

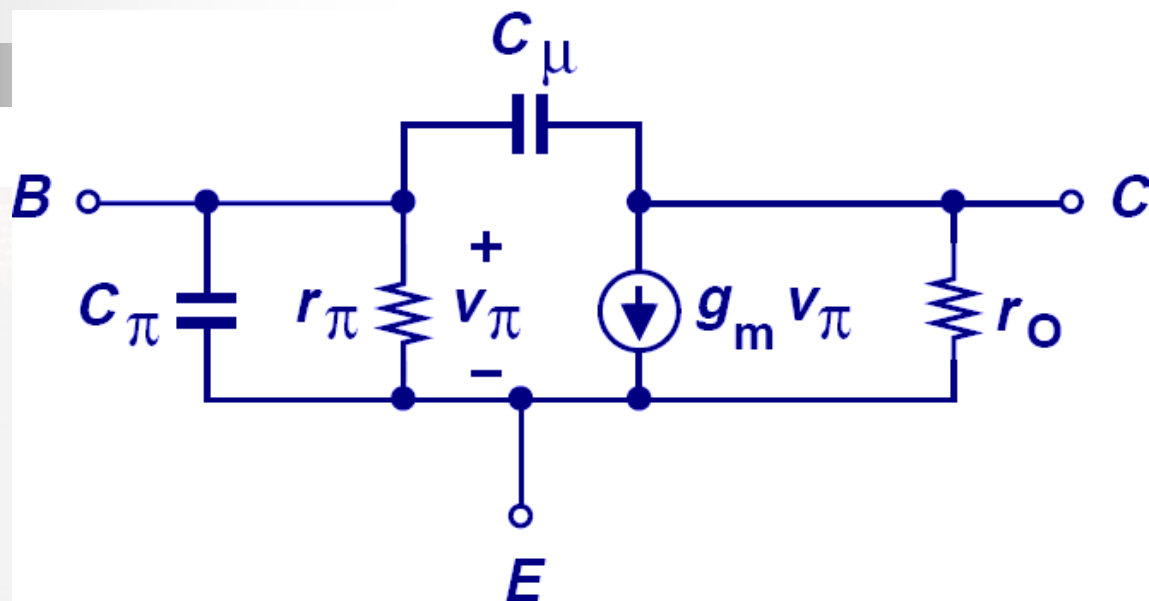
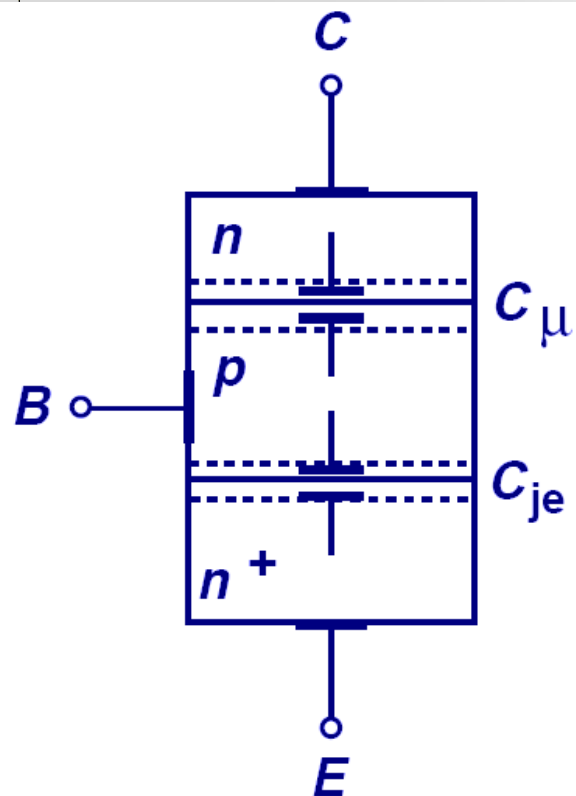
## Lecture 24 – Frequency Response, Part II



# 课程纲要

- 11.1 传输函数和波特图
  - 11.1.1 放大器传输函数的定义
  - 11.1.2 幅度和相位波特图的绘制
- 11.2 放大器频率响应分析
  - 11.2.1 共射和共源放大器低频响应
  - 11.2.2 共射和共源放大器高频响应（包括晶体管小信号高频等效模型、密勒定理）

## 三极管的高频模型

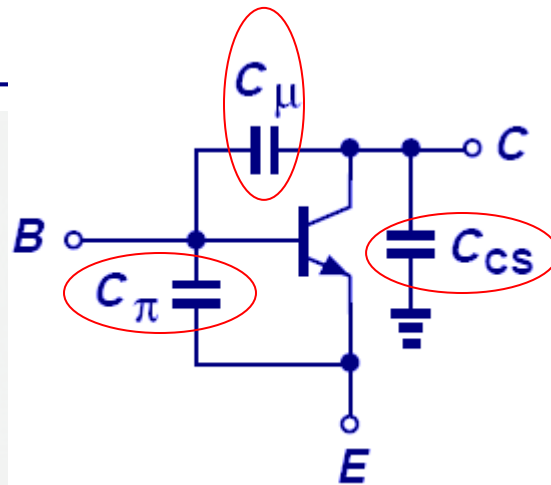
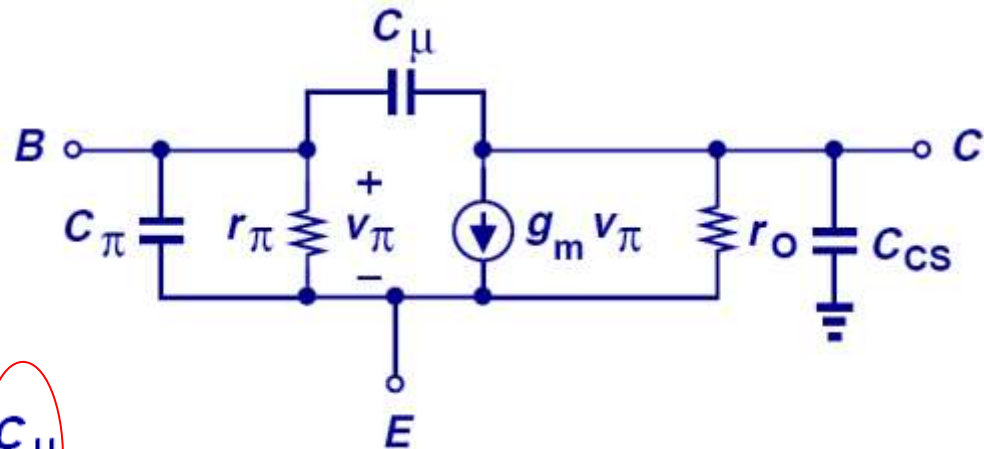
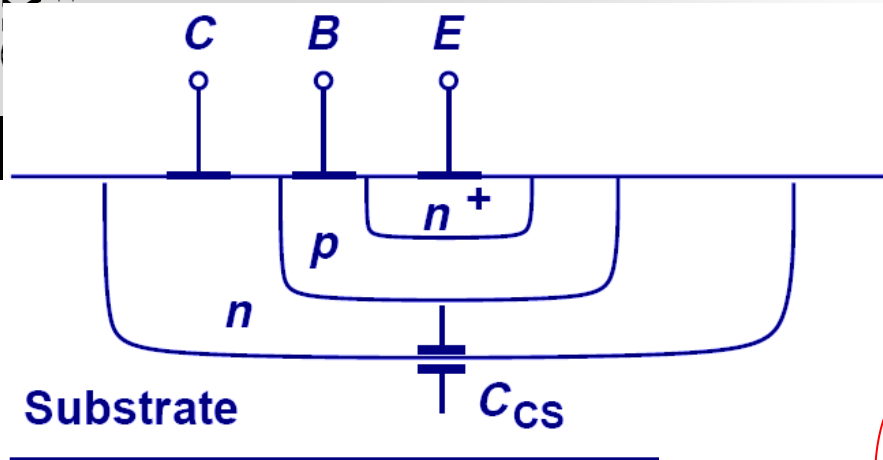


$$C_{\pi} = C_b + C_{je}$$

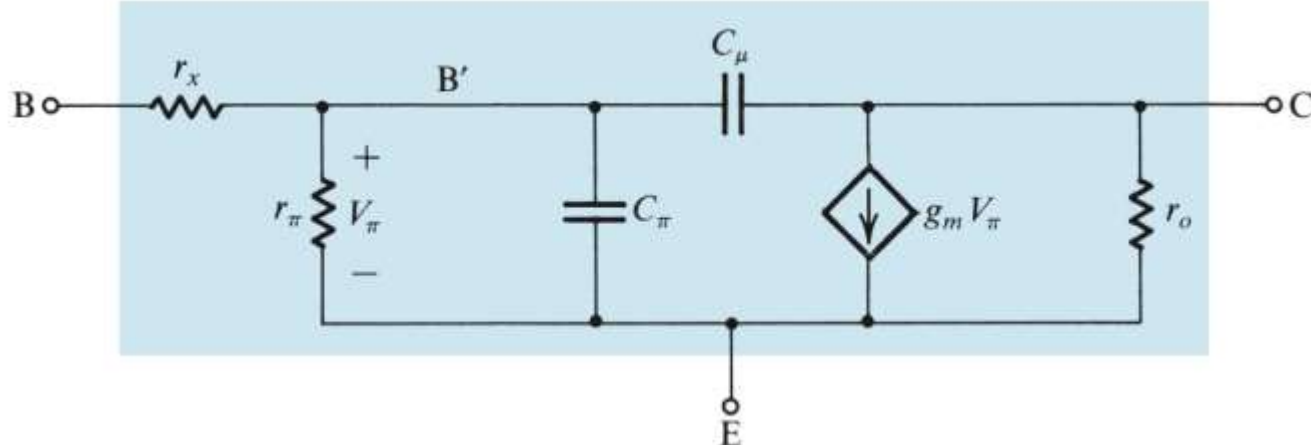
- 高频时，电容的影响不可忽略；
- $C_b$  为正向偏置PN结的扩散电容,  $C_{\mu}$  和  $C_{je}$  分别为耗尽层势垒电容；

# High-Frequency Model of Integrated Bipolar

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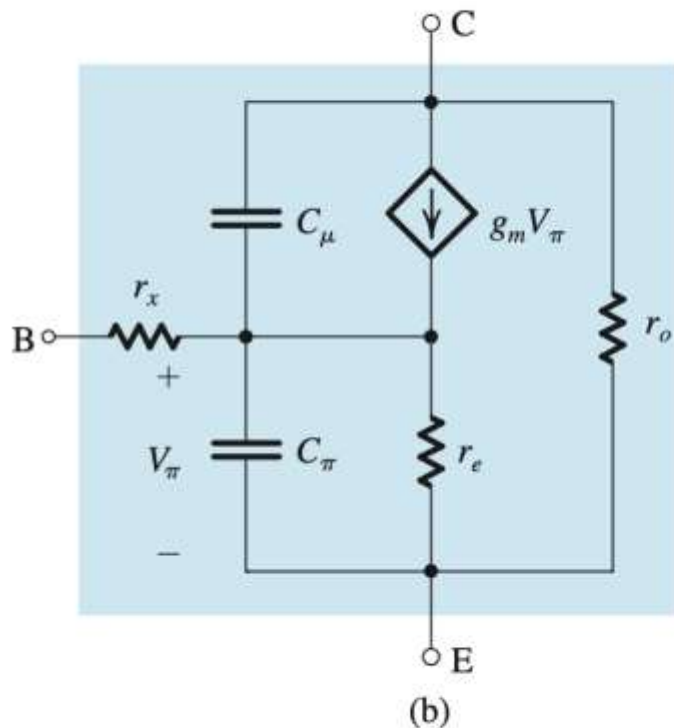


- 三极管的高频模型包含三个电容.【教材中把 $C_{cs}$ 也忽略了】
- 发射极与衬底之间无电容！可从结构图理解；



(a)

添加了基极电阻  $r_x$ ,  $r_x \ll r_\pi$ ;  
在低频时,  $r_x$  影响不大, 但高频时需要考虑



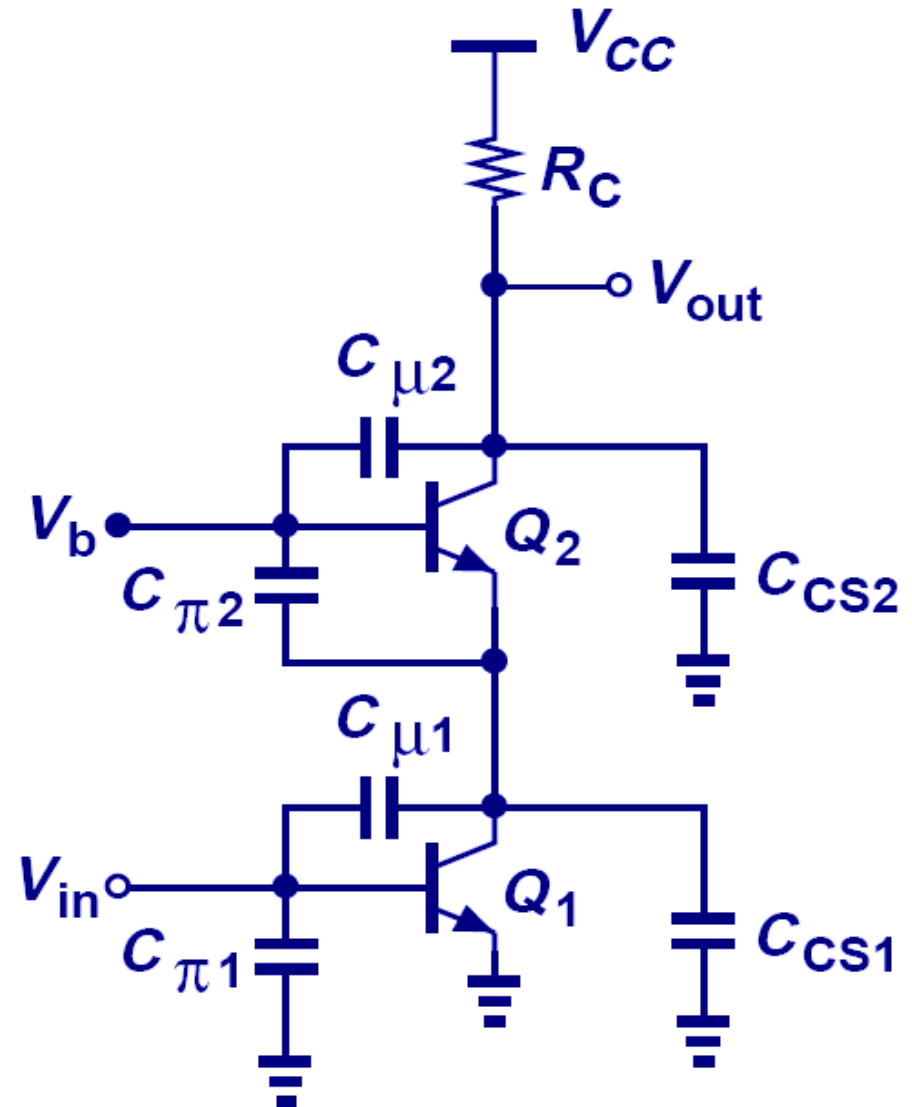
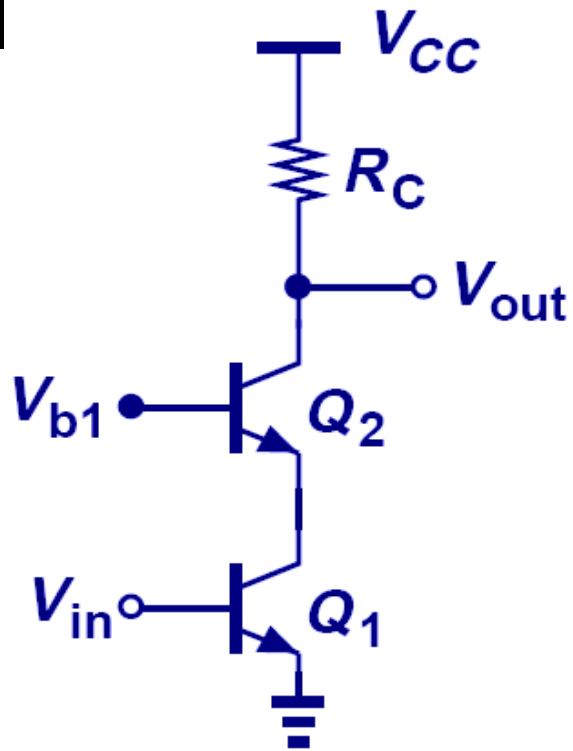
(b)

