电子电路基础

第二讲：电路分析的基本方法和定理~part1

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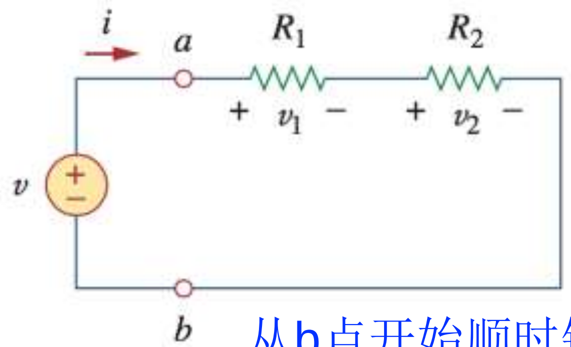
电路分析的基本方法和定理

- 2.1 电阻电路的一般分析方法
  - 2.1.1 电阻的串联和并联
  - 2.1.2 电阻的混联和Y-Δ等效变换
- 2.2 电容和电感的串联和并联
- 2.3 电路定理
  - 2.3.1 节点、支路、回路和网孔基本概念
  - 2.3.2 网孔电流法和节点电压法（包括不含受控源和含受控源的电路的分析）
  - 2.3.3 叠加定理和替代定理
  - 2.3.4 戴维南定理和诺顿定理
  - 2.3.5 最大功率传递定理
- 2.4 电路等效和输入电阻

电阻串联、电阻分压

The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.



从b点开始顺时针写KVL

Figure 2.29
A single-loop circuit with two resistors in series.

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

$$\frac{v_1}{v_2} = \frac{R_1}{R_2}$$

$$\left.\begin{aligned} -v + v_1 + v_2 &= 0 \\ v_1 = iR_1, \qquad v_2 &= iR_2 \end{aligned}\right\} \quad i = \frac{v}{R_1 + R_2}$$

电流相等

电阻串联：熟练掌握两电阻串联

- 等效电阻 = 各电阻之和
- 电阻电压：按阻值比例**分压**

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

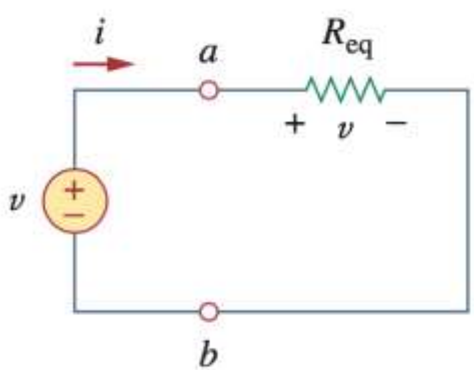


Figure 2.30
Equivalent circuit of the Fig. 2.29 circuit.

电阻并联、电阻分流

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

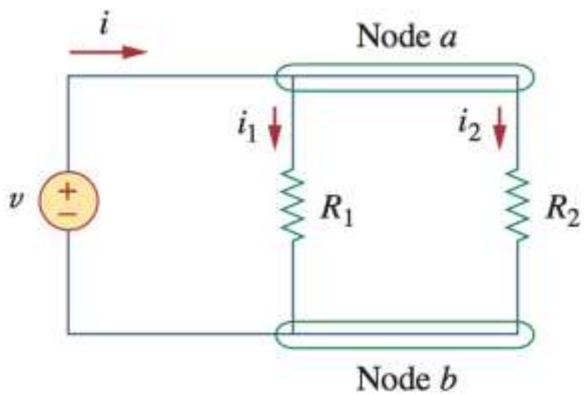


Figure 2.31
Two resistors in parallel.

$$i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

对a点写KCL $i = i_1 + i_2$

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2}$$

$$v = iR_{eq} = \frac{iR_1 R_2}{R_1 + R_2}$$

电压相等

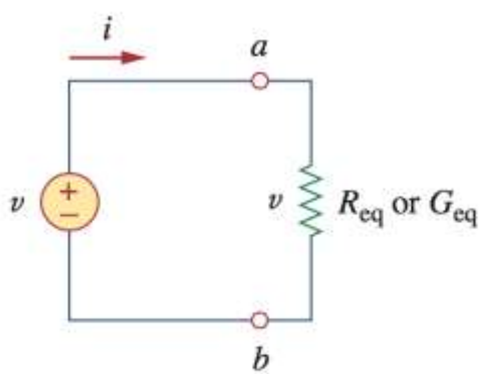


Figure 2.32
Equivalent circuit to Fig. 2.31.

电阻并联：熟练掌握两电阻并联

- 等效电导 = 各电导之和
- **等效电阻 = 两电阻之积/两电阻之和**
- 电阻电流：按比例**分流**，阻值越大，电流越小

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

Find R_{eq} for the circuit shown in Fig. 2.34.

Example 2.9

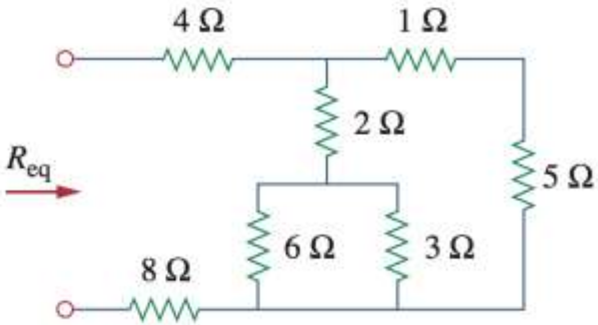


Figure 2.34
For Example 2.9.

Example 2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

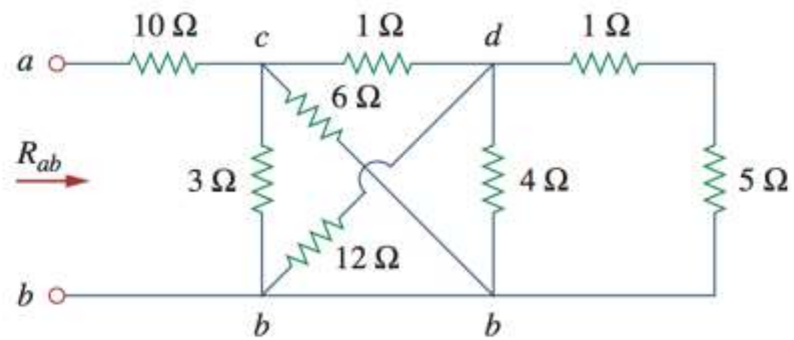
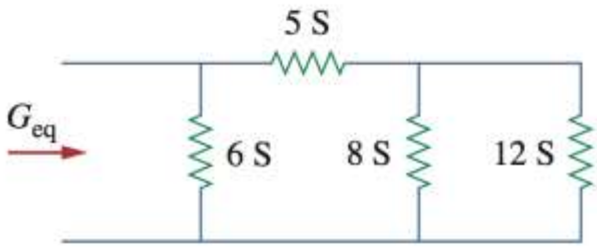


Figure 2.37
For Example 2.10.

Find the equivalent conductance G_{eq} for the circuit in Fig. 2.40(a).

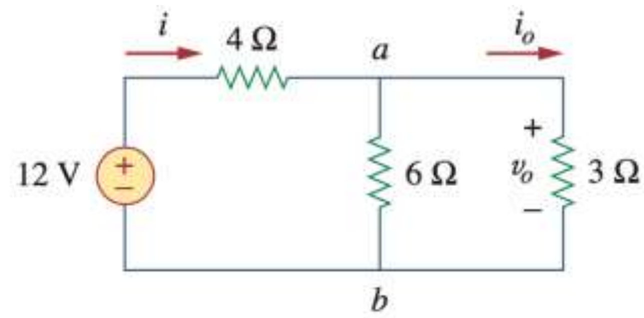
Example 2.11



(a)

Example 2.12

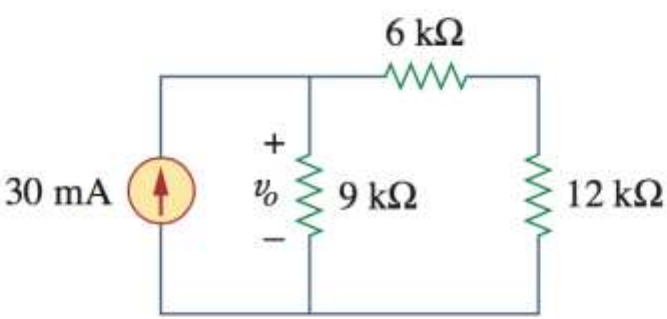
Find i_o and v_o in the circuit shown in Fig. 2.42(a). Calculate the power dissipated in the 3-Ω resistor.



(a)

Example 2.13

For the circuit shown in Fig. 2.44(a), determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.



