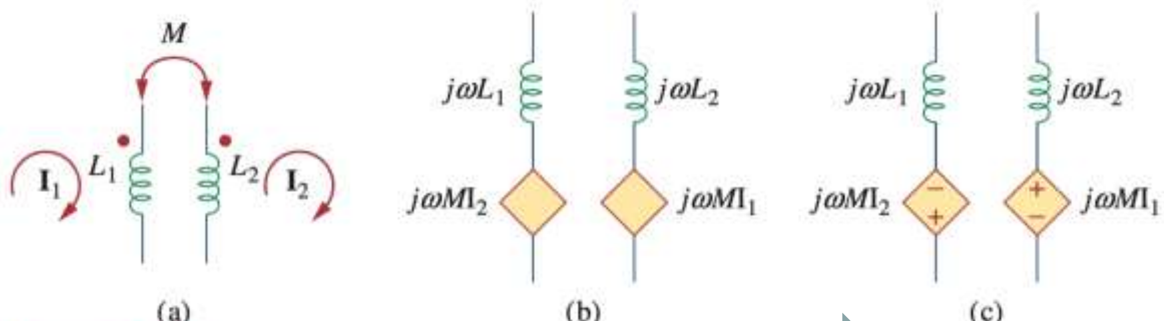


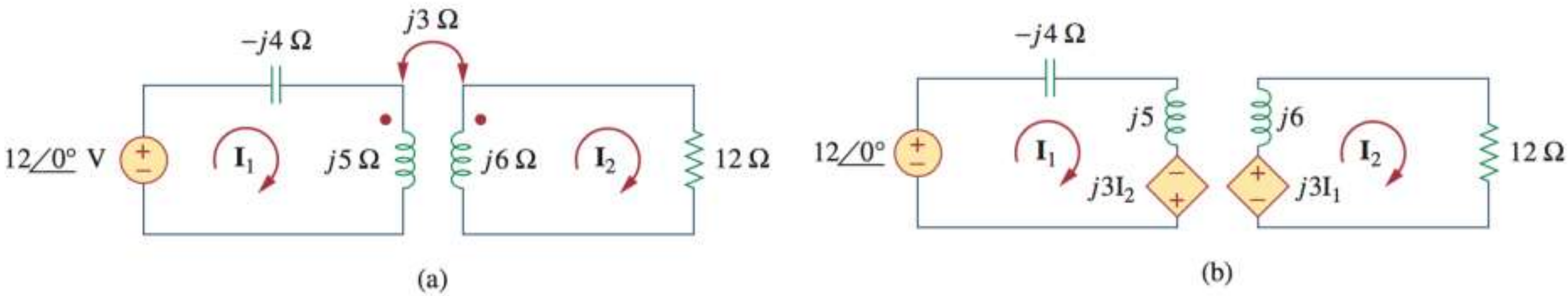
Figure 13.8
Model that makes analysis of mutually coupled easier to solve.

去耦

根据同名端的定义

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.9.

Example 13.1



注意：I_1和I_2不是相邻网孔

For loop 1, KVL gives

$$-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$$

For loop 2, KVL gives

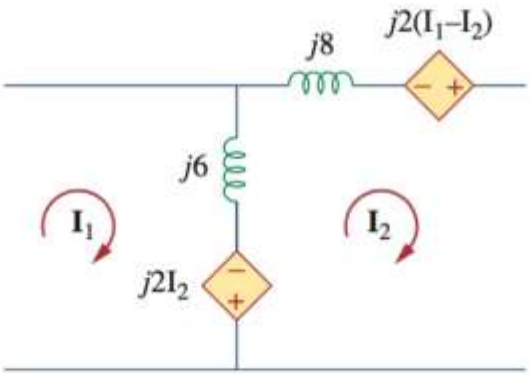
$$-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$$

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\angle 14.04°\ \text{A}$$

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\angle -63.43°)(2.91\angle 14.04°) = 13.01\angle -49.39°\ \text{A}$$

Example 13.2

Calculate the mesh currents in the circuit of Fig. 13.11.



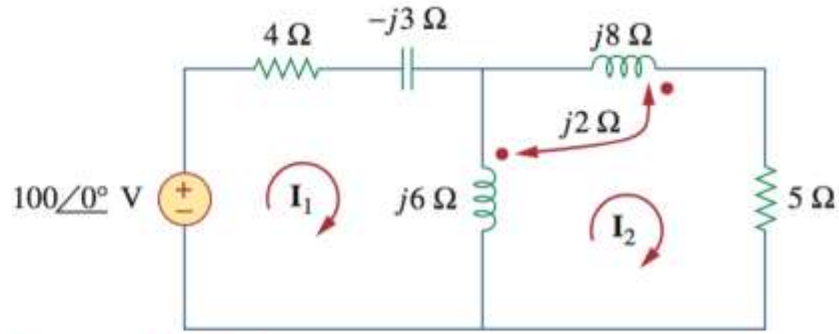


Figure 13.11
For Example 13.2.

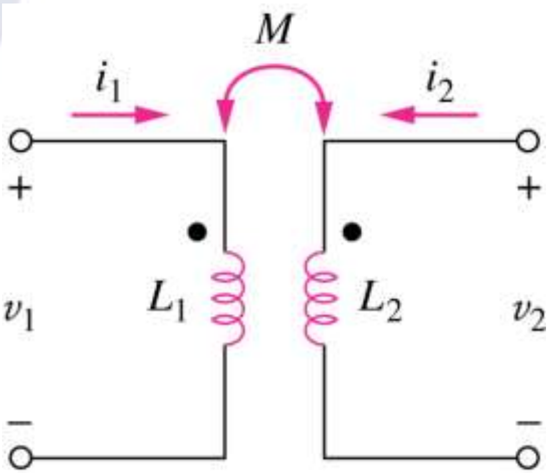
先分析流过两个电感的电流分别是多少

$$-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$$

$$0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$$

13.3 Energy in a Coupled Circuit

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \le k \le 1$. 磁耦合的程度，用耦合系数 k 表示



$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\boxed{M = k\sqrt{L_1 L_2}}$$

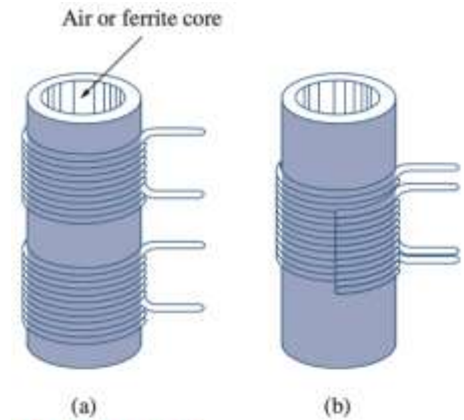


Figure 13.15
Windings: (a) loosely coupled, (b) tightly coupled; cutaway view demonstrates both windings.

- The instantaneous energy stored in the circuit is given by 互感线圈的总储能（瞬时值）

$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

电流都从同名端流入（或流出），则为“＋”；一进一出则为“－”；

Example 13.3

Consider the circuit in Fig. 13.16. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time $t = 1$ s if $v = 60\cos(4t + 30^\circ)$ V.

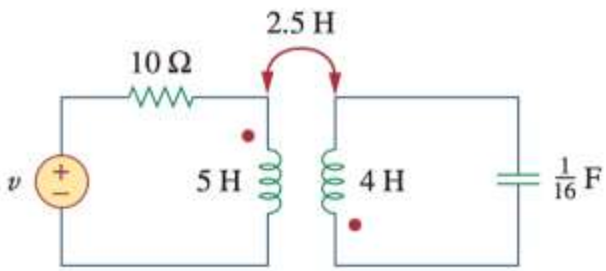
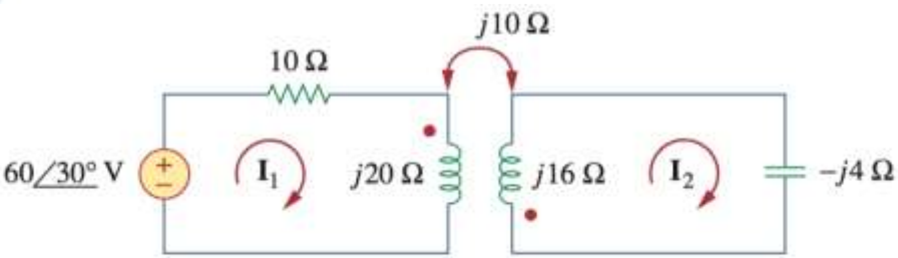


Figure 13.16

Solution:

The coupling coefficient is

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$



$$\begin{aligned}
60\cos(4t + 30^\circ) \quad &\Rightarrow \quad 60\angle 30^\circ, \quad \omega = 4 \text{ rad/s} \\
5\text{ H} \quad &\Rightarrow \quad j\omega L_1 = j20\ \Omega \\
2.5\text{ H} \quad &\Rightarrow \quad j\omega M = j10\ \Omega \\
4\text{ H} \quad &\Rightarrow \quad j\omega L_2 = j16\ \Omega \\
\frac{1}{16}\text{ F} \quad &\Rightarrow \quad \frac{1}{j\omega C} = -j4\ \Omega
\end{aligned}$$