为简单起见, 我们教材上不考虑C到衬底的寄生电容, 所以三极管的高频寄生电容只需考虑  $C_\pi$  和  $C_\mu$

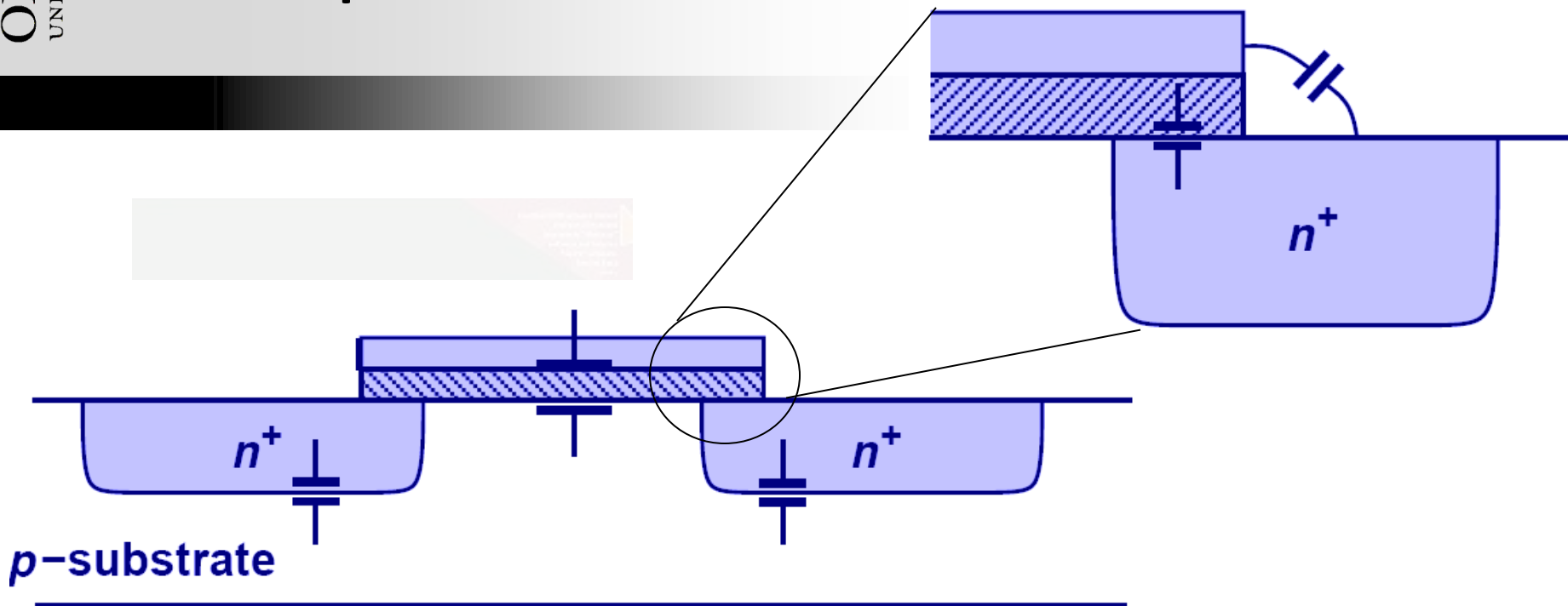
在教材第八版中,  $r_x$  也忽略了, 不作考虑, 但中文教材中仍保留, 所以需要了解下

**Figure 10.14** The high-frequency models of the BJT: (a) hybrid- $\pi$  model and (b) T model.

# Example: Capacitance Identification

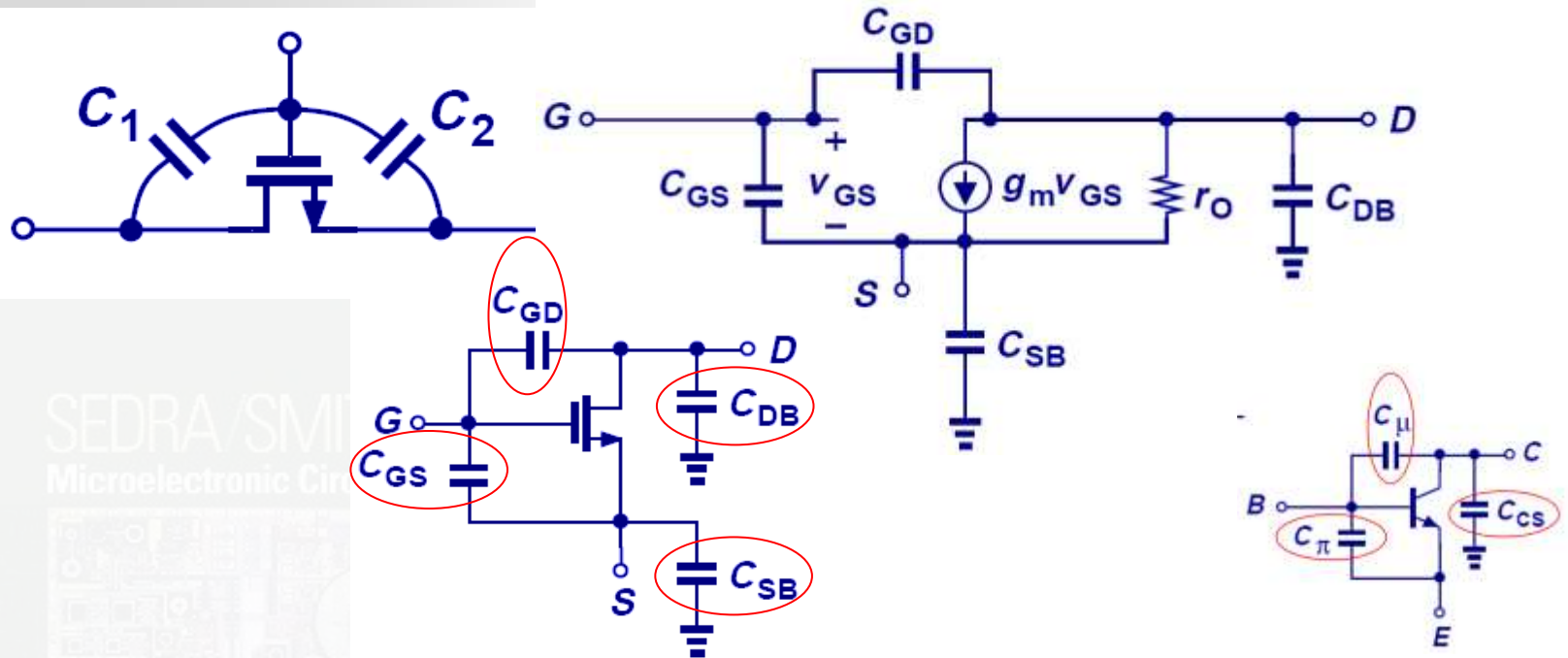


# MOS Intrinsic Capacitances



- 对于MOS, 有三处寄生电容：①栅极和沟道之间的电容；②源极/漏极与衬底之间的电容；③栅极与源极/漏极重合部分之间的电容

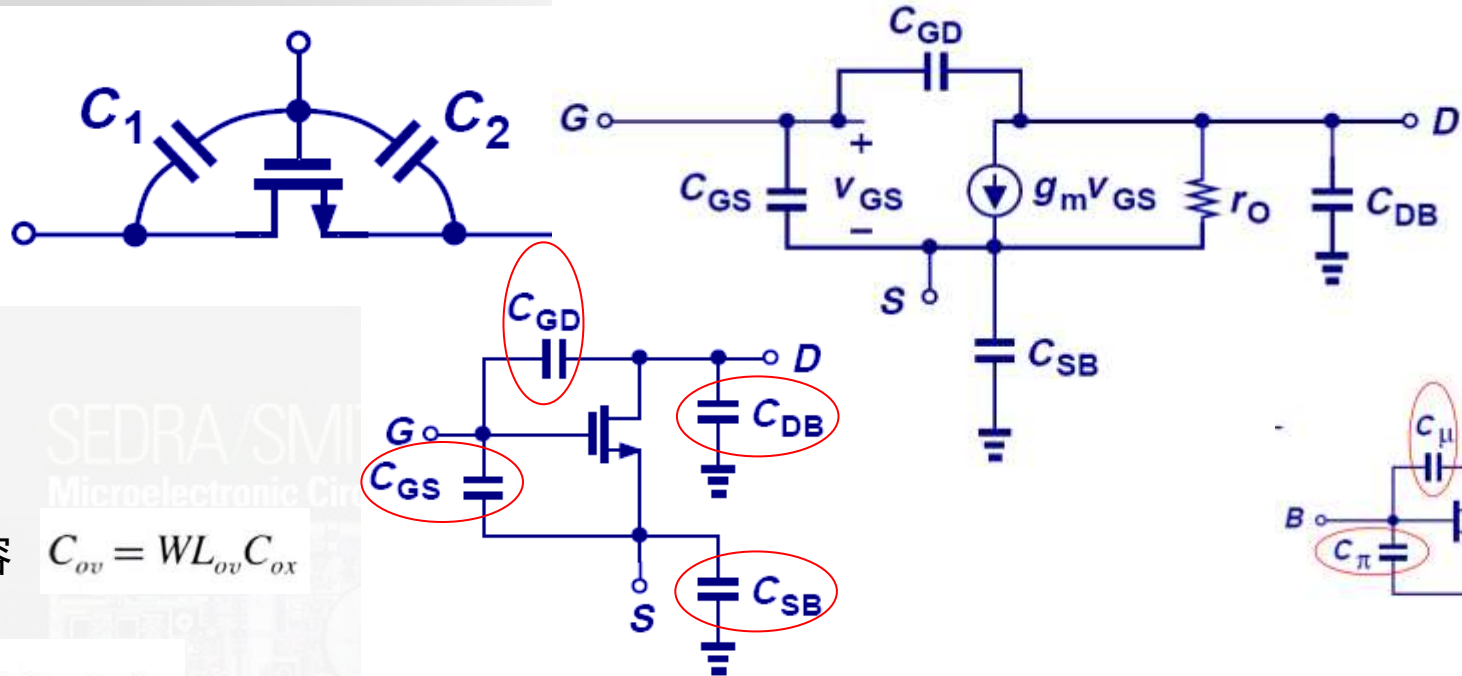
# Gate Oxide Capacitance Partition and Full Model



- 因为MOS模型中无沟道端点，所以栅极和沟道之间的电容 $C_{gate}$ 被分配到栅极与源极、栅极与漏极之间的电容 $C_1$ 、 $C_2$ 表示；



# Gate Oxide Capacitance Partition and Full Model



Overlap电容

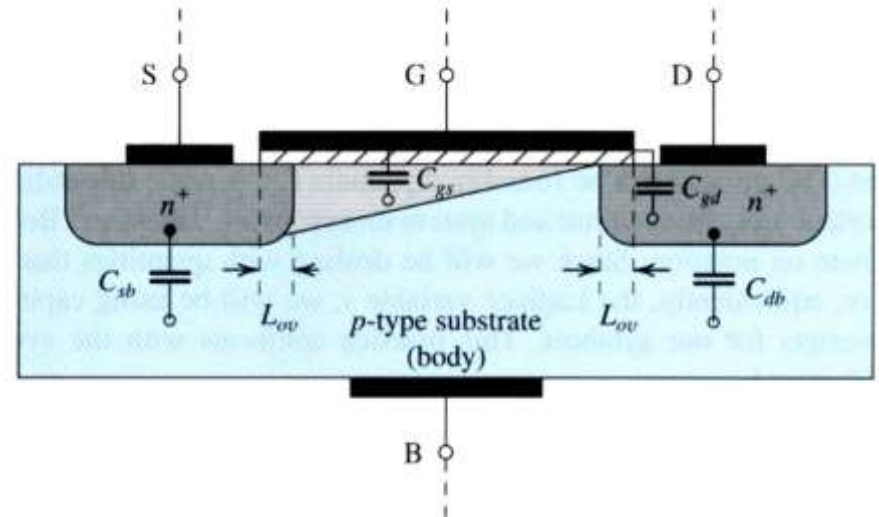
$$C_{ov} = WL_{ov}C_{ox}$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + C_{ov}$$

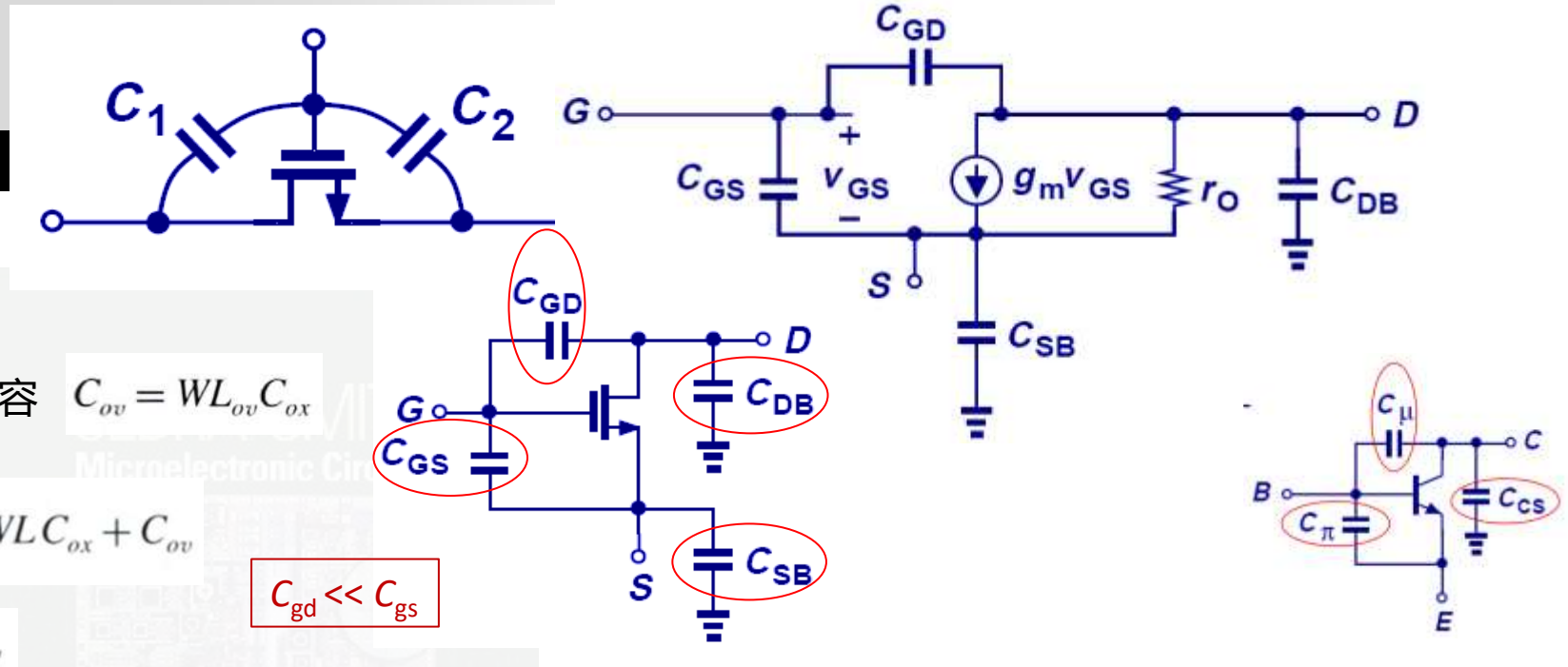
$$C_{gd} = C_{ov}$$

$$C_{gd} \ll C_{gs}$$

- 工作在饱和区时  $C_1 \sim C_{gate}$ ,  $C_2 \sim 0$ . 它们与栅极源极、栅极漏极之间的overlap电容合并, 形成  $C_{gs}$  和  $C_{gd}$ .

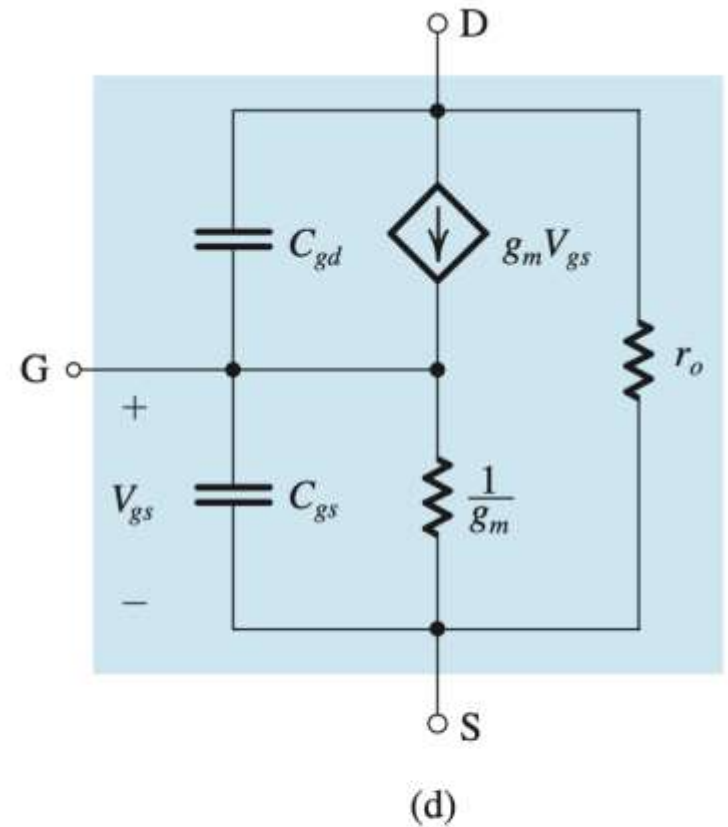
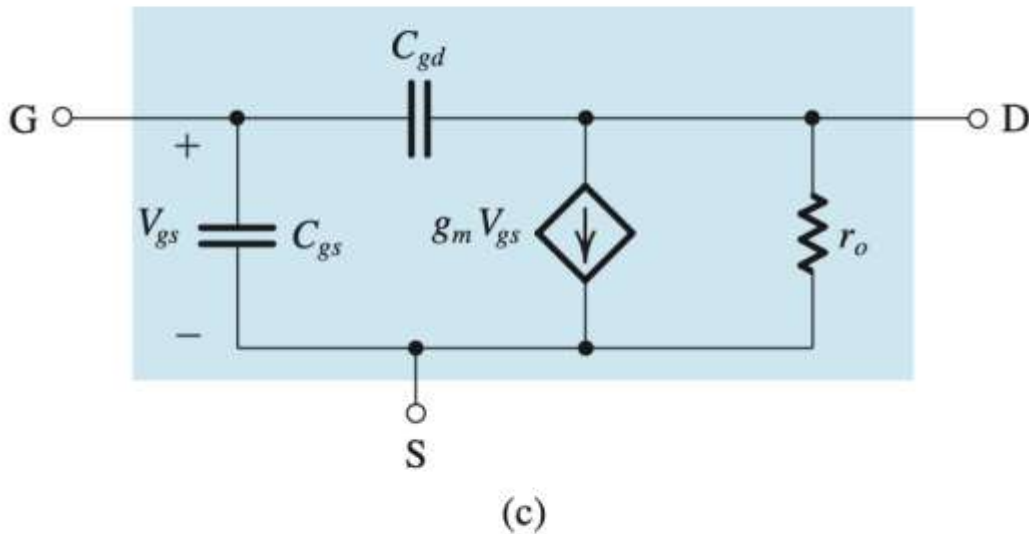


