电子电路基础

第七讲~正弦交流电路-part2

课程纲要

- 3.3 交流功率分析
  - 3.3.1 重要物理量：瞬时功率、平均功率、无功功率、有功功率、视在功率、复功率和功率因数等
  - 3.3.2 交流功率守恒
  - 3.3.3 功率因数的校正
  - 3.3.4 最大功率传递定理
- 3.4 三相交流电的基本概念
  - 3.4.1 平衡三相交流电的概念
  - 3.4.2 简单了解Y-Y、Y-Δ、Δ-Δ和Δ-Y四种连接方式
  - 3.4.3 了解线电流、相电流、线电压和相电压的定义

AC Power Analysis
Chapter 11

11.1 Instantaneous and Average Power 瞬时功率 & 平均功率
11.2 Maximum Average Power Transfer 最大平均功率传递
11.3 Effective or RMS Value 有效值（均方值）
11.4 Apparent Power and Power Factor 视在功率 & 功率因数
11.5 Complex Power 复功率
11.6 Conservation of AC Power 交流功率守恒
11.7 Power Factor Correction 功率因数的校正
11.8 Power Measurement 功率测量

11.1 Instantaneous and Average Power 瞬时功率 & 平均功率

- **瞬时功率**：元件两端瞬时电压与流经元件瞬时电流的乘积。

The **instantaneous power** (in watts) is the power at any instant of time.

$$p(t) = v(t)i(t)$$

单位是 W

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power — **Sinusoidal power at 2ωt**

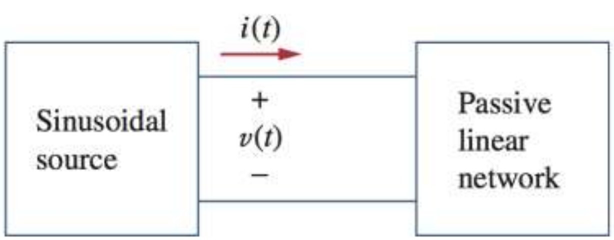
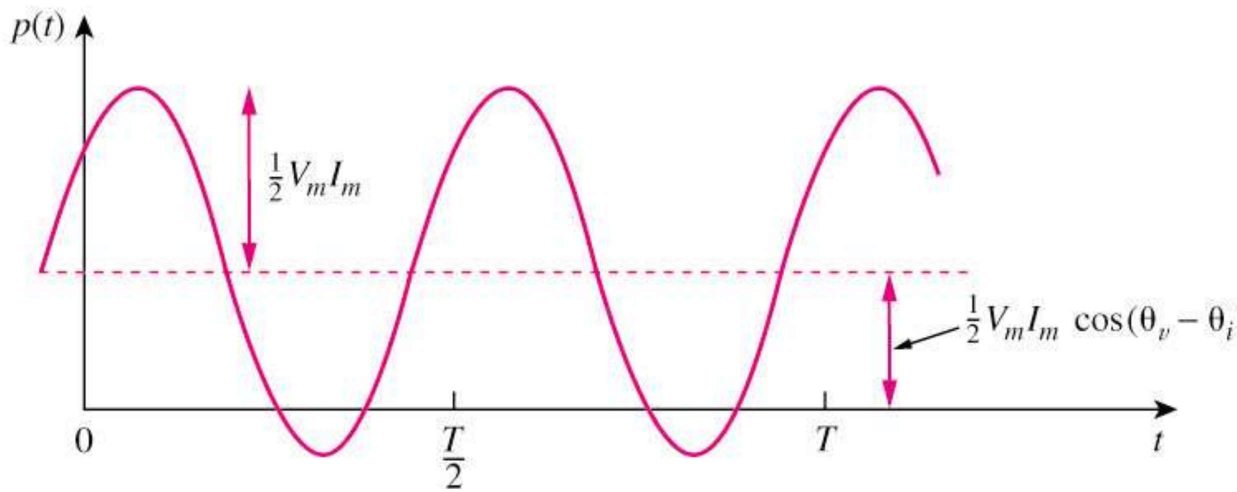


Figure 11.1
Sinusoidal source and passive linear circuit.



$p(t) > 0$: power is absorbed by the circuit; $p(t) < 0$: power is absorbed by the source.

11.1 Instantaneous and Average Power

单位是 W

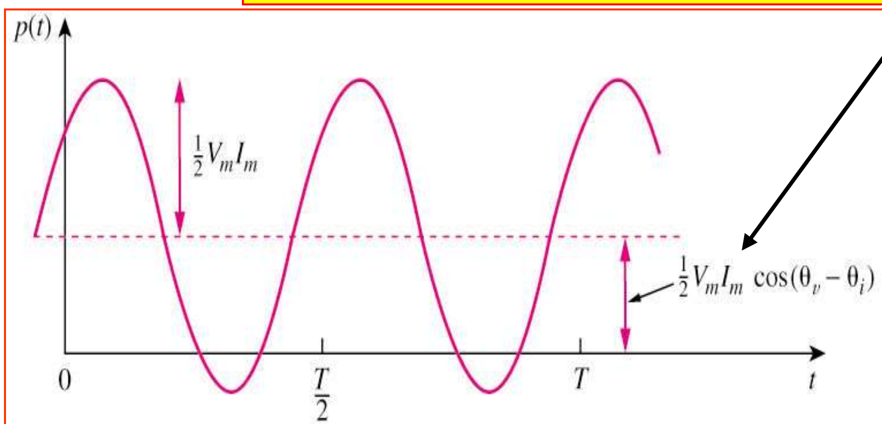
- **平均功率：**瞬时功率在一个周期内的平均值

The **average power**, in watts, is the average of the instantaneous power over one period.

$$
\begin{aligned}
P &= \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt \\
&\quad + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt \\
&= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt \\
&\quad + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt
\end{aligned}
$$

瞬时功率由两部分构成：常数部分+正弦部分，其中常数部分就是平均功率。（因为正弦函数的平均值为0）

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}$$



A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90°$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

平均功率的相量求法

- 瞬时功率，是时变的，必须知道 v(t) 和 i(t)；
- 平均功率，是常数，可以从v(t) 和 i(t) 求出，也可以从相量**V**和**I**求出；

$$\mathbf{V} = V_m\underline{/\theta_v} \text{ and } \mathbf{I} = I_m\underline{/\theta_i} \xrightarrow{?} P = \frac{1}{2}V_mI_m\cos(\theta_v - \theta_i)$$

$$\frac{1}{2}\mathbf{VI}^* = \frac{1}{2}V_mI_m\underline{/\theta_v - \theta_i}$$

$$= \frac{1}{2}V_mI_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

1. 直接法，根据V_m，I_m，θ_v, θ_i直接写出。但对应什么数学运算呢？

$$\boxed{P = \frac{1}{2}\text{Re}[\mathbf{VI}^*] = \frac{1}{2}V_mI_m\cos(\theta_v - \theta_i)}$$

2. 数学运算法

Example 11.1

Given that

$$v(t) = 120\cos(377t + 45°)\ \text{V} \quad \text{and} \quad i(t) = 10\cos(377t - 10°)\ \text{A}$$

find the instantaneous power and the average power absorbed by the passive linear network of Fig. 11.1.

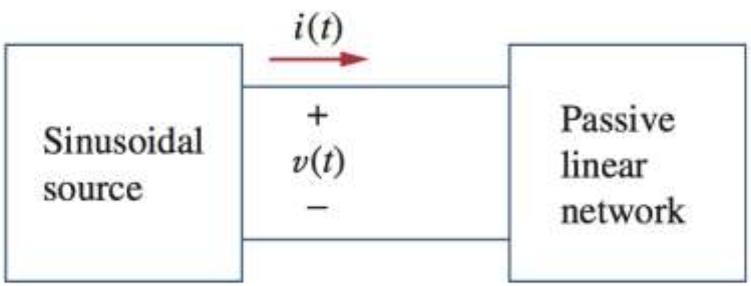


Figure 11.1
Sinusoidal source and passive linear circuit.

平均功率可以根据公式直接写出

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Example 11.2

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\underline{/0^\circ}$ is applied across it.

小心功率不是线性的！！！

Example 11.3

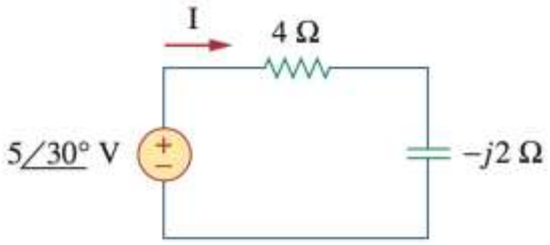
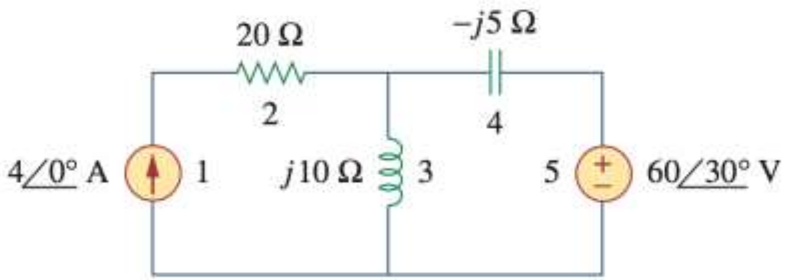


Figure 11.3
For Example 11.3.

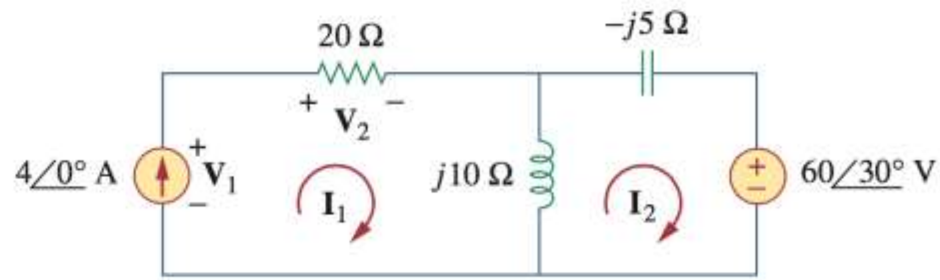
For the circuit shown in Fig. 11.3, find the average power supplied by the source and the average power absorbed by the resistor.

Example 11.4

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit of Fig. 11.5(a).



(a)



(b)

$$(j10 - j5)\mathbf{I}_2 - j10\mathbf{I}_1 + 60\underline{/30^\circ} = 0, \qquad \mathbf{I}_1 = 4\text{ A} \quad \Longrightarrow \quad \mathbf{I}_2 = 10.58\underline{/79.1^\circ}\text{ A}$$

$$P_5 = \frac{1}{2}(60)(10.58)\cos(30^\circ - 79.1^\circ) = 207.8\text{ W}$$

$$\mathbf{V}_1 = 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 80 + j10(4 - 2 - j10.39)$$
$$= 183.9 + j20 = 184.984\underline{/6.21^\circ}\text{ V}$$

$$P_1 = -\frac{1}{2}(184.984)(4)\cos(6.21^\circ - 0) = -367.8\text{ W}$$

$$P_2 = \frac{1}{2}(80)(4) = 160\text{ W}$$

For the capacitor, the current through it is $\mathbf{I}_2 = 10.58\underline{/79.1^\circ}$ and the voltage across it is $-j5\mathbf{I}_2 = (5\underline{/-90^\circ})(10.58\underline{/79.1^\circ}) = 52.9\underline{/79.1^\circ - 90^\circ}$. The average power absorbed by the capacitor is

$$P_4 = \frac{1}{2}(52.9)(10.58)\cos(-90^\circ) = 0$$

For the inductor, the current through it is $\mathbf{I}_1 - \mathbf{I}_2 = 2 - j10.39 = 10.58\underline{/-79.1^\circ}$. The voltage across it is $j10(\mathbf{I}_1 - \mathbf{I}_2) = 10.58\underline{/-79.1^\circ + 90^\circ}$. Hence, the average power absorbed by the inductor is

$$P_3 = \frac{1}{2}(105.8)(10.58)\cos 90^\circ = 0$$

$$P_1 + P_2 + P_3 + P_4 + P_5 = -367.8 + 160 + 0 + 0 + 207.8 = 0$$

最大平均功率传递

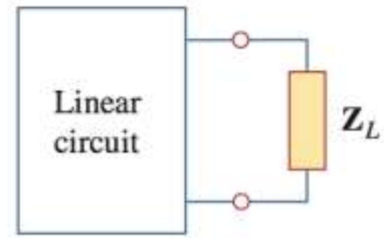
$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L$$

$$\Downarrow$$

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

电阻上消耗的功率见上式；
电抗元件上消耗的功率为零

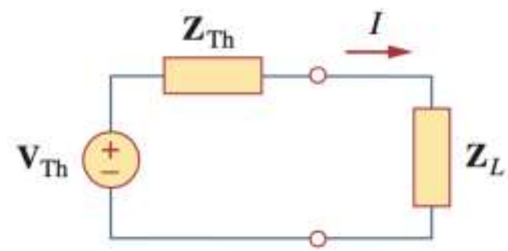


(a)

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\text{Th}}}{(R_{\text{Th}} + jX_{\text{Th}}) + (R_L + jX_L)}$$

$$P = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{|\mathbf{V}_{\text{Th}}|^2 R_L/2}{(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2}$$

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}$$



$$\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*$$

$$\frac{\partial P}{\partial X_L} = -\frac{|\mathbf{V}_{\text{Th}}|^2 R_L (X_{\text{Th}} + X_L)}{[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2} \rightarrow X_L = -X_{\text{Th}}$$

$$\frac{\partial P}{\partial R_L} = \frac{|\mathbf{V}_{\text{Th}}|^2[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2 - 2R_L(R_{\text{Th}} + R_L)]}{2[(R_{\text{Th}} + R_L)^2 + (X_{\text{Th}} + X_L)^2]^2} \rightarrow R_L = \sqrt{R_{\text{Th}}^2 + (X_{\text{Th}} + X_L)^2}$$

共轭匹配

For **maximum average power transfer**, the load impedance $\mathbf{Z}_L$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.