For mesh 1,

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\angle 30^\circ$$

For mesh 2,

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

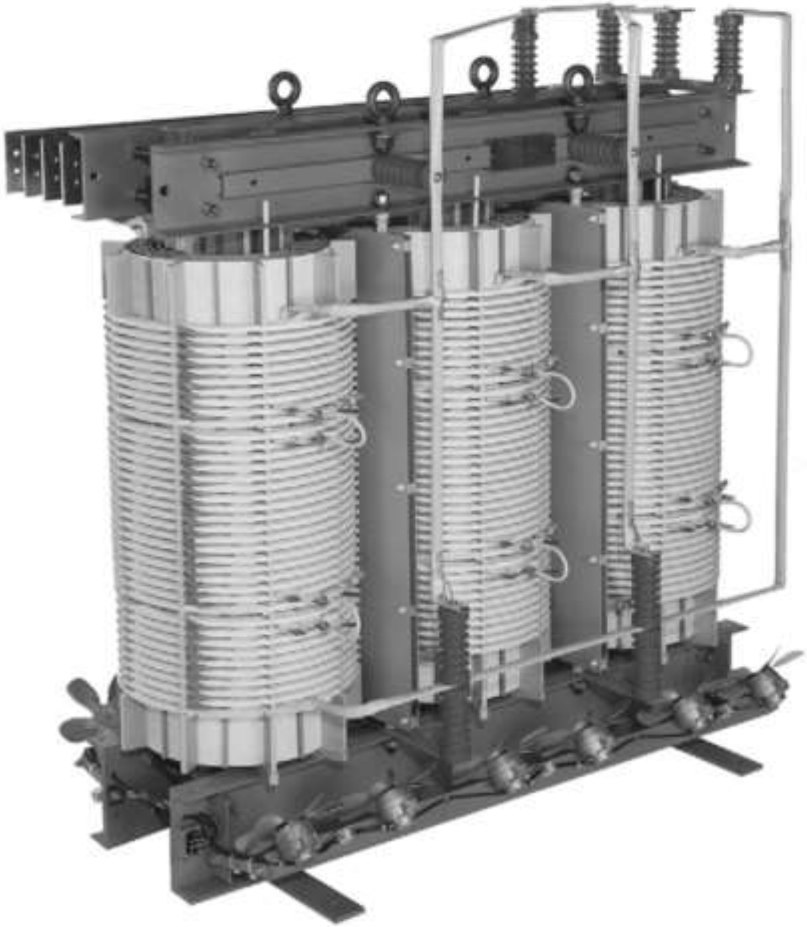
$$\mathbf{I}_2 = 3.254\angle 160.6^\circ \text{ A}$$

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\angle -19.4^\circ \text{ A}$$

$$i_1 = 3.905\cos(4t - 19.4^\circ)$$

$$i_2 = 3.254\cos(4t + 160.6^\circ)$$

$$\begin{aligned}
w &= \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 \\
&= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}
\end{aligned}$$



(a)

Figure 13.20

Different types of transformers: (a) copper wound dry power transforme

Courtesy of: (a) Electric Service Co., (b) Jensen Transformers.

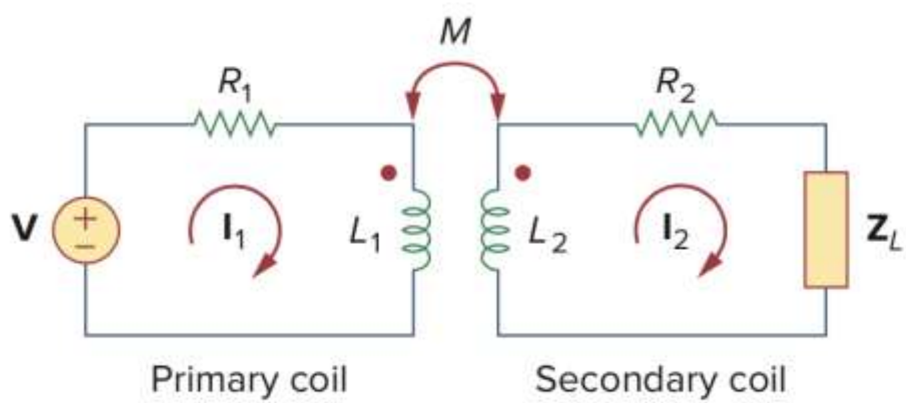
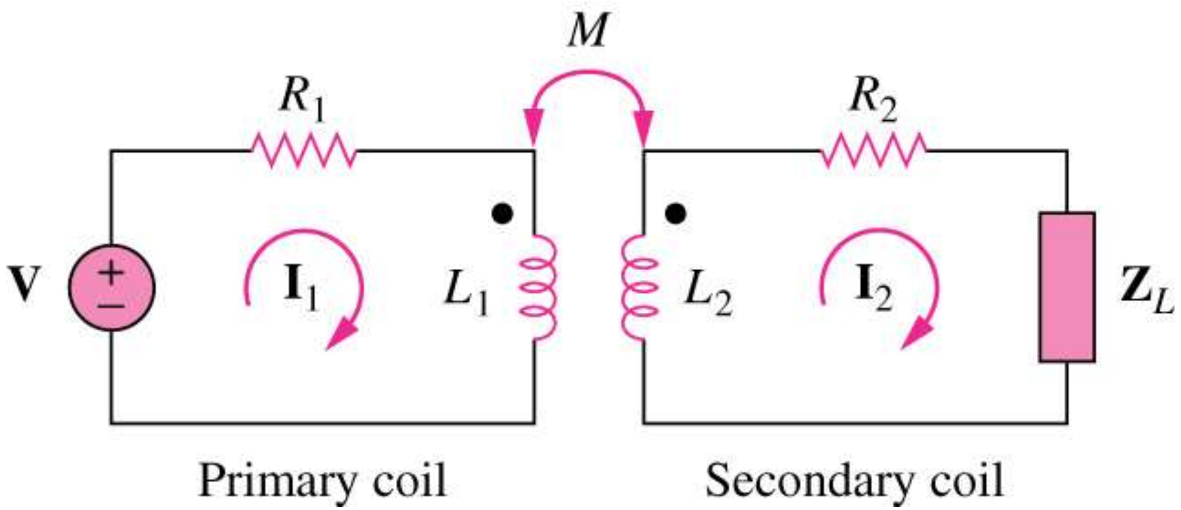


Figure 13.19

A linear transformer.

13.4 Linear Transformer 线性变压器

- It is generally a four-terminal device comprising two (or more) magnetically coupled coils



线性变压器：线圈绕在“磁线性材料”上，该材料的磁导率为常数，如空气、塑料、电工胶木等

同名端标记不影响结果, $M^2=(-M)^2$

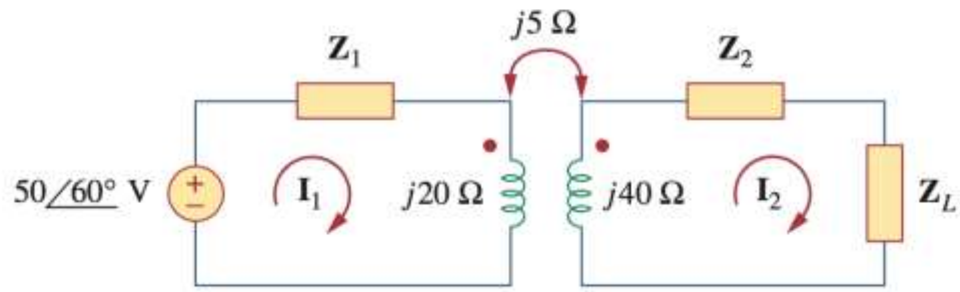
相当于将Z_L折算到输入端的阻抗

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$
$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

$$\boxed{\mathrm{Z_{in}} = \frac{\mathrm{V}}{\mathrm{I_1}} = R_1 + j\omega L_1 + \mathrm{Z}_R\,,\ \mathrm{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathrm{Z}_L}\ \text{is } \textit{reflected impedance}}$$

Example 13.4

In the circuit of Fig. 13.24, calculate the input impedance and current $\mathbf{I}_1$. Take $\mathbf{Z}_1 = 60 - j100\ \Omega$, $\mathbf{Z}_2 = 30 + j40\ \Omega$, and $\mathbf{Z}_L = 80 + j60\ \Omega$.



$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + j20 + \frac{(5)^2}{j40 + \mathbf{Z}_2 + \mathbf{Z}_L}$$

$$= 60 - j100 + j20 + \frac{25}{110 + j140}$$

$$= 60 - j80 + 0.14\underline{/-51.84^\circ}$$

$$= 60.09 - j80.11 = 100.14\underline{/-53.1^\circ}\ \Omega$$

线性变压器的去耦合等效法

耦合线性变压器电路
→ 非耦合电感电路

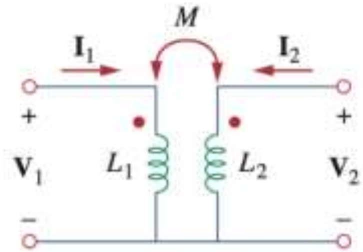


Figure 13.21
Determining the equivalent circuit of a linear transformer.

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}j\omega L_1 & j\omega M\\ j\omega M & j\omega L_2\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}\dfrac{L_2}{j\omega(L_1L_2-M^2)} & \dfrac{-M}{j\omega(L_1L_2-M^2)}\\ \dfrac{-M}{j\omega(L_1L_2-M^2)} & \dfrac{L_1}{j\omega(L_1L_2-M^2)}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

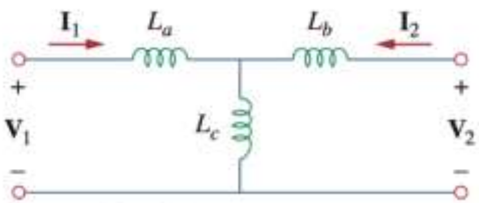


Figure 13.22
An equivalent T circuit.