# Gate Oxide Capacitance Partition and Full Model



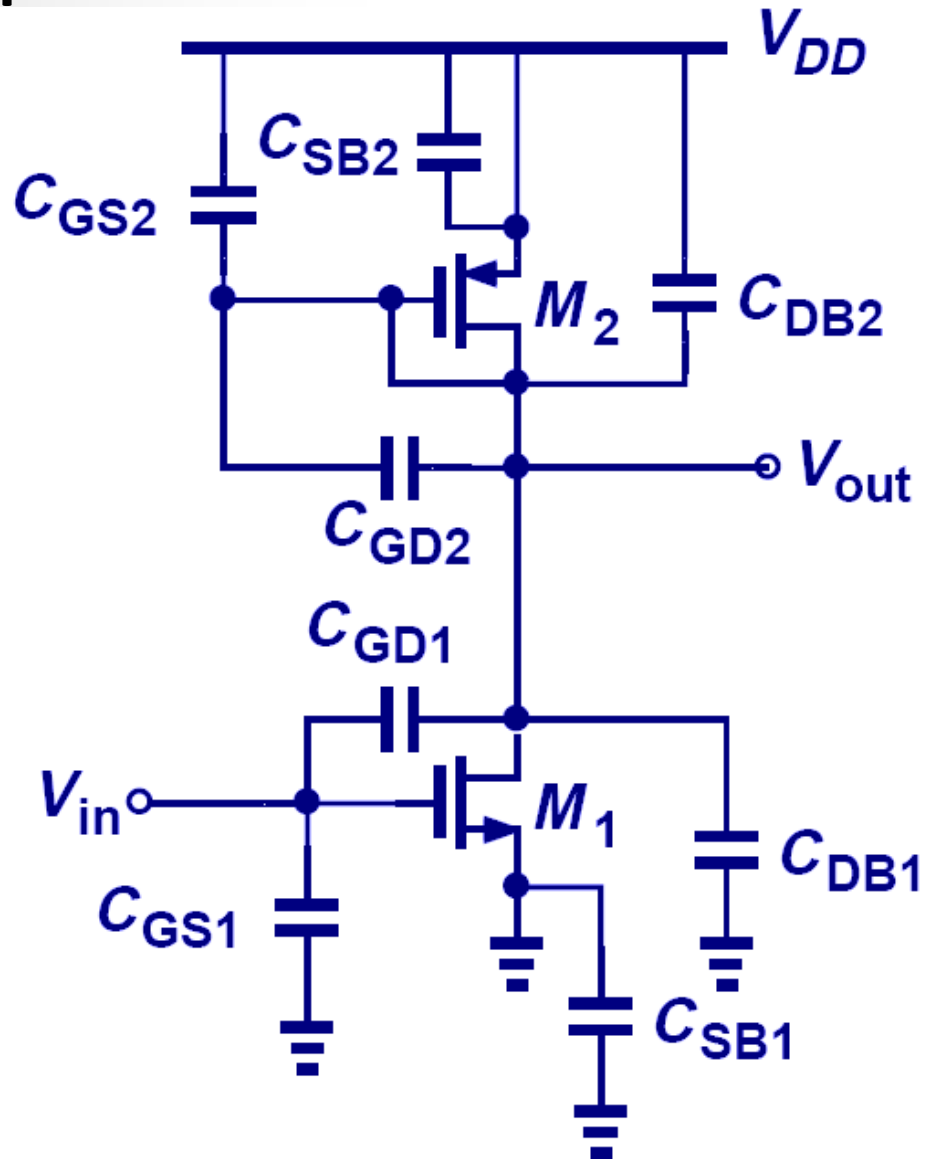
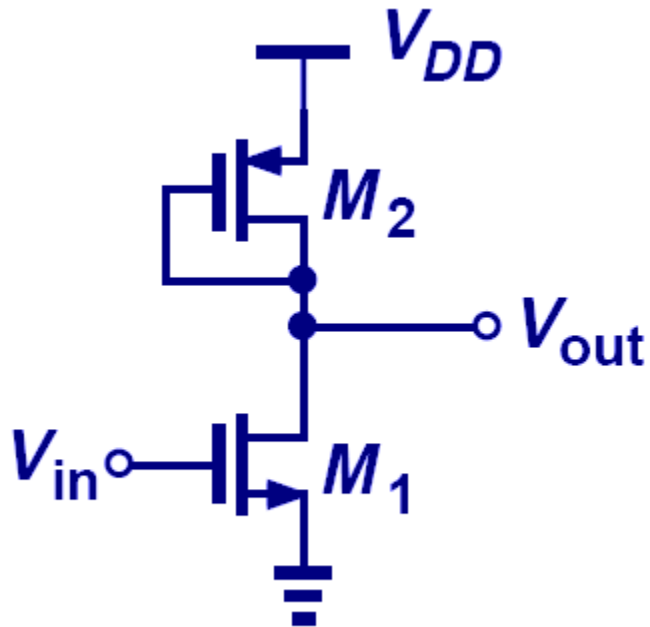
- 因为MOS模型中无沟道端点，所以栅极和沟道之间的电容 $C_{gate}$ 被分配到栅极与源极、栅极与漏极之间的电容 $C_1$ 、 $C_2$ 表示；
- 工作在饱和区时 $C_1 \sim C_{gate}$ ， $C_2 \sim 0$ 。它们与栅极源极、栅极漏极之间的重合电容合并，形成 $C_{GS}$ 和 $C_{GD}$ 。
- MOS高频模型共有4个电容（三极管是3个，无 $C_{EB}$ ）；注意三极管里衬底用S表示，MOS里衬底用B表示

为简单起见，也可以不考虑D和S到衬底的寄生电容，所以最简单的MOSFET高频寄生电容模型只需考虑 $C_{gs}$ 和 $C_{gd}$



**Figure 10.12** (a) High-frequency, equivalent-circuit model for the MOSFET. (b) The equivalent circuit for the case in which the source is connected to the substrate (body). (c) The equivalent-circuit model of (b) with  $C_{db}$  neglected (to simplify analysis). (d) The simplified high-frequency T model.

## Example: Capacitance Identification



# Transit Frequency (特征频率)

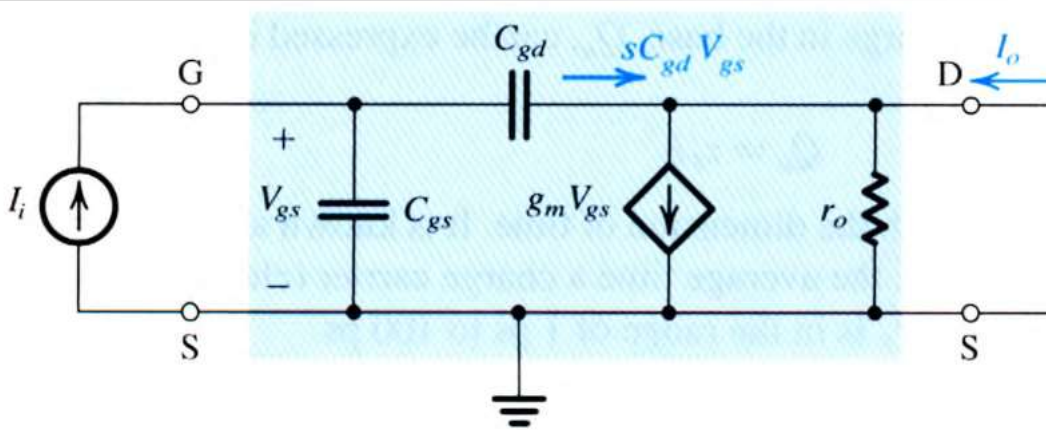


Figure 10.5 Determining the short-circuit current gain  $I_o/I_i$ .

$$I_o = g_m V_{gs} - sC_{gd} V_{gs}$$

$$C_{gd} \text{ is small, } \Rightarrow I_o \simeq g_m V_{gs}$$

$$V_{gs} = \frac{I_i}{s(C_{gs} + C_{gd})}$$

$$\Rightarrow \frac{I_o}{I_i} = \frac{g_m}{s(C_{gs} + C_{gd})}$$

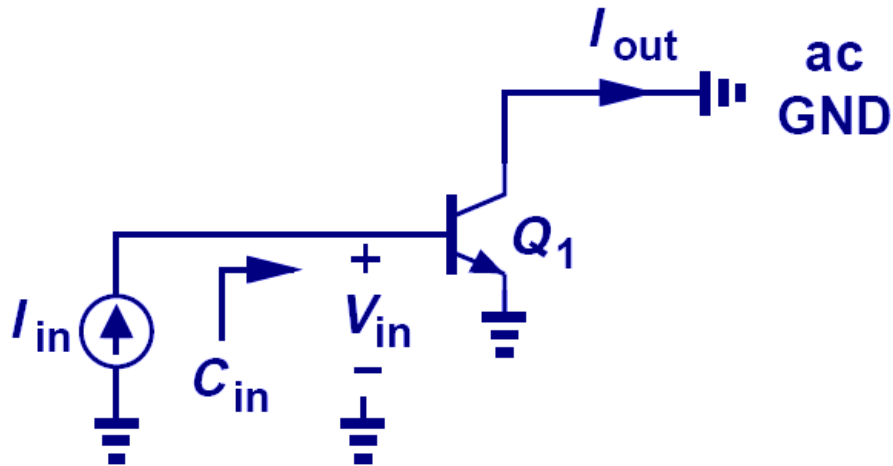
$$\left| \frac{I_o}{I_i} \right| = \frac{g_m}{\omega(C_{gs} + C_{gd})}$$

$$\Rightarrow \omega_T = g_m / (C_{gs} + C_{gd})$$

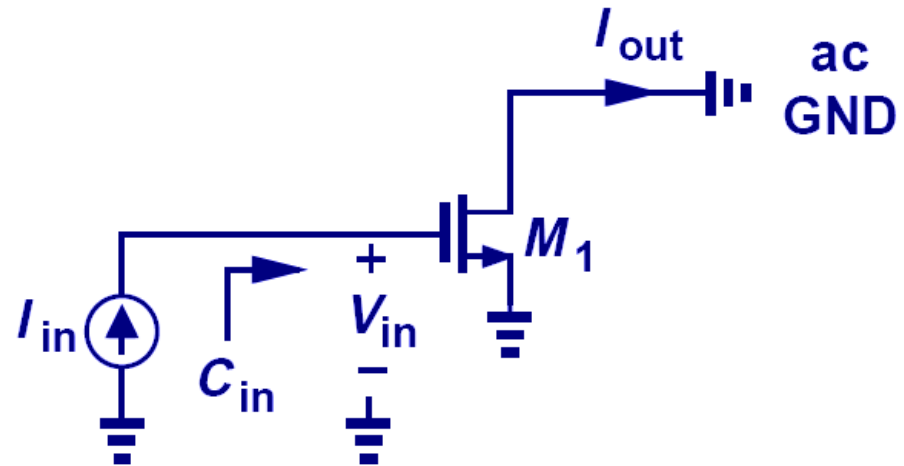
$$\Rightarrow f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

- 特征频率 ( $f_T$ )：也称为unity-gain frequency, 定义为当 Common Source结构的短路电流增益降到1时的频率

# Transit Frequency (特征频率)



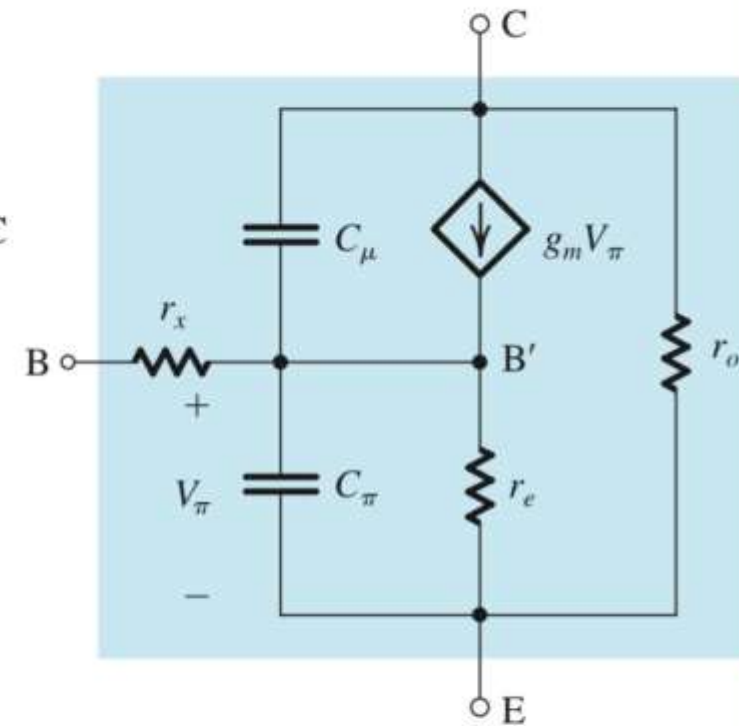
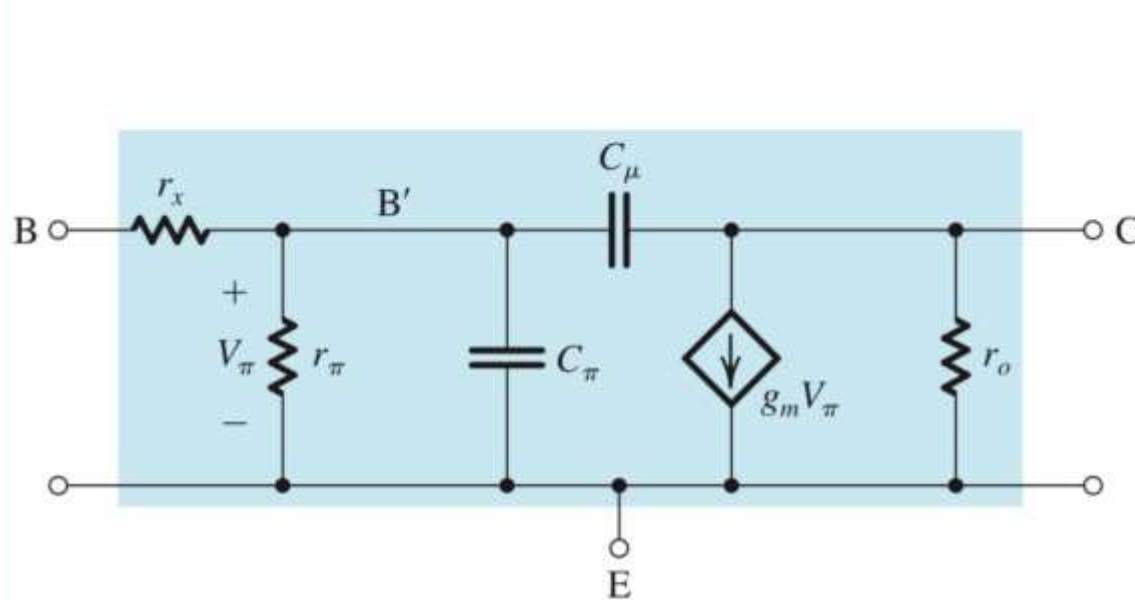
$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$