Example 11.5

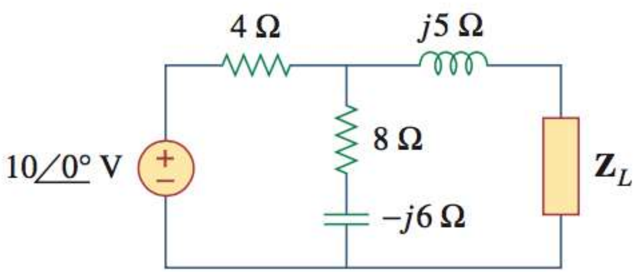


Figure 11.8
For Example 11.5.

Determine the load impedance $\mathbf{Z}_L$ that maximizes the average power drawn from the circuit of Fig. 11.8. What is the maximum average power?

将左边电路用戴维南定理等效

Example 11.6

In the circuit in Fig. 11.11, find the value of R_L that will absorb the maximum average power. Calculate that power.

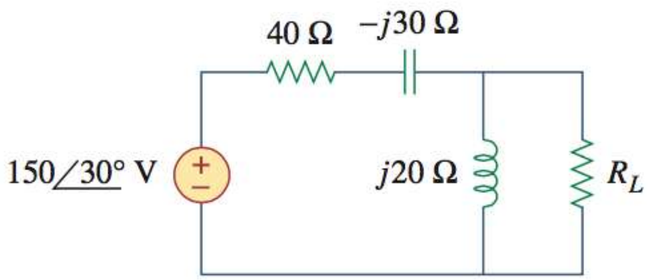


Figure 11.11
For Example 11.6.

将左边电路用戴维南定理等效负载为纯电阻情况

Solution:

We first find the Thevenin equivalent at the terminals of R_L.

$$\mathbf{Z}_{\text{Th}} = (40 - j30) \parallel j20 = \frac{j20(40 - j30)}{j20 + 40 - j30} = 9.412 + j22.35\ \Omega$$

By voltage division,

$$\mathbf{V}_{\text{Th}} = \frac{j20}{j20 + 40 - j30}(150\underline{/30^\circ}) = 72.76\underline{/134^\circ}\text{ V}$$

The value of R_L that will absorb the maximum average power is

$$R_L = |\mathbf{Z}_{\text{Th}}| = \sqrt{9.412^2 + 22.35^2} = 24.25\ \Omega$$

注意不是 $R_L=R_{Th}$

$$R_L = \sqrt{R_{\text{Th}}^2 + (X_{\text{Th}} + X_L)^2}$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + R_L} = \frac{72.76\underline{/134^\circ}}{33.66 + j22.35} = 1.8\underline{/100.42^\circ}\text{ A}$$

The maximum average power absorbed by R_L is

$$P_{\max} = \frac{1}{2}|\mathbf{I}|^2 R_L = \frac{1}{2}(1.8)^2(24.25) = 39.29\text{ W}$$

若负载指定为纯电阻，则共轭匹配不适用！匹配阻值为戴维南阻抗的幅度

What is root mean square (rms)

- In **statistics** and its applications, the root mean square (abbreviated RMS or rms) is defined as the **square root of the mean square** (the arithmetic mean of the squares of a set of numbers)

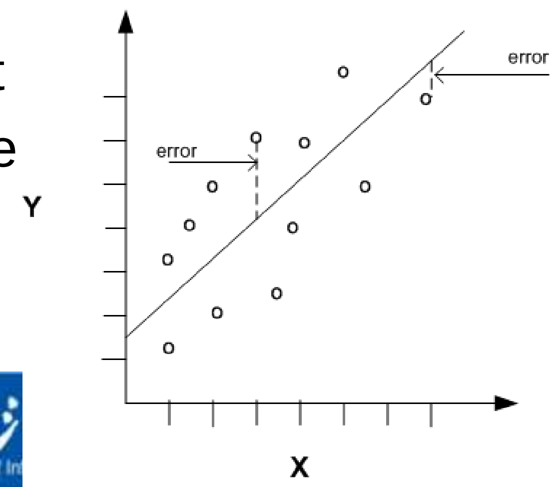
$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}.$$

- For **alternating electric current**, RMS is equal to the value of the direct current that would produce the same average power dissipation in a resistive load.

$$f_{\mathrm{rms}} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} [f(t)]^2\, dt}.$$

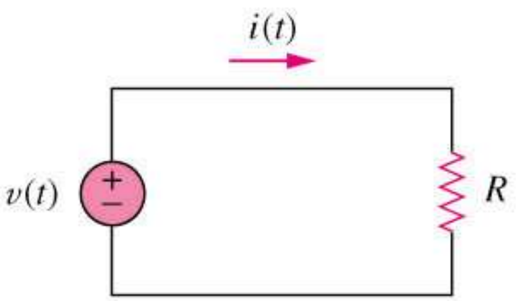
- In estimation theory of **machine learning**, the root mean square error of an estimator is a measure of the imperfection of the fit of the estimator to the data

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(Predicted_i - Actual_i\right)^2}{N}}$$

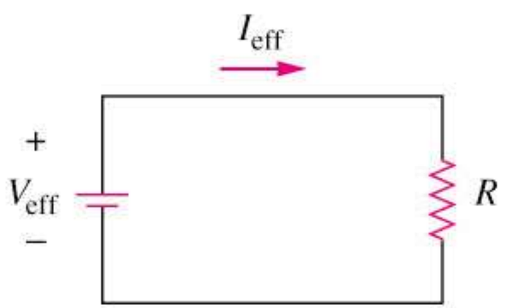


11.3 Effective or RMS Value 有效值

The effective of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.



(a)



(b)

The total power dissipated by R is given by:

$$P=\frac{1}{T}\int_0^T i^2 R dt=\frac{R}{T}\int_0^T i^2 dt=I_{rms}^2 R$$

Hence, I_{eff} is equal to:

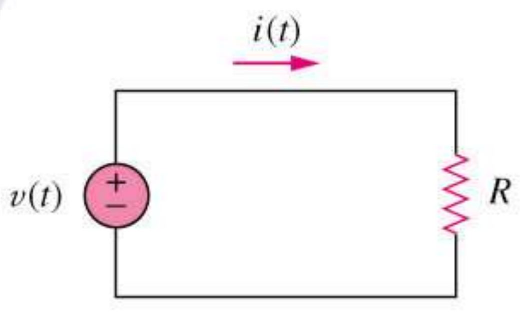
$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}$$

$$X_{rms}=\sqrt{\frac{1}{T}\int_0^T x^2 dt}$$

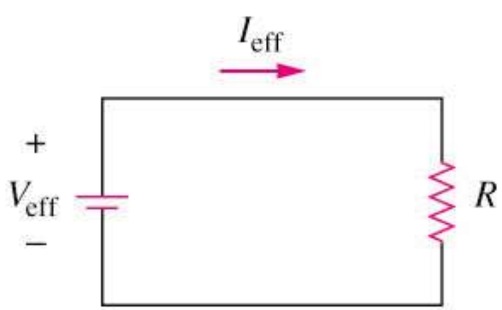
The **effective value** of a periodic signal is its root mean square (rms) value.

The rms value is a constant, which depending on the shape of the function.

11.3 Effective or RMS Value



(a)



(b)

The rms value of a sinusoid $i(t) = I_m\cos(\omega t)$ is given by:

$$\boxed{I_{\text{rms}} = \frac{I_m}{\sqrt{2}}} \qquad \boxed{V_{\text{rms}} = \frac{V_m}{\sqrt{2}}}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2\omega t\, dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$$

仅对正弦信号而言！

The average power can be written in terms of the rms values: 振幅（峰值）

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

回顾平均功率的表达式

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

有效值

消耗在电阻上的平均功率：$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$

在电力行业，习惯于使用rms值，比如家用电 220V 指的是电压幅度的有效值（而非幅值）为 220V，采用有效值后，功率的计算公式就如直流时一样了

Note: If you express amplitude of a phasor source(s) in rms, then all the answer as a result of this phasor source(s) must also be in rms value.

Example 11.7

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.

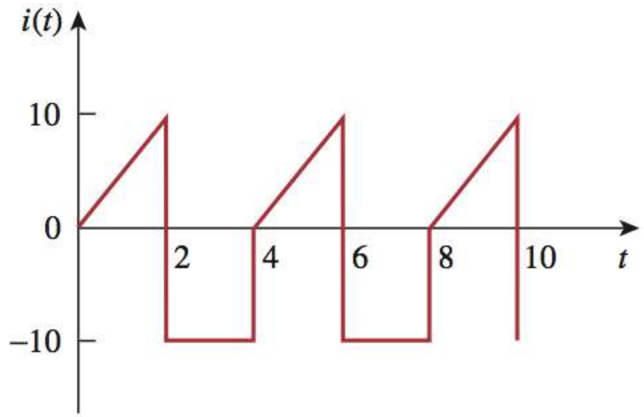


Figure 11.14
For Example 11.7.

Solution:

The period of the waveform is $T = 4$. Over a period, we can write the current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\Bigg|_0^2 + 100t\Bigg|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3\text{ W}$$

周期信号。有效值：计算一个周期内的均方根

Example 11.8

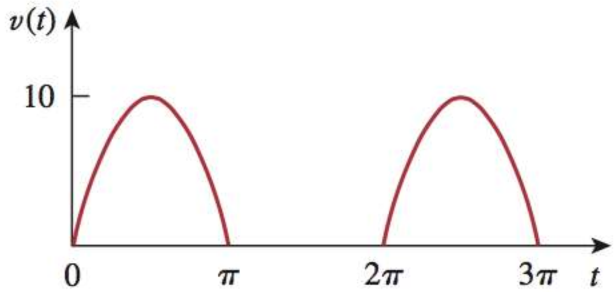


Figure 11.16
For Example 11.8.

周期信号。有效值：计算一个周期内的均方根

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.

Solution:

The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^\pi (10 \sin t)^2\,dt + \int_\pi^{2\pi} 0^2\,dt\right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^\pi \frac{100}{2}(1 - \cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^\pi$$

$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5\text{ V}$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\text{ W}$$

11.4 Apparent Power and Power Factor 视在功率 & 功率因数

- Apparent Power（视在功率）S, is the product of the **rms** values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power that is measured in watts.

单位是VA

视在功率不是真实功率，所以单位不用W，而用VA

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = \underbrace{S}_{\text{Apparent Power, } S} \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}$$

平均功率 = 视在功率 x 功率因数

$$\boxed{S = V_{rms} I_{rms}}$$

定义　　对于正弦信号

$$\boxed{\text{pf} = \frac{P}{S}} = \cos(\theta_v - \theta_i)$$

- Power factor （功率因数）is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

功率因数是电压电流相位差（也即负载相位）的cos

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i$$

11.4 Apparent Power and Power Factor

Purely resistive load (R)	$\theta_v - \theta_i = 0$, pf = 1	$P/S = 1$, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	$P = 0$, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ 感性 $\theta_v - \theta_i < 0$ 容性	• pf Lagging - inductive load，电流 lags 电压 • pf Leading - capacitive load，电流 leads 电压

leading means current leads voltage ~ capacitive load;

Example 11.9

平均功率 = 视在功率 x 功率因数

$$S = V_{\text{rms}} I_{\text{rms}}$$

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

A series-connected load draws a current $i(t) = 4\cos(100\pi t + 10°)$ A when the applied voltage is $v(t) = 120\cos(100\pi t - 20°)$ V. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Solution:

The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20° - 10°) = 0.866 \qquad \text{(leading)}$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\underline{/-20°}}{4\underline{/10°}} = 30\underline{/-30°} = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30°) = 0.866 \qquad \text{(leading)}$$

The load impedance $\mathbf{Z}$ can be modeled by a 25.98-Ω resistor in series with a capacitor with

$$X_C = -15 = -\frac{1}{\omega C}$$

or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

Example 11.10

Determine the power factor of the entire circuit of Fig. 11.18 as seen by the source. Calculate the average power delivered by the source.

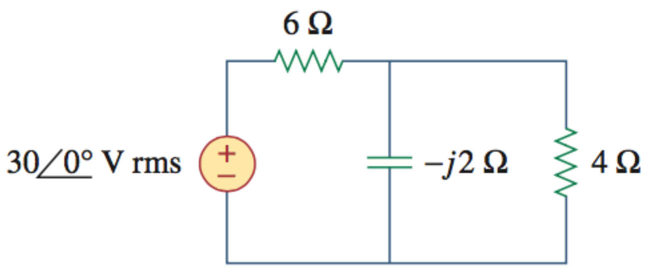


Figure 11.18
For Example 11.10.