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}=\begin{bmatrix}j\omega(L_a+L_c) & j\omega L_c\\ j\omega L_c & j\omega(L_b+L_c)\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}$$

⇓

$$\boxed{L_a=L_1-M,\qquad L_b=L_2-M,\qquad L_c=M}$$

注意同名端标记，若不在同一侧，则M → -M

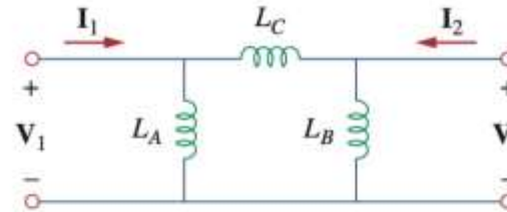


Figure 13.23
An equivalent Π circuit.

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}\dfrac{1}{j\omega L_A}+\dfrac{1}{j\omega L_C} & -\dfrac{1}{j\omega L_C}\\ -\dfrac{1}{j\omega L_C} & \dfrac{1}{j\omega L_B}+\dfrac{1}{j\omega L_C}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\end{bmatrix}$$

⇓

$$\boxed{L_A=\frac{L_1L_2-M^2}{L_2-M},\qquad L_B=\frac{L_1L_2-M^2}{L_1-M}\qquad L_C=\frac{L_1L_2-M^2}{M}}$$

Example 13.5

Determine the T-equivalent circuit of the linear transformer in Fig. 13.26(a).

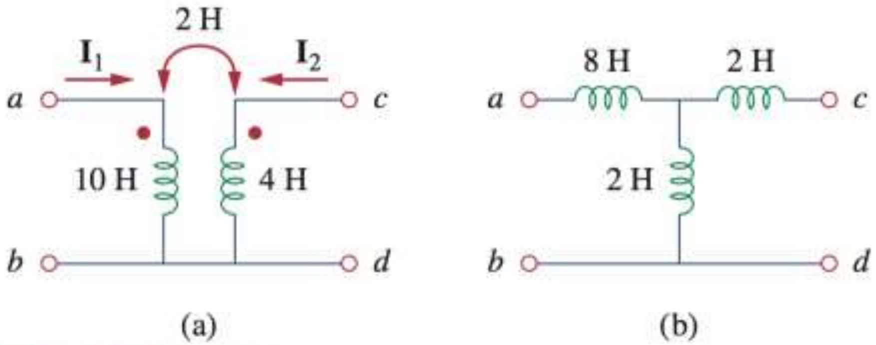
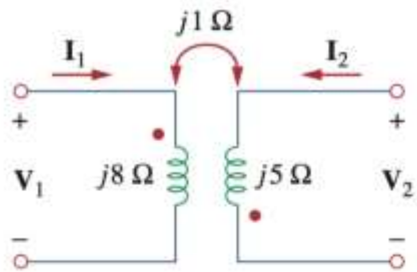


Figure 13.26
For Example 13.5: (a) a linear transformer, (b) its T-equivalent circuit.

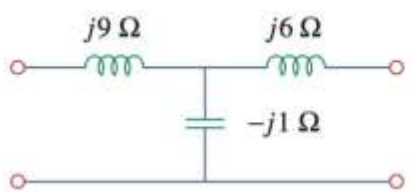
Example 13.6

Solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{V}_o$ in Fig. 13.27 (the same circuit as for Practice Prob. 13.1) using the T-equivalent circuit for the linear transformer.

同名端标记不在同一侧，M → -M



(a)



(b)

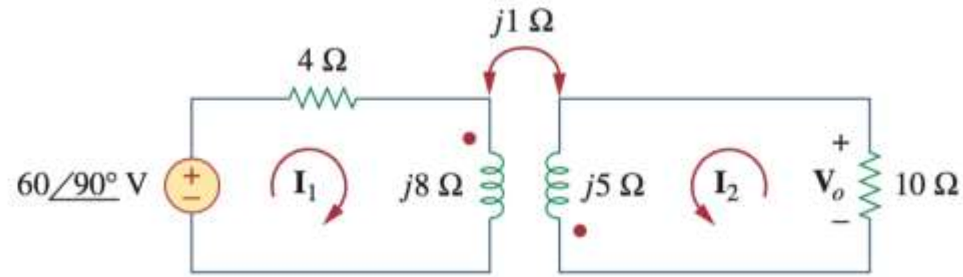
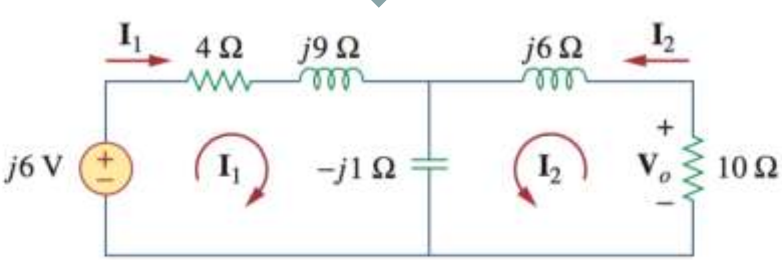


Figure 13.27
For Example 13.6.

$$j6 = \mathbf{I}_1(4 + j9 - j1) + \mathbf{I}_2(-j1)$$

$$0 = \mathbf{I}_1(-j1) + \mathbf{I}_2(10 + j6 - j1)$$



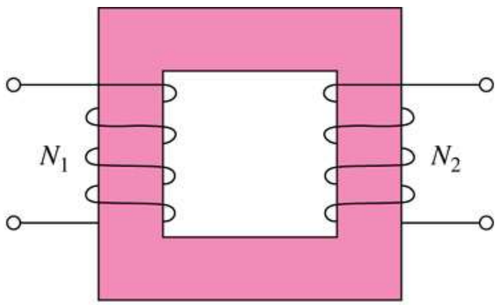
$$\mathbf{I}_2 = \frac{j6}{100} = j0.06 = 0.06\underline{/90^\circ}\ \text{A}$$

$$\mathbf{I}_1 = (5 - j10)j0.06 = 0.6 + j0.3\ \text{A}$$

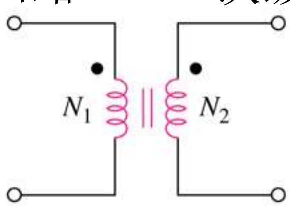
$$\mathbf{V}_o = -10\mathbf{I}_2 = -j0.6 = 0.6\underline{/-90^\circ}\ \text{V}$$

13.5 Ideal Transformer理想变压器

- An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.
- 理想变压器：耦合系数 $k = 1$、无耗、自感 L 无穷大；



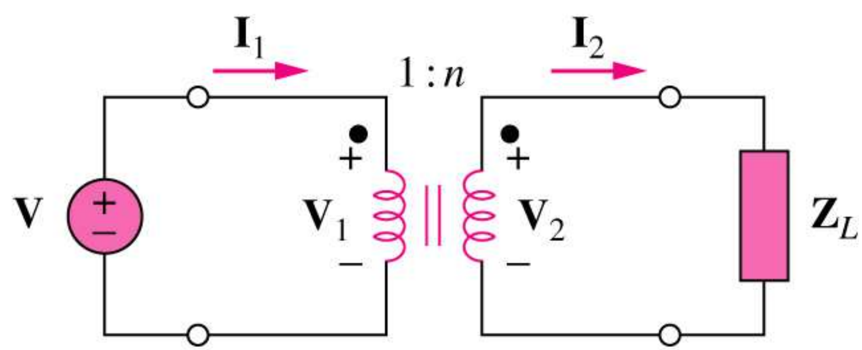
初级回路 (a) 次级回路



||表示磁芯
N 为线圈匝数

(b)

(a) Ideal Transformer
(b) Circuit symbol

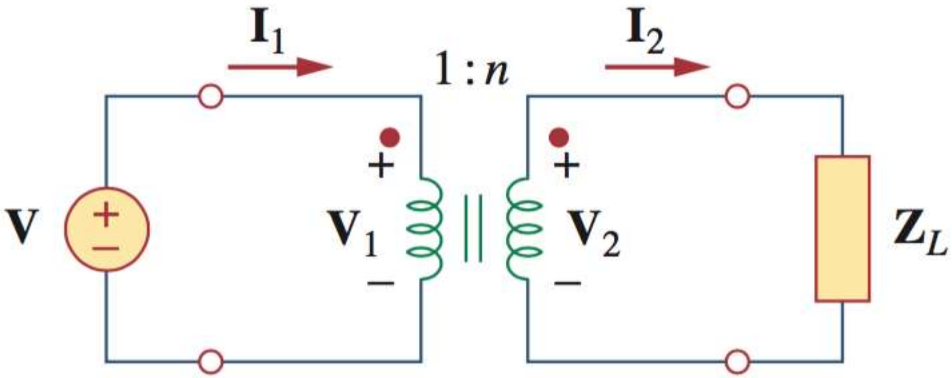


$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n \qquad (13.52)$$

理想变压器的“电压电流约束关系”

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n} \qquad (13.55)$$

$V_2 > V_1 \rightarrow$ step-up transformer 升压
$V_2 < V_1 \rightarrow$ step-down transformer 降压