$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

- 特征频率 ( $f_T$ )：也称为unity-gain frequency, 定义为当Common Source结构的短路电流增益降到1时的频率

**Table 10.2** The BJT High-Frequency Model



$$g_m = I_C / V_T$$

$$r_o = |V_A| / I_C$$

$$r_\pi = \beta_0 / g_m$$

$$r_e = r_\pi / (\beta + 1)$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T}$$

$$C_\pi = C_{de} + C_{je}$$

$$C_{de} = \tau_F g_m$$

$$C_{je} \simeq 2C_{je0}$$

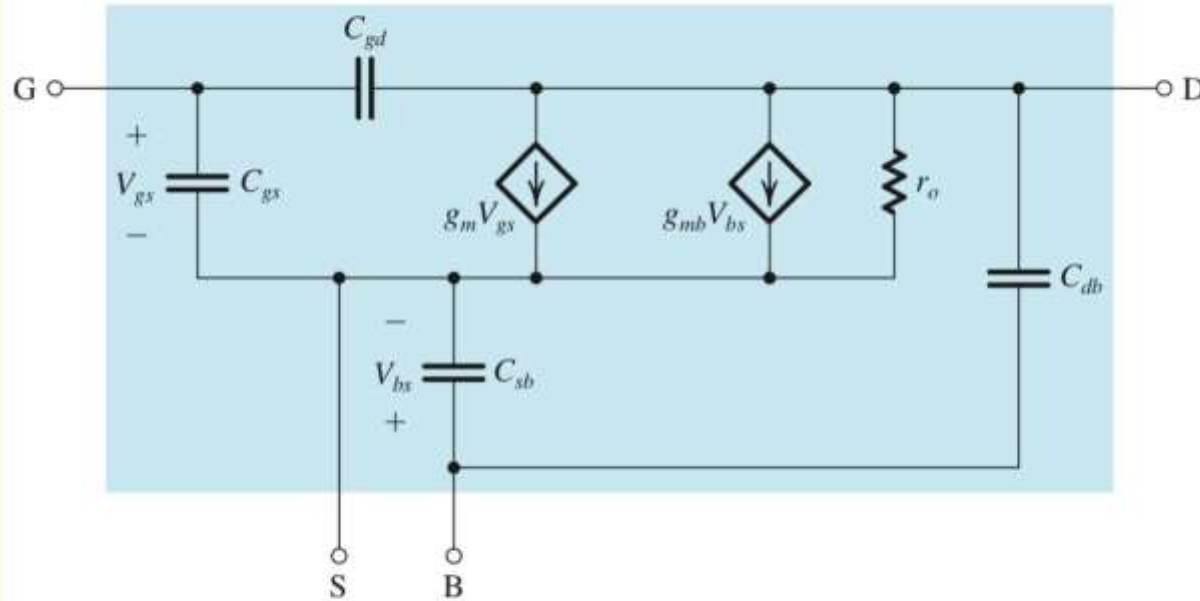
$$C_\mu = C_{jc0} / \left( 1 + \frac{|V_{CB}|}{V_{0c}} \right)^m$$

$$m = 0.3 - 0.5$$



**Table 10.1** The MOSFET High-Frequency Model

**Model**



**Model Parameters**

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$

$$g_{mb} = \chi g_m, \quad \chi = 0.1 \text{ to } 0.2$$

$$r_o = |V_A|/I_D$$

$$C_{gs} = \frac{2}{3} WLC_{ox} + WL_{ov} C_{ox}$$

$$C_{gd} = WL_{ov} C_{ox}$$

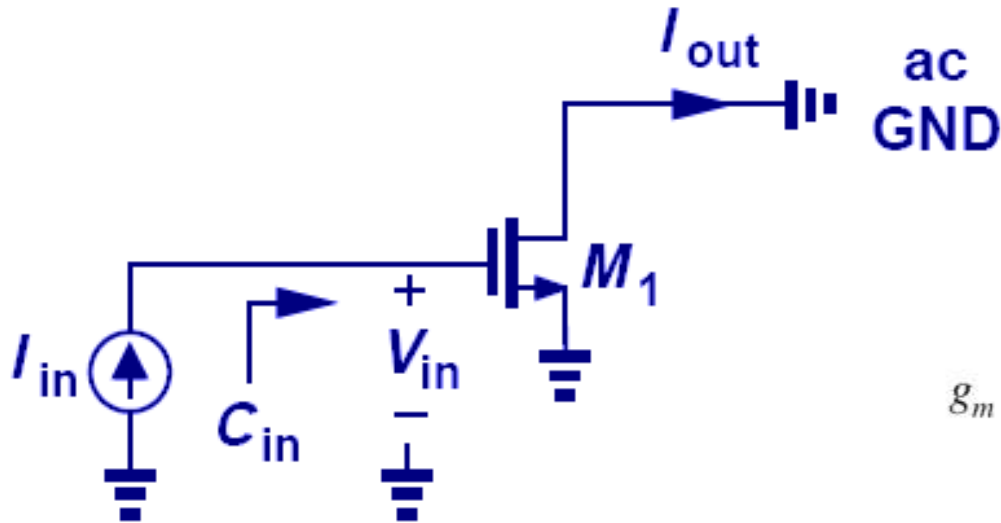
$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{|V_{SB}|}{V_0}}}$$

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{|V_{DB}|}{V_0}}}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$



# Example: Transit Frequency Calculation



$$L = 65nm$$

$$V_{GS} - V_{TH} = 100mV$$

$$\mu_n = 400cm^2/(V.s)$$

$$f_T = 226GHz$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} |V_{OV}| = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{|V_{OV}|}$$



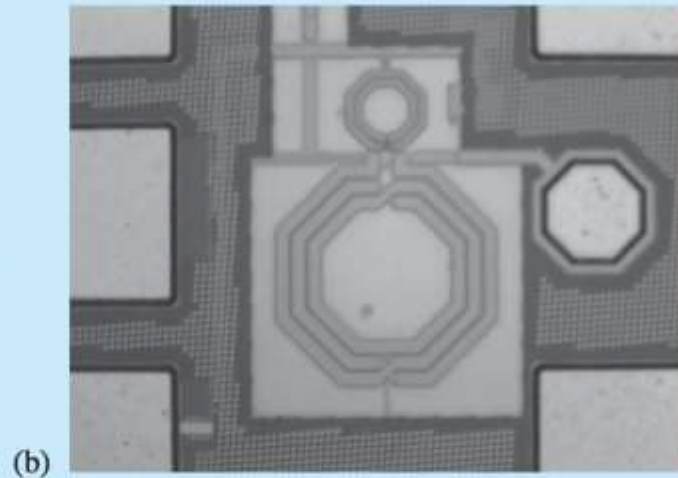
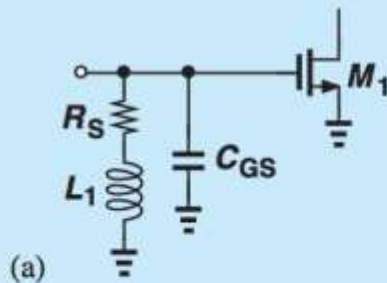
$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

└越小（工艺约先进），特征频率越高

# 可用电感来消除输入 寄生电容

## Did you know?

If the  $f_T$  of 65 nm MOSFETs is around 220 GHz, is it possible to operate such a device at a higher frequency? Yes, indeed. The key is to use inductors to cancel the effect of capacitors. Suppose as shown in Fig. (a), we place inductor  $L_1$  in parallel with  $C_{GS}$ . (Realized as a metal spiral on the chip, the inductor has some resistance,  $R_S$ .) At the resonance frequency,  $\omega_0 = 1/\sqrt{L_1 C_{GS}}$ , the parallel combination reduces to a single resistor, almost as if  $M_1$  had no gate-source capacitance! For this reason, the use of on-chip inductors has become common in high-frequency design. Figure (b) shows the chip photograph of a 300 GHz oscillator designed by the author in 65 nm technology. Such high frequencies find application in medical imaging.



(a) Use of resonance to cancel transistor capacitance, (b) chip photograph of a 300 GHz CMOS oscillator.

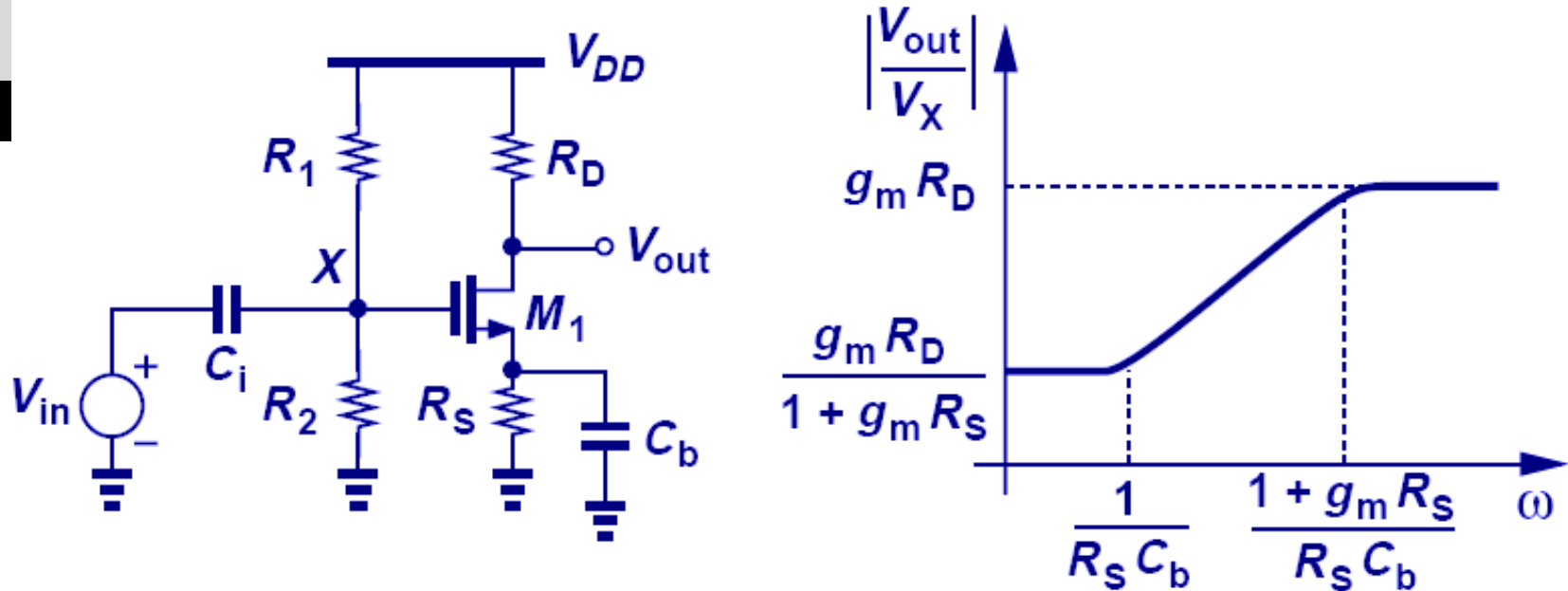
## Analysis Summary

- 频率响应：传输函数幅度、相位随频率变化的关系；
- 若已知传输函数的零极点，用Bode近似可以快速画出频率响应（零点增加20dB/dec，极点减小20dB/dec）；
- 一般而言，信号通路上的“结点”都关联着传输函数的一个“极点”；
- Miller's 定理可将浮接的电容转化为到地的电容；
- Bipolar 有三个寄生电容，MOS 有四个寄生电容，这些电容影响着电路的高频特性；

# 高频电路的分析步骤

- ①分析哪个电容影响着低频响应，并计算其低频极点（此时可忽略晶体管的寄生电容），**计算下限截止频率  $f_L$**
- ②计算通带增益/中频增益（理想化：电路电容用短路替代，并忽略晶体管的寄生电容）
- ③将晶体管的寄生电容考虑进来
- ④合并到地的电容；
- ⑤分析高频零极点，**计算上限截止频率  $f_H$**
- ⑥使用“波特近似”或者“精确分析”画出频率响应曲线；

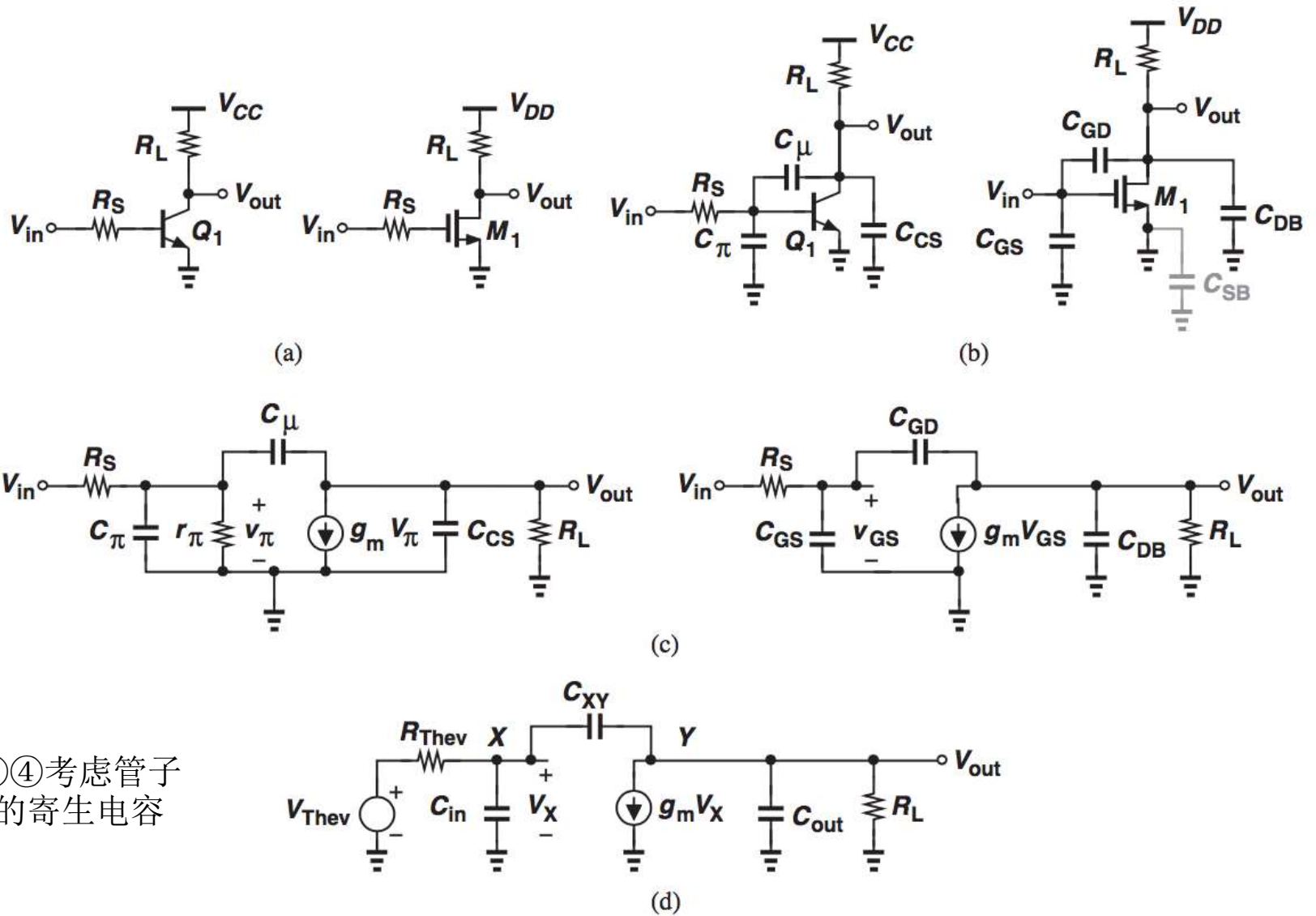
## Frequency Response of CS Stage with Bypassed Degeneration



①②低频分析:

$$\left| \frac{V_{out}}{V_X}(s) \right| = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

- 为了增加通带增益, 添加一旁路电容  $C_b$ ;
- 极点频率必须低于最小的信号频率;
- 注: 输入  $C_i$  引起的高通结构在前面已分析, 这里从  $X$  点开始分析;

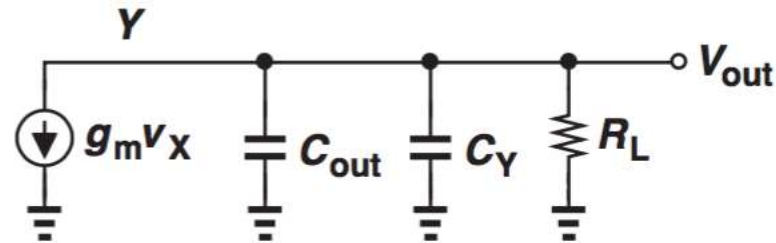
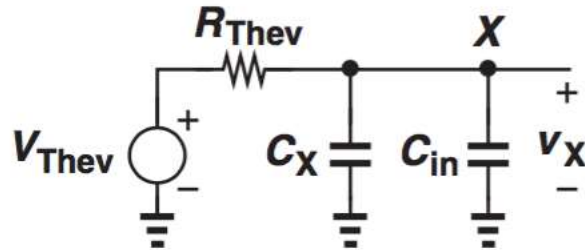
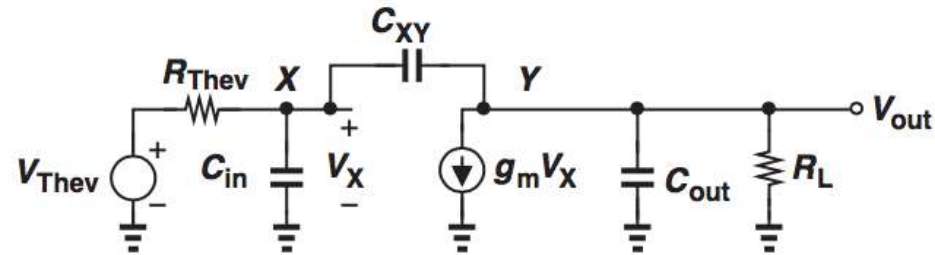
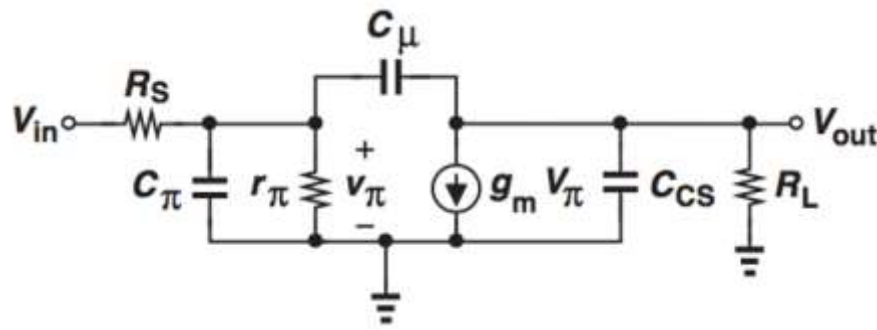


③④考虑管子的寄生电容

**Figure 11.29** (a) CE and CS stages, (b) inclusion of transistor capacitances, (c) small-signal equivalents, (d) unified model of both circuits.