(a)

下面电路如何计算？

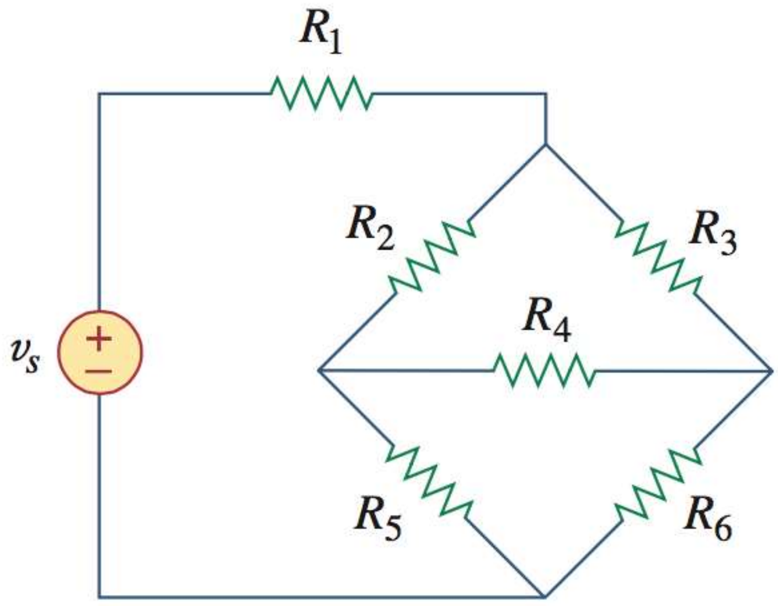
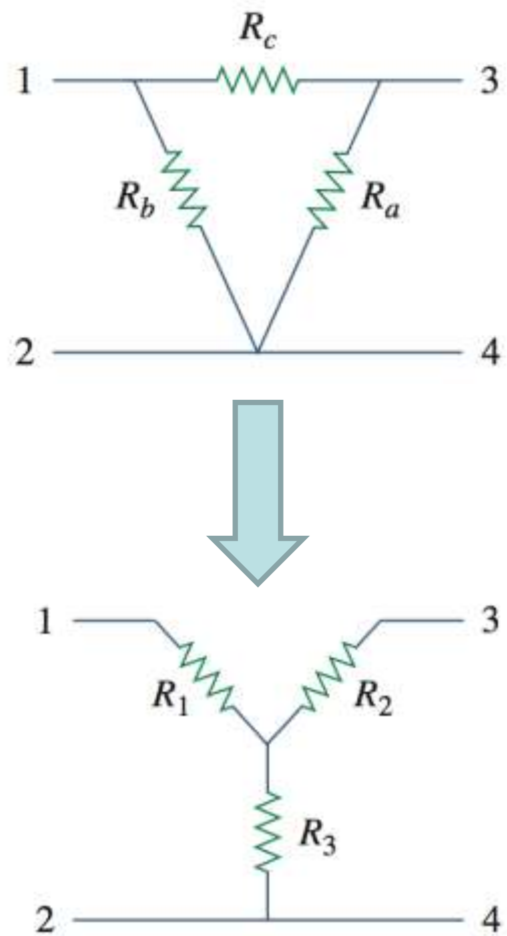


Figure 2.46
The bridge network.



△-Y变换（Delta - Wye）

abc *123*

△ → Y

$$\frac{\text{相邻之积}}{\text{周长之和}}$$

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

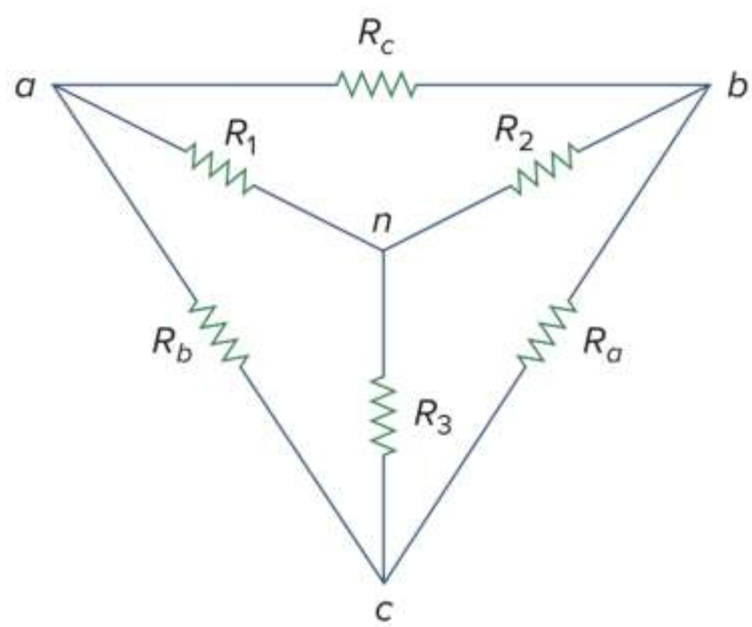


Figure 2.49
Superposition of Y and Δ networks as an aid in transforming one to the other.

$$R_\Delta = 3R_Y$$

阻值相等 → $R_{a,b,c} = 3R_{1,2,3}$

123 *abc*

Y → △

$$\frac{\text{两两相乘之和}}{\text{对面电阻}}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

*** Insight：△的阻值比较大；【直观理解： △ 更像并联，所以阻值大；Y更像串联】**

Convert the Δ network in Fig. 2.50(a) to an equivalent Y network.

Example 2.14

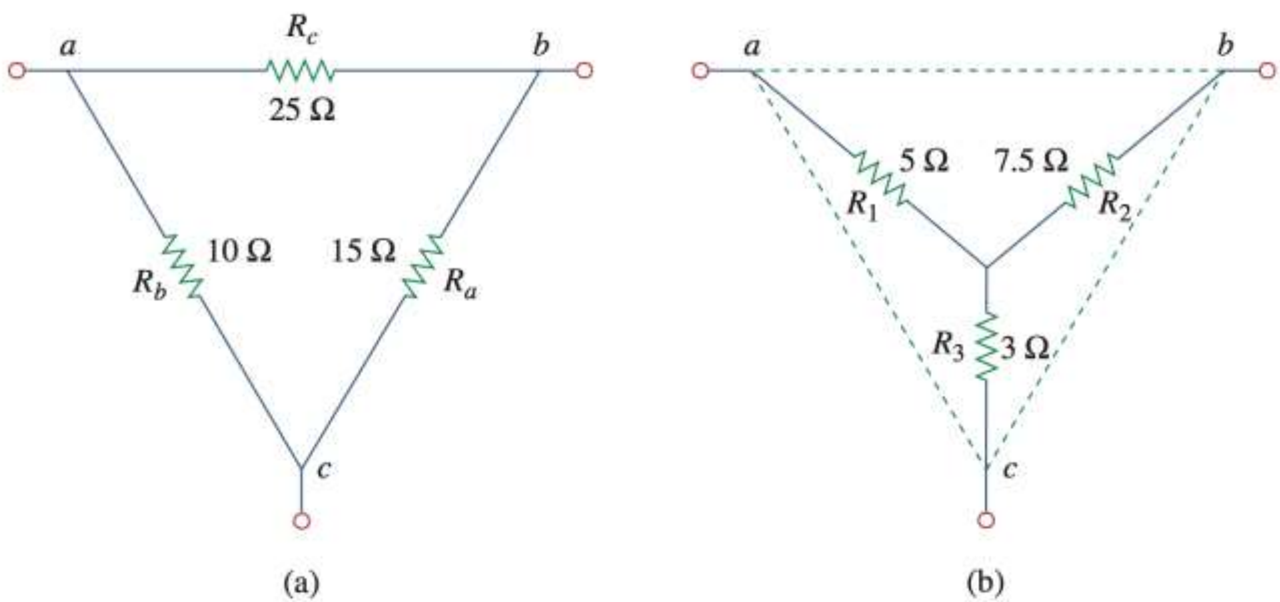


Figure 2.50
For Example 2.14: (a) original Δ network, (b) Y equivalent network.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

Example 2.15

Obtain the equivalent resistance R_{ab} for the circuit in Fig. 2.52 and use it to find current i.

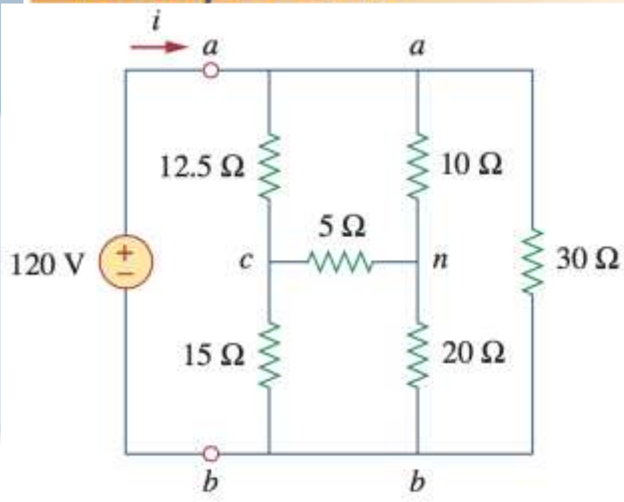


Figure 2.52
For Example 2.15.