功率因数 = 负载相位的cos
平均功率 = 视在功率 x 功率因数

计算技巧！！！掌握！

Solution:
The total impedance is

$$\mathbf{Z} = 6 + 4 \| (-j2) = 6 + \frac{-j2 \times 4}{4 - j2} = 6.8 - j1.6 = 7\underline{/-13.24^\circ}\ \Omega$$

The power factor is

$$\text{pf} = \cos(-13.24) = 0.9734 \qquad \text{(leading)}$$

since the impedance is capacitive. The rms value of the current is

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\underline{/0^\circ}}{7\underline{/-13.24^\circ}} = 4.286\underline{/13.24^\circ}\ \text{A}$$

The average power supplied by the source is

$$P = V_{\text{rms}} I_{\text{rms}}\, \text{pf} = (30)(4.286)0.9734 = 125\ \text{W}$$

or

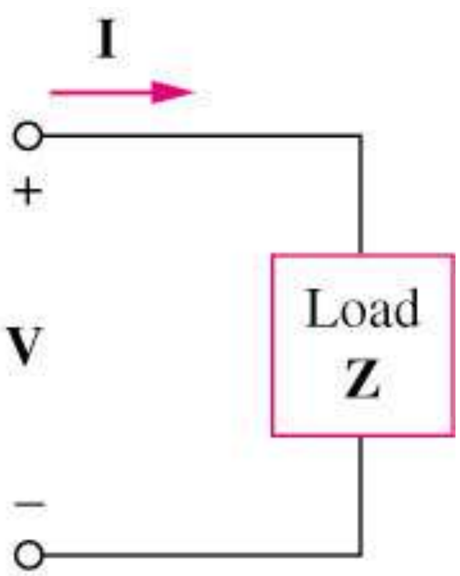
$$P = I_{\text{rms}}^2 R = (4.286)^2(6.8) = 125\ \text{W}$$

where R is the resistive part of $\mathbf{Z}$.

11.5 Complex Power 复功率

Complex power **S** is the product of the voltage and the complex **conjugate** of the current:

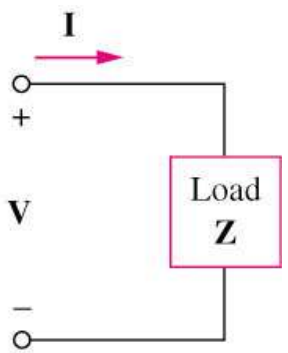
复功率 = 0.5 x 电压相量（振幅） × 电流相量（振幅）的共轭
= 电压相量（有效值）× 电流相量（有效值）的共轭



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = V_{rms} I_{rms} \underline{/\theta_v - \theta_i}$$

11.5 Complex Power



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\boxed{\mathbf{S} = I_{rms}^2 \mathbf{Z} = \frac{V_{rms}^2}{\mathbf{Z}^*} = \mathbf{V}_{rms} \mathbf{I}_{rms}^*}$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}$$

$$\mathbf{S} = P + j\, Q$$

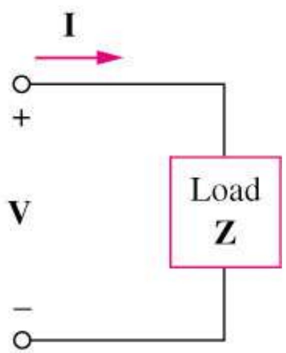
P: is the average power in watts delivered to a load and it is the only useful power. 有功功率，即平均功率，单位**W**

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR. 无功功率，复功率的虚部，单位**VAR**(VA reactive)

- $Q = 0$ for *resistive loads* (unity pf).
- $Q < 0$ for *capacitive loads* (leading pf).
- $Q > 0$ for *inductive loads* (lagging pf).

Q 的正负与 *Z* 相同

11.5 Complex Power



$$\Rightarrow S = \underbrace{V_{rms}\ I_{rms}\cos(\theta_v - \theta_i)}_{P} + j\,\underbrace{V_{rms}\ I_{rms}\sin(\theta_v - \theta_i)}_{Q}$$

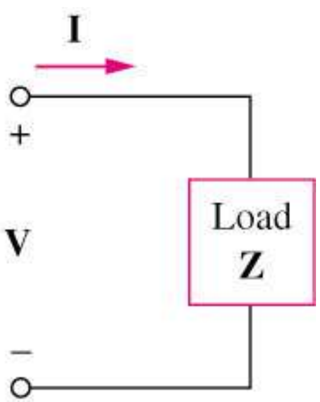
$$\mathbf{S} = P + j\,Q$$

复功率是重要概念，包含了所有描述功率的量

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{rms}(\mathbf{I}_{rms})^* \quad \textbf{VA}$$
$$= |\mathbf{V}_{rms}|\ |\mathbf{I}_{rms}| \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{rms}|\ |\mathbf{I}_{rms}| = \sqrt{P^2 + Q^2} \qquad (11.51)$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

1）它的模：视在功率 **VA**
2）它的相位：→ 功率因数 无量纲
3）它的实部：平均功率（有功功率）**W**
4）它的虚部：无功功率 **VAR**

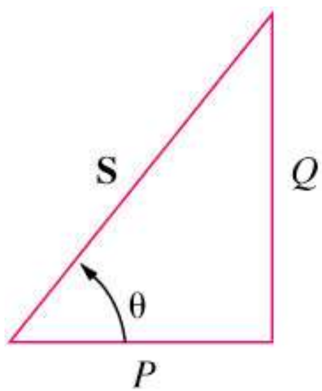
11.5 Complex Power

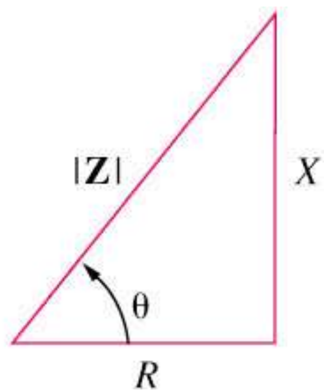


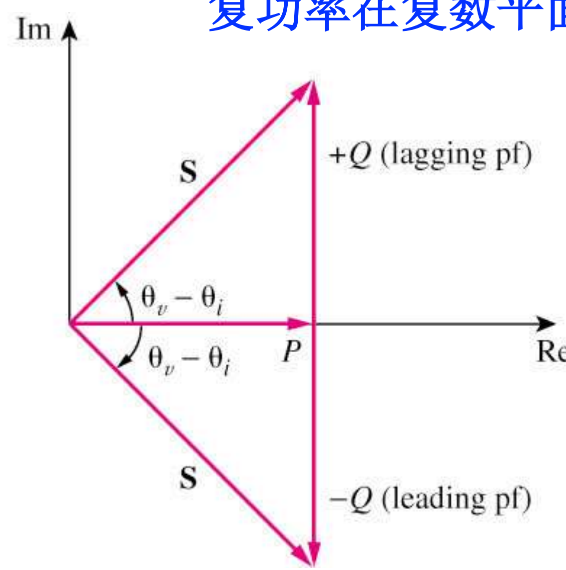
$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$$\mathbf{S} = P + j\,Q$$

复功率在复数平面的表示







感性负载

容性负载

Power Triangle **Impedance Triangle** **Power Factor**

The voltage across a load is $v(t) = 60\cos(\omega t - 10°)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50°)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Solution:

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\angle -10°, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\angle +50°$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^* = \left(\frac{60}{\sqrt{2}}\angle -10°\right)\left(\frac{1.5}{\sqrt{2}}\angle -50°\right) = 45\angle -60° \text{ VA}$$

The apparent power is

$$S = |\mathbf{S}| = 45 \text{ VA}$$

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\angle -60° = 45[\cos(-60°) + j\sin(-60°)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is

$$P = 22.5 \text{ W}$$

while the reactive power is

$$Q = -38.97 \text{ VAR}$$

(c) The power factor is

$$\text{pf} = \cos(-60°) = 0.5 \text{ (leading)}$$

It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\angle -10°}{1.5\angle +50°} = 40\angle -60° \ \Omega$$

which is a capacitive impedance.

①写出电压、电流的相量形式（用有效值表示）

②写出复功率

$$\mathbf{S} = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$

③依次写出其他功率量

Example 11.12

- 单位 VA，说明是“视在功率”
- lagging → 电流 lags 电压 → 感性负载

①根据视在功率和功率因数，可以写出复功率，以及复功率的实部（平均功率）、虚部（无功功率）

②有了复功率，根据电压有效值，可以写出电流有效值

③根据电压电流求阻抗

A load $\mathbf{Z}$ draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source. Calculate: (a) the average and reactive powers delivered to the load, (b) the peak current, and (c) the load impedance.

Solution:

(a) Given that $\text{pf} = \cos\theta = 0.856$, we obtain the power angle as $\theta = \cos^{-1} 0.856 = 31.13°$. If the apparent power is $S = 12{,}000$ VA, then the average or real power is

$$P = S\cos\theta = 12{,}000 \times 0.856 = 10.272 \text{ kW}$$

while the reactive power is

$$Q = S\sin\theta = 12{,}000 \times 0.517 = 6.204 \text{ kVA}$$

(b) Since the pf is lagging, the complex power is

$$\mathbf{S} = P + jQ = 10.272 + j6.204 \text{ kVA}$$

From $\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$, we obtain

$$\mathbf{I}^*_{\text{rms}} = \frac{\mathbf{S}}{\mathbf{V}_{\text{rms}}} = \frac{10{,}272 + j6204}{120\underline{/0°}} = 85.6 + j51.7 \text{ A} = 100\underline{/31.13°} \text{ A}$$

Thus $\mathbf{I}_{\text{rms}} = 100\underline{/-31.13°}$ and the peak current is

$$I_m = \sqrt{2}I_{\text{rms}} = \sqrt{2}(100) = 141.4 \text{ A}$$

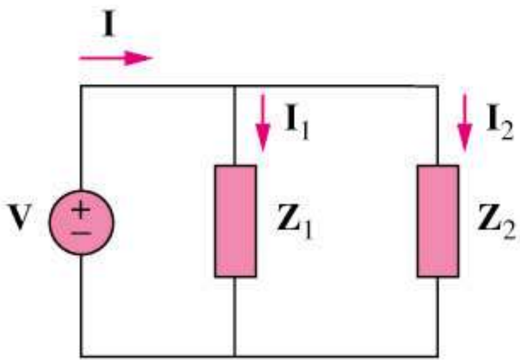
(c) The load impedance

$$\mathbf{Z} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{120\underline{/0°}}{100\underline{/-31.13°}} = 1.2\underline{/31.13°}\ \Omega$$

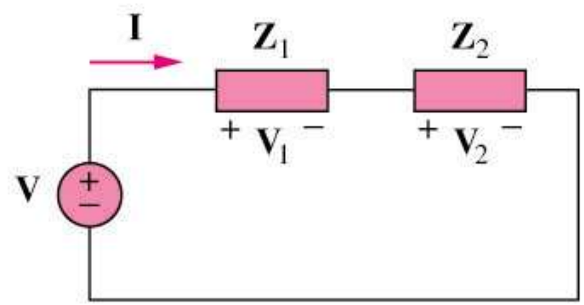
which is an inductive impedance.

11.6 Conservation of AC Power 交流功率守恒

The complex, real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.



(a)



(b)

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = \mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \mathbf{V}\mathbf{I}_1^* + \mathbf{V}\mathbf{I}_2^* = \mathbf{S}_1 + \mathbf{S}_2$$

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = (\mathbf{V}_1 + \mathbf{V}_2)\mathbf{I}^* = \mathbf{V}_1\mathbf{I}^* + \mathbf{V}_2\mathbf{I}^* = \mathbf{S}_1 + \mathbf{S}_2$$

$$\boxed{\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N}$$

复数的加减是实部虚部的加减，不是幅度的加减

复功率、有功功率、无功功率满足相加性，视在功率不满足

Example 11.13

Figure 11.24 shows a load being fed by a voltage source through a transmission line. The impedance of the line is represented by the $(4 + j2)\ \Omega$ impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.

supplied by

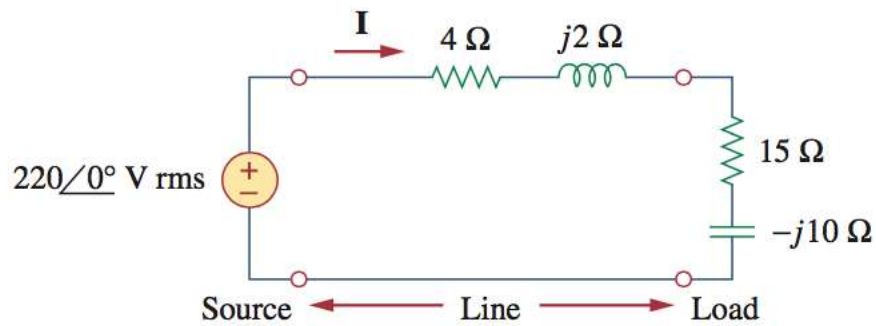


Figure 11.24
For Example 11.13.

$$\mathbf{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62\underline{/-22.83^\circ}\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{220\underline{/0^\circ}}{20.62\underline{/-22.83^\circ}} = 10.67\underline{/22.83^\circ}\ \text{A rms}$$

$$\begin{aligned}\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* &= (220\underline{/0^\circ})(10.67\underline{/-22.83^\circ})\\ &= 2347.4\underline{/-22.83^\circ} = (2163.5 - j910.8)\ \text{VA}\end{aligned}$$

$$\begin{aligned}\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* &= (47.72\underline{/49.4^\circ})(10.67\underline{/-22.83^\circ})\\ &= 509.2\underline{/26.57^\circ} = 455.4 + j227.7\ \text{VA}\end{aligned}$$

$$\boxed{\mathbf{S}_s = \mathbf{S}_{\text{line}} + \mathbf{S}_L,}$$

$$\begin{aligned}\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* &= (192.38\underline{/-10.87^\circ})(10.67\underline{/-22.83^\circ})\\ &= 2053\underline{/-33.7^\circ} = (1708 - j1139)\ \text{VA}\end{aligned}$$

Example 11.14

In the circuit of Fig. 11.26, $\mathbf{Z}_1 = 60\angle{-30^\circ}\ \Omega$ and $\mathbf{Z}_2 = 40\angle{45^\circ}\ \Omega$. Calculate the total: (a) apparent power, (b) real power, (c) reactive power, and (d) pf, supplied by the source and seen by the source.