BJT和MOSFET的统一模型

# Unified Model Using Miller's Theorem



## CE Stage

$$V_{\text{Thev}} = V_{\text{in}} \frac{r_{\pi}}{r_{\pi} + R_S}$$

$$R_{\text{Thev}} = R_S \parallel r_{\pi}$$

$$C_X = C_{\mu} (1 + g_m R_L)$$

$$C_Y = C_{\mu} \left( 1 + \frac{1}{g_m R_L} \right)$$

## CS Stage

$$V_{\text{Thev}} = V_{\text{in}}$$

$$R_{\text{Thev}} = R_S$$

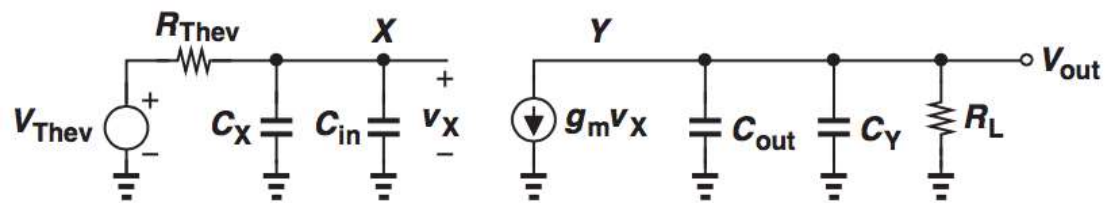
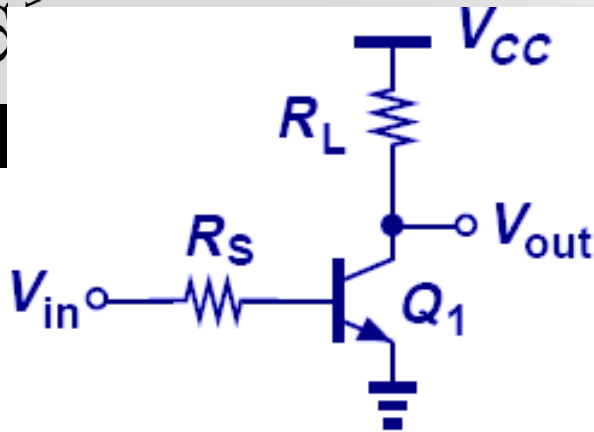
$$C_X = C_{\text{GD}} (1 + g_m R_L)$$

$$C_Y = C_{\text{GD}} \left( 1 + \frac{1}{g_m R_L} \right)$$

Parameters in unified model of CE and CS stages with Miller's approximation.



## Example: CE Stage



$$|\omega_{p,in}| = \frac{1}{R_{Thev}[C_{in} + (1 + g_m R_L)C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[ C_{out} + \left( 1 + \frac{1}{g_m R_L} \right) C_{XY} \right]}$$

⑤分析高频  
零极点

信号通路结点  
极点估算法

$$R_S = 200\Omega$$

$$I_C = 1mA$$

$$\beta = 100$$

$$C_\pi = 100fF$$

$$C_\mu = 20fF$$

$$C_{CS} = 30fF$$

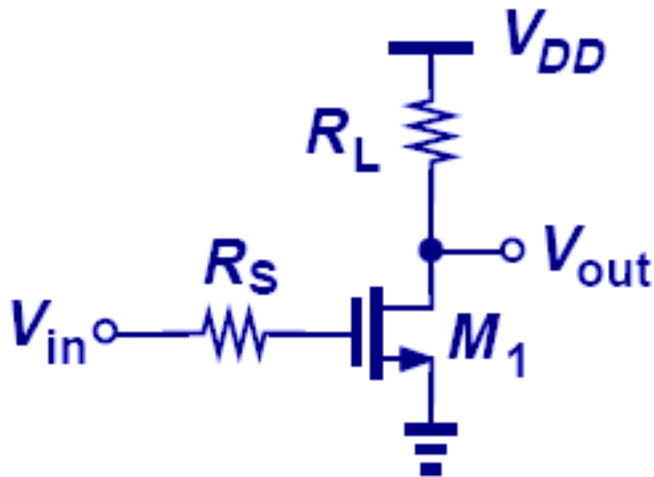
$$|\omega_{p,in}| = 2\pi \times (516MHz)$$

$$|\omega_{p,out}| = 2\pi \times (1.59GHz)$$

➤ 输入极点是瓶颈



若MOS管的宽度减半、偏置电流也减半，如何？



$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$$

$$W \downarrow 2X$$

$$|\omega_{p,in}| = \frac{1}{R_{Thev} [C_{in} + (1 + g_m R_L) C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[ C_{out} + \left( 1 + \frac{1}{g_m R_L} \right) C_{XY} \right]}$$



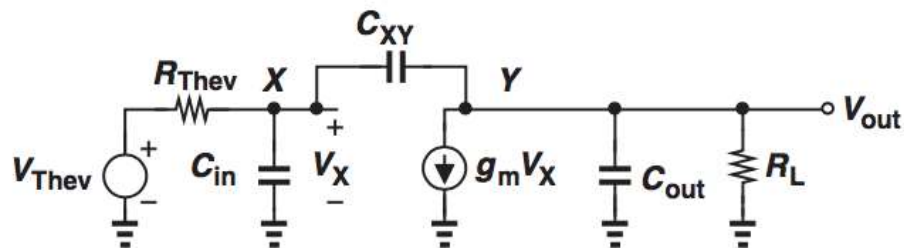
$$|\omega_{p,in}| = \frac{1}{R_S \left[ \frac{C_{in}}{2} + \left( 1 + \frac{g_m R_L}{2} \right) \frac{C_{XY}}{2} \right]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[ \frac{C_{out}}{2} + \left( 1 + \frac{2}{g_m R_L} \right) \frac{C_{XY}}{2} \right]}$$

# Direct Analysis of CE and CS Stages, 求上限截止频率

At Node X:  $(V_{out} - V_X)C_{XY}s = V_X C_{in}s + \frac{V_X - V_{Thev}}{R_{Thev}}$

At Node Y:  $(V_X - V_{out})C_{XY}s = g_m V_X + V_{out} \left( \frac{1}{R_L} + C_{out}s \right)$



⑥精确分析,  
并画出频响  
曲线

求上限截止频率  $f_H$  的**方法1 (解析法)**: 将  $s$  用  $j\omega$  替代, 得到  $\frac{V_{out}}{V_{Thev}}(\omega)$ , 令其值等于  $\frac{g_m R_L}{\sqrt{2}}$ , 求得的  $\omega$  即为  $\omega_H$

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

“精确分析法”

$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out}).$$

该情况下  $\omega_{p1}$  即为上限截止频率

假设  $\omega_{p2} \gg \omega_{p1}$

$$b = \frac{1}{\omega_{p1}}$$

$$|\omega_{p1}| = \frac{1}{(1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})}$$

$$= \frac{1}{\sum_i C_i R_i}$$

**方法2 (主极点法)**: 存在主极点的情况下, 可先用中频增益做Miller等效, 然后求X、Y点电容的RC之和

$$\begin{aligned} as^2 + bs + 1 &= \left( \frac{s}{\omega_{p1}} + 1 \right) \left( \frac{s}{\omega_{p2}} + 1 \right) \\ &= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left( \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right)s + 1 \end{aligned}$$

$$|\omega_{p2}| = \frac{b}{a}$$

$$= \frac{(1 + g_m R_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})}{R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})}$$

# Direct Analysis of CE and CS Stages (直接精 确分析)

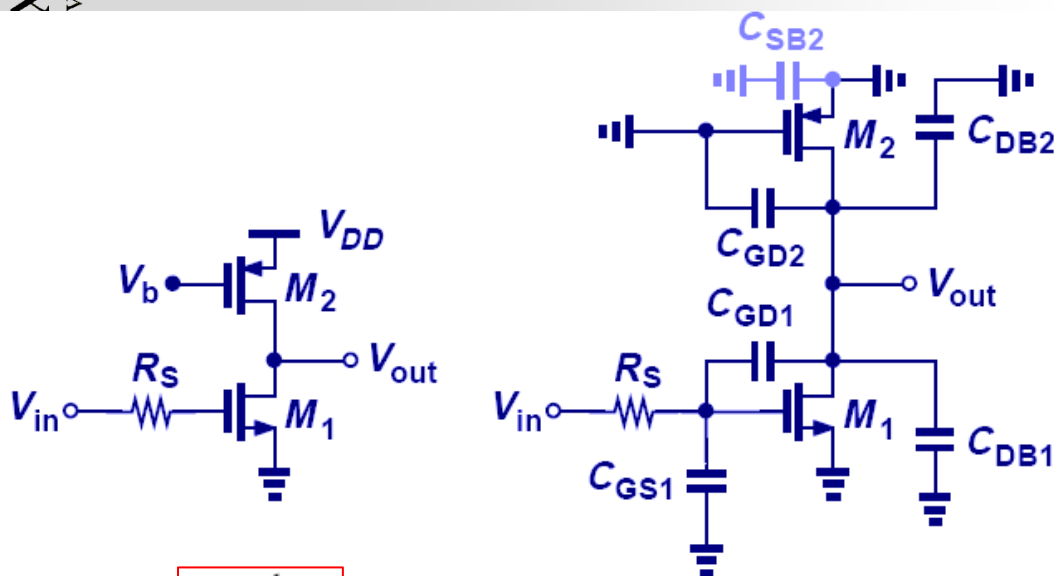
$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$|\omega_{p1}| = \frac{1}{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}$$

$$|\omega_{p2}| = \frac{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

➤ 直接分析可以看出传输函数还有一个零点，但 $C_{xy}$ 比较小，该零点的频率往往很高，不影响电路性能

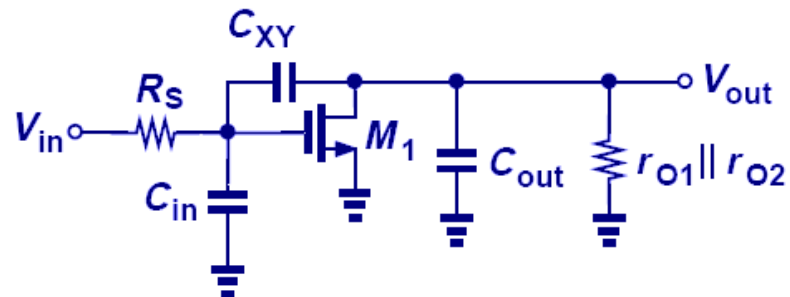
## Example: CE and CS Direct Analysis



$$C_{in} = C_{GS1}$$

$$C_{XY} = C_{GD1}$$

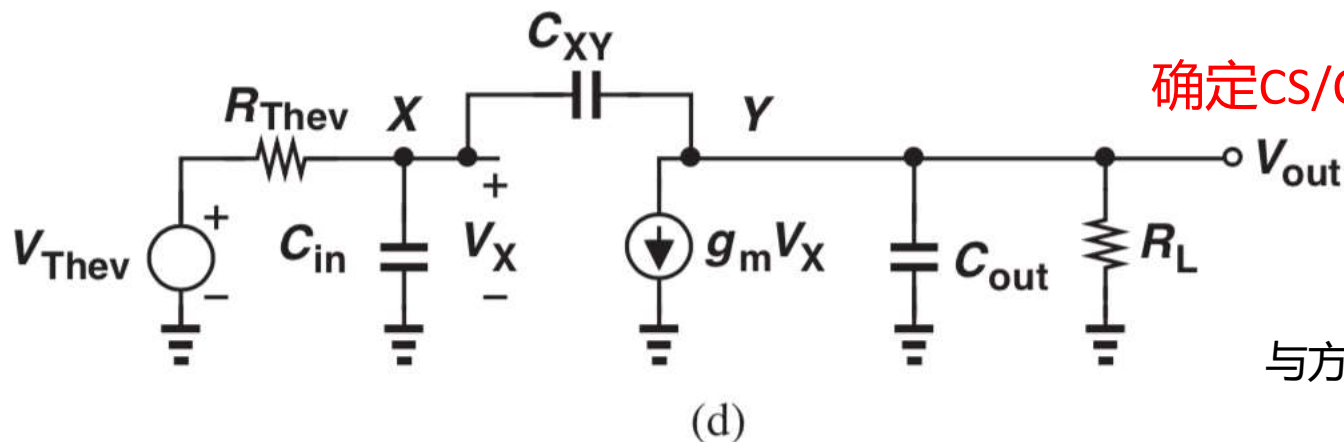
$$C_{out} = C_{DB1} + C_{GD2} + C_{DB2}$$



$$= \frac{1}{\sum_i C_i R_i}$$

$$\omega_{p1} \approx \frac{1}{[1 + g_{m1}(r_{O1} \parallel r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{[1 + g_{m1}(r_{O1} \parallel r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}{R_S (r_{O1} \parallel r_{O2})(C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$



确定CS/CE结构的上限截止频率

与方法2的区别是不做求和运算

**方法3 (Miller近似)**：将 $C_{XY}$  Miller等效到输入X和输出Y，再用信号通路极点法得到两个极点，**因X点处关联的极点频率往往较小，所以其为上限截止频率 $f_H$** ：

- Miller近似将主极点分裂成了输入X和输出Y处的两个极点；
- Miller近似是有较大误差的，但有助于直观分析；
- 当Miller近似得到的X、Y两处的极点相隔较远时，误差较小
- **若由Miller近似计算出的两极点频率接近时，则需用方法2**

方法3，用Miller等效获得X处的极点，作为 $f_H$

$$|\omega_{p1}| = \frac{1}{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in}}$$

方法2和方法3的关系

$$|\omega_{p1}| = \frac{1}{\underbrace{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in}}_{\text{输入X处的极点}} + \underbrace{R_L (C_{XY} + C_{out})}_{\text{输入Y处的极点}}}$$

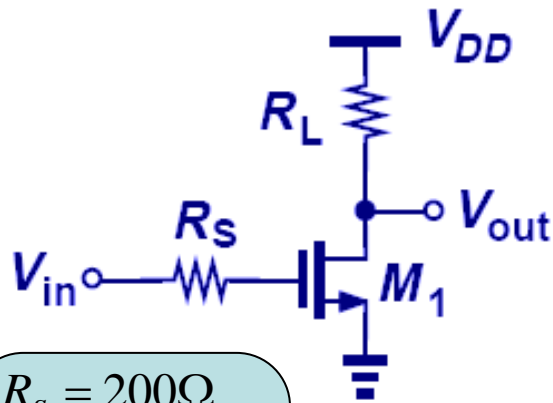
方法2

输入X处的极点

输入Y处的极点

输入X点处的极点是瓶颈  
(RC最大，频率最低)

# Example: Comparison Between Different Methods



$$R_S = 200\Omega$$

$$C_{GS} = 250\text{fF}$$

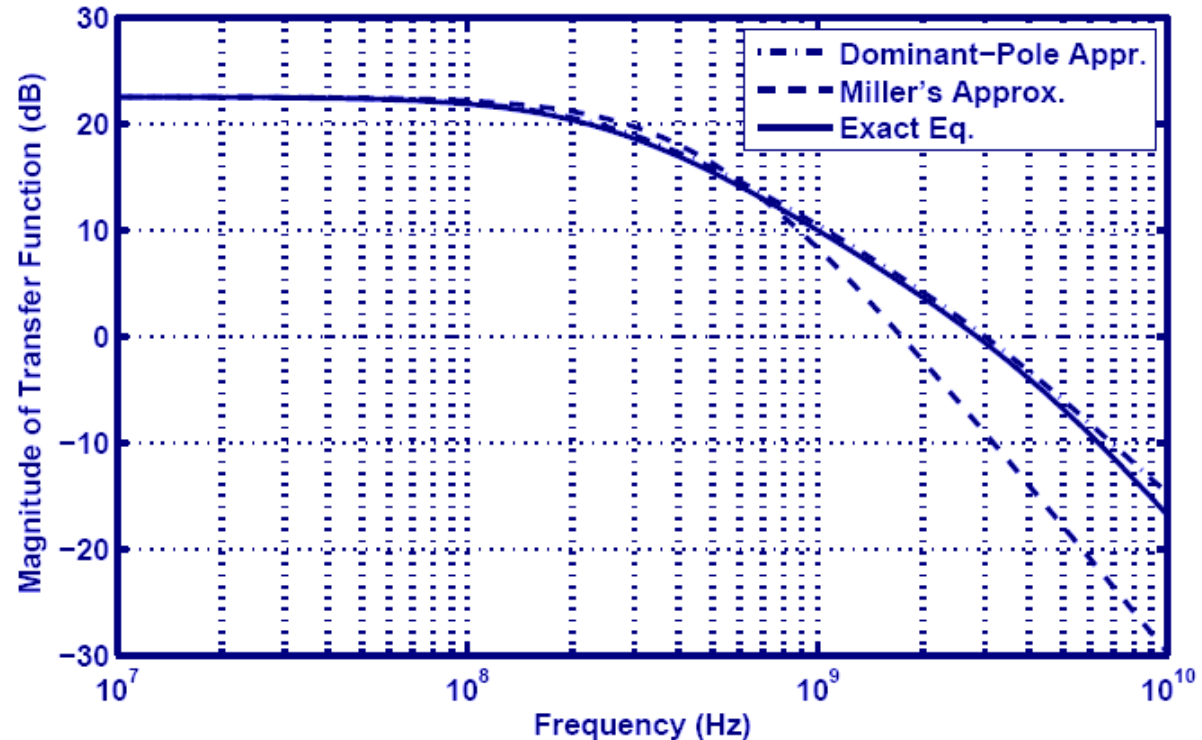
$$C_{GD} = 80\text{fF}$$

$$C_{DB} = 100\text{fF}$$

$$g_m = (150\Omega)^{-1}$$

$$\lambda = 0$$

$$R_L = 2\text{K}\Omega$$



## Exact

$$|\omega_{p,in}| = 2\pi \times (264\text{MHz})$$

$$|\omega_{p,out}| = 2\pi \times (4.53\text{GHz})$$

方法1: 精确分析法

## Dominant Pole

$$|\omega_{p,in}| = 2\pi \times (249\text{MHz})$$

$$|\omega_{p,out}| = 2\pi \times (4.79\text{GHz})$$

方法2: 主极点法

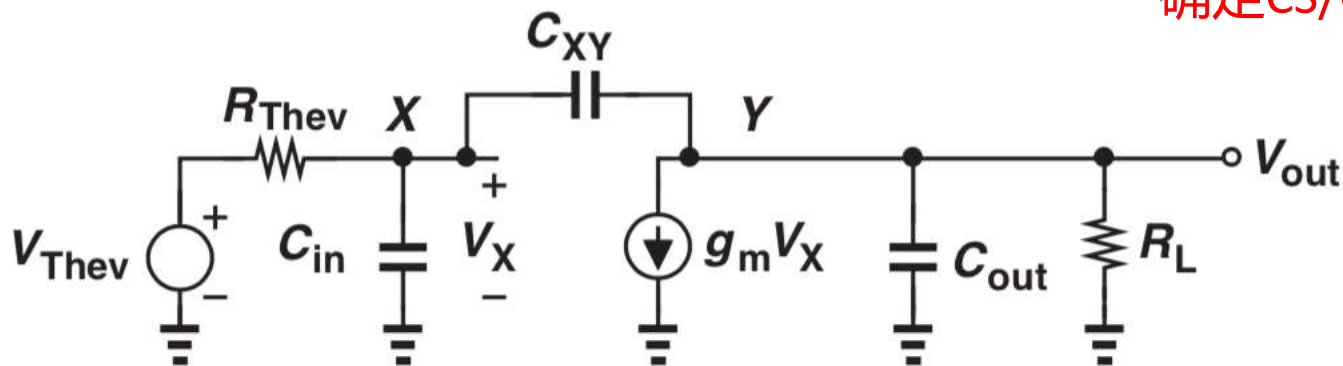
## Miller's

$$|\omega_{p,in}| = 2\pi \times (571\text{MHz})$$

$$|\omega_{p,out}| = 2\pi \times (428\text{MHz})$$

方法3: Miller近似法

## 确定CS/CE结构的上限截止频率



(d)