Y → △

$$\frac{\text{两两相乘之和}}{\text{对面电阻}}$$

△ → Y

$$\frac{\text{相邻之积}}{\text{周长之和}}$$

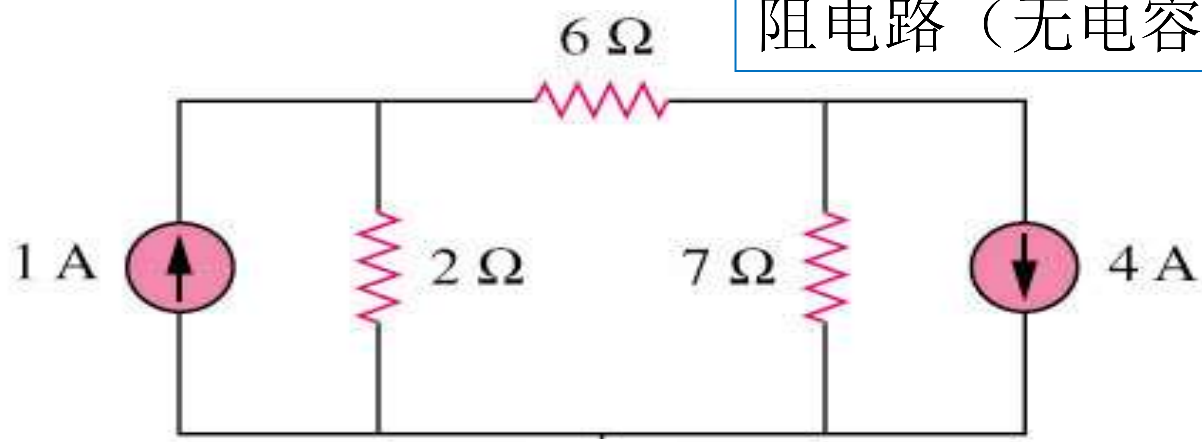
节点电压法 & 网孔电流法

- 3.1 Motivation
- 3.2 Nodal analysis （节点电压法）.
- 3.3 Nodal analysis with voltage sources.
- 3.4 Mesh analysis （网孔电流法）.
- 3.5 Mesh analysis with current sources.
- ~~3.6 Nodal and mesh analysis by inspection.~~
- 3.7 Nodal versus mesh analysis.

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the **voltage** across each resistor, (2) **current** through each resistor. (3) power generated by each current source, etc.

如何分析较为复杂的纯电阻电路（无电容、电感）



What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

- Kirchhoff's Current Laws (KCL)
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

我们所拥有的武器

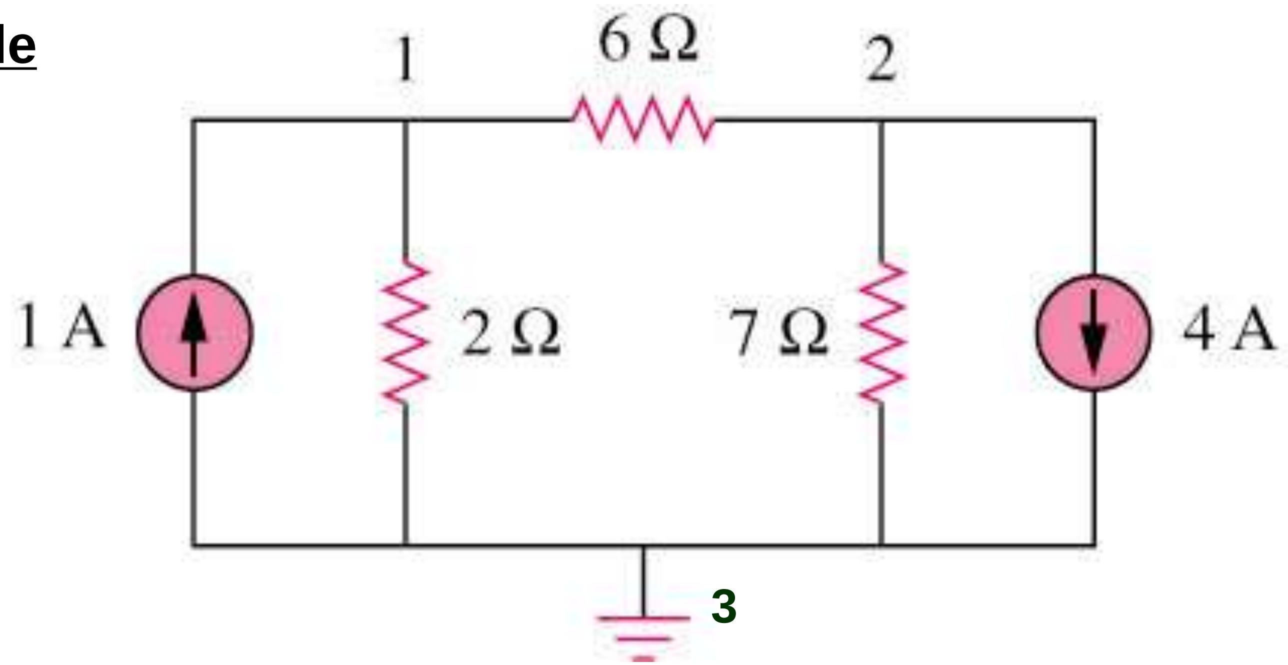
How should we apply these laws to determine the answers?

3.2 节点电压法Nodal Analysis (1)

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

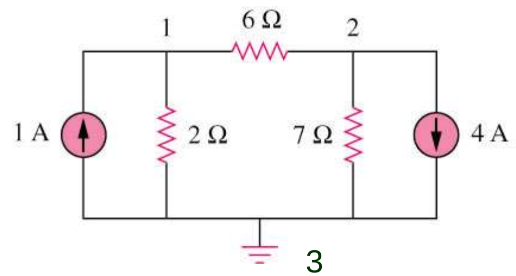
节点电压法：采用电路中的节点电压作为变量（n-1个），知道了各节点电压，就可以求出流过各元件的电流

Example



使用节点电压（1、2）作为变量 → 然后可以求出流过3个电阻的电流

3.2 Nodal Analysis (2)



Steps to determine the node voltages:

节点电压法的计算步骤：

1. Select a node as the reference node（选择参考地，一般情况下电路中都已标明的）.
2. Assign voltages v_1, v_2 ,…, v_{n-1} to the remaining n-1 nodes. The voltages are referenced with respect to the reference node. (**n-1**个节点电压设为变量)
3. Apply **KCL** to each of the n-1 non-reference nodes. Use **Ohm's law** to express the branch currents in terms of node voltages. （每个支路根据欧姆定律写出流过的电流；**n-1**个节点写出**KCL**方程，共得到**n-1**个方程；）
4. Solve the resulting simultaneous equations to obtain the unknown node voltages. （求解方程，变量数与方程数相等，可解）