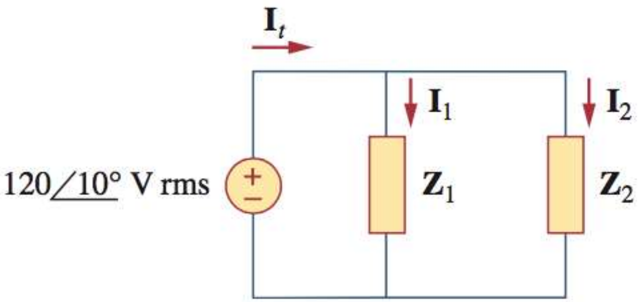


Figure 11.26 根据*VI*写出复功率
For Example 11.14.

Solution:

The current through $\mathbf{Z}_1$ is

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_1} = \frac{120\angle{10^\circ}}{60\angle{-30^\circ}} = 2\angle{40^\circ}\ \text{A rms}$$

while the current through $\mathbf{Z}_2$ is

$$\mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_2} = \frac{120\angle{10^\circ}}{40\angle{45^\circ}} = 3\angle{-35^\circ}\ \text{A rms}$$

The complex powers absorbed by the impedances are

$$\mathbf{S}_1 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_1^*} = \frac{(120)^2}{60\angle{30^\circ}} = 240\angle{-30^\circ} = 207.85 - j120\ \text{VA}$$

$$\mathbf{S}_2 = \frac{V_{\text{rms}}^2}{\mathbf{Z}_2^*} = \frac{(120)^2}{40\angle{-45^\circ}} = 360\angle{45^\circ} = 254.6 + j254.6\ \text{VA}$$

The total complex power is

$$\mathbf{S}_t = \mathbf{S}_1 + \mathbf{S}_2 = 462.4 + j134.6\ \text{VA}$$

(a) The total apparent power is

$$|\mathbf{S}_t| = \sqrt{462.4^2 + 134.6^2} = 481.6\ \text{VA.}$$

(b) The total real power is

$$P_t = \text{Re}(\mathbf{S}_t) = 462.4\ \text{W or } P_t = P_1 + P_2.$$

(c) The total reactive power is

$$Q_t = \text{Im}(\mathbf{S}_t) = 134.6\ \text{VAR or } Q_t = Q_1 + Q_2.$$

(d) The pf $= P_t/|\mathbf{S}_t| = 462.4/481.6 = 0.96$ (lagging).

We may cross check the result by finding the complex power $\mathbf{S}_s$ supplied by the source.

$$\begin{aligned}\mathbf{I}_t = \mathbf{I}_1 + \mathbf{I}_2 &= (1.532 + j1.286) + (2.457 - j1.721)\\ &= 4 - j0.435 = 4.024\angle{-6.21^\circ}\ \text{A rms}\\ \mathbf{S}_s = \mathbf{V}\mathbf{I}_t^* &= (120\angle{10^\circ})(4.024\angle{6.21^\circ})\\ &= 482.88\angle{16.21^\circ} = 463 + j135\ \text{VA}\end{aligned}$$

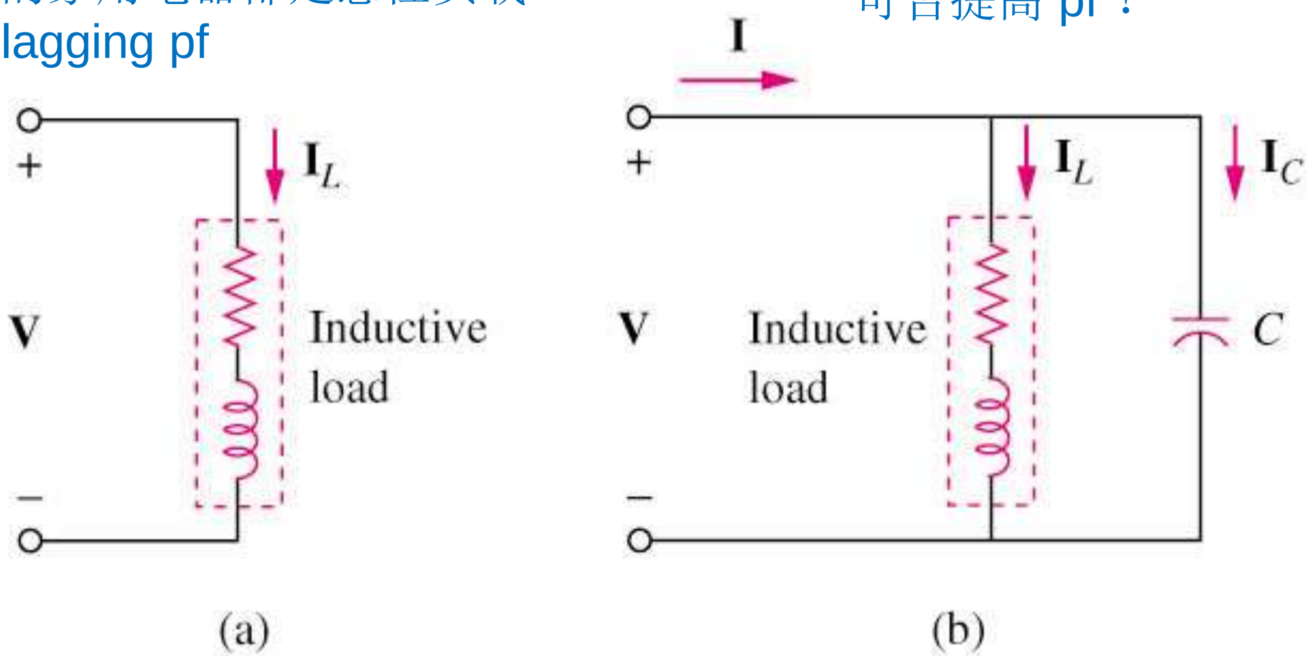
which is the same as before.

11.7 Power Factor Correction 功率因数校正

Power factor correction is the process of increasing the power factor without altering the voltage or current to the original load.

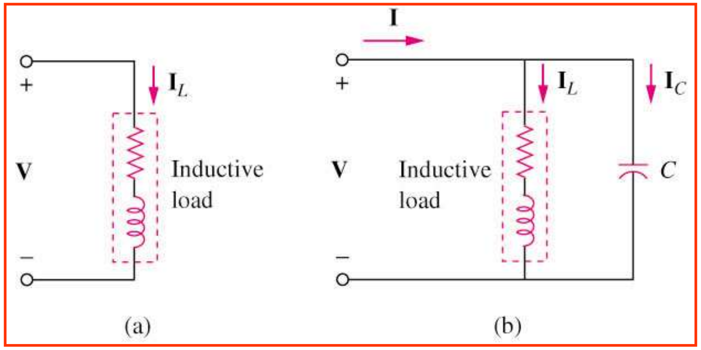
大部分的家用电器都是感性负载
→ low lagging pf

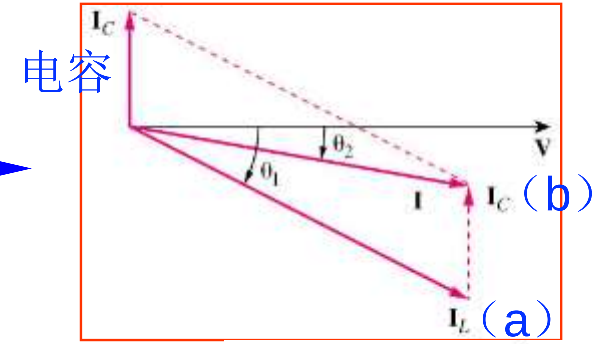
可否提高 pf？



Power factor correction is necessary for economic reason.
国家电网是根据**视在功率**收费的

11.7 Power Factor Correction



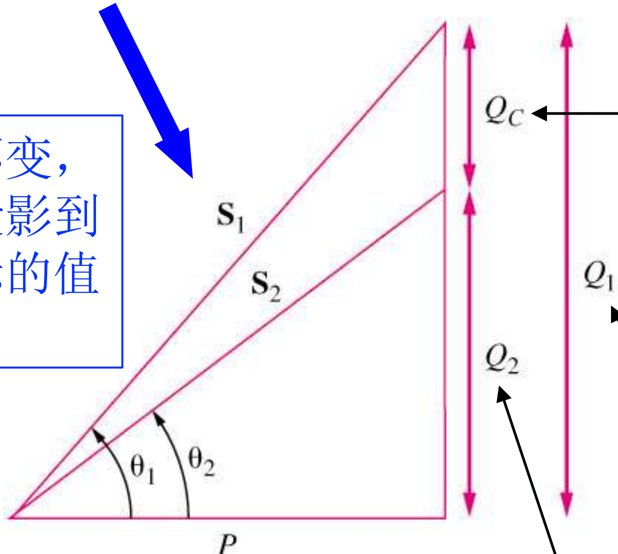


$$\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*}$$

$$Y_2 = Y_1 + j\omega C$$

$$S_2 = S_1 - V_{rms}^2 \cdot j\omega C$$

电阻不变，所以投影到横坐标的值不变



$\mathbf{Q_c = Q_1 - Q_2}$

$\mathbf{= P\,(\tan\theta_1 - \tan\theta_2)}$

$$Q_C = V_{\text{rms}}^2 / X_C = \omega C V_{\text{rms}}^2$$

$\mathbf{Q_1 = S_1 \sin\theta_1}$

$\mathbf{= P \tan\theta_1}$

$\mathbf{Q_2 = P \tan\theta_2}$

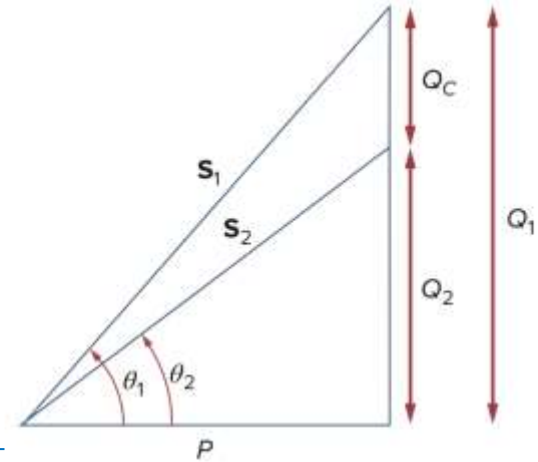
$\mathbf{P = S_1 \cos\theta_1}$

$$C = \frac{Q_c}{\omega V_{\text{rms}}^2} = \frac{P\,(\tan\theta_1 - \tan\theta_2)}{\omega\, V_{\text{rms}}^2}$$

Example 11.15

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

单位 kW，说明是平均功率



Solution:

If the pf = 0.8, then

$$\cos\theta_1 = 0.8 \quad \Rightarrow \quad \theta_1 = 36.87^\circ$$

where θ_1 is the phase difference between voltage and current. We obtain the apparent power from the real power and the pf as

$$S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

When the pf is raised to 0.95,

$$\cos\theta_2 = 0.95 \quad \Rightarrow \quad \theta_2 = 18.19^\circ$$

The real power P has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The new reactive power is

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

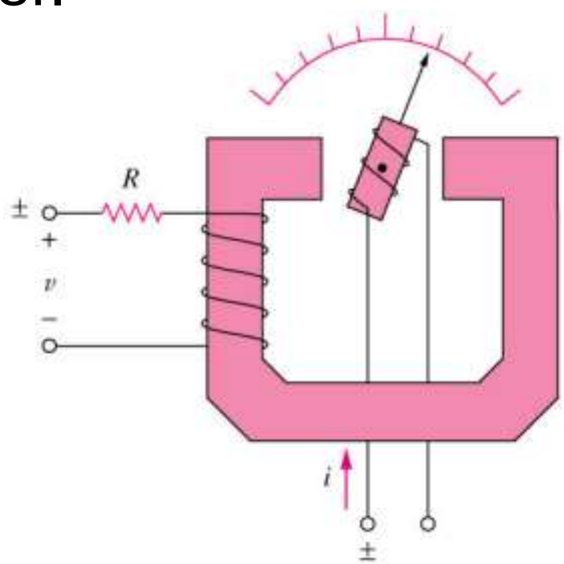
and

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu\text{F}$$

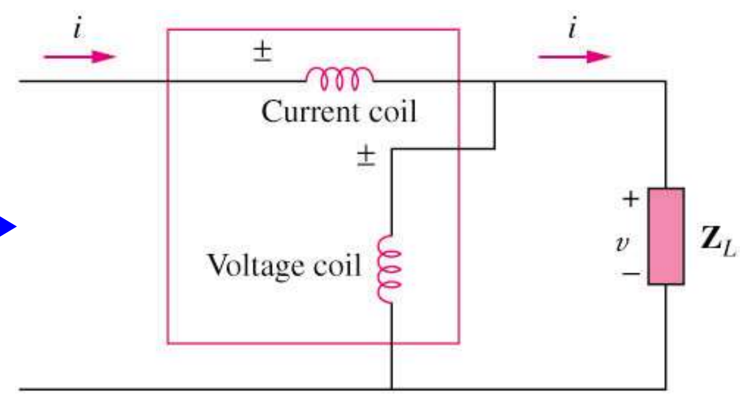
11.8 Power Measurement

测量得到的是平均功率（有功功率）

The wattmeter is the instrument for measuring the average power.