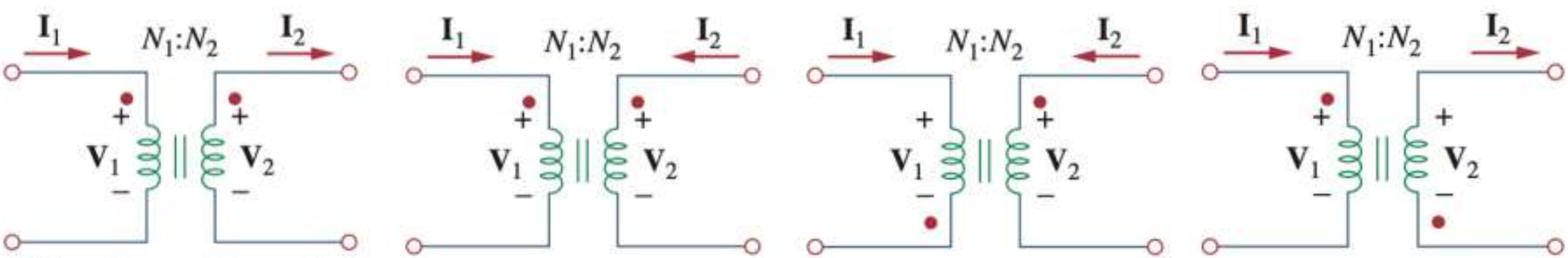
理想变压器记住此图示

电压“+”在同名端（比值为正）

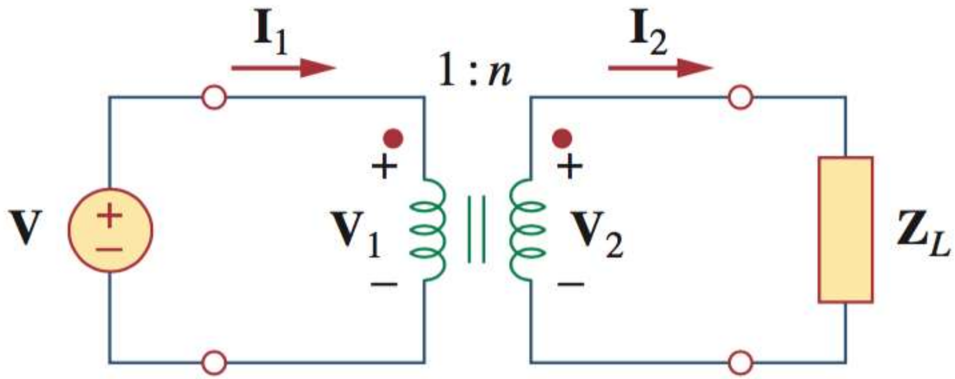
电流从同名端一进一出 （比值为正）

与此相反：比值为负

1. If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* positive or both negative at the dotted terminals, use $+n$ in Eq. (13.52). Otherwise, use $-n$.
2. If $\mathbf{I}_1$ and $\mathbf{I}_2$ *both* enter into or both leave the dotted terminals, use $-n$ in Eq. (13.55). Otherwise, use $+n$.



$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1}$, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$	$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1}$, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$	$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}$, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$	$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1}$, $\frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$



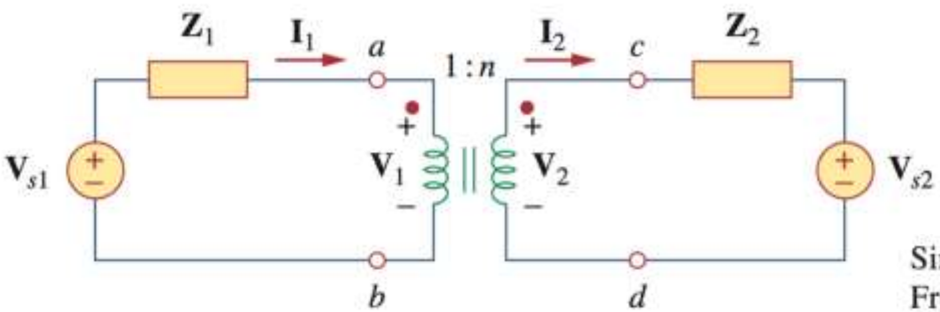
The complex power in the primary winding is

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

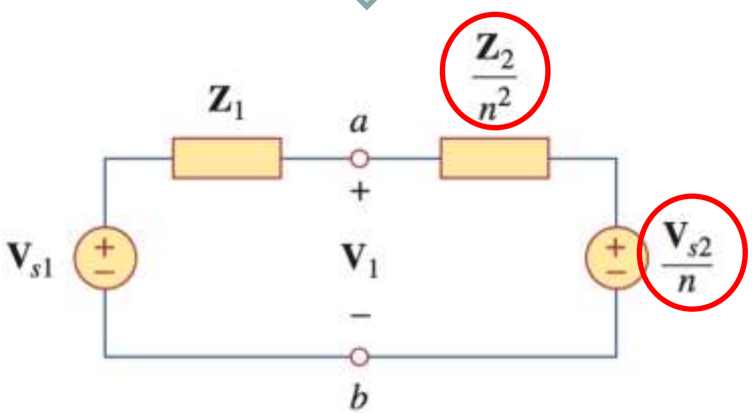
$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2} \Rightarrow \boxed{\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}}$$

可实现阻抗变换

- 若理想变压器的初级回路和次级回路没有额外的连接，那么，次级回路可以折算到初级回路；初级回路也可以折算到次级回路



对ab右侧电路做戴维南等效



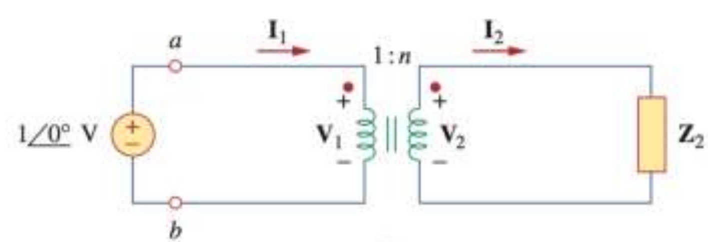
电压按比例折算；
阻抗按比例平方折算

Since terminals a-b are open, $\mathbf{I}_1 = 0 = \mathbf{I}_2$ so that $\mathbf{V}_2 = \mathbf{V}_{s2}$. Hence, From Eq. (13.56),

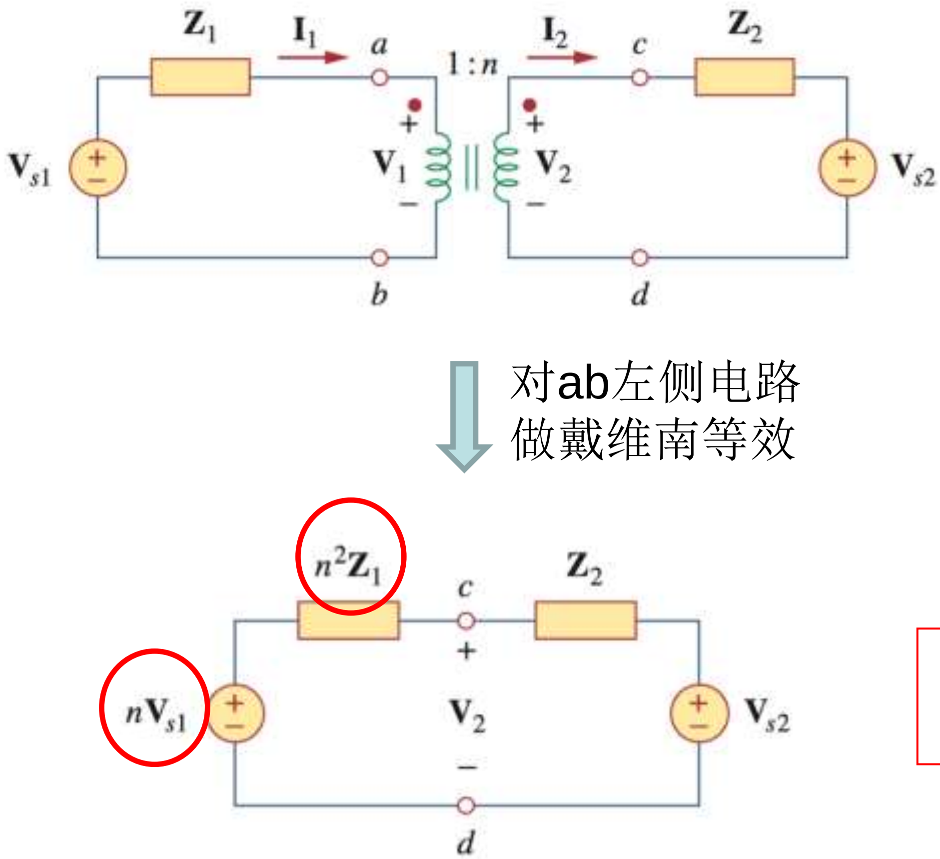
$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{s2}}{n} \tag{13.61}$$

To get $\mathbf{Z}_{\mathrm{Th}}$, we remove the voltage source in the secondary winding and insert a unit source at terminals a-b, as in Fig. 13.34(b). From Eqs. (13.56) and (13.57), $\mathbf{I}_1 = n\mathbf{I}_2$ and $\mathbf{V}_1 = \mathbf{V}_2/n$, so that

$$\mathbf{Z}_{\mathrm{Th}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2 \tag{13.62}$$



(b)



电压按比例折算；
阻抗按比例平方折算

Example 13.7

- 电压标称：初级/次级
- kVA → 视在功率

An ideal transformer is rated at 2400/120 V, 9.6 kVA, and has 50 turns on the secondary side. Calculate: (a) the turns ratio, (b) the number of turns on the primary side, and (c) the current ratings for the primary and secondary windings.