Q1: 可见节点电压法的前提是根据节点电压能写出支路电流。那么若支路含电压源怎么办？电流源呢？

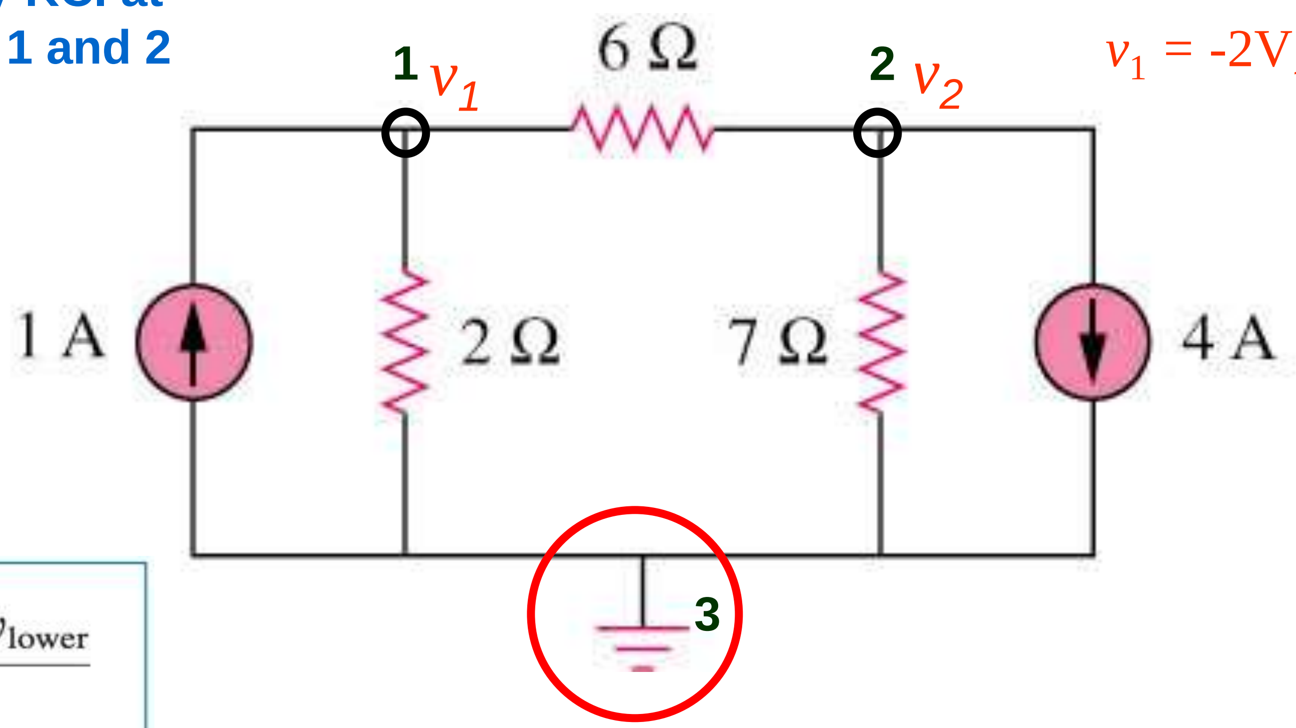
Q2: 线性方程组如何解？

3.2 Nodal Analysis (3)

Example – circuit **independent current source only ①仅含独立电流源**

请同学们根据节点电压法的步骤分析下面的电路（1分钟）

Apply KCl at node 1 and 2



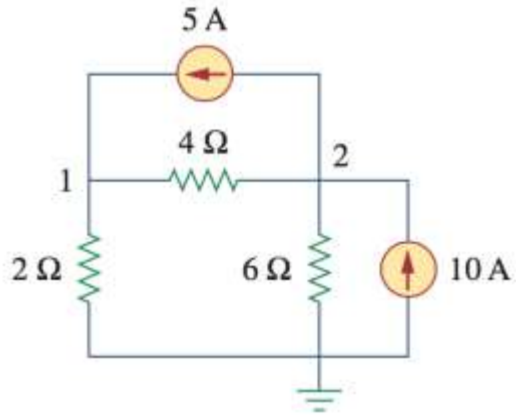
v_1 = -2V, v_2 = -14V

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Current flows from a higher potential to a lower potential in a resistor.

Example 3.1

Calculate the node voltages in the circuit shown in Fig. 3.3(a).

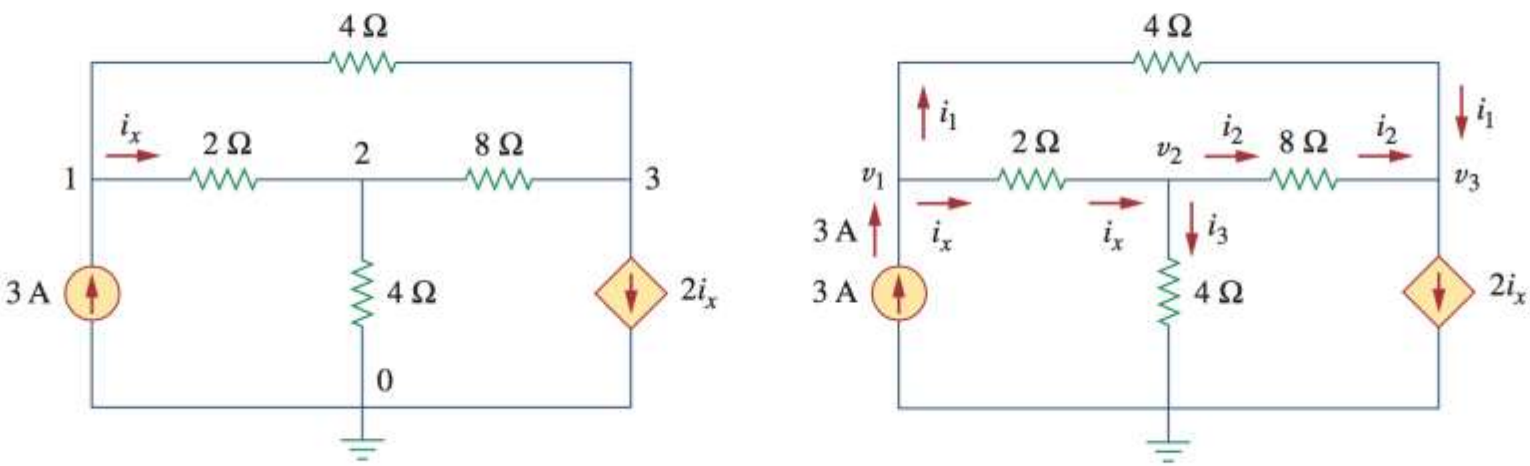


(a)

3.2 Nodal Analysis (4)

<u>Example 3.2</u> – current with dependant current source ②含受控电流源

受控电流源的处理方式与独立电流源一样



(a) (b)

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$\Rightarrow \begin{cases} 3v_1 - 2v_2 - v_3 = 12 \\ -4v_1 + 7v_2 - v_3 = 0 \\ 2v_1 - 3v_2 + v_3 = 0 \end{cases} \Rightarrow \begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

解方程的三种方法

- **METHOD 1** Using the elimination technique, 消除法
- **METHOD 2** To use Cramer's rule 克莱姆法则

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = \frac{\Delta_1}{\Delta}, \qquad v_2 = \frac{\Delta_2}{\Delta}, \qquad v_3 = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = \begin{array}{c|ccc|c} & 3 & -2 & -1 & \\ & -4 & 7 & -1 & \\ & 2 & -3 & 1 & \\ - & 3 & -2 & -1 & + \\ - & -4 & 7 & -1 & + \\ - & & & & + \end{array}$$

$$= 21 - 12 + 4 + 14 - 9 - 8 = 10$$

$$\Delta_1 = \begin{array}{c|ccc|c} & 12 & -2 & -1 & \\ & 0 & 7 & -1 & \\ & 0 & -3 & 1 & \\ - & 12 & -2 & -1 & + \\ - & 0 & 7 & -1 & + \\ - & & & & + \end{array} = 84 + 0 + 0 - 0 - 36 - 0 = 48$$

$$\Delta_2 = \begin{array}{c|ccc|c} & 3 & 12 & -1 & \\ & -4 & 0 & -1 & \\ & 2 & 0 & 1 & \\ - & 3 & 12 & -1 & + \\ - & -4 & 0 & -1 & + \\ - & & & & + \end{array} = 0 + 0 - 24 - 0 - 0 + 48 = 24$$

$$\Delta_3 = \begin{array}{c|ccc|c} & 3 & -2 & 12 & \\ & -4 & 7 & 0 & \\ & 2 & -3 & 0 & \\ - & 3 & -2 & 12 & + \\ - & -4 & 7 & 0 & + \\ - & & & & + \end{array} = 0 + 144 + 0 - 168 - 0 - 0 = -24$$

解方程的三种方法

METHOD 3 We now use *MATLAB* to solve the matrix. Equation (3.2.6) can be written as

数值软件

$$\mathbf{AV} = \mathbf{B} \quad \Rightarrow \quad \mathbf{V} = \mathbf{A}^{-1}\mathbf{B}$$

where **A** is the 3 by 3 square matrix, **B** is the column vector, and **V** is a column vector comprised of v_1, v_2, and v_3 that we want to determine. We use *MATLAB* to determine **V** as follows:

```
>>A = [3  -2  -1;  -4  7  -1;  2  -3  1];
>>B = [12  0  0]';
>>V = inv(A) * B
            4.8000
      V =   2.4000
           -2.4000
```

$$\boxed{\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}}$$

Thus, $v_1 = 4.8$ V, $v_2 = 2.4$ V, and $v_3 = -2.4$ V, as obtained previously.

也可以采用 Python 语言

```
import numpy as np

# A*v == B
A = np.mat([[3, -2, -1],
          [-4, 7, -1],
          [2, -3, 1]])

B = np.mat([12, 0, 0]).T

v = A.I*B

print(f"v1 = {v[0, 0]:.4f} V; v2 = {v[1, 0]:.4f} V; v3 = {v[2, 0]:.4f} V.")
```

```
v1 = 4.8000 V; v2 = 2.4000 V; v3 = -2.4000 V.
```

Nodal Analysis with Voltage Source

Example –circuit with independent voltage source ③含独立电压源

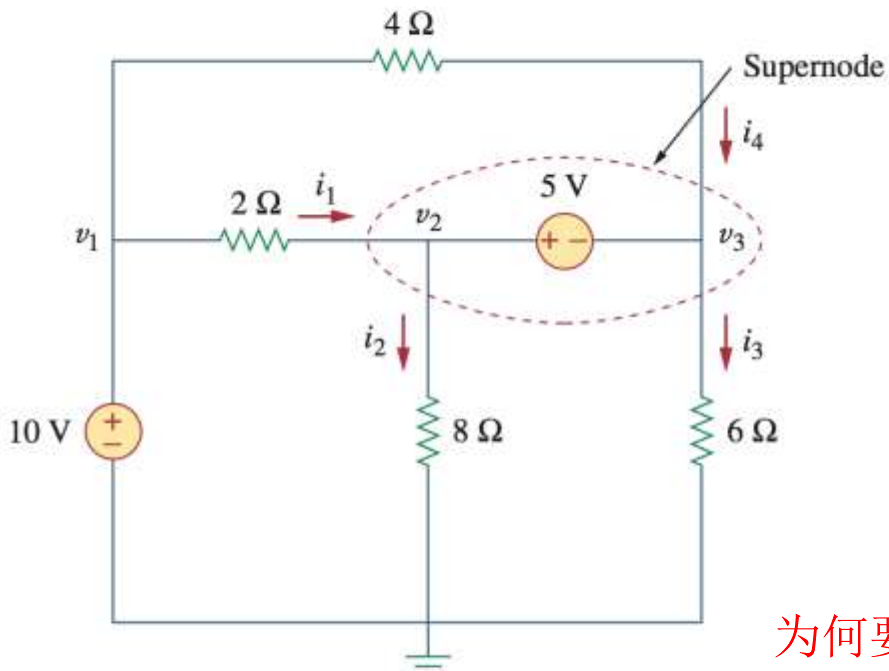


Figure 3.7
A circuit with a supernode.