The basic structure



Equivalent Circuit with load

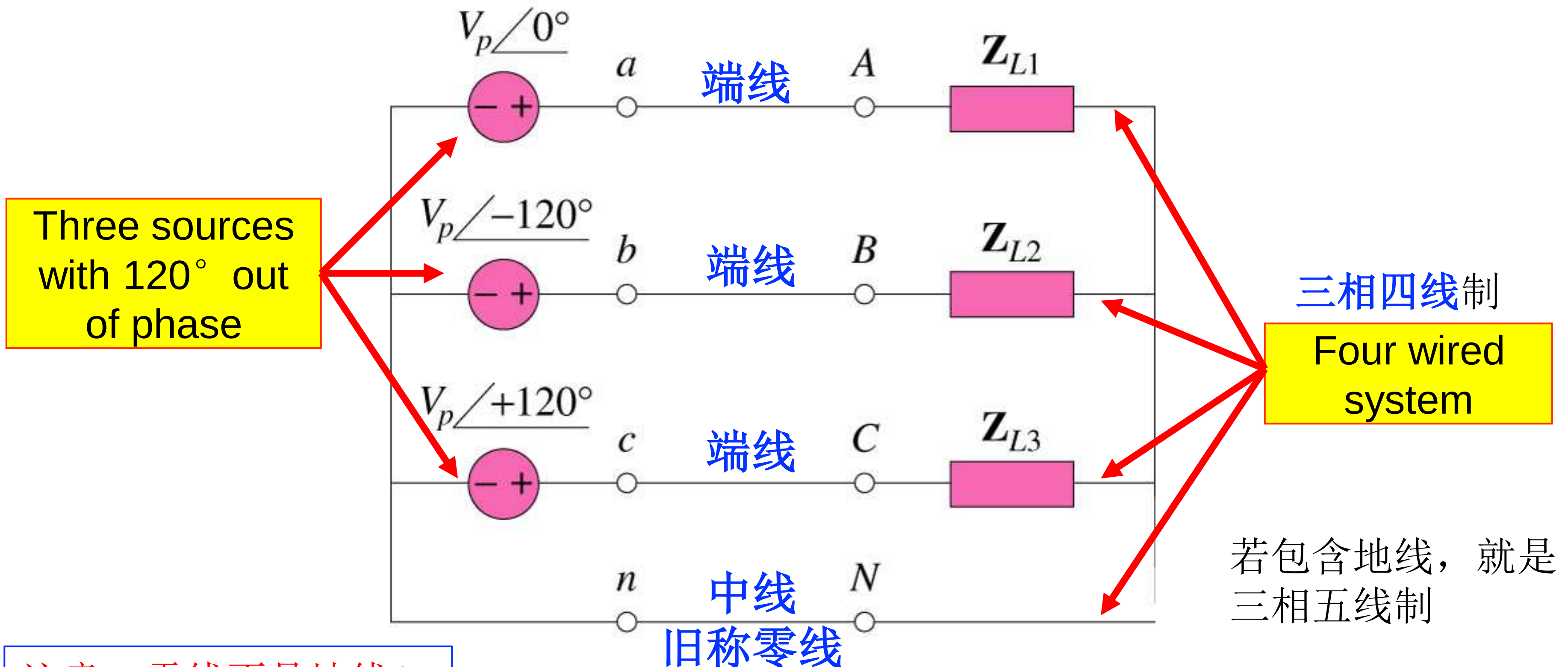
If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Three-Phase Circuits
Chapter 12 三相电路

12.1 What is a Three-Phase Circuit?

- It is a system produced by a generator consisting of **<u>three sources</u>** having the same amplitude and frequency but **<u>out of phase</u>** with each other by 120°.

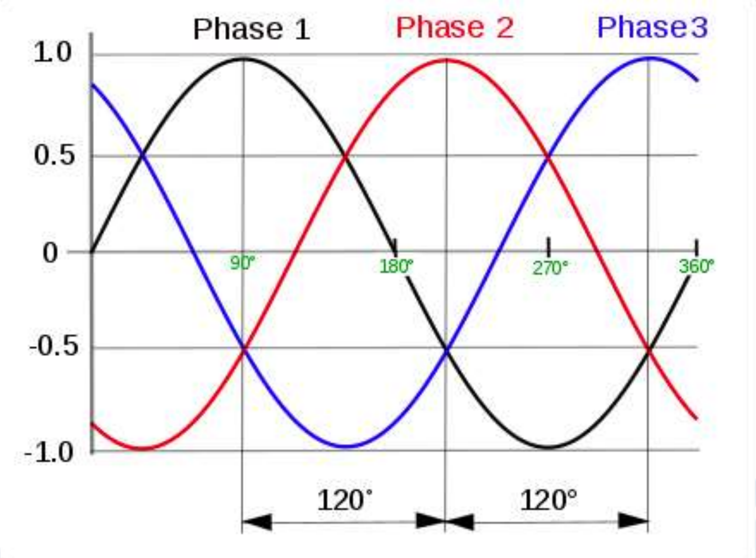


注意：零线不是地线！

12.1 What is a Three-Phase Circuit?

Advantages:

1. Most of the electric power is generated and distributed in three-phase.
2. The instantaneous power in a three-phase system can be constant. 瞬时功率和为常数
3. The amount of power, the three-phase system is more economical than the single-phase. 发电机容易做
4. In fact, the amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.



12.2 Balanced Three-Phase Voltages

- A three-phase generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).

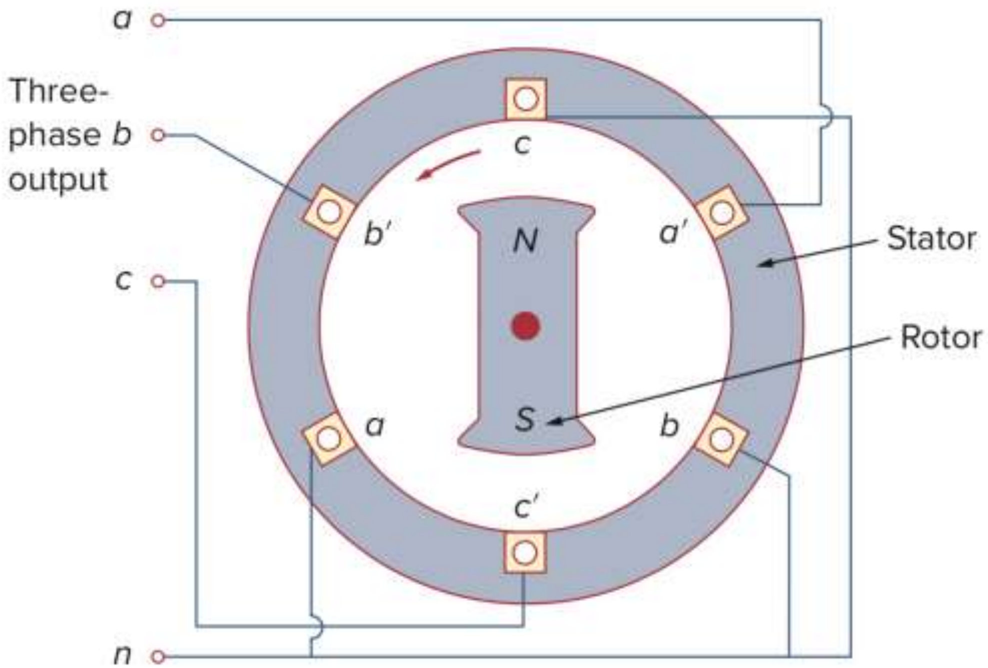
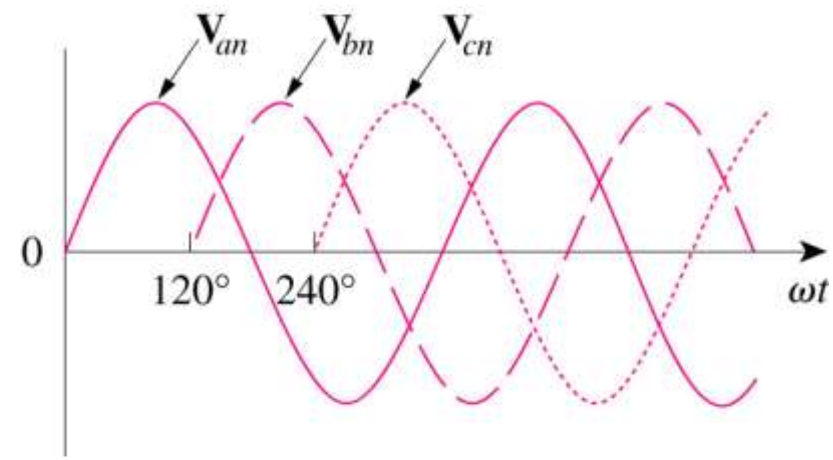


Figure 12.4
A three-phase generator.



The generated voltages

12.2 Balanced Three-Phase Voltages

- Two possible configurations:

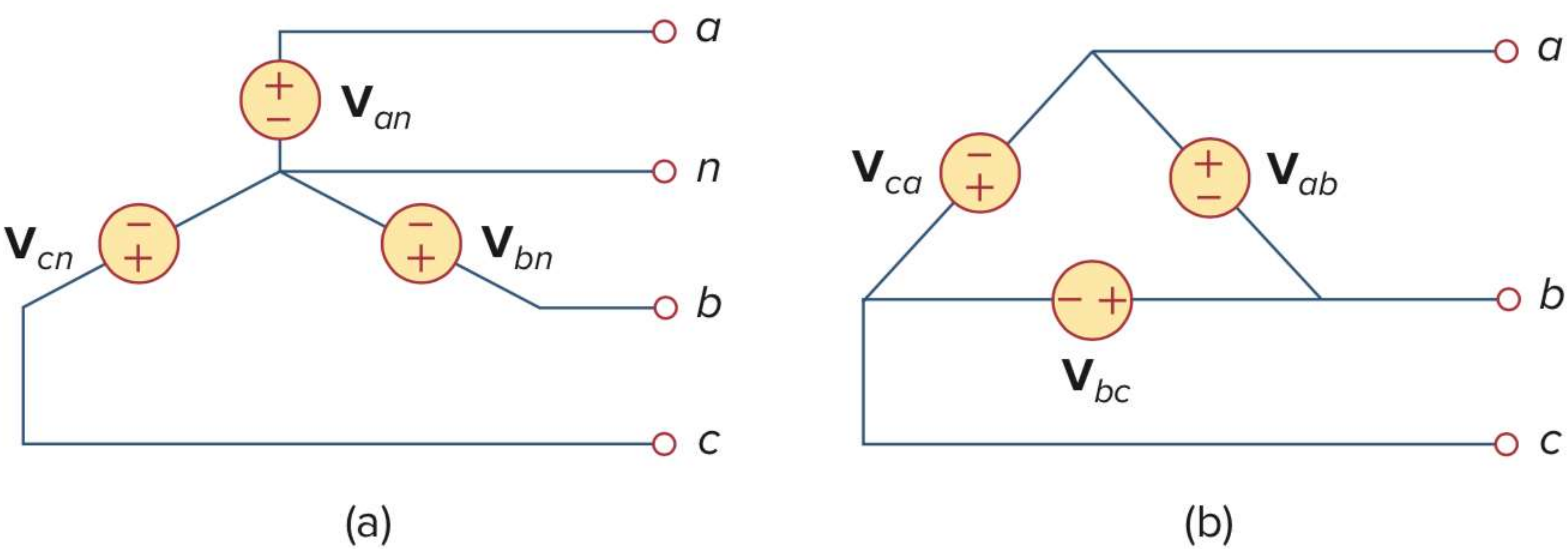
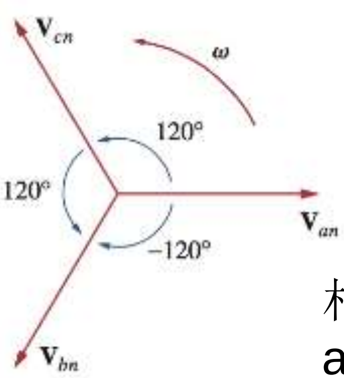


Figure 12.6
Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

12.2 Balanced Three-Phase Voltages

- ***Balanced phase voltages***（平衡相电压） are equal in magnitude and are out of phase with each other by 120°.

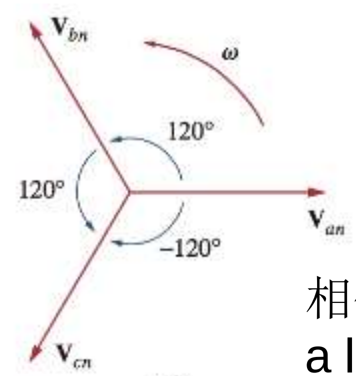
- The ***phase sequence***（相序） is the time order in which the voltages pass through their respective maximum values.