Solution:

(a) This is a step-down transformer, since $V_1 = 2{,}400\text{ V} > V_2 = 120\text{ V}$.

$$n = \frac{V_2}{V_1} = \frac{120}{2{,}400} = 0.05$$

(b)

$$n = \frac{N_2}{N_1} \quad \Rightarrow \quad 0.05 = \frac{50}{N_1}$$

or

$$N_1 = \frac{50}{0.05} = 1{,}000 \text{ turns}$$

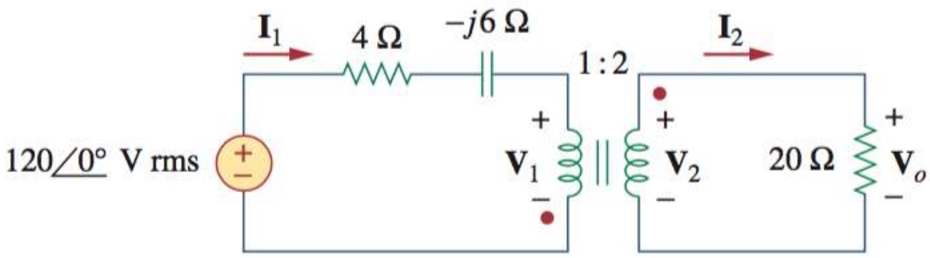
(c) $S = V_1 I_1 = V_2 I_2 = 9.6$ kVA. Hence,

$$I_1 = \frac{9{,}600}{V_1} = \frac{9{,}600}{2{,}400} = 4\text{ A}$$

$$I_2 = \frac{9{,}600}{V_2} = \frac{9{,}600}{120} = 80\text{ A} \qquad \text{or} \qquad I_2 = \frac{I_1}{n} = \frac{4}{0.05} = 80\text{ A}$$

Example 13.8

For the ideal transformer circuit of Fig. 13.37, find: (a) the source current $\mathbf{I}_1$, (b) the output voltage $\mathbf{V}_o$, and (c) the complex power supplied by the source.



(a) The 20-Ω impedance can be reflected to the primary side and we get

$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

将次级折算到初级

Thus,

$$\mathbf{Z}_{\text{in}} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\underline{/-33.69^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{120\underline{/0^\circ}}{\mathbf{Z}_{\text{in}}} = \frac{120\underline{/0^\circ}}{10.82\underline{/-33.69^\circ}} = 11.09\underline{/33.69^\circ}\ \text{A}$$

(b) Since both $\mathbf{I}_1$ and $\mathbf{I}_2$ leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\underline{/33.69^\circ}\ \text{A}$$

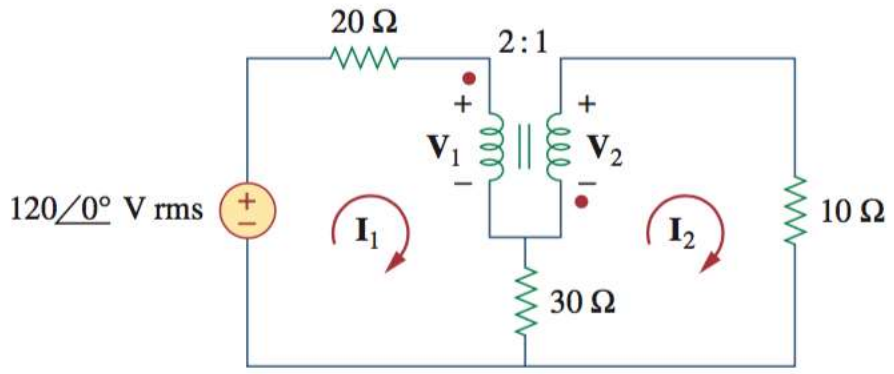
$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\underline{/213.69^\circ}\ \text{V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\underline{/0^\circ})(11.09\underline{/-33.69^\circ}) = 1{,}330.8\underline{/-33.69^\circ}\ \text{VA}$$

Example 13.9

Calculate the power supplied to the 10-Ω resistor in the ideal transformer circuit of Fig. 13.39.



利用理想变压器的“电压电流约束关系”

$$-120 + (20 + 30)\mathbf{I}_1 - 30\mathbf{I}_2 + \mathbf{V}_1 = 0$$

$$-\mathbf{V}_2 + (10 + 30)\mathbf{I}_2 - 30\mathbf{I}_1 = 0$$

$$\mathbf{V}_2 = -\frac{1}{2}\mathbf{V}_1$$

$$\mathbf{I}_2 = -2\mathbf{I}_1$$

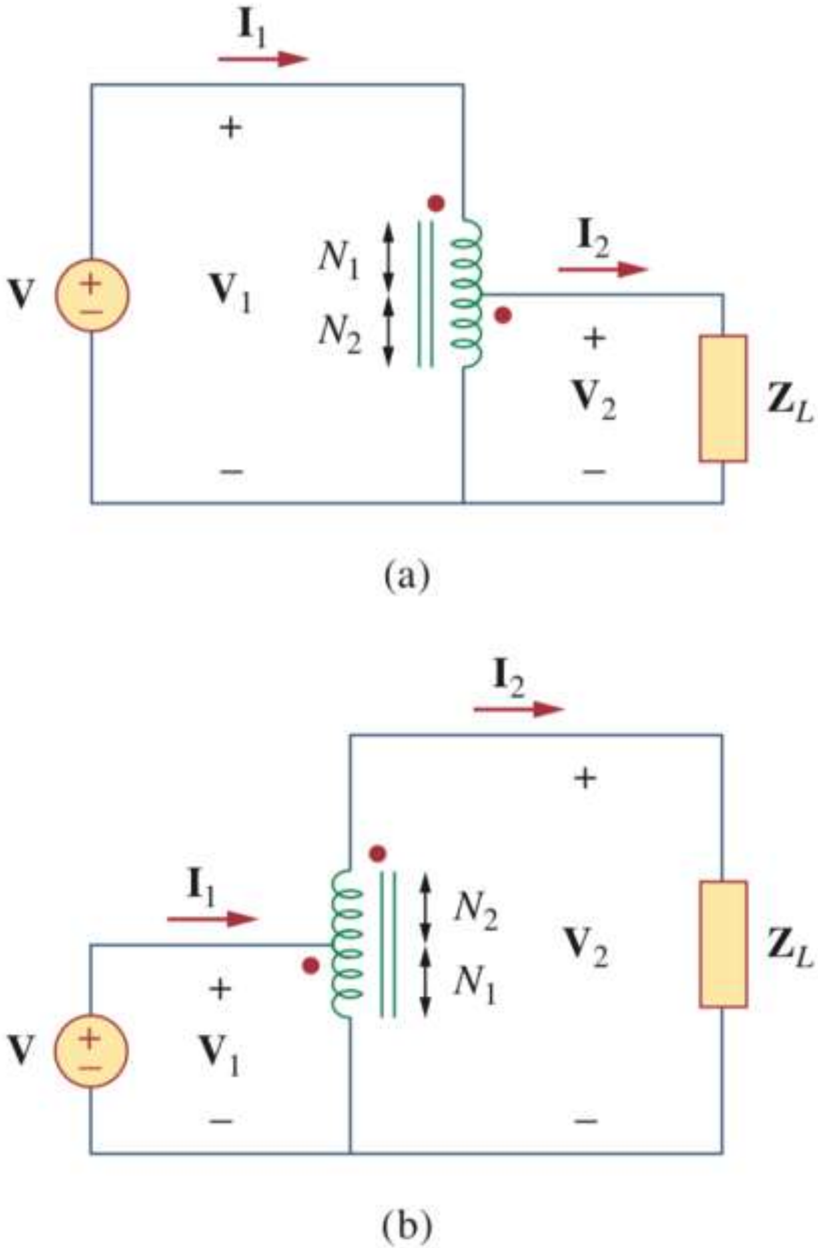
$$\Rightarrow$$

$$\mathbf{I}_2 = -\frac{120}{165} = -0.7272 \text{ A}$$

$$P = (-0.7272)^2(10) = 5.3 \text{ W}$$

Ideal Autotransformer 自耦变压器

An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding.



$$\boxed{\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1 + N_2}{N_2} = 1 + \frac{N_1}{N_2}}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_2}{N_1 + N_2}$$

Figure 13.42

(a) Step-down autotransformer, (b) step-up autotransformer.

Example 13.11

Refer to the autotransformer circuit in Fig. 13.44. Calculate: (a) $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$ if $\mathbf{Z}_L = 8 + j6\ \Omega$, and (b) the complex power supplied to the load.