case1: 若电压源一端为“地”→ 直接得到v_1值；

case2: 若电压源两端都不是地 → 把电压源当做一个广义节点（**supernode**）看待
① 对该广义节点实施KCL；
② 独立方程减少了一个！怎么办？
③ 广义节点内的**电压源决定了其两端的电压差；**

为何要这么处理？
节点电压法需要知道流入各节点的电流，
而流经电压源的电流无法直接确定

A **supernode** is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

与电压源并联的电阻是无意义的，因为其两端电压差完全由电压源决定！

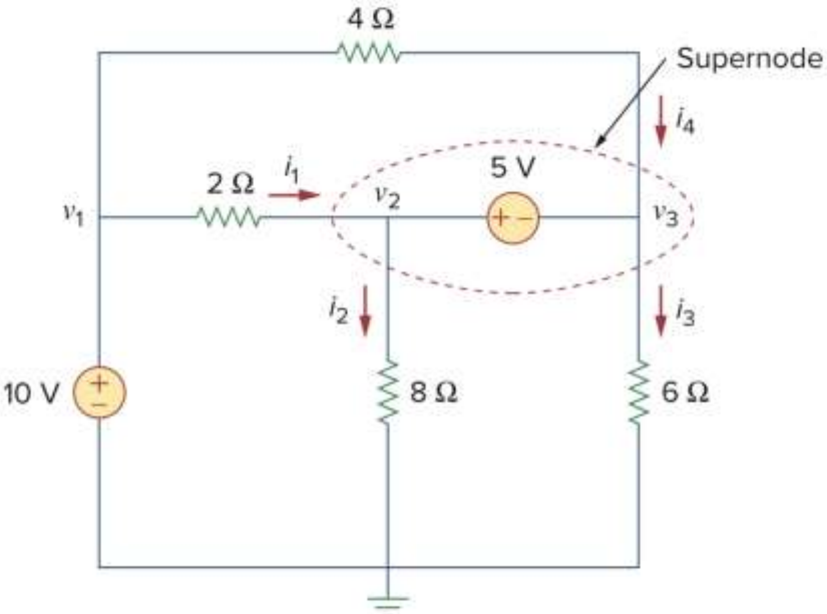


Figure 3.7
A circuit with a supernode.

方程1 $i_1 + i_4 = i_2 + i_3$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

方程2 $v_2 - v_3 = 5$

以及 $v_1 = 10$ V

Example 3.3

For the circuit shown in Fig. 3.9, find the node voltages.

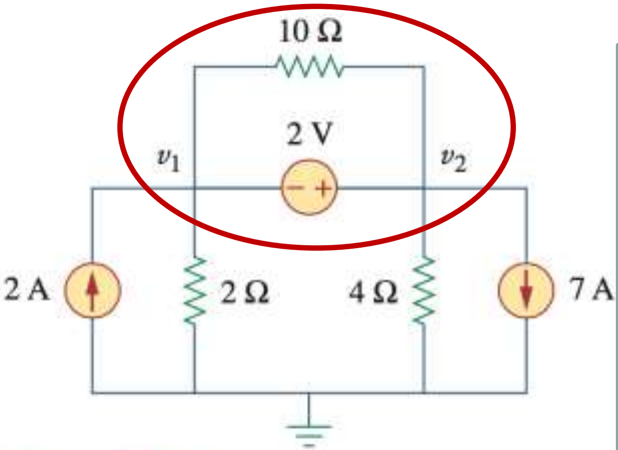


Figure 3.9
For Example 3.3.

Find the node voltages in the circuit of Fig. 3.12.

Example 3.4

④含受控电压源　　受控电压源的处理方式与独立电压源一样

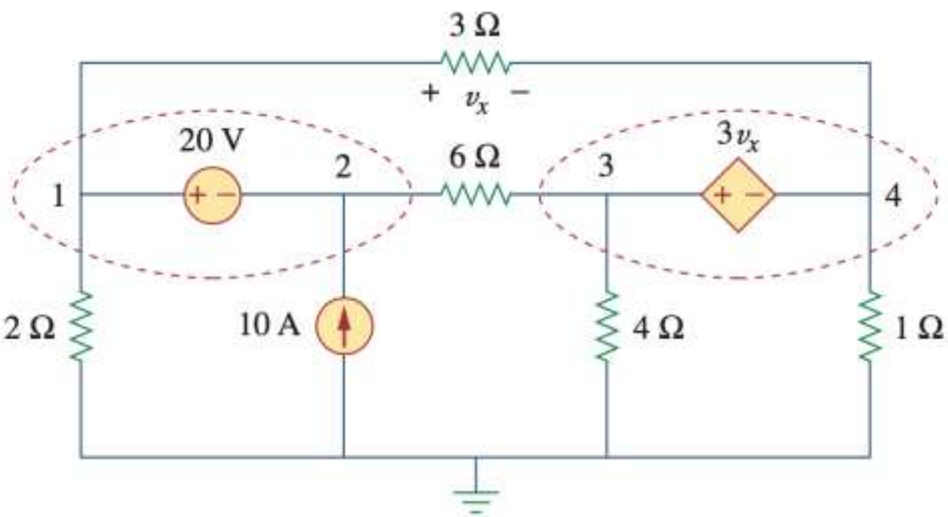


Figure 3.12
For Example 3.4.

3.4 网孔电流法Mesh Analysis (1)

1. A mesh is a loop that has no other loops inside it. 网孔即最小的回路

2. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.（网孔电流作为变量）

3. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

3.4 Mesh Analysis (2)

Steps to determine the mesh currents:

网孔电流法的计算步骤：

1. Assign mesh currents $i_1, i_2, \ldots, i_l$ to the l meshes. （*l*个网孔电流变量, *l* 是网孔数，也是独立回路数）

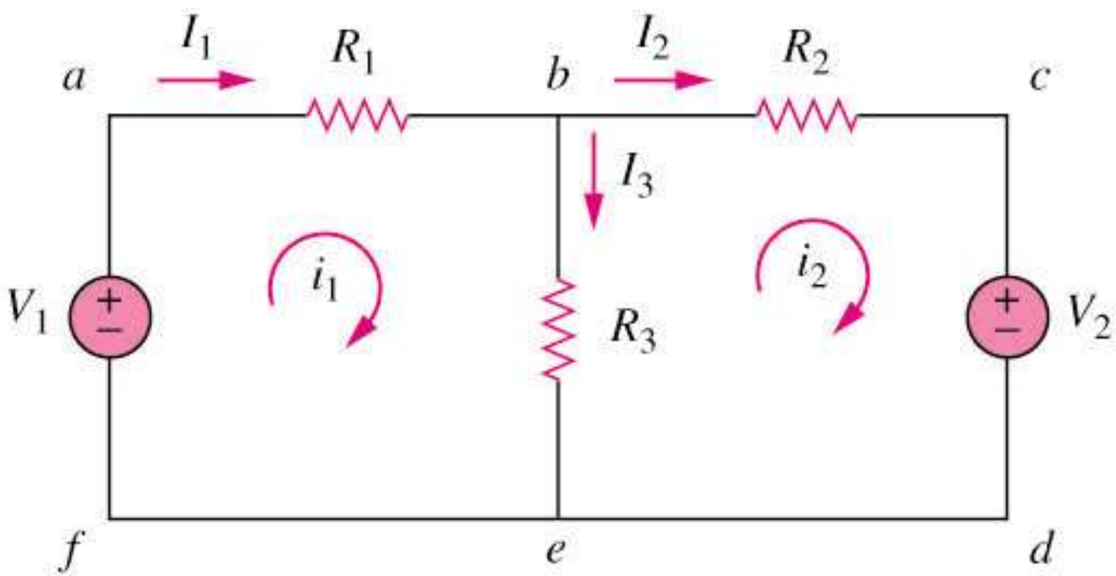
2. Apply KVL to each of the l meshes. Use Ohm's law to express the voltages in terms of the mesh currents. （*l* 个独立方程，注意，节点电流法是n-1个变量）

3. Solve the resulting n simultaneous equations to get the mesh currents.（求解方程，变量数与方程数相等，可解）

Q: 可见网孔电流法的前提是根据网孔电流写出网孔中各元件的电压。那么若网孔含电流源怎么办？电压源呢？

3.4 Mesh Analysis (3)

<u>Example</u> – circuit with independent voltage sources ①仅含独立电压源



注意：要用网孔电流作为变量，而非支路电流

<u>Note</u>:

i_1 and i_2 are mesh current (**imaginative**, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

Example 3.5

For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.