(a)

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

相位关系：
a leads b, b leads c
故称为**abc序列**，或顺序【电力系统常用】



(b)

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

相位关系：
a leads c, c leads b
故称为**acb序列**，或逆序

•A ***balanced load*** （平衡负载） is one in which the phase impedances are equal in magnitude and in phase

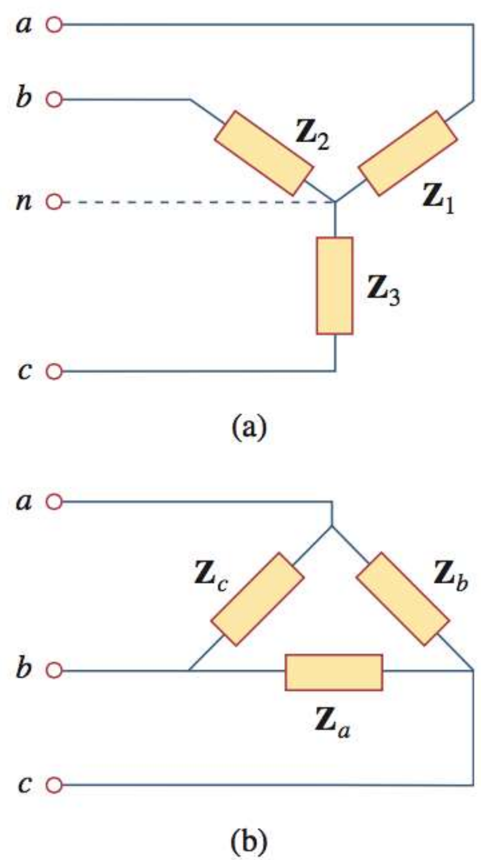


Figure 12.8
Two possible three-phase load configurations: (a) a Y-connected load, (b) a Δ-connected load.

For a *balanced* wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$

where $\mathbf{Z}_Y$ is the load impedance per phase. For a *balanced* delta-connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta \tag{12.7}$$

where $\mathbf{Z}_\Delta$ is the load impedance per phase in this case. We recall from Eq. (9.69) that

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \tag{12.8}$$

Example 12.1

Determine the phase sequence of the set of voltages

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ), \qquad v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200\underline{/10^\circ}\text{ V}, \qquad \mathbf{V}_{bn} = 200\underline{/-230^\circ}\text{ V}, \qquad \mathbf{V}_{cn} = 200\underline{/-110^\circ}\text{ V}$$

We notice that $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120° and $\mathbf{V}_{cn}$ in turn leads $\mathbf{V}_{bn}$ by 120°. Hence, we have an *acb* sequence.

12.3 Balanced Three-Phase Connection

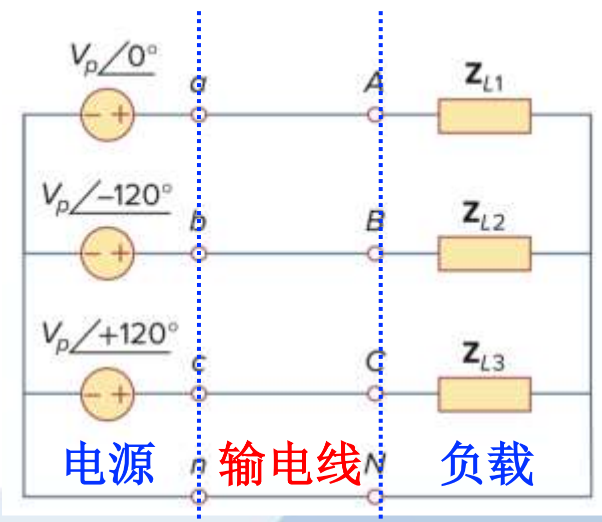
- Four possible connections

  1. Y-Y connection (Y-connected source with a Y-connected load) 最容易理解
  2. Y-Δ connection (Y-connected source with a Δ-connected load) **最常用**
  3. Δ-Δ connection
  4. Δ-Y connection

电源端若是Δ连接，则不能接中线，所以一般电源端都是Y连接

三相电的电压电流概念

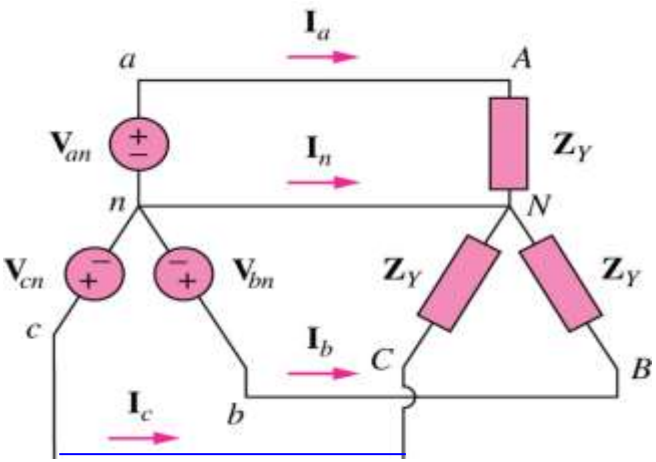
- **线电压、线电流**
  - 流经输电线中的电流，称为线电流
  - 各输电线之间的电压，称为线电压
- **相电压、相电流**
  - 三相电源或三相负载中，每一相的电压，称为相电压
  - 三相电源或三相负载中，每一相的电流，称为相电流



实际三相电路中，三相电源是对称的，三条端线阻抗是相等的，但负载阻抗不一定是相等的。
负载阻抗相等的情况，称为平衡三相电

12.3 Balanced Three-Phase Connection

- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

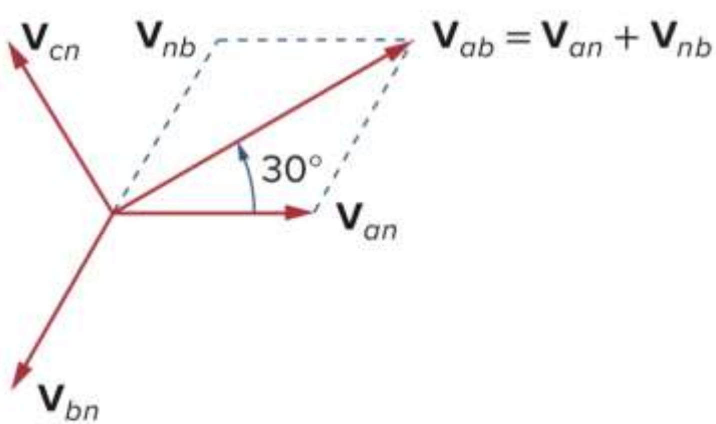


Phase voltage，相电压，即电源或负载中每一相的电压，相线（火线，abc）与中线（零线，n）之间的电压

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

Line-to-line voltage，线电压，即输电线 abc 两两之间的电压（ab,bc,ca）



$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$

$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ$$

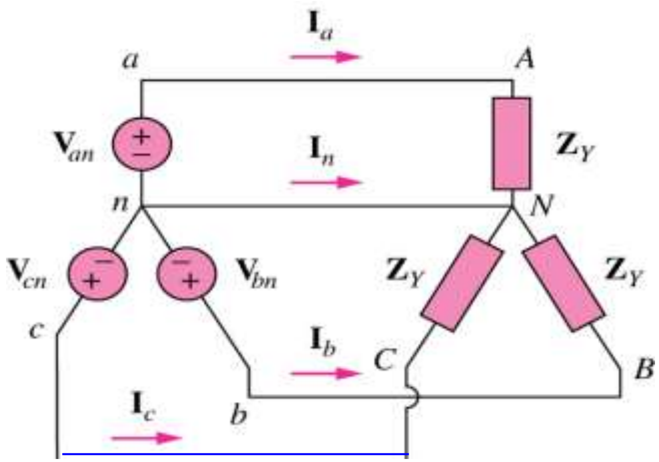
$$V_L = \sqrt{3}V_p \text{, where}$$

$$V_p = |\mathrm{V}_{an}| = |\mathrm{V}_{bn}| = |\mathrm{V}_{cn}|$$

$$V_L = |\mathrm{V}_{ab}| = |\mathrm{V}_{bc}| = |\mathrm{V}_{ca}|$$

12.3 Balanced Three-Phase Connection

- A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



While the line current is the current in each line, the *phase* current is the current in each phase of the source or load. In the Y-Y system, the line current is the same as the phase current. We will use single subscripts

线电流：流经输电线中的电流；相电流：电源或负载中每一相的电流；对于Y-Y连接，线电流 = 相电流

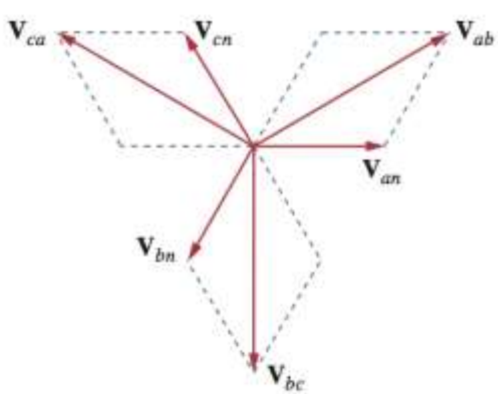
$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -120^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -120^\circ$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle -240^\circ}{\mathbf{Z}_Y} = \mathbf{I}_a\angle -240^\circ$$

$$\Downarrow$$

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \qquad \mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0$$

中线几乎无电流，线可以细一点，省材料



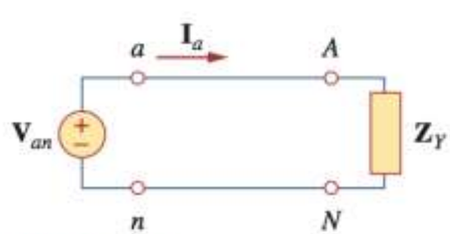


Figure 12.12
A single-phase equivalent circuit.

可以单独拿出一相作分析

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

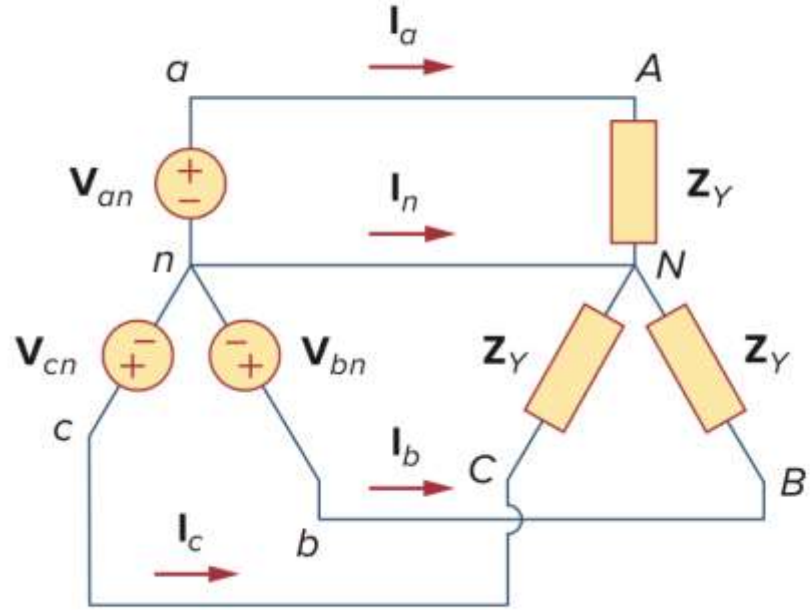


Figure 12.10
Balanced Y-Y connection.

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$

Calculate the line currents in the three-wire Y-Y system of Fig. 12.13.