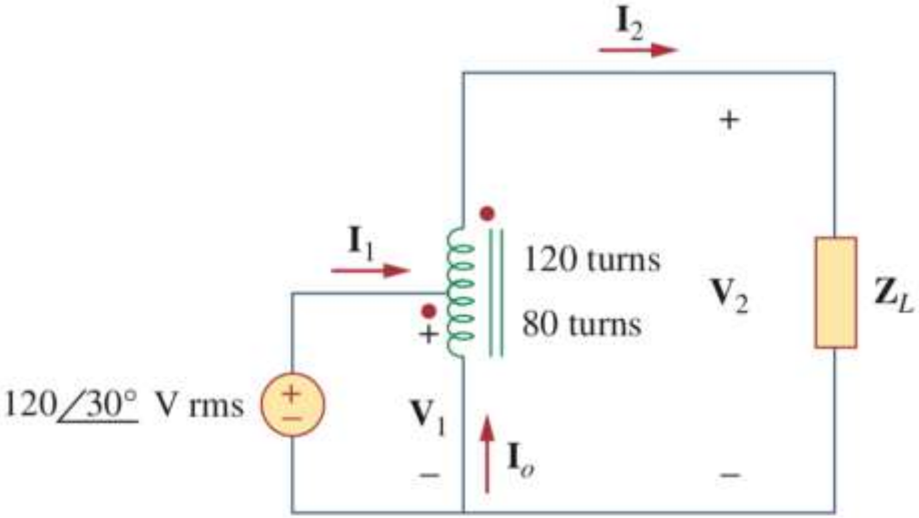


Figure 13.44
For Example 13.11.

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_1 + N_2} = \frac{80}{200}$$

$$\mathbf{V}_2 = \frac{200}{80}\mathbf{V}_1 = \frac{200}{80}(120\underline{/30^\circ}) = 300\underline{/30^\circ}\ \text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_L} = \frac{300\underline{/30^\circ}}{8 + j6} = \frac{300\underline{/30^\circ}}{10\underline{/36.87^\circ}} = 30\underline{/-6.87^\circ}\ \text{A}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{N_1 + N_2}{N_1} = \frac{200}{80}$$

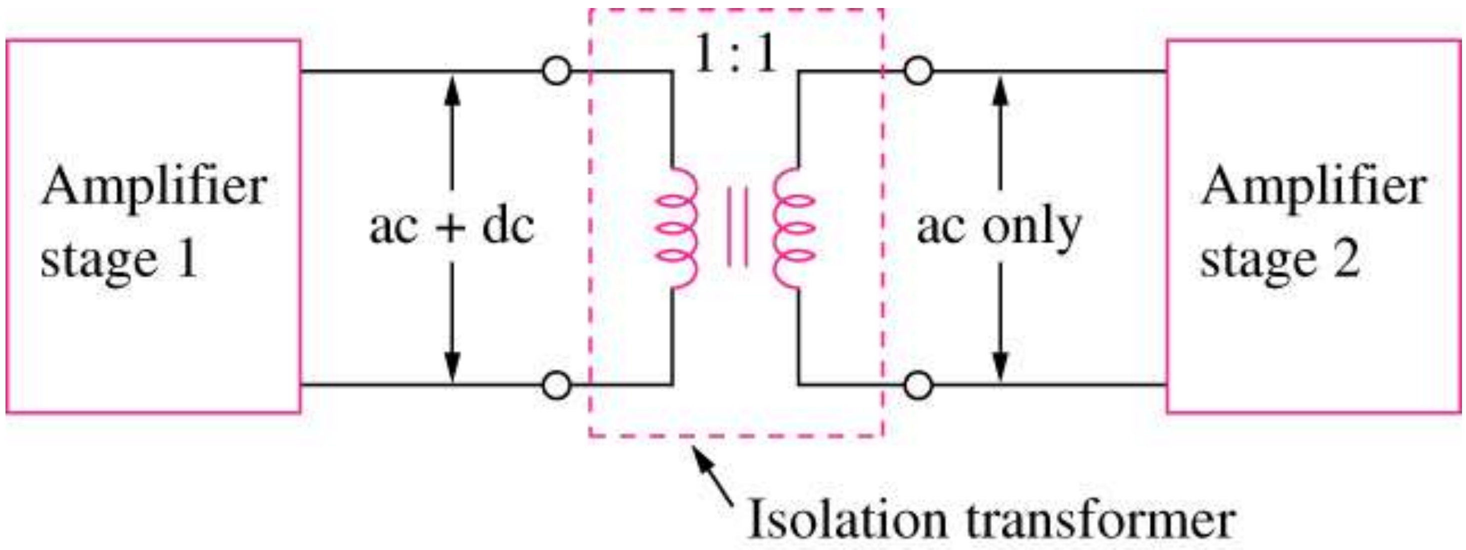
$$\mathbf{I}_1 = \frac{200}{80}\mathbf{I}_2 = \frac{200}{80}(30\underline{/-6.87^\circ}) = 75\underline{/-6.87^\circ}\ \text{A}$$

$$\mathbf{I}_o = \mathbf{I}_2 - \mathbf{I}_1 = 30\underline{/-6.87^\circ} - 75\underline{/-6.87^\circ} = 45\underline{/173.13^\circ}\ \text{A}$$

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = |\mathbf{I}_2|^2\mathbf{Z}_L = (30)^2(10\underline{/36.87^\circ}) = 9\underline{/36.87^\circ}\ \text{kVA}$$

13.6 Applications

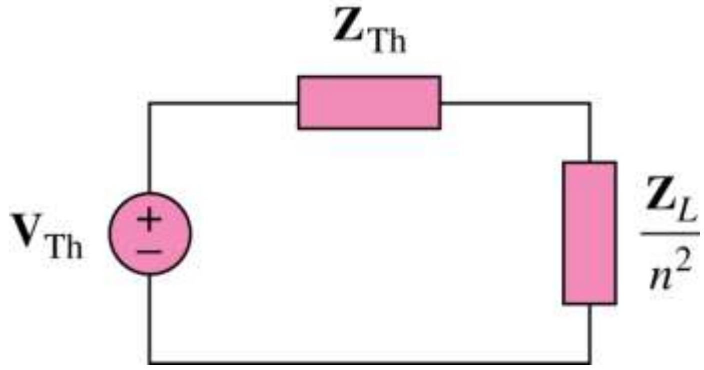
- Transformer as an Isolation Device to isolate dc between two amplifier stages. 应用之一：隔离直流