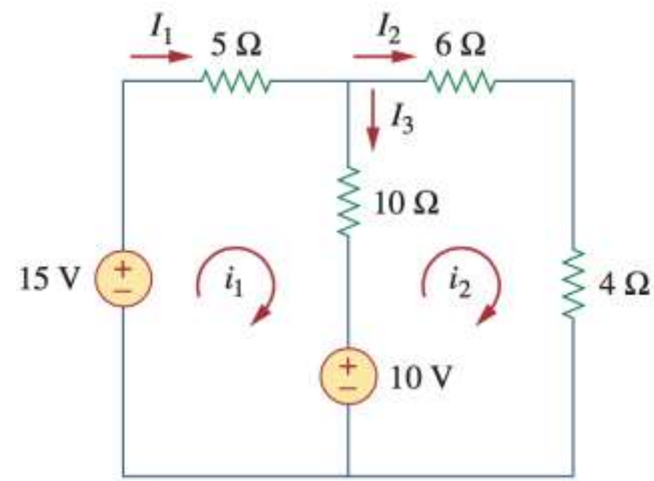


Figure 3.18
For Example 3.5.

Example 3.6

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

②含受控电压源

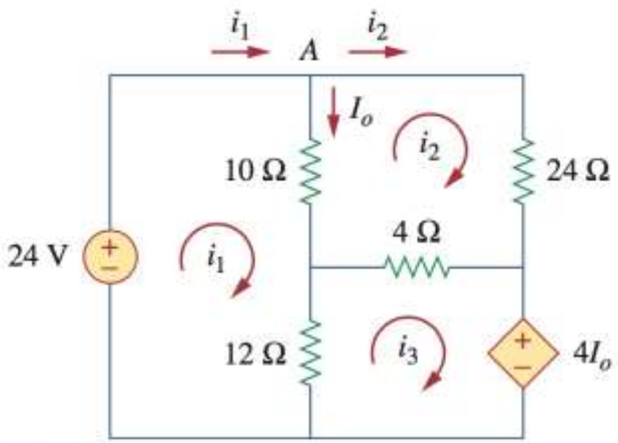
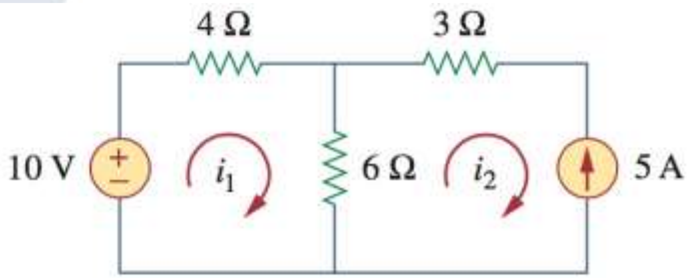
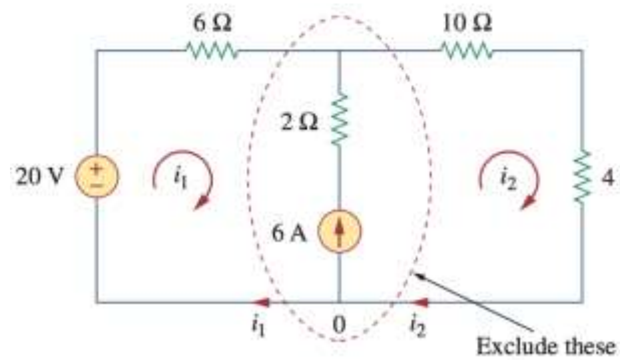


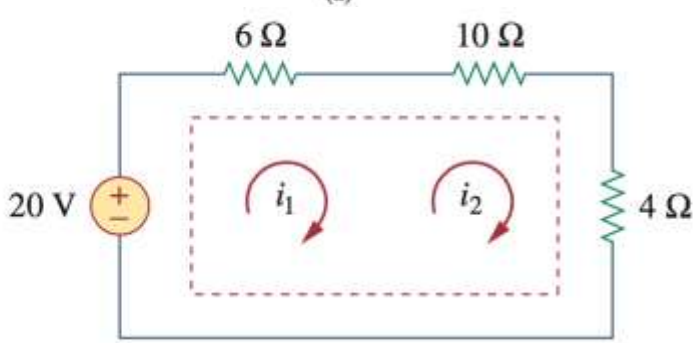
Figure 3.20
For Example 3.6.

3.5 Mesh Analysis with Current Source (1)





(a)



③含独立电流源

Case1. 若电流源仅在一个mesh中→ 直接得到该mesh的电流值；

Case2. 若电流源被两个mesh共享 → 把电流源及与其**串联**的元件去掉，形成一个超级网孔（supermesh）

① 对该超级网孔实施KVL；

② 独立方程减少了一个！怎么办？

③ **该电流源的值决定了相邻两个mesh的电流差**

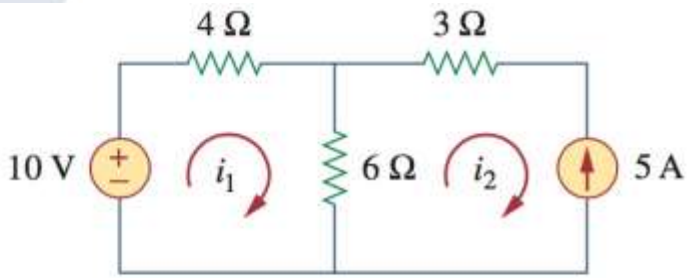
为何要这么处理？

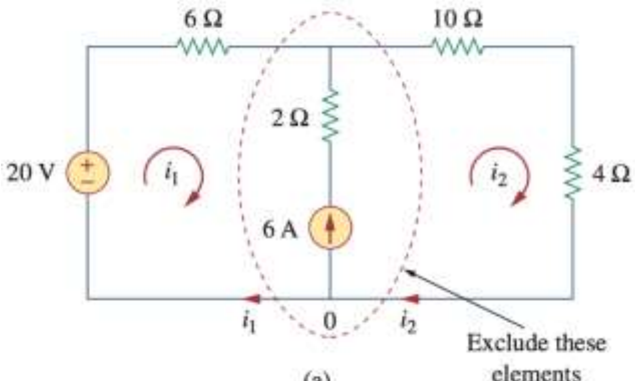
网孔电流法需要知道网孔各支路的两端电压，而电流源两端的电压无法直接确定

注意：超级网孔的网孔电流仍保持原先的$i_1 i_2$

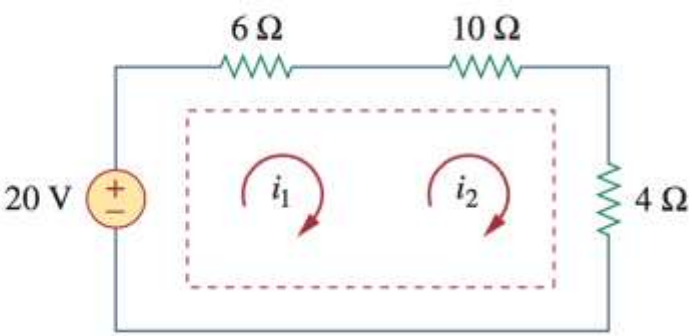
A **supermesh** results when two meshes have a (dependent or independent) current source in common.

与电流源串联的电阻是无意义的，因为其流经电流完全由电流源决定！





(a)



$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6 = i_2 - i_1$$

$$\Downarrow$$

$$i_1 = -3.2\ \text{A}, \qquad i_2 = 2.8\ \text{A}$$

For the circuit in Fig. 3.24, find i_1 to i_4 using mesh analysis.