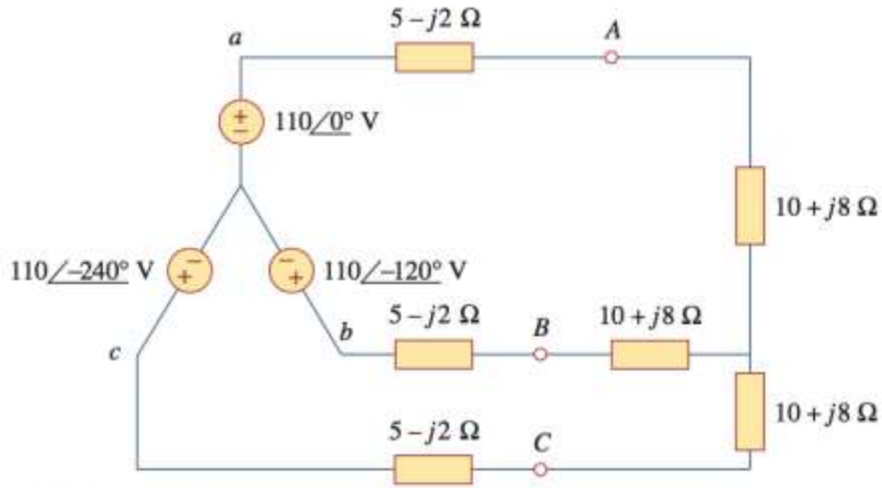
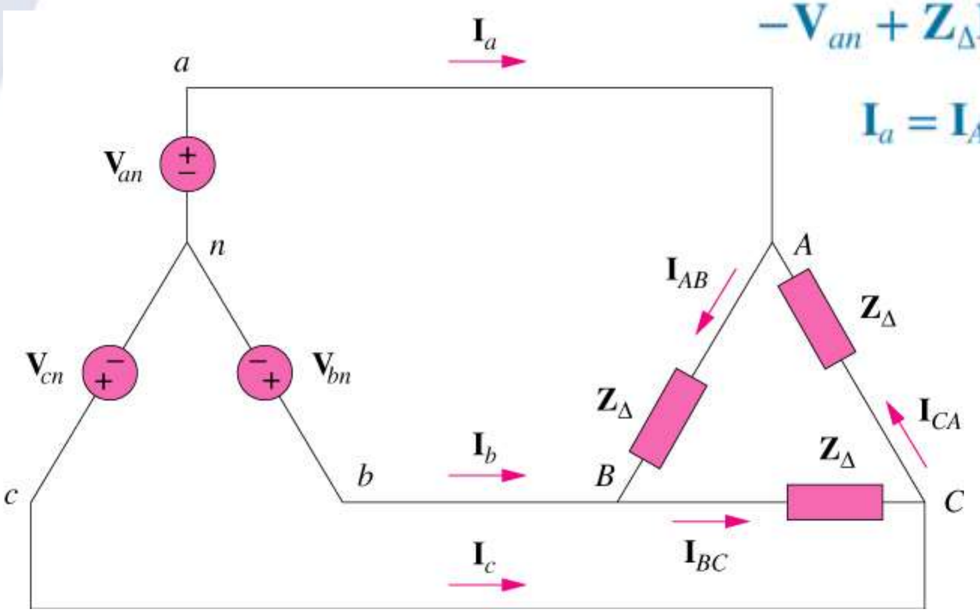


Figure 12.13
Three-wire Y-Y system; for Example 12.2.

Y-Y情况下，
把中线添加回去，简化分析

12.3 Balanced Three-Phase Connection

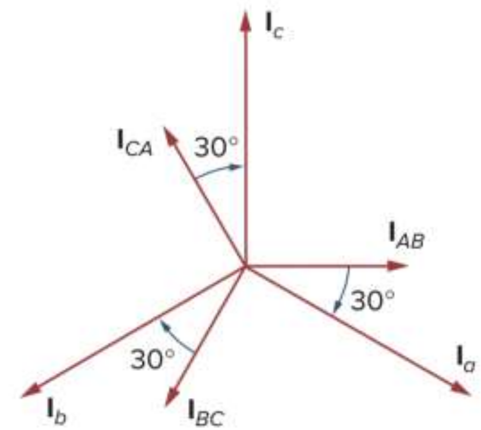
- A balanced Y-Δ system is a three-phase system with a balanced Y-connected source and a balanced Δ-connected load.



$$-\mathbf{V}_{an} + \mathbf{Z}_\Delta \mathbf{I}_{AB} + \mathbf{V}_{bn} = 0 \qquad \mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle{-240^\circ})$$

$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}$$



- 相电压看源端 $\mathbf{V}_{an} = V_p\angle{0^\circ}$ $\quad \mathbf{V}_{ab} = \sqrt{3}V_p\angle{30^\circ} = \mathbf{V}_{AB}$
- 相电流看负载端
- 线电压线电流无歧义

$$I_L = \sqrt{3} I_p \text{, where}$$

$$I_L = |\mathrm{I}_a| = |\mathrm{I}_b| = |\mathrm{I}_c|$$

$$I_p = |\mathrm{I}_{AB}| = |\mathrm{I}_{BC}| = |\mathrm{I}_{CA}|$$

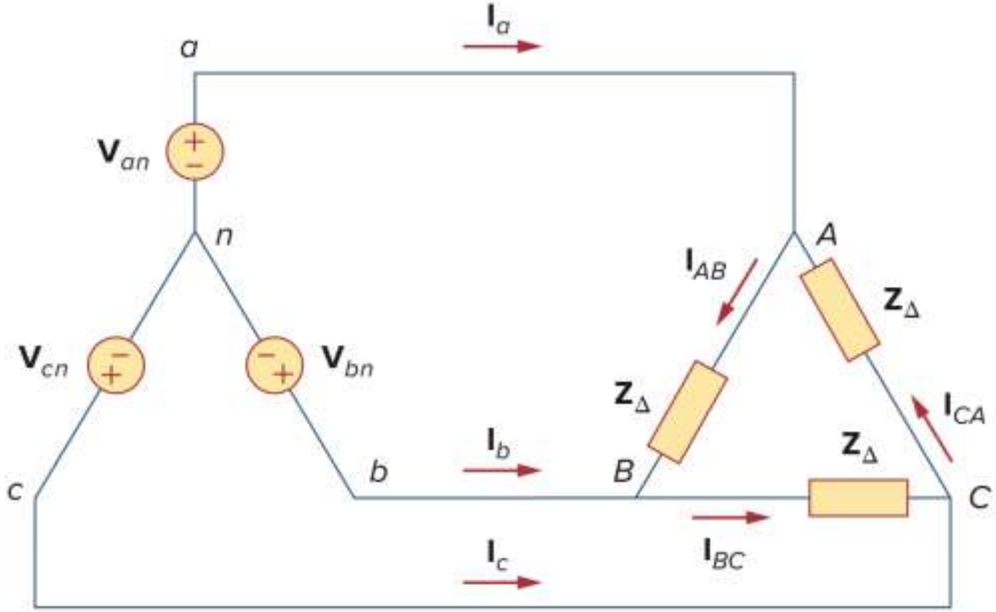
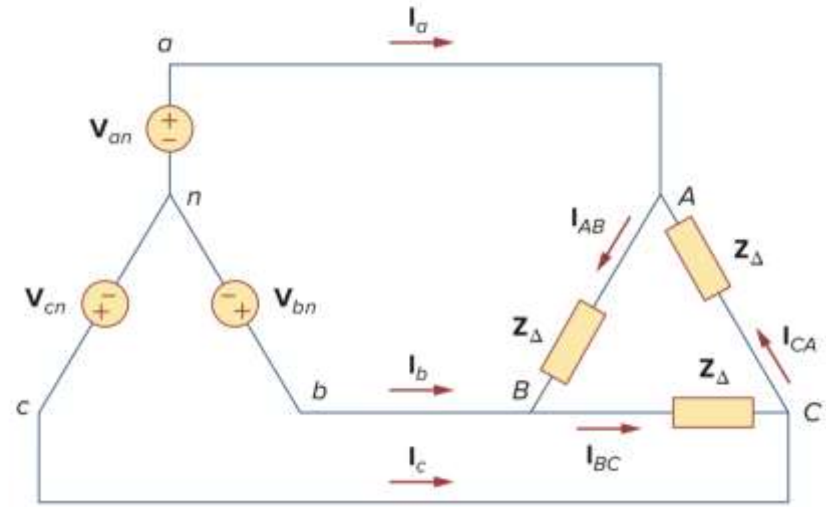


Figure 12.14
Balanced Y-Δ connection.

Connection	Phase voltages/currents	Line voltages/currents
Y-Δ	$\mathbf{V}_{an} = V_p\underline{/0°}$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}\,V_p\underline{/30°}$
	$\mathbf{V}_{bn} = V_p\underline{/-120°}$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\underline{/-120°}$
	$\mathbf{V}_{cn} = V_p\underline{/+120°}$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\underline{/+120°}$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30°}$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\underline{/-120°}$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\underline{/+120°}$

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\underline{/10^\circ}$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

Example 12.3



把负载的Δ转换成Y后，用Y-Y的单相计算

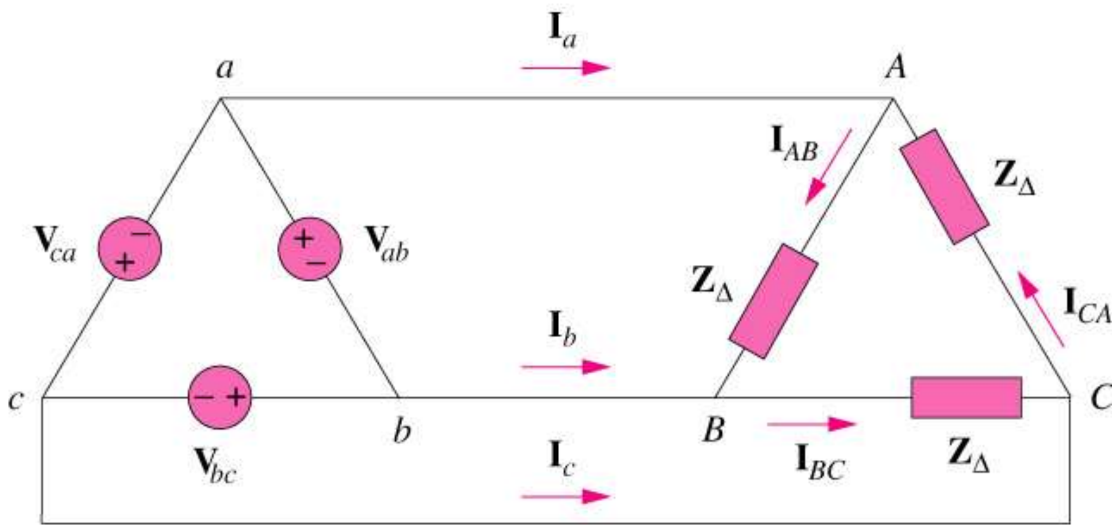
METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

12.3 Balance Three-Phase Connection (6)

- A balanced Δ-Δ system is a three-phase system with a balanced Δ-connected source and a balanced Δ-connected load.



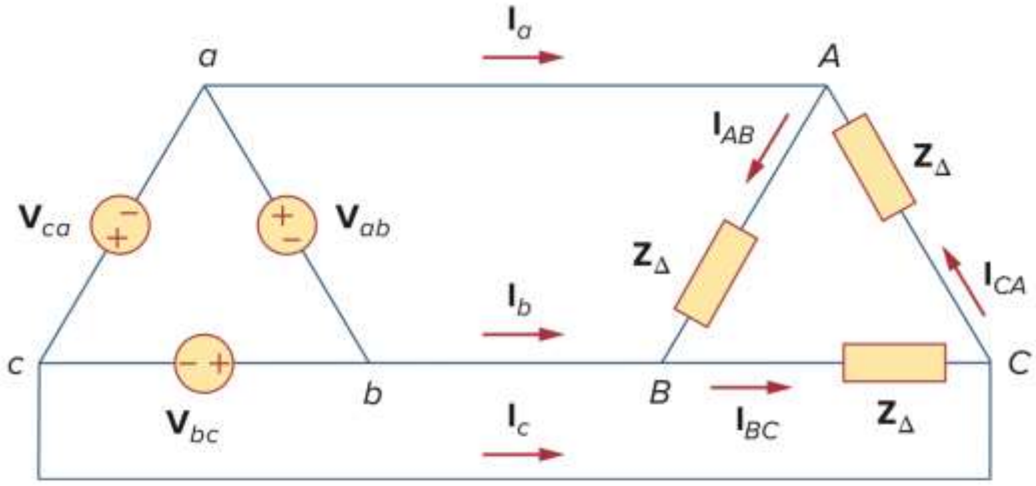
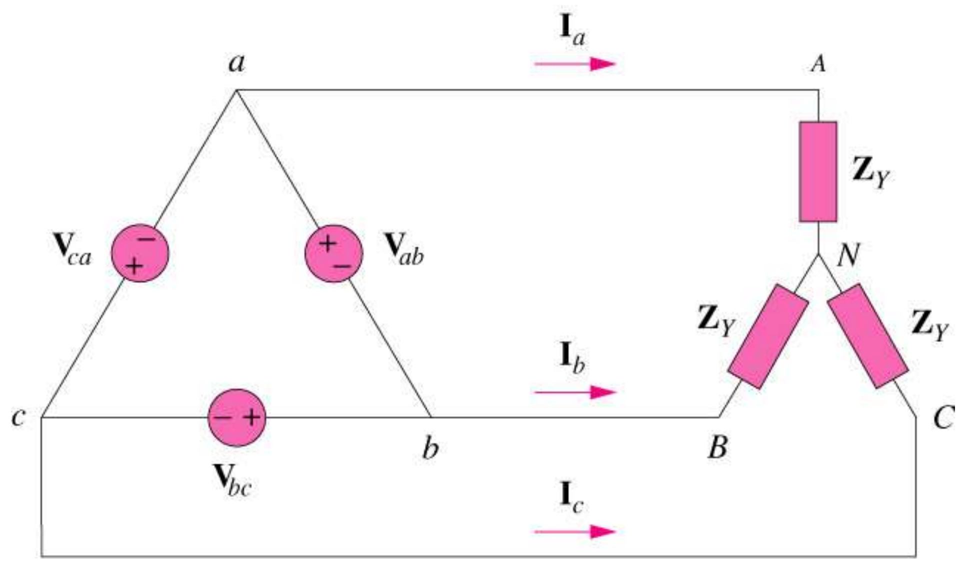


Figure 12.17
A balanced Δ-Δ connection.

Connection	Phase voltages/currents	Line voltages/currents
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

12.3 Balance Three-Phase Connection (8)

- A balanced Δ-Y system is a three-phase system with a balanced Δ-connected source and a balanced Y-connected load.