13.6 Applications

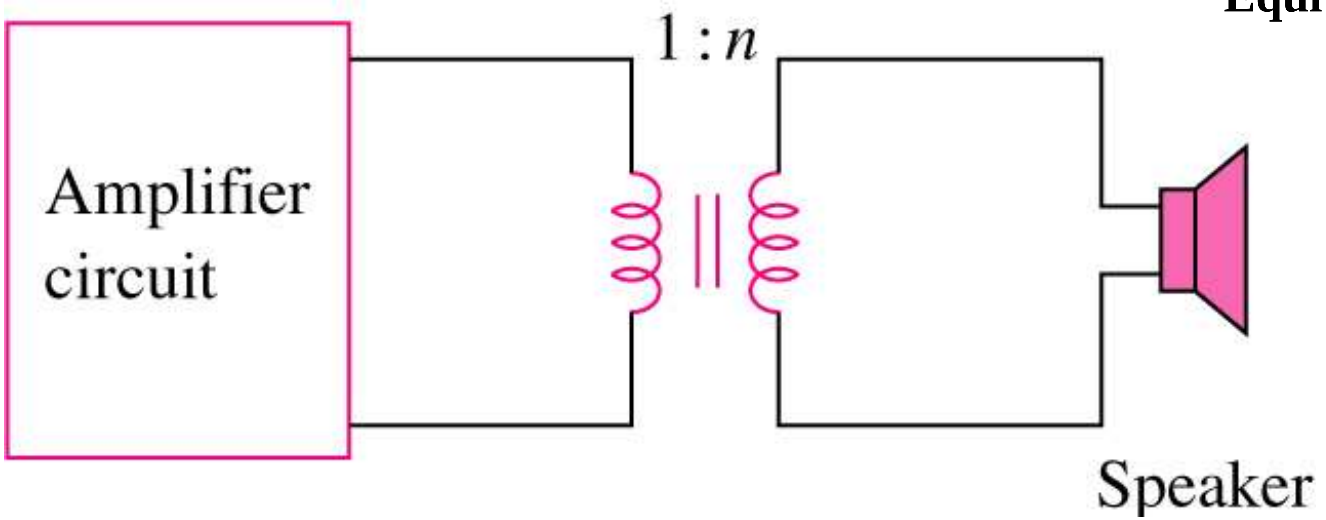
- Transformer as a Matching Device

  应用之二：阻抗匹配



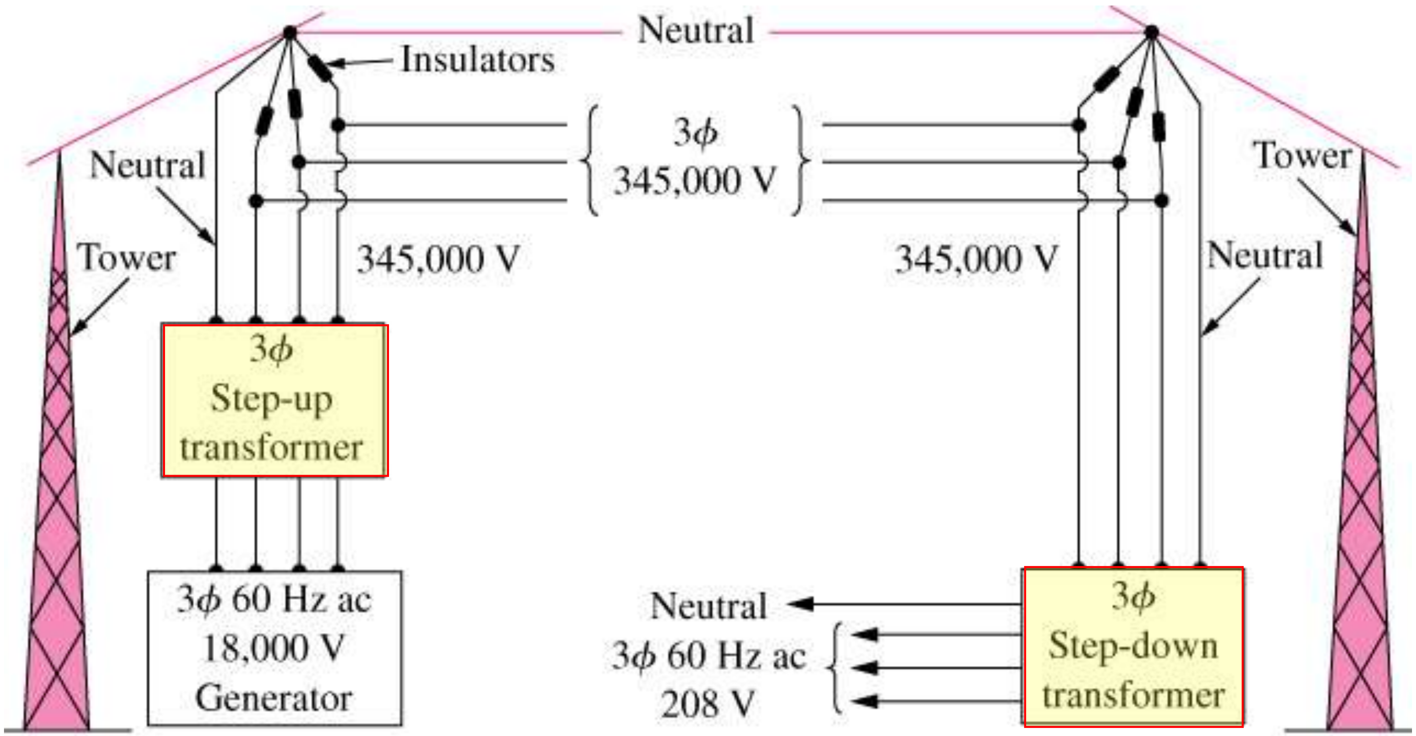
Equivalent circuit

Using an ideal transformer to match the speaker to the amplifier



13.6 Applications

- A typical power distribution system应用之三：供电系统



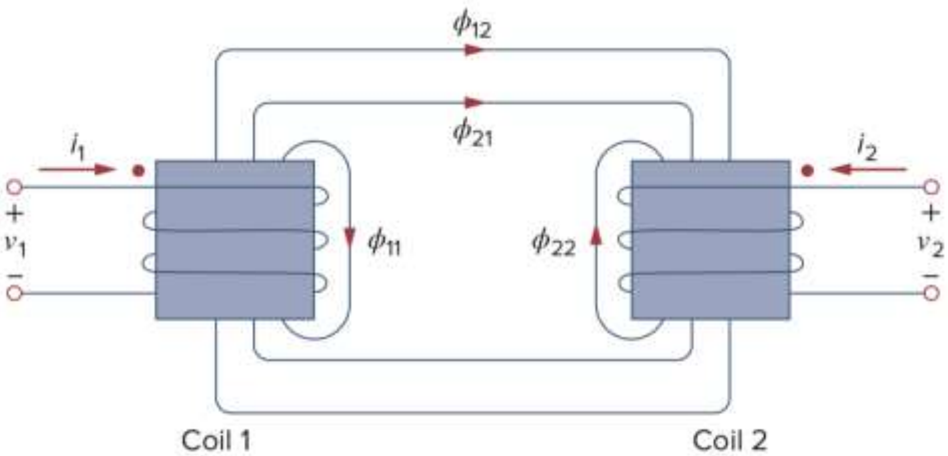
小结

- 耦合线圈 → 产生互感

$$\boxed{v_2 = M_{21}\frac{di_1}{dt}} \qquad \boxed{v_1 = M_{12}\frac{di_2}{dt}}$$

$$M_{12} = M_{21} = M$$

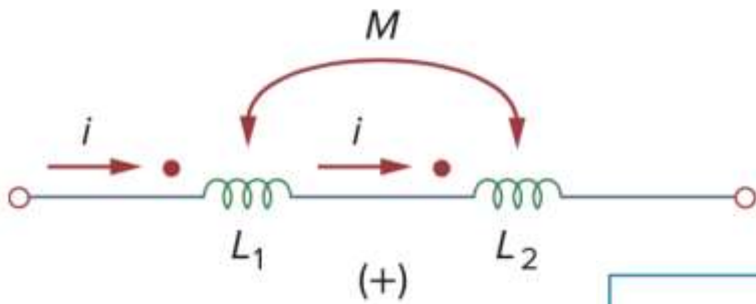
$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 \qquad \mathbf{V}_1 = j\omega M\mathbf{I}_2$$



- 同名端（一对标记端）
  - 标红点的是一对，不标红点的也是一对
  - 电流从一个线圈的标记端流入，在另一线圈产生的感应电压的正极在相应的标记端

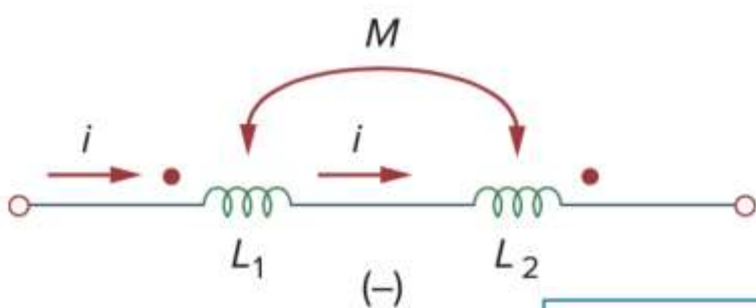
小结

- 互感线圈的串联: (a) 同向串联；(b)反向串联



(a)

$$L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}$$



(b)

$$L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}$$

小结

- 含互感线圈的电路分析
  - 电流从标记端流入 → 在另一线圈的标记端同向串联感应电压源

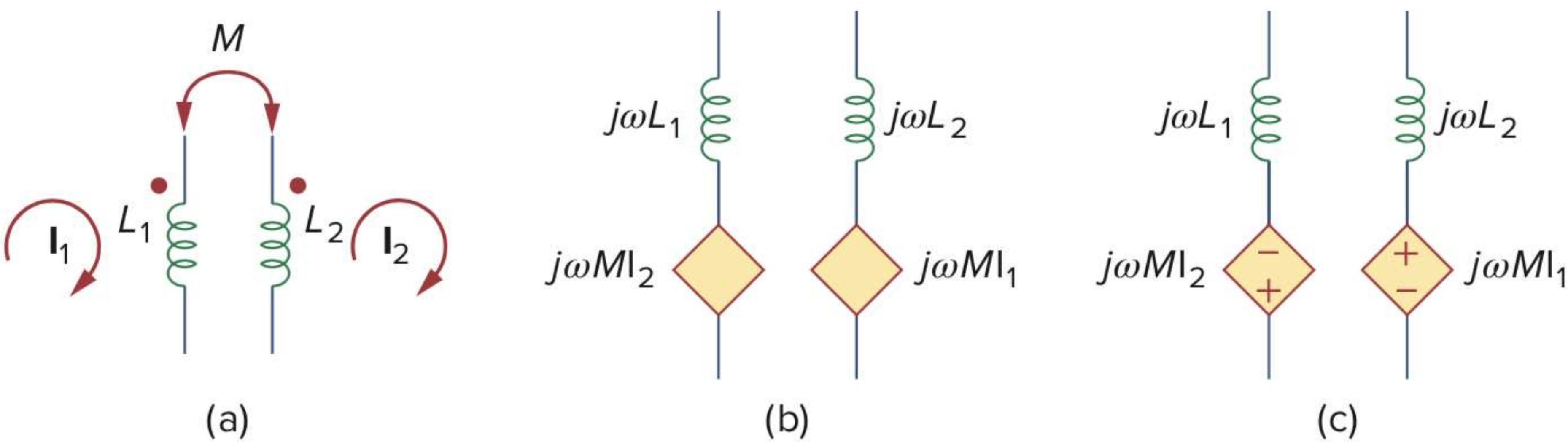


Figure 13.8
Model that makes analysis of mutually coupled easier to solve.