## 教材中用的方法，考试时请用此方法

**方法4（开路时间常数法）：**每次考虑一个电容，将其余电容**开路**（理想化）

- 该方法是分析  $f_L$  时采用的“短路时间常数”法的对偶
- 请注意，分析  $f_L$  将其他电容短路（理想化）；分析  $f_H$  将其他电容开路（也是理想化）

**步骤如下**

- ① 输入信号需置零
- ② 一次考虑一个电容，分析时将其他电容**开路**（理想化）
- ③ 求电容两端的等效电阻，并得到相应的时间常数  $RC$
- ④ 求和计算上限截止频率

$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$$

\* ①存在主极点时适用；②不存在主极点时精度也还不错③当电路中极点不能直观地看出时适用



# 确定CS/CE结构的上限截止频率

## 方法4 (开路时间常数) 举例

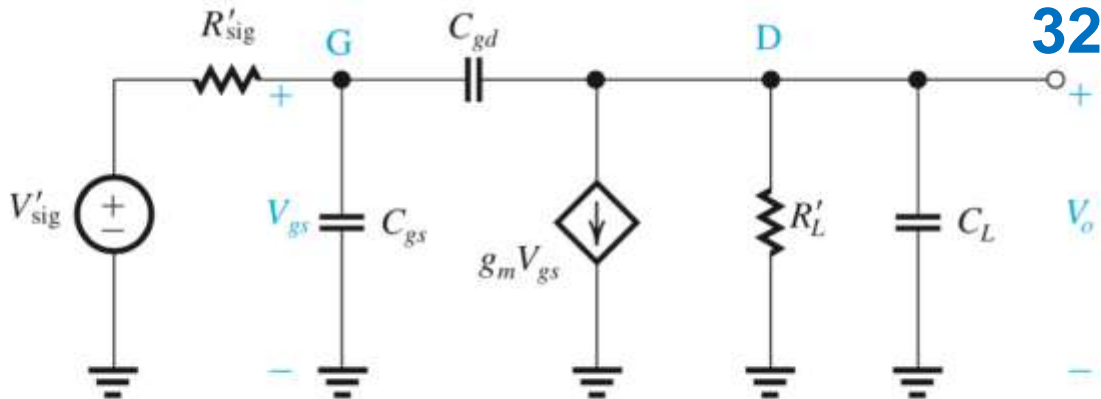
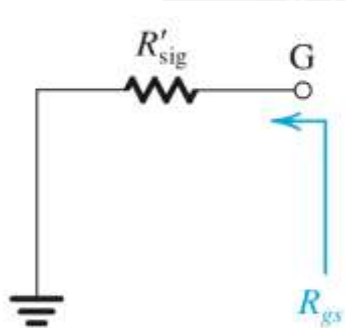


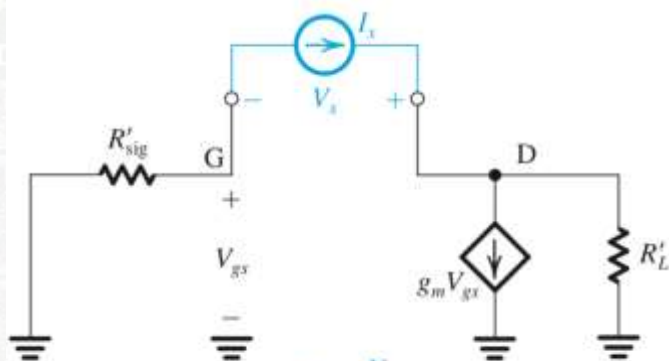
Figure 10.24 Generalized high-frequency equivalent circuit for the CS amplifier.

①只考虑 $C_{gs}$



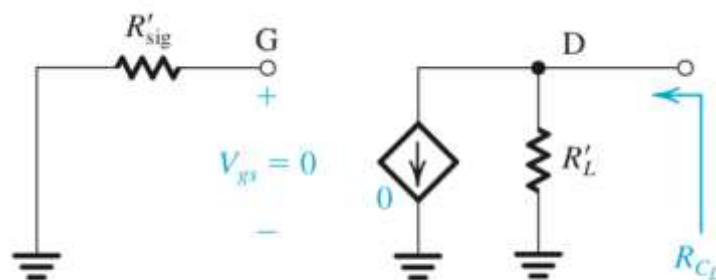
$$R_{gs} = R'_{sig}$$

②只考虑 $C_{gd}$



$$R_{gd} \equiv \frac{V_x}{I_x} = R'_{sig}(1 + g_m R'_L) + R'_L$$

③只考虑 $C_L$



$$R_{CL} = R'_L$$

$$f_H = \frac{1}{2\pi\tau_H}$$

$$\tau_H = C_{gs}R'_{sig} + C_{gd}[R'_{sig}(1 + g_m R'_L) + R'_L] + C_L R'_L \quad \text{方法4}$$



$$\tau_H = [C_{gs} + C_{gd}(1 + g_m R'_L)]R'_{sig} + (C_{gd} + C_L)R'_L$$

也可以Miller等效后再计算



Find the midband gain  $A_M$  and the upper 3-dB frequency  $f_H$  of a CS amplifier fed with a signal source having an internal resistance  $R_{sig} = 100 \text{ k}\Omega$ . The amplifier has  $R_G = 4.7 \text{ M}\Omega$ ,  $R_D = R_L = 15 \text{ k}\Omega$ ,  $g_m = 1 \text{ mA/V}$ ,  $r_o = 150 \text{ k}\Omega$ ,  $C_{gs} = 1 \text{ pF}$ , and  $C_{gd} = 0.4 \text{ pF}$ . Also, find the frequency of the transmission zero.

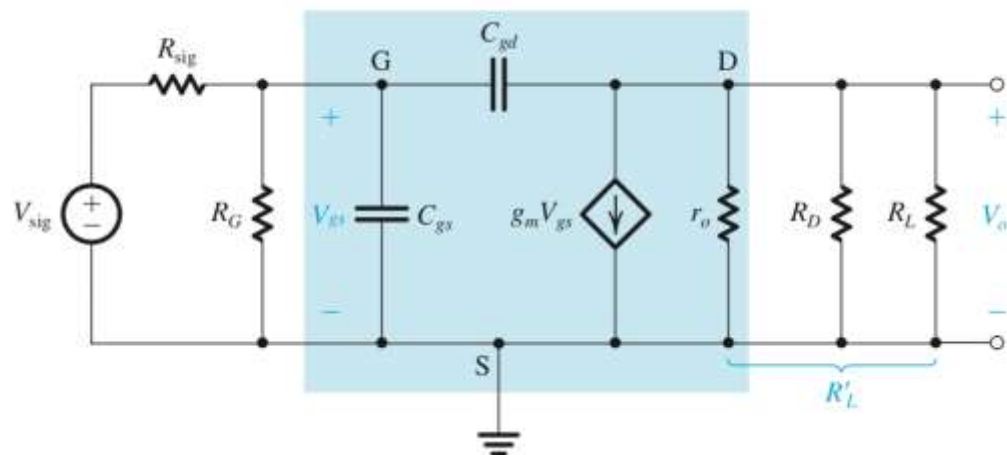
### ① 计算中频增益

$$A_M = -\frac{R_G}{R_G + R_{sig}} g_m R'_L$$

$$R'_L = r_o \parallel R_D \parallel R_L = 150 \parallel 15 \parallel 15 = 7.14 \text{ k}\Omega$$

$$g_m R'_L = 1 \times 7.14 = 7.14 \text{ V/V}$$

$$A_M = -\frac{4.7}{4.7 + 0.1} \times 7.14 = -7 \text{ V/V}$$



### ② Miller等效, 将 $C_{gd}$ 分别折算到G和D, 开路时间常数法计算每个RC

$$C_{eq} = (1 + g_m R'_L) C_{gd}$$

$$= (1 + 7.14) \times 0.4 = 3.26 \text{ pF}$$

$$C_{in} = C_{gs} + C_{eq} = 1 + 3.26 = 4.26 \text{ pF}$$

$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$$

$$(R_{sig} \parallel R_G) \times C_{in} = 0.417 \mu\text{s} \quad R'_L \times C_{gd} = 0.00285 \mu\text{s}$$

### ③ 两个RC相差较大, 可忽略D结点的

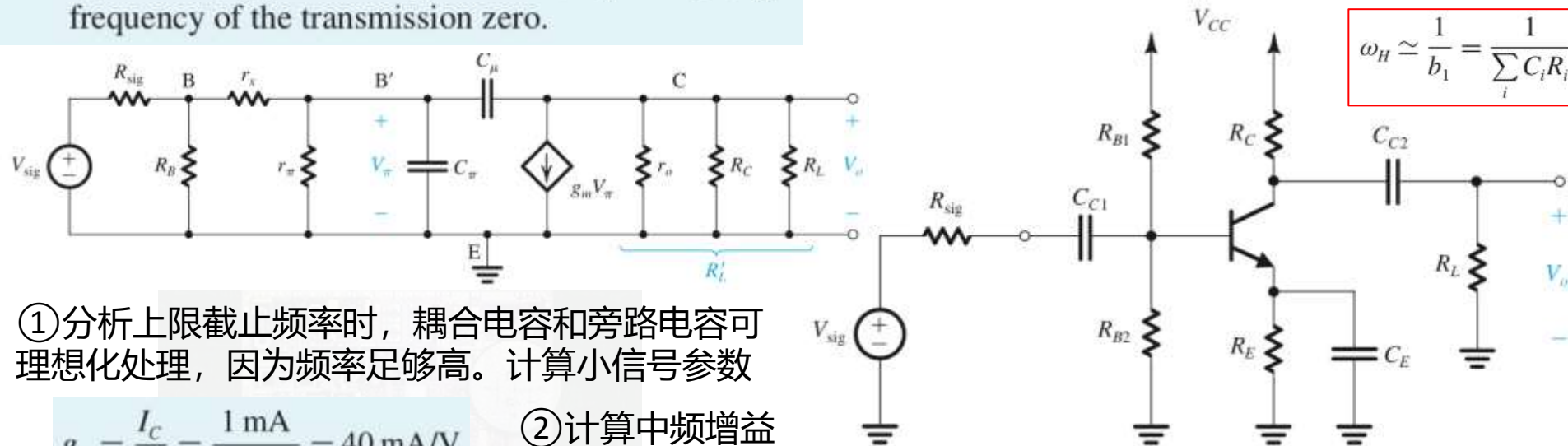
$$f_H = \frac{1}{2\pi C_{in} (R_{sig} \parallel R_G)} = 382 \text{ kHz}$$

### ④ 零点远高于上限截止频率

$$f_z = \frac{g_m}{2\pi C_{gd}} = \frac{1 \times 10^{-3}}{2\pi \times 0.4 \times 10^{-12}} = 398 \text{ MHz}$$

## Example 10.4

It is required to find the midband gain and the upper 3-dB frequency of the common-emitter amplifier of Fig. 10.9(a) for the following case:  $I_E = 1$  mA,  $R_B = R_{B1} \parallel R_{B2} = 100$  k $\Omega$ ,  $R_C = 8$  k $\Omega$ ,  $R_{sig} = 5$  k $\Omega$ ,  $R_L = 5$  k $\Omega$ ,  $\beta_0 = 100$ ,  $V_A = 100$  V,  $C_\mu = 1$  pF,  $f_T = 800$  MHz, and  $r_x = 50$   $\Omega$ . Also, determine the frequency of the transmission zero.



①分析上限截止频率时，耦合电容和旁路电容可理想化处理，因为频率足够高。计算小信号参数

$$g_m = \frac{I_C}{V_T} = \frac{1 \text{ mA}}{25 \text{ mV}} = 40 \text{ mA/V}$$

$$r_\pi = \frac{\beta_0}{g_m} = \frac{100}{40 \text{ mA/V}} = 2.5 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_C} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega$$

②计算中频增益

$$A_M = -\frac{R_B}{R_B + R_{sig}} \frac{r_\pi}{r_\pi + r_x + (R_B \parallel R_{sig})} g_m R'_L$$

$$R'_L = r_o \parallel R_C \parallel R_L = 3 \text{ k}\Omega$$

$$g_m R'_L = 40 \times 3 = 120 \text{ V/V}$$

$$A_M = -39 \text{ V/V}$$

$V_{sig}$ ,  $R_{sig}$ ,  $R_B$ 戴维南等效

④开路时间常数法计算每个RC

$$C_{in} = C_\pi + C_\mu (1 + g_m R'_L) = 128 \text{ pF}$$

$$R'_{sig} = r_\pi \parallel [r_x + (R_B \parallel R_{sig})] = 1.65 \text{ k}\Omega$$

$$R'_{sig} \times C_{in} = 0.211 \mu\text{s}$$

$$R'_L \times C_\mu = 0.003 \mu\text{s}$$

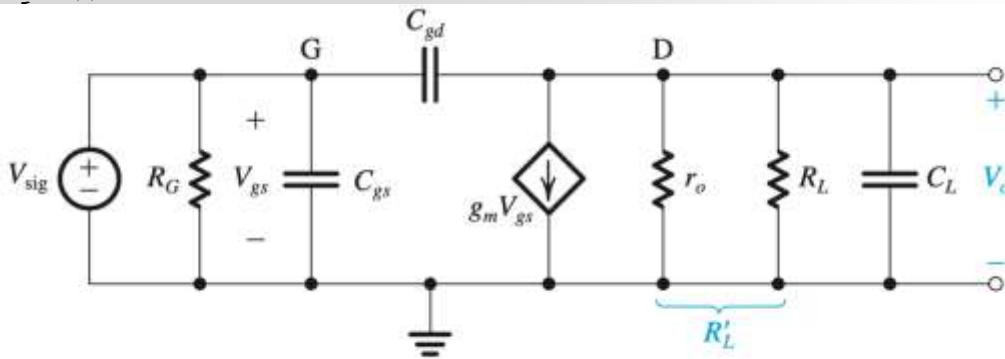
⑤两个RC相差较大，可忽略负载端

$$f_H = \frac{1}{2\pi C_{in} R'_{sig}} = 754 \text{ kHz}$$

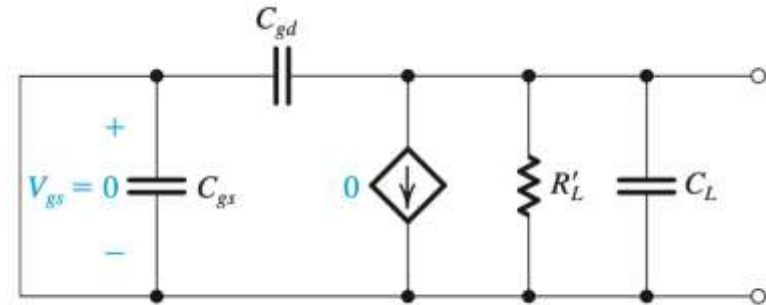
$$C_\pi + C_\mu = \frac{g_m}{\omega_T} = \frac{40 \times 10^{-3}}{2\pi \times 800 \times 10^6} = 8 \text{ pF}$$

$$C_\pi = 7 \text{ pF}$$

## 10.3.4 Frequency Response of the CS Amplifier When $R_{sig}$ Is Low

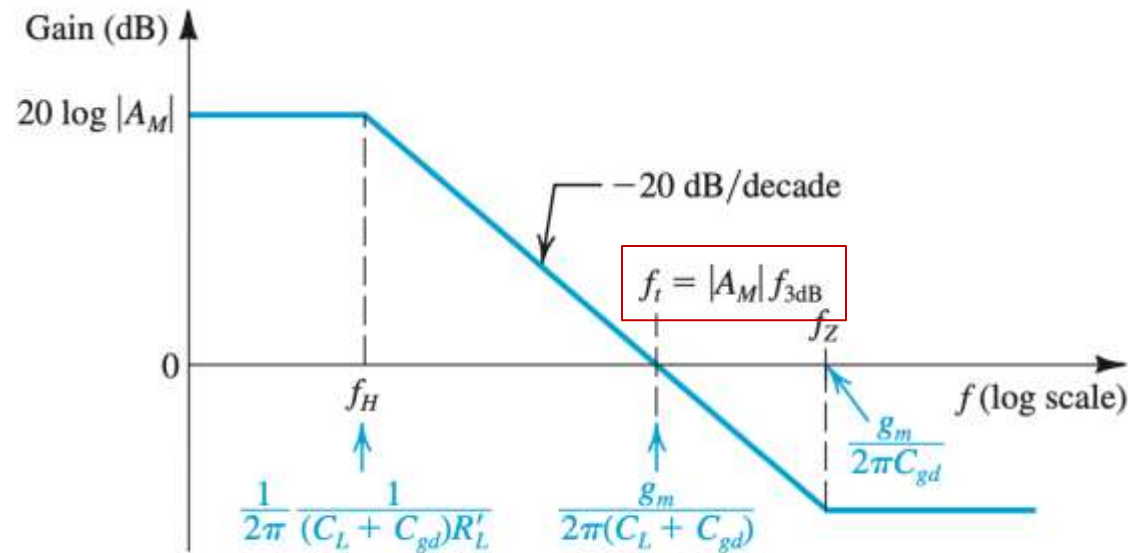


(a)



(b)

当 $R_{sig}$ 很小时，上限截止频率由输出结点决定



(c)

$$\begin{cases} V_{gs} = V_{sig} \\ I_{gd} = sC_{gd}(V_{gs} - V_o) \\ I_{gd} = g_m V_{gs} + \frac{V_o}{R'_L} + sC_L V_o \end{cases}$$

$$\Rightarrow \frac{V_o}{V_{sig}} = -g_m R'_L \frac{1 - s(C_{gd}/g_m)}{1 + s(C_L + C_{gd})R'_L}$$

注意：管子的特征频率  $f_T$  是大写，电路的unity gain 频率  $f_t$  是小写

Consider an IC CS amplifier fed with a source having  $R_{sig} = 0$  and having an effective load resistance  $R'_L$  composed of  $r_o$  of the amplifier transistor in parallel with an equal resistance  $r_o$  of the current-source load. Let  $g_m = 1.25 \text{ mA/V}$ ,  $r_o = 20 \text{ k}\Omega$ ,  $C_{gs} = 20 \text{ fF}$ ,  $C_{gd} = 5 \text{ fF}$ , and  $C_L = 25 \text{ fF}$ . Find  $A_M$ ,  $f_H$ ,  $f_t$ , and  $f_z$ . If the amplifying transistor is to be operated at twice the original overdrive voltage while  $W$  and  $L$  remain unchanged, by what factor must the bias current be changed? What are the new values of  $A_M$ ,  $f_H$ ,  $f_t$ , and  $f_z$ ?