Example 3.7

④含受控电流源

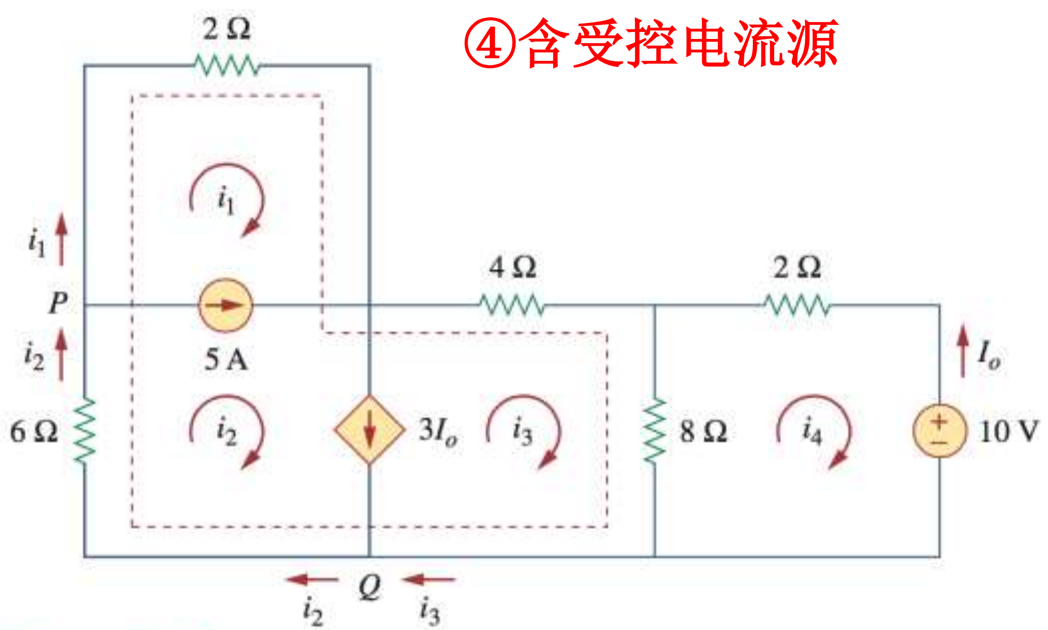


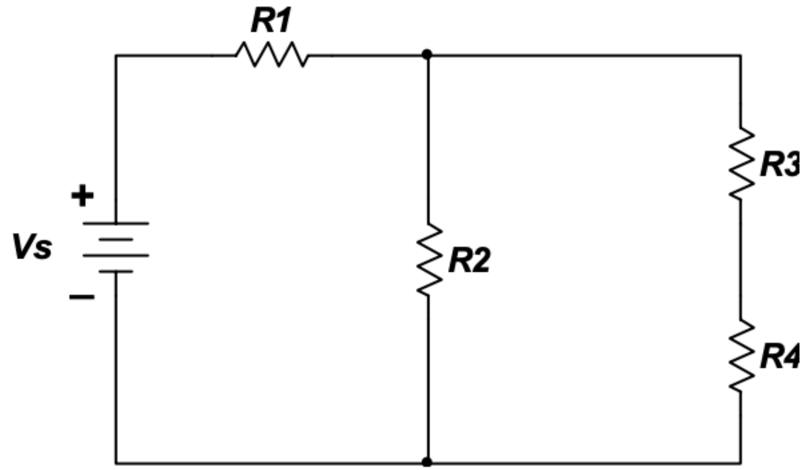
Figure 3.24
For Example 3.7.

3.7 节点电压法 vs 网孔电流法

- 两种方法都可以解决问题，都可以用，没有谁对谁错
  - 知道了节点电压，可以求出支路电流；反之亦然；

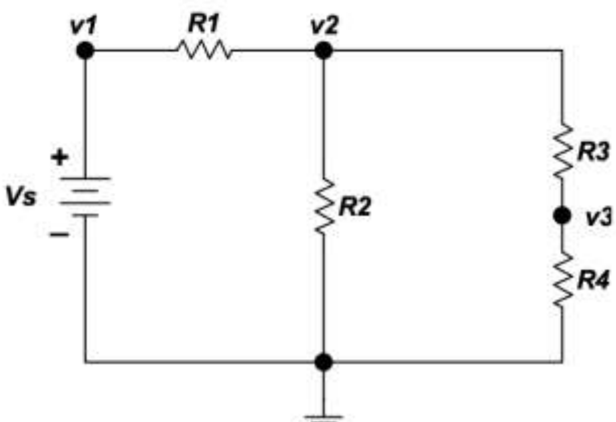
- 区别
  - 独立方程数不同：节点电压法对 n-1 个节点立KCL方程；网孔电流法对 l 个独立回路立KVL方程【回顾图论，b 个支路，n-1个树枝，l 个连枝（独立回路），$b = n\text{-}1 + l$】，选择方程数少的方法有利于手动计算，但对计算机计算无所谓；

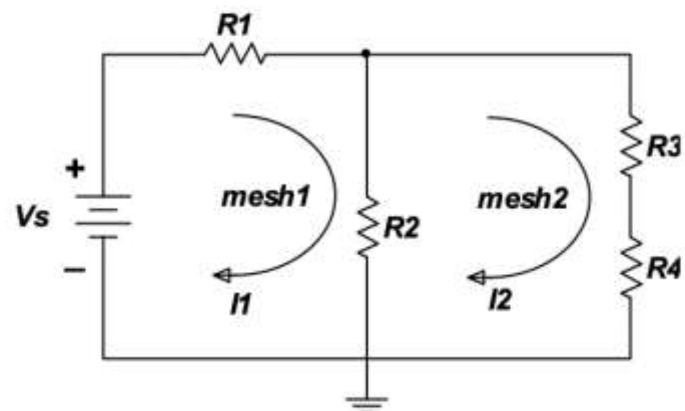
  - 根据目标导向来选择：若需要求解的是节点电压，则用节点电压法直接一些；若需要求解的是支路电流，则用网孔电流法直接一些；