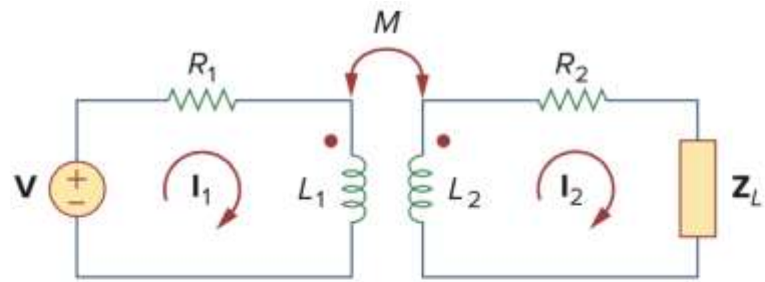
小结

- 磁耦合系数 $k = \dfrac{M}{\sqrt{L_1 L_2}}$ $0 \le k \le 1$
- 磁耦合线圈的能量 $$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

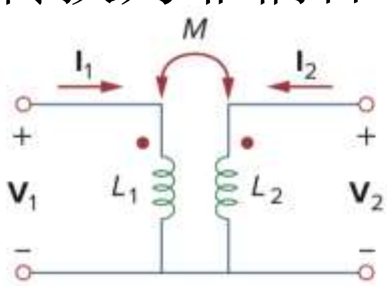
电流都从同名端流入，则为“＋”；一进一出则为“－”；

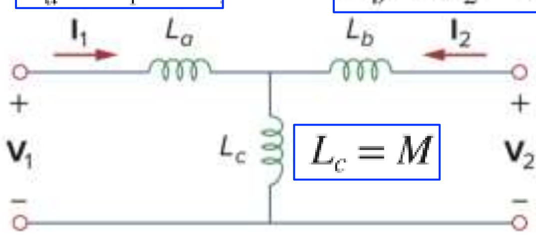
- 线性变压器的次级线圈总阻抗折算到主线圈后的增加阻抗

$$\boxed{\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}}$$



- 转换为非耦合电路



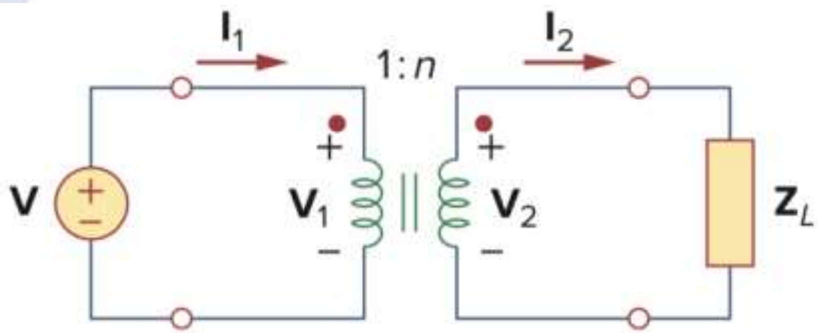


$\boxed{L_a = L_1 - M}$ $\boxed{L_b = L_2 - M}$ $\boxed{L_c = M}$

同名端在不同侧，则M → -M

小结

- 理想变压器的“电压电流约束关系”



$$\frac{\mathbf{V}_2}{\mathbf{V}_1}=\frac{N_2}{N_1}=n$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1}=\frac{N_1}{N_2}=\frac{1}{n}$$

- 如何区分是磁耦合电路，还是理想变压器电路？
  - 看标注的是互感，还是线圈匝数的比例

作业

Determine the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.13.

Practice Problem 13.2

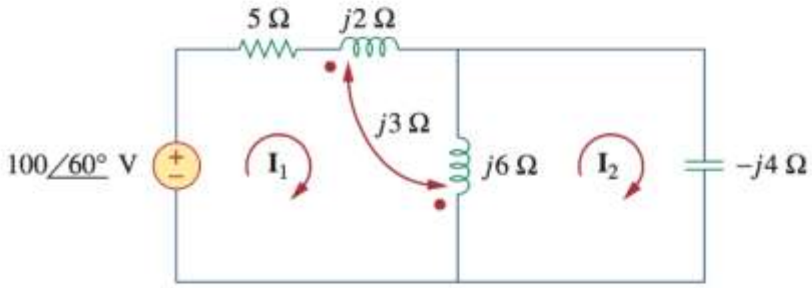


Figure 13.13
For Practice Prob. 13.2.

互感的概念~稍复杂情况
Focus on 流过线圈的电流

Answer: $I_1 = 17.889\underline{/86.57^\circ}$ A, $I_2 = 26.83\underline{/86.57^\circ}$ A.

For the circuit in Fig. 13.18, determine the coupling coefficient and the energy stored in the coupled inductors at $t = 1.5$ s.

Practice Problem 13.3

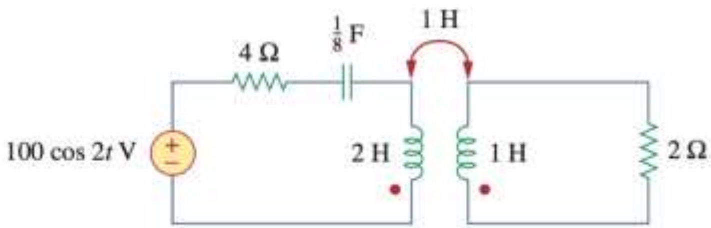


Figure 13.18
For Practice Prob. 13.3.

磁耦合系数和储能

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

Answer: 0.7071, 246.2 J.

Practice Problem 13.4

Find the input impedance of the circuit in Fig. 13.25 and the current from the voltage source.

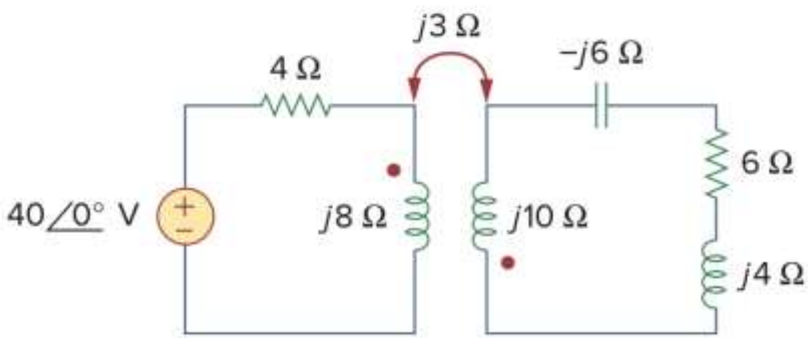


Figure 13.25
For Practice Prob. 13.4.

阻抗折算

$$\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

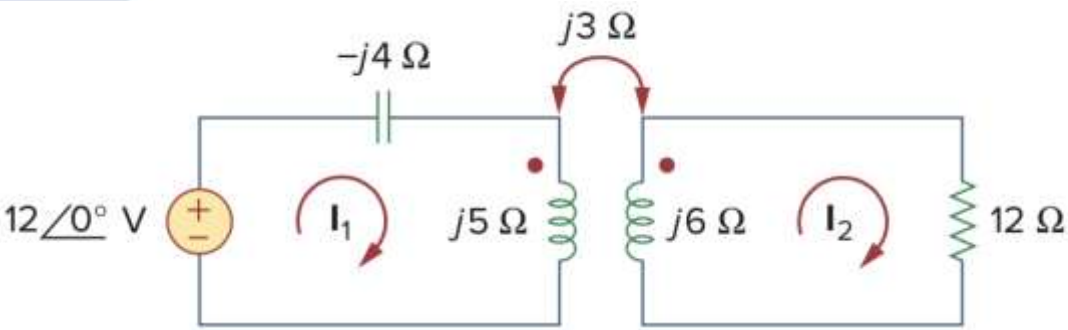
Answer: $8.58\underline{/58.05^\circ}\ \Omega$, $4.662\underline{/-58.05^\circ}$ A.

Solve the problem in Example 13.1 (see Fig. 13.9) using the T-equivalent model for the magnetically coupled coils.

Practice Problem 13.6

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit

Answer: $13\angle -49.4^\circ$ A, $2.91\angle 14.04^\circ$ A.



去耦合等效法

Practice Problem 13.8

In the ideal transformer circuit of Fig. 13.38, find $\mathbf{V}_o$ and the complex power supplied by the source.

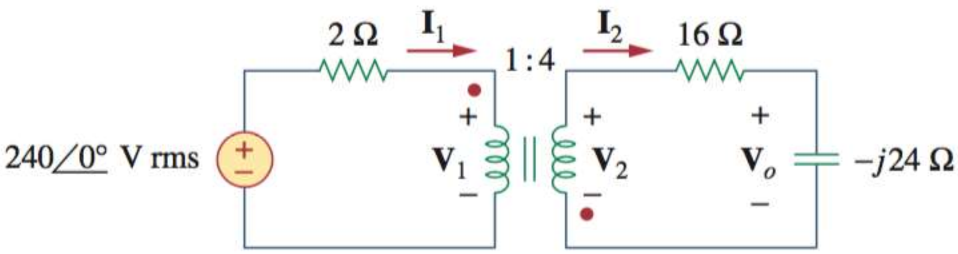


Figure 13.38
For Practice Prob. 13.8.

理想变压器的折算法简化

Answer: $429.4\angle 116.57^\circ$ V, $17.174\angle -26.57^\circ$ kVA.

Practice Problem 13.9

Find $\mathbf{V}_o$ in the circuit of Fig. 13.40.

折算法简化不能应用时，
采用传统电路分析法

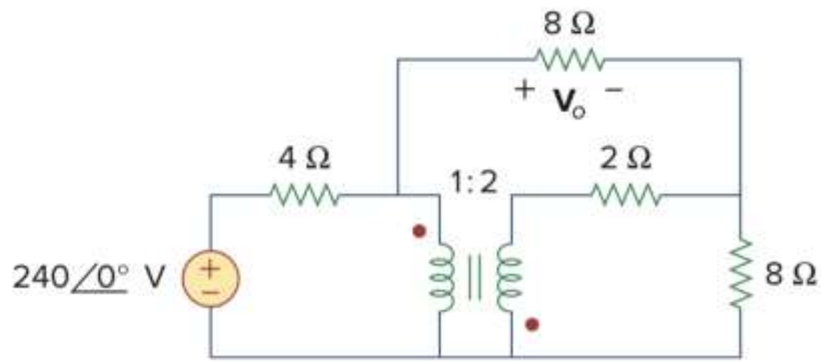


Figure 13.40
For Practice Prob. 13.9.

Answer: 96 V.