① 计算中频增益

$$A_M = -g_m R'_L = -g_m (r_o \parallel r_o) = -12.5 \text{ V/V}$$

② 开路时间常数法计算每个RC

$$C_{in} = C_{gs} + (1 + g_m R'_L) \times C_{gd} = 87.5 \text{ fF}$$

$$R_{sig} \text{ 很小, 所以 } (R_{sig} \parallel R_G) \times C_{in} = 0 \text{ s}$$

$$R'_L \times (C_{gd} + C_L) = 10 \text{ k}\Omega \times 30 \text{ fF} = 0.3 \text{ ns}$$

③ 两个RC相差较大, 可忽略一个

$$f_H = \frac{1}{2\pi (C_L + C_{gd}) R'_L} = 530.5 \text{ MHz}$$



## Example 10.8

An integrated-circuit CS amplifier has  $g_m = 1.25 \text{ mA/V}$ ,  $C_{gs} = 20 \text{ fF}$ ,  $C_{gd} = 5 \text{ fF}$ ,  $C_L = 25 \text{ fF}$ ,  $R'_{sig} = 10 \text{ k}\Omega$ , and  $R'_L = 10 \text{ k}\Omega$ . Determine  $f_H$  and the frequency of the transmission zero  $f_Z$  caused by  $C_{gd}$ .

①方法4, 直接用开路时间常数法

$$R_{gs} = R'_{sig} = 10 \text{ k}\Omega$$

$$\begin{aligned} R_{gd} &= R'_{sig}(1 + g_m R'_L) + R'_L \\ &= 10(1 + 1.25 \times 10) + 10 = 145 \text{ k}\Omega \end{aligned}$$

$$R_{CL} = R'_L = 10 \text{ k}\Omega$$

$$\tau_{gs} = C_{gs} R_{gs} = 20 \times 10^{-15} \times 10 \times 10^3 = 200 \text{ ps}$$

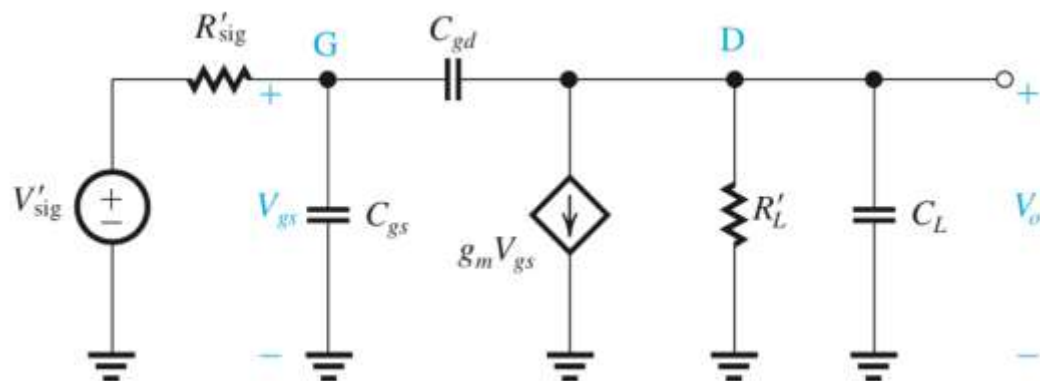
$$\tau_{gd} = C_{gd} R_{gd} = 5 \times 10^{-15} \times 145 \times 10^3 = 725 \text{ ps}$$

$$\tau_{CL} = C_L R_{CL} = 25 \times 10^{-15} \times 10 \times 10^3 = 250 \text{ ps}$$

$$\tau_H = \tau_{gs} + \tau_{gd} + \tau_{CL}$$

$$= 200 + 725 + 250 = 1175 \text{ ps}$$

$$f_H = \frac{1}{2\pi \tau_H} = \frac{1}{2\pi \times 1175 \times 10^{-12}} = 135.5 \text{ MHz}$$



②方法2, 先用Miller等效, 再用开路时间常数法  
(更直观)

$$C_{in} = C_{gs} + (1 + g_m R'_L) \times C_{gd} = 87.5 \text{ fF}$$

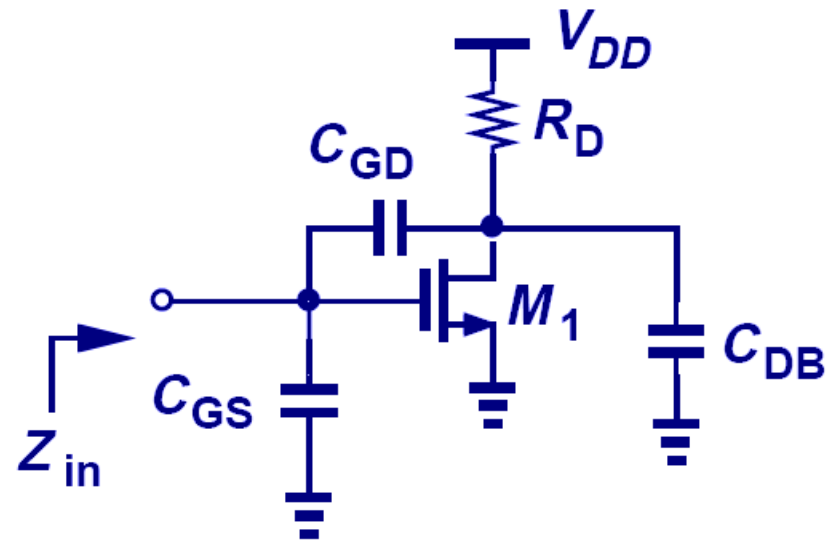
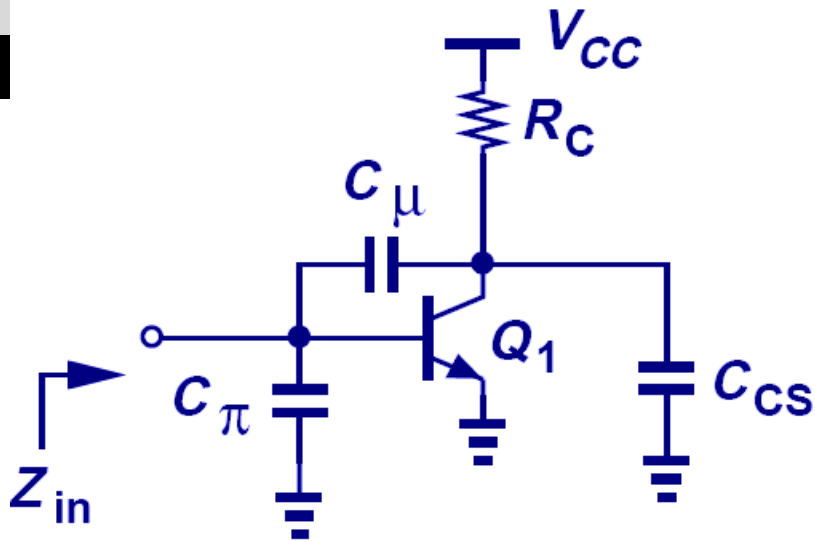
$$R'_{sig} \times C_{in} = 875 \text{ ps}$$

$$C_{out} = C_{gd} || C_L = 30 \text{ fF}$$

$$R'_L \times C_{out} = 300 \text{ ps}$$

$$\sum_i R_i C_i = 1175 \text{ ps} \quad \text{与①相等}$$

# 输入阻抗



$$Z_{in} \approx \frac{1}{[C_\pi + (1 + g_m R_C)C_\mu]s} \parallel r_\pi$$

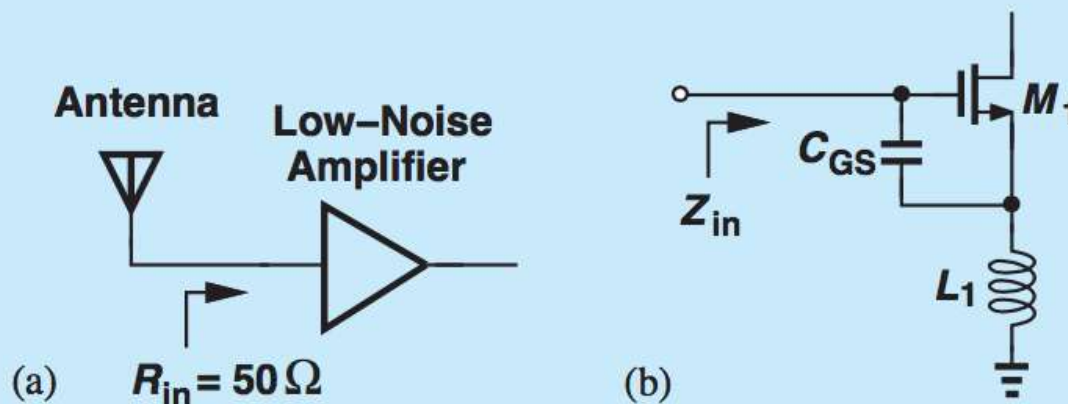
$$Z_{in} \approx \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

## Did you know?

Most RF receivers incorporate a common-source or common-emitter amplifier at their front end. This “low-noise” amplifier must present an input resistance of  $50\ \Omega$  so as to “match” the impedance of the antenna [Fig. (a)]. But how could a CS stage have such a low input resistance? A clever technique is to add an inductor in series with the source of the transistor [Fig. (b)]. It can be shown that the input impedance is given by

$$Z_{in}(s) = \frac{1}{C_{GS}s} + L_1s + \frac{L_1g_m}{C_{GS}}.$$

Note that the last term is a real quantity, representing a resistance. Proper choice of  $L_1$ ,  $g_m$ , and  $C_{GS}$  provides a value of  $50\ \Omega$ . Next time you turn on your cell phone or your GPS, you may be receiving an RF signal through an inductively-degenerated CS amplifier.



Input impedance matching in a receiver.

About 13,100 results (0.12 sec)

## Noise optimization of an inductively degenerated CMOS low noi

P Andreani, H Sjolund - ... Transactions on Circuits and Systems II ..., 2001 - ieeexplor

This paper presents a technique for substantially reducing the **noise** of a **CMOS low no amplifier** implemented in the **inductive source degeneration** topology. The effects of tl induced current **noise** on the **noise** performance are taken into account, and the total o

☆ 77 Cited by 310 Related articles All 6 versions

## A 1.5-V, 1.5-GHz CMOS low noise amplifier

DK Shaeffer, TH Lee - IEEE Journal of solid-state circuits, 1997 - ieeexplore.ieee.org

... The interconnect can be routed in a metal layer that possesses significantly **lower sh** and hence is ... to that of base resistance in bipolar devices, the gate resistanc significant in ... SHAEFFER AND LEE: 1.5-V, 1.5-GHz **CMOS LOW NOISE AM**

☆ 77 Cited by 1998 Related articles All 23 versions

## An ultrawideband CMOS low-noise amplifier for 3.1-10.6-G receivers

A Bevilacqua, AM Niknejad - IEEE Journal of solid-state circuits, 2004 - ieeexplor

... BEVILACQUA AND NIKNEJAD: AN ULTRAWIDEBAND **CMOS** LNA FOR 3.1-10.6-G WIRELESS RECEIVERS 2265 ... The maximum observed spread of the param 1 dB ... from the use of a ladder-filter input network, a structure well known for i

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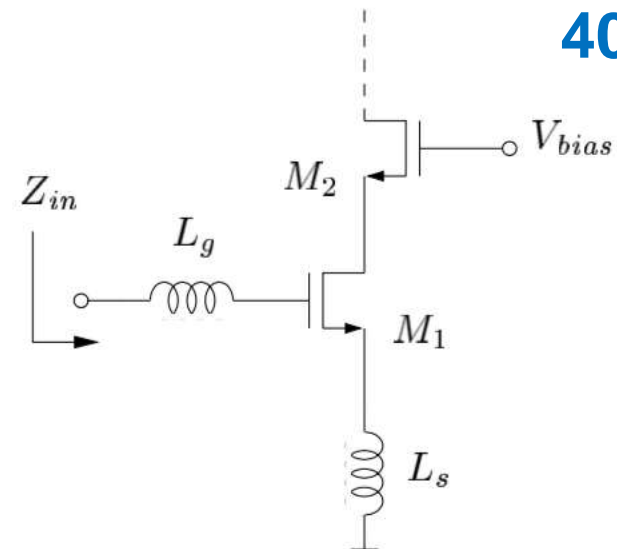
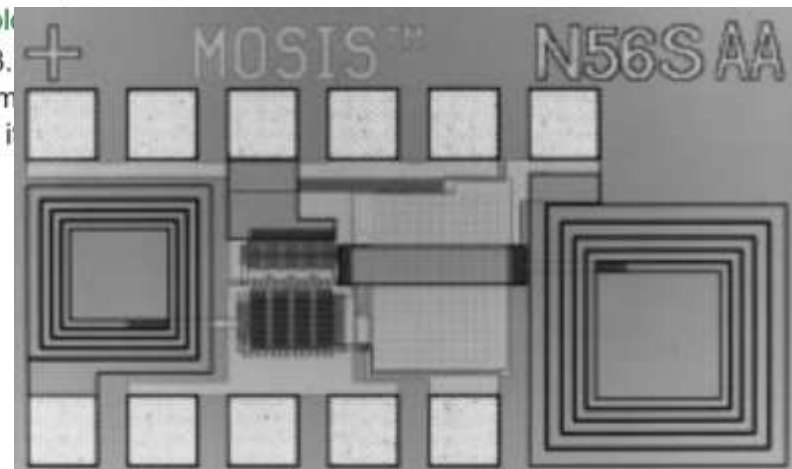


Fig. 3. Common-source input stage.