节点电压法

网孔电流法





变量：**3**个节点电压

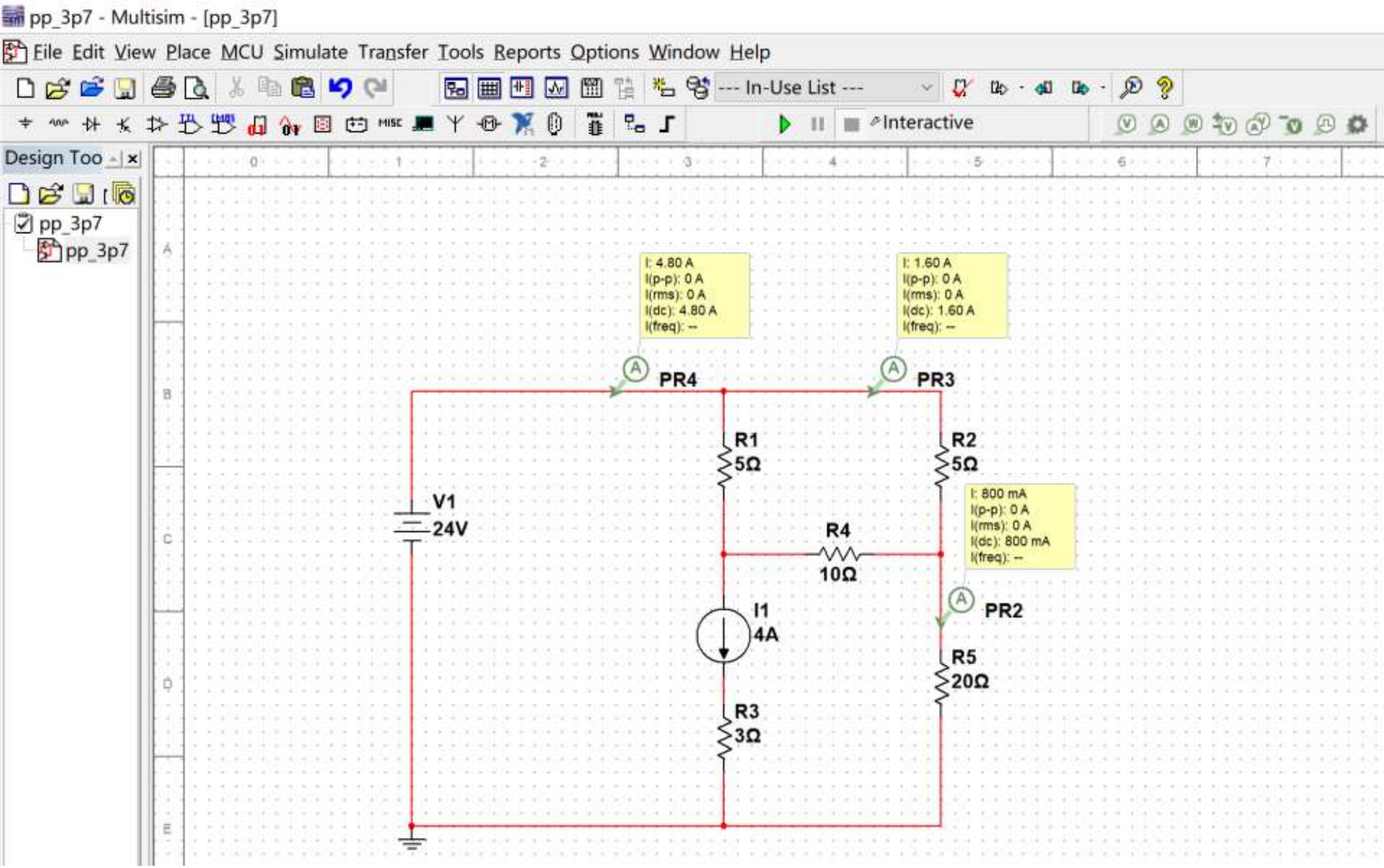
变量：**2**个网孔电流

请熟悉以下软件的使用

- Matlab：解电路方程；

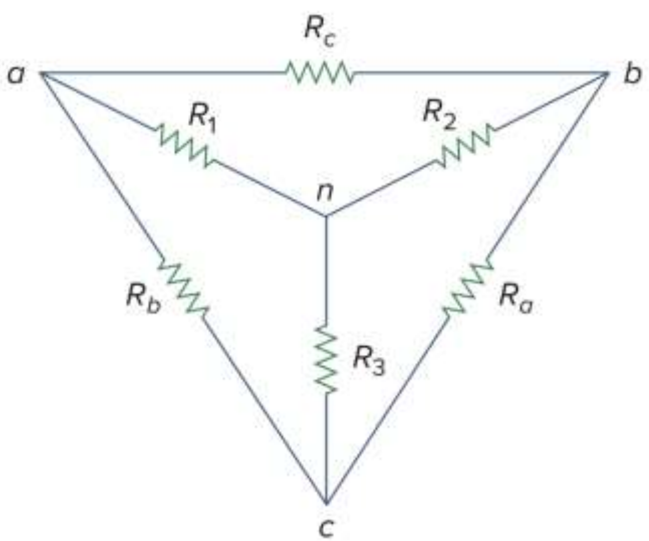
1. 学习**软件自带的入门教程**
2. 用好 Google

- PSpice 或 Multisim（更友好）：文本描述电路拓扑，或绘制电路图，计算机自动求解节点电压


pp_3p7 - Multisim - [pp_3p7]
File Edit View Place MCU Simulate Transfer Tools Reports Options Window Help
--- In-Use List ---
Interactive
Design Too
pp_3p7
pp_3p7
I: 4.80 A
I(p-p): 0 A
I(rms): 0 A
I(dc): 4.80 A
I(freq): --
I: 1.60 A
I(p-p): 0 A
I(rms): 0 A
I(dc): 1.60 A
I(freq): --
PR4
PR3
R1
5Ω
R2
5Ω
I: 800 mA
I(p-p): 0 A
I(rms): 0 A
I(dc): 800 mA
I(freq): --
V1
24V
R4
10Ω
PR2
I1
4A
R5
20Ω
R3
3Ω

小结

- **电阻串联：**分压
- **电阻并联：**分流 $$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$
- **Wye-Delta 变换：**Delta阻值较大，三电阻相等时，$R_\Delta = 3R_Y$



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_cR_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

小结

- **节点电压法：**n-1个节点电压为变量，对n-1个节点应用KCL列出n-1个独立方程

- **网孔电流法：**l 个网孔电流为变量（网孔数=独立回路数=连枝数），对 l 个网孔应用KVL列出 l 个独立方程

作业

Practice Problem 2.15

For the bridge network in Fig. 2.54, find R_{ab} and i.

Answer: 60 Ω, 4 A.

△ → Y: $\boxed{\dfrac{\text{相邻之积}}{\text{周长之和}}}$ 或 Y → △: $\boxed{\dfrac{\text{两两相乘之和}}{\text{对面电阻}}}$

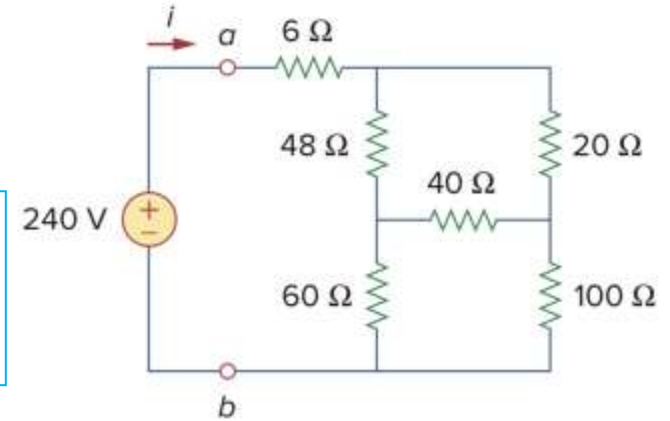


Figure 2.54
For Practice Prob. 2.15.

Practice Problem 3.2

Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

Answer: $v_1 = 32$ V, $v_2 = -25.6$ V, $v_3 = 62.4$ V.

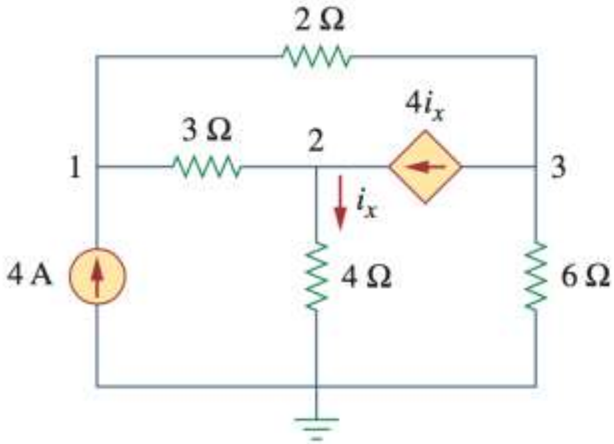


Figure 3.6
For Practice Prob. 3.2.

节点电压法，②含受控电流源情况

Practice Problem 3.4

Find v_1, v_2, and v_3 in the circuit of Fig. 3.14 using nodal analysis.

Answer: $v_1 = 7.608$ V, $v_2 = -17.39$ V, $v_3 = 1.6305$ V.

节点电压法，④含受控电压源情况

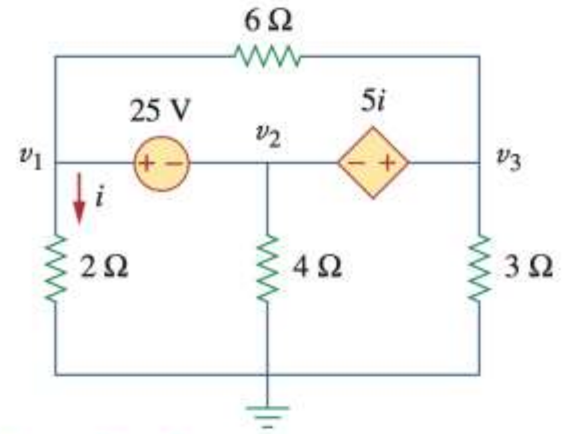


Figure 3.14
For Practice Prob. 3.4.

Practice Problem 3.6

Using mesh analysis, find I_o in the circuit of Fig. 3.21.

Answer: -4 A.

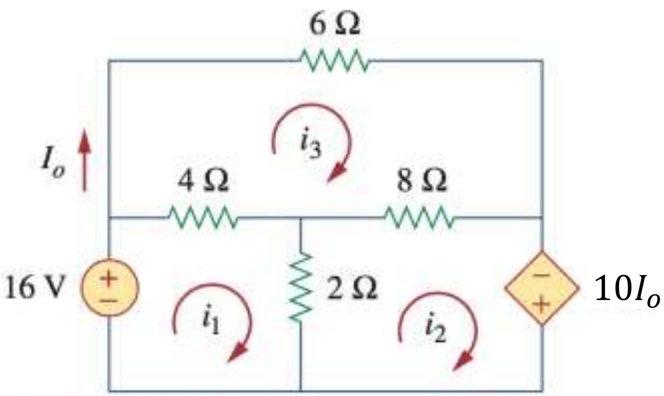


Figure 3.21
For Practice Prob. 3.6.

网孔电流法，②含受控电压源情况

Practice Problem 3.7

Use mesh analysis to determine i_1, i_2, and i_3 in Fig. 3.25.

$i_1 = 4.8\text{ A}$ $i_2 = 0.8\text{ A}$ $i_3 = 1.6\text{ A}$

书上答案有错

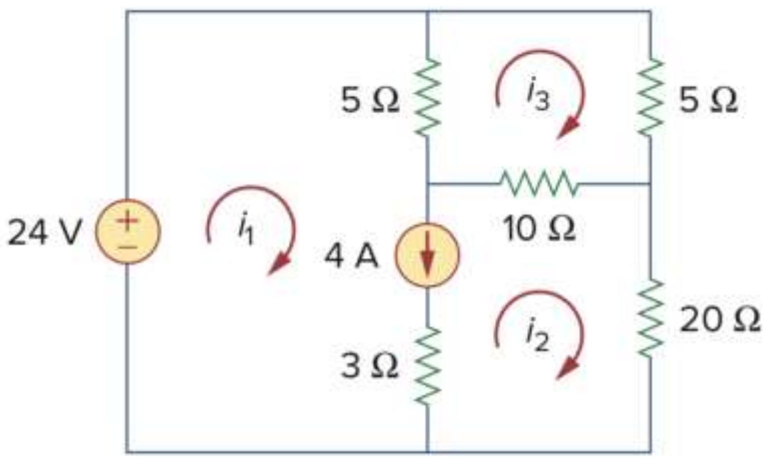


Figure 3.25
For Practice Prob. 3.7.

网孔电流法，③含电流源情况