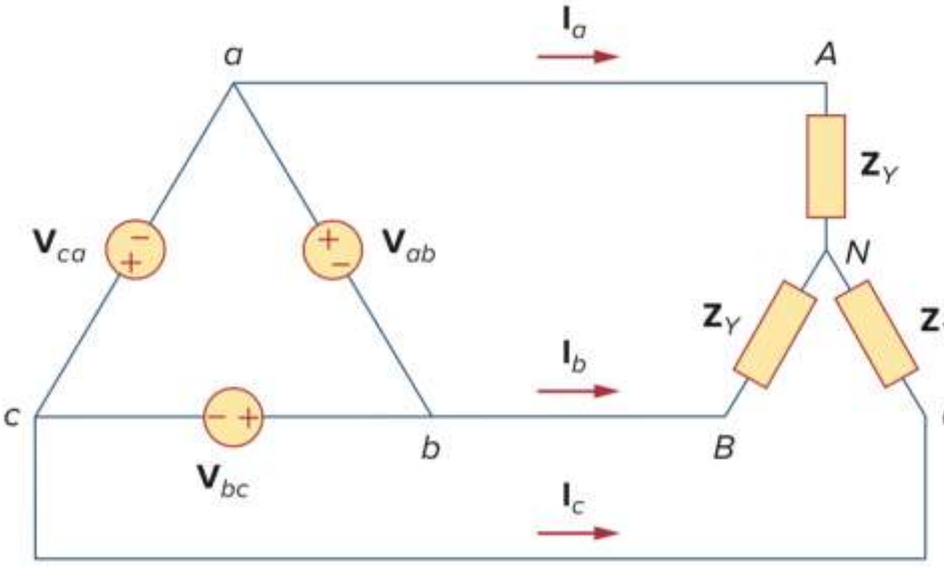


Figure 12.18
A balanced Δ-Y connection.

把负载的Y 转换成Δ再分析

Connection	Phase voltages/currents	Line voltages/currents
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$	Same as phase voltages
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

TABLE 12.1

Summary of phase and line voltages/currents for balanced three-phase systems.[1]

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

小结

- 瞬时功率

$$p(t) = v(t)i(t)$$

- 平均功率

$$P = \frac{1}{2}\mathrm{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

- 最大功率传输（共轭匹配）

$$\mathbf{Z}_L = R_L + jX_L = R_{\mathrm{Th}} - jX_{\mathrm{Th}} = \mathbf{Z}_{\mathrm{Th}}^*$$

$$P_{\max} = \frac{|\mathbf{V}_{\mathrm{Th}}|^2}{8R_{\mathrm{Th}}}$$

  - 纯阻性负载情况

$$R_L = \sqrt{R_{\mathrm{Th}}^2 + X_{\mathrm{Th}}^2} = |\mathbf{Z}_{\mathrm{Th}}|$$

小结

- 周期信号的有效值：信号的均方根，对于正弦信号 $V_{\mathrm{rms}} = \dfrac{V_m}{\sqrt{2}}$

$$X_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}$$

$$P = I_{\mathrm{rms}}^2 R = \frac{V_{\mathrm{rms}}^2}{R}$$

- 视在功率 & 功率因数

平均功率 = 视在功率 x 功率因数

$$S = V_{\mathrm{rms}} I_{\mathrm{rms}}$$

$$\mathrm{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

复功率

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

$$\mathbf{S} = \mathbf{V}_{\mathrm{rms}}\mathbf{I}_{\mathrm{rms}}^*$$

小结

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^* \quad \text{VA}$$
$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2} \qquad (11.51)$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

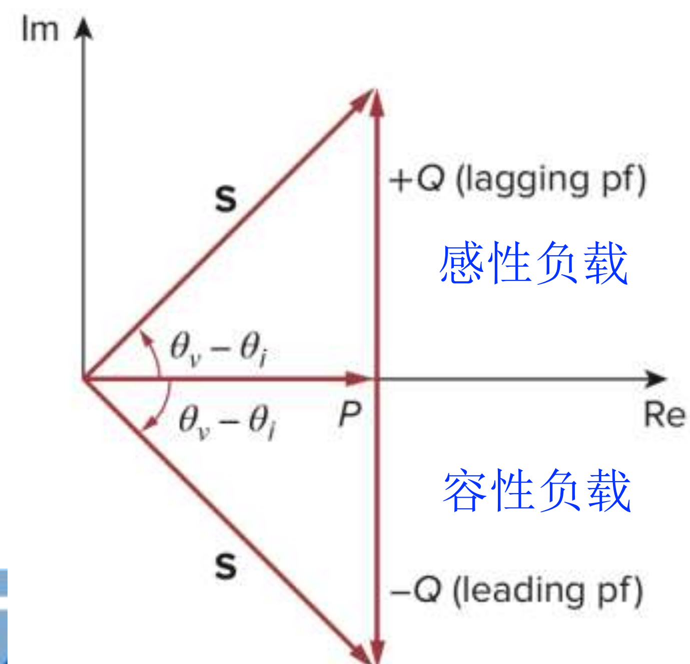
1）它的模：视在功率 VA
2）它的相位：→ 功率因数 无量纲
3）它的实部：平均功率（有功功率）W
4）它的虚部：无功功率 VAR

- 从复功率导出所有功率量

$$\mathbf{S} = I_{\text{rms}}^2\mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^*$$

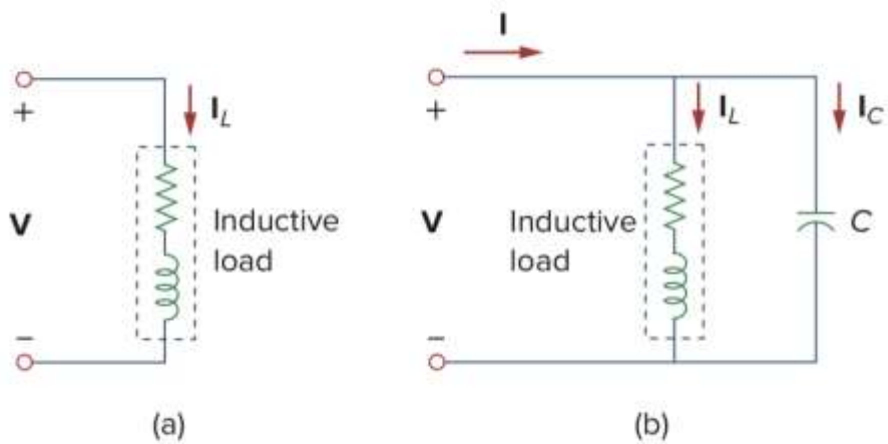
- 复功率守恒
  - 实部、虚部可相加，幅度不行

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \cdots + \mathbf{S}_N$$

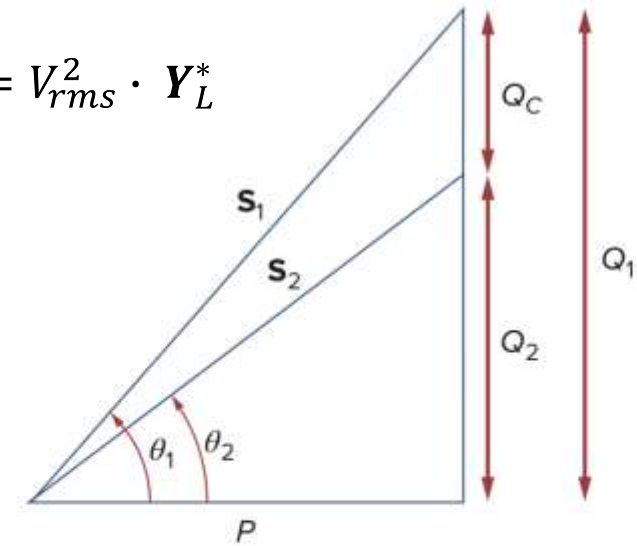


小结

- 功率因数校准



$\mathbf{S} = V_{rms}^2 \cdot \boldsymbol{Y}_L^*$



  - 家用电器一般是感性负载
  - 并联一个合适的C，可以增加功率因数

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

小结

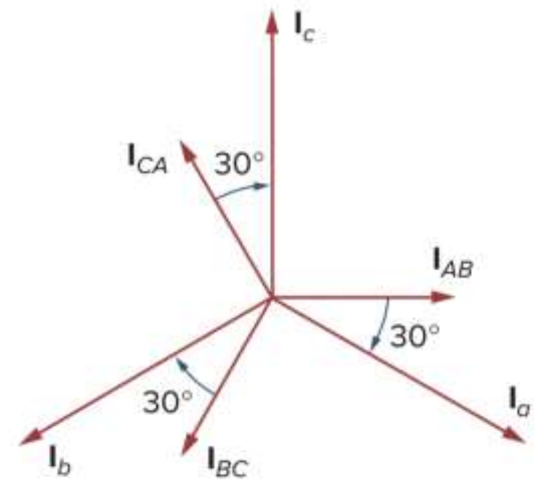
- 三相电传输
  - 三相四线制（ABC三端线 + 中线）
  - 三相五线制（ABC三端线 + 中线 + 地线）

- 电力系统常用abc顺序
  - 相位 a 领先 b 120度，b 领先 c 120度

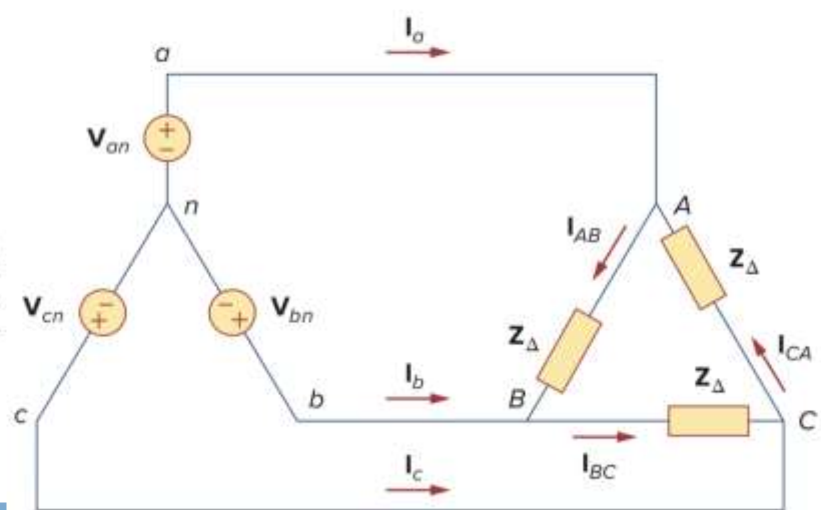
- 线电流、线电压、相电流、相电压的概念
  - 输电线上的电流——线电流；输电线之间的电压——线电压
  - 电源或负载端，每一相的电压——相电压；每一相的电流——相电流。相电压看电源端；相电流看负载端。

小结

- 三相电的常用接法：电源端Y，负载端Y或Δ
- 三相电的分析技巧
  - 将负载端的Y或Δ进行转换，有时可简化分析；
  - 添加中线，有时可简化分析；
  - 电压电流相加减：矢量运算



Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30°$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$



作业

Practice Problem 11.1

Calculate the instantaneous power and average power absorbed by the passive linear network of Fig. 11.1 if

$$v(t) = 330\cos(10t + 20^\circ)\text{ V} \quad \text{and} \quad i(t) = 33\sin(10t + 60^\circ)\text{ A}$$

Answer: $3.5 + 5.445\cos(20t - 10^\circ)$ kW, 3.5 kW.

$$\boxed{p(t) = v(t)i(t)}$$

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

瞬时功率和平均功率的定义

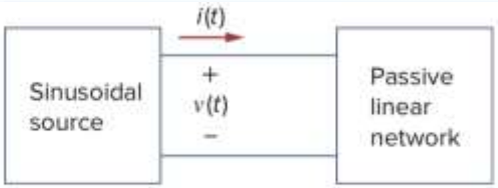


Figure 11.1
Sinusoidal source and passive linear circuit.

Practice Problem 11.5

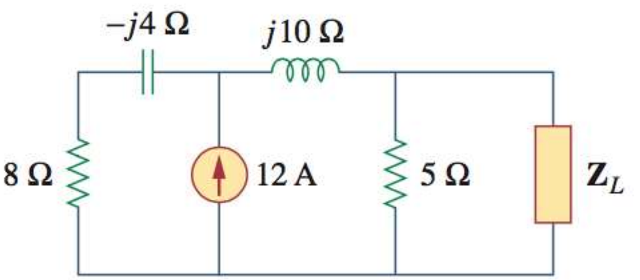


Figure 11.10
For Practice Prob. 11.5.

For the circuit shown in Fig. 11.10, find the load impedance $\mathbf{Z}_L$ that absorbs the maximum average power. Calculate that maximum average power.

共轭匹配

Answer: $3.415 - j0.7317\ \Omega$, 51.47 W.

Practice Problem 11.11

For a load, $\mathbf{V}_{rms} = 110\underline{/85^\circ}$ V, $\mathbf{I}_{rms} = 3\underline{/15^\circ}$ A. Determine: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

从复功率导出其他所有功率量

Answer: (a) $330\underline{/70^\circ}$ VA, 330 VA (b) 112.87 W, 310.1 VAR, (c) 0.342 lagging, $(12.541 + j34.46)$ Ω.

Practice Problem 11.15

Find the value of parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume that the load is sup - plied by a 220-V (rms), 60-Hz line.

功率因数校正

$$C = \frac{Q_C}{\omega V_{\mathrm{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\mathrm{rms}}^2}$$

Answer: 7.673 mF.