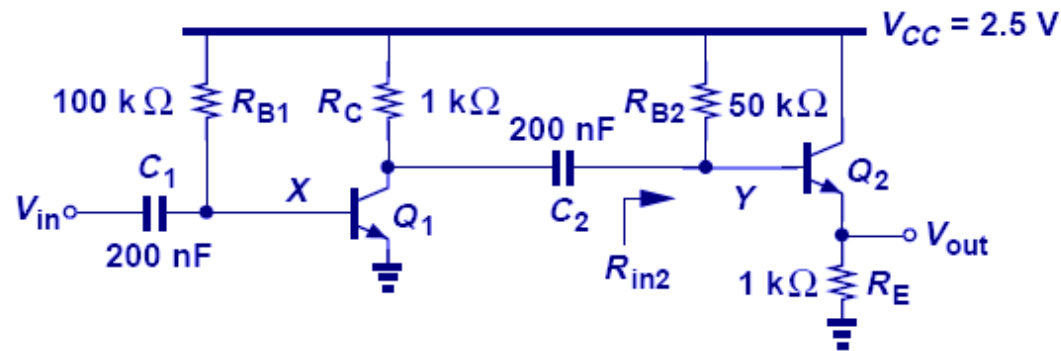
$$Z_{in} = s(L_s + L_g) + \frac{1}{sC_{gs}} + \left( \frac{g_{m1}}{C_{gs}} \right) L_s$$

$$\approx \omega_T L_s \quad (\text{at resonance})$$

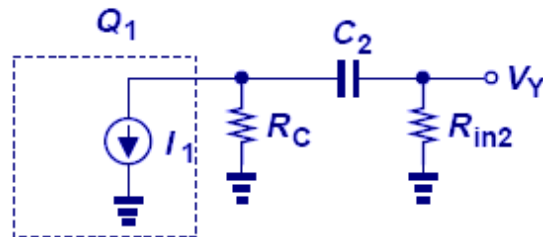




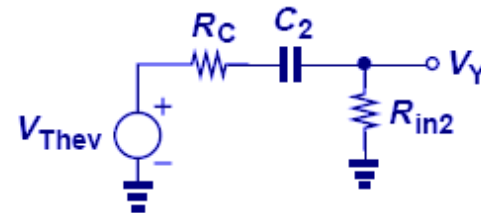
# 综合实例1: Capacitive Coupling



(a)



(b)



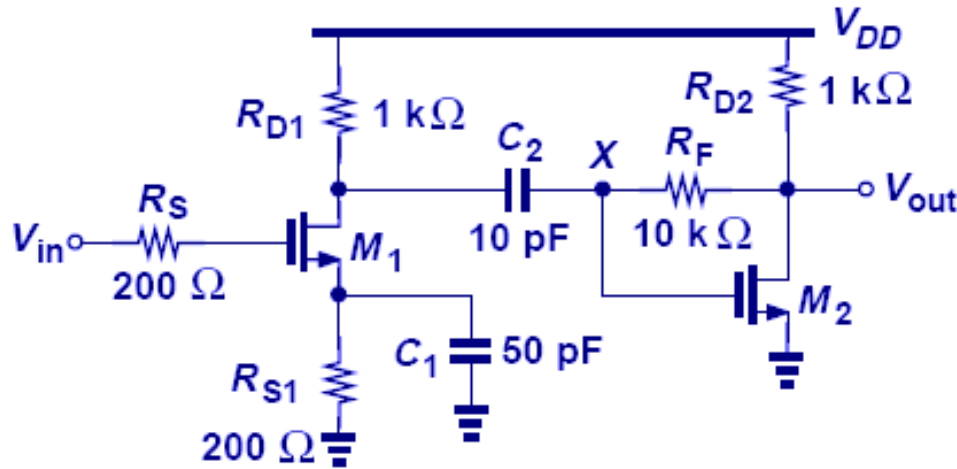
(c)

$$R_{in2} = R_{B2} \parallel [r_{\pi2} + (\beta + 1)R_E]$$

$$\omega_{L1} = \frac{1}{(r_{\pi1} \parallel R_{B1})C_1} = 2\pi \times (542\text{Hz})$$

$$\omega_{L2} = \frac{1}{(R_C + R_{in2})C_2} = \pi \times (22.9\text{Hz})$$

## 综合实例2: ① Low Frequency Design

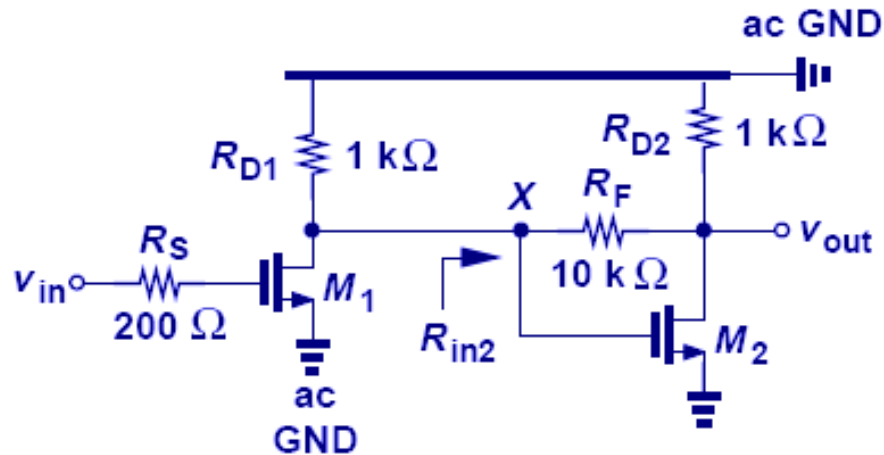


$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

$$\omega_{L1} = \frac{g_{m1}R_{S1} + 1}{R_{S1}C_1} = 2\pi \times (42.4 \text{ MHz})$$

$$\omega_{L2} = \frac{1}{(R_{D1} + R_{in2})C_2} = 2\pi \times (6.92 \text{ MHz})$$

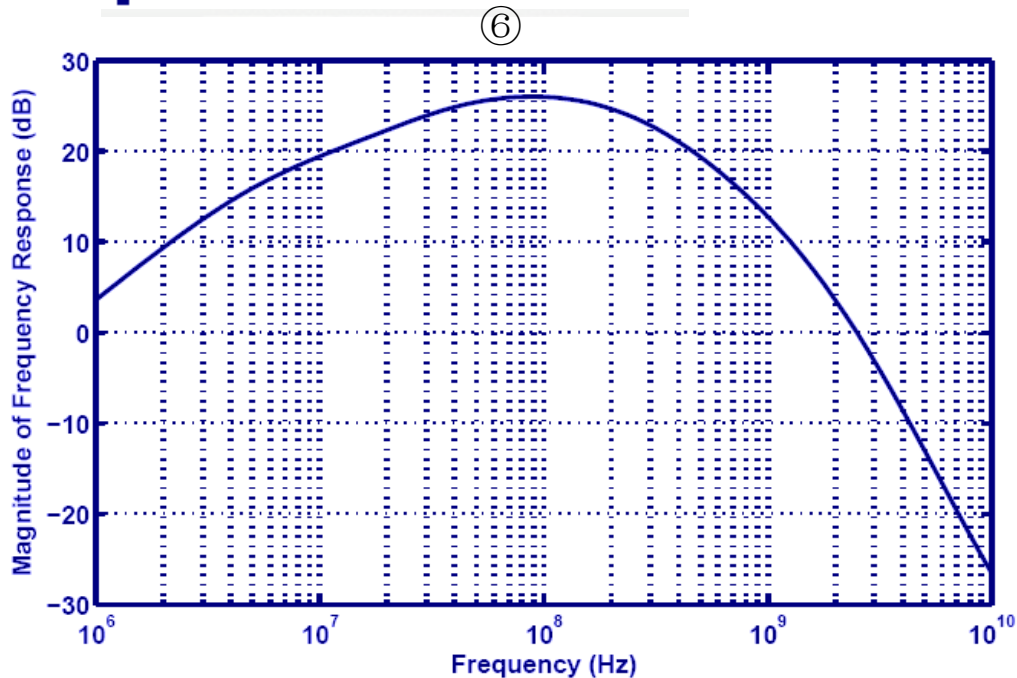
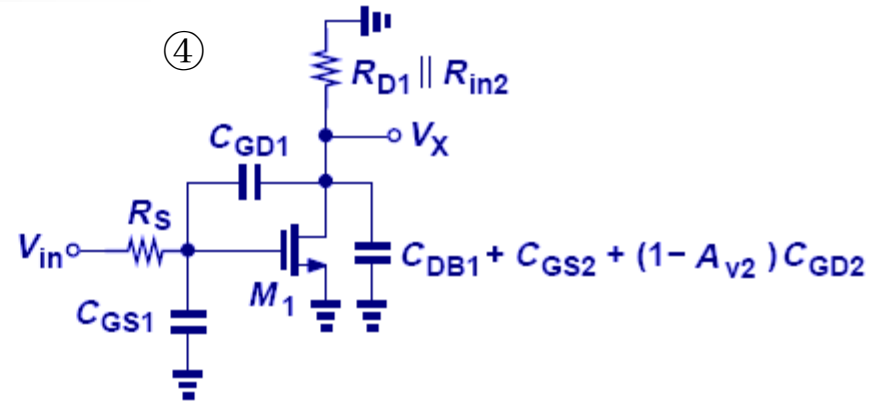
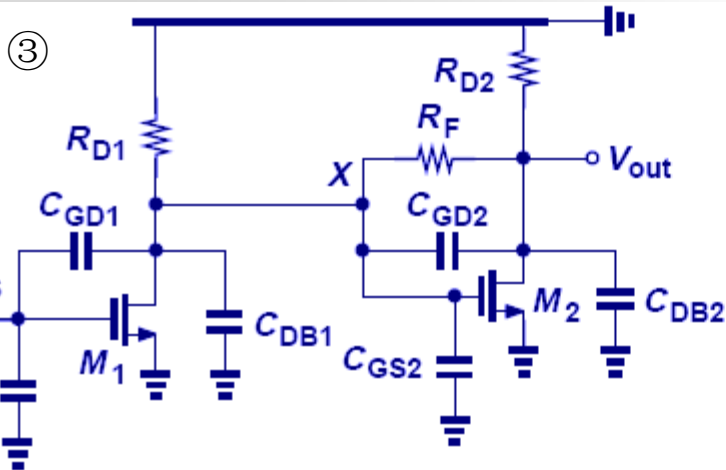
## 综合实例: ② Midband Design



$$\frac{v_X}{v_{in}} = -g_{m1}(R_{D1} \parallel R_{in2}) = -3.77$$

The voltage gain from node  $X$  to the output is approximately equal to  $-g_{m2}R_{D2}$

# 综合实例：③~⑥ High Frequency Design



⑤ 第一级放大器贡献两个极点  
 $|\omega_{p1}| = 2\pi \times (242 \text{ MHz})$

$$|\omega_{p2}| = 2\pi \times (2.74 \text{ GHz}).$$

第二级放大器贡献一个极点

$$|\omega_{p3}| = \frac{1}{R_{L2}(1.15C_{GD2} + C_{DB2})}$$

$$= 2\pi \times (0.829 \text{ GHz}).$$

# 放大器频率响应分析 小结

- Step 1: 观察电路中的电容，将它们按低频衰减和高频衰减分类
  - 隔直电容、旁路电容贡献低频极点；
  - 信号通路上到地的电容（管子寄生电容、负载电容）贡献高频极点；
  - 若有跨接电容，用Miller等效；
- Step 2: 低频特性（下限截止频率的计算）
  - 采用短路时间常数法，信号源置零；
- Step3: 高频特性（上限截止频率的计算）
  - 采用开路时间常数法，信号源置零， Miller等效
- Step4: 如有必要，根据Bode规则，勾勒出幅频特性
- 若要考虑零点，需写出传递函数表达式

$$\omega_L \simeq \sum_{i=1}^n \frac{1}{C_i R_i}$$

主极点RC最小

$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_i C_i R_i}$$

主极点RC最大

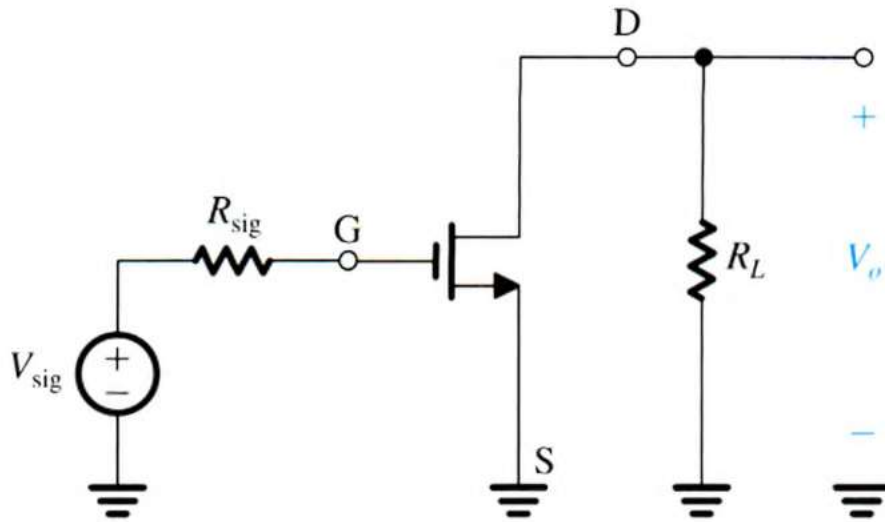
# 作业

**10.6** For the CS amplifier specified in Example 10.1, find the values of  $A_M$  and  $f_H$  that result when the load resistance is reduced to 10 k $\Omega$ .

**Ans.** -13.3 V/V; 86.9 MHz

## Example 10.1

Find the midband gain  $A_M$  and the upper 3-dB frequency  $f_H$  of an integrated-circuit CS amplifier fed with a signal source having an internal resistance  $R_{sig} = 20$  k $\Omega$ . The amplifier has  $R_L = 20$  k $\Omega$ ,  $g_m = 2$  mA/V,  $r_o = 20$  k $\Omega$ ,  $C_{gs} = 20$  fF, and  $C_{gd} = 5$  fF. Also, find the frequency of the transmission zero.



(a)

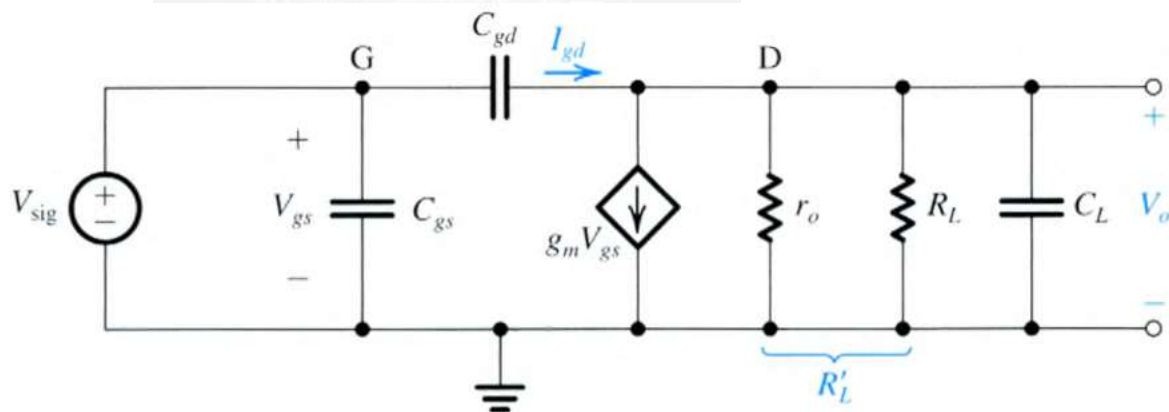


**10.8** For the CS amplifier considered in Example 10.2 operating at the original values of  $V_{OV}$  and  $I_D$ , find the value to which  $C_L$  should be increased to place  $f_t$  at 3 GHz.

**Ans.** 101 fF

### Example 10.2

Consider an IC CS amplifier fed with a source having  $R_{sig} = 0$  and having an effective load resistance  $R'_L$  composed of  $r_o$  of the amplifier transistor in parallel with an equal resistance  $r_o$  of the current-source load. Let  $g_m = 2$  mA/V,  $r_o = 20$  k $\Omega$ ,  $C_{gs} = 20$  fF,  $C_{gd} = 5$  fF, and  $C_L = 25$  fF. Find  $A_M$ ,  $f_H$ ,  $f_t$ , and  $f_z$ . If the amplifying transistor is to be operated at twice the original overdrive voltage while  $W$  and  $L$  remain unchanged, by what factor must the bias current be changed? What are the new values of  $A_M$ ,  $f_H$ ,  $f_t$ , and  $f_z$ ?

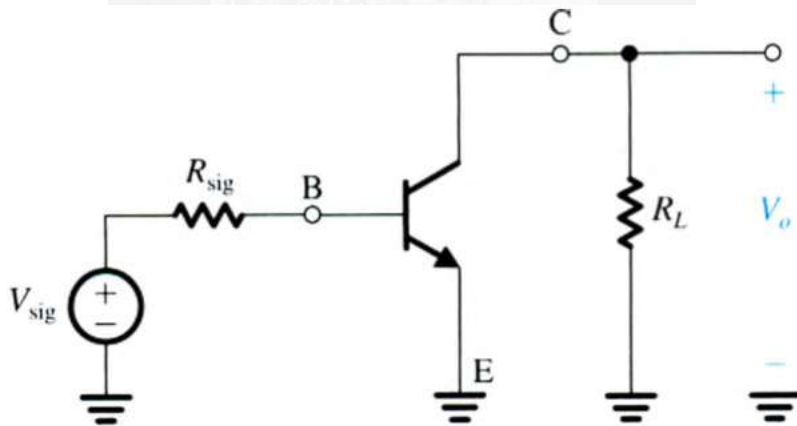


**10.9** For the amplifier in Example 10.3, find the value of  $R_L$  that reduces the midband gain to half the value found. What value of  $f_H$  results? Note the trade-off between gain and bandwidth.

**Ans.** 2.44 k $\Omega$ ; 923 kHz

### Example 10.3

Find the midband gain and the upper 3-dB frequency of the common-emitter amplifier of Fig. 10.13(a) for the following case:  $I_E = 1$  mA,  $R_{\text{sig}} = 5$  k $\Omega$ ,  $R_L = 5$  k $\Omega$ ,  $\beta_0 = 100$ ,  $V_A = 100$  V,  $C_\mu = 1$  pF, and  $f_T = 800$  MHz. Also, determine the frequency of the transmission zero.



**10.14** Consider a bipolar active-loaded CE amplifier having the load current source implemented with a *pnp* transistor. Let the circuit be operating at a 1-mA bias current. The transistors are specified as follows:  $\beta(npn) = 200$ ,  $V_{An} = 130$  V,  $|V_{Ap}| = 50$  V,  $C_\pi = 16$  pF,  $C_\mu = 0.3$  pF, and  $C_L = 5$  pF. The amplifier is fed with a signal source having a resistance of 36 k $\Omega$ . Determine: (a)  $A_M$ ; (b)  $C_{in}$  and  $f_H$  using the Miller effect; (c)  $f_H$  using open-circuit time constants; (d)  $f_z$ ; and (e) the gain–bandwidth product.

**Ans.** (a)  $-176$  V/V; (b) 450 pF, 80.6 kHz; (c) 73.5 kHz; (d) 21.2 GHz; (e) 12.9 MHz

