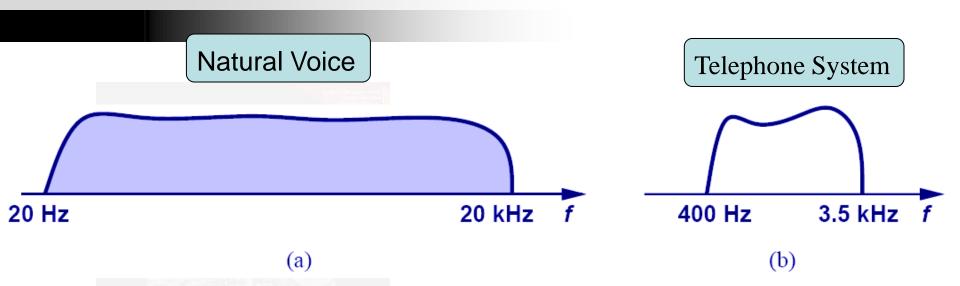
Lecture 23 – Frequency Response, Part I

Microelectronic Circuits

课程纲要

- 11.1 传输函数和波特图
- 11.1.1 放大器传输函数的定义
- 11.1.2 幅度和相位波特图的绘制
- 11.2 放大器频率响应分析
- 11.2.1 共射和共源放大器低频响应
- 11.2.2 共射和共源放大器高频响应(包括晶体管小信号高频等效模型、密勒定理)

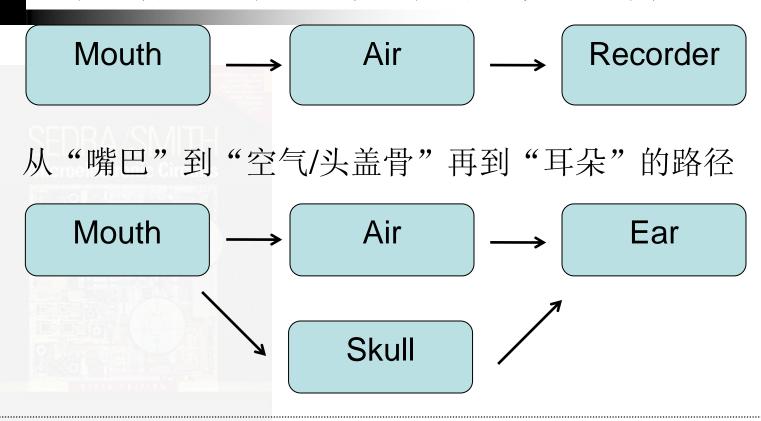
举例:人类的声音信号 |



- 人自然的声音频率在 20Hz 到 20kHz 之间, 但是电话系统的频率通常是 400Hz 到 3.5kHz.
- 因此,在电话里听到的声音往往与实际面对面交谈时 听到的声音不一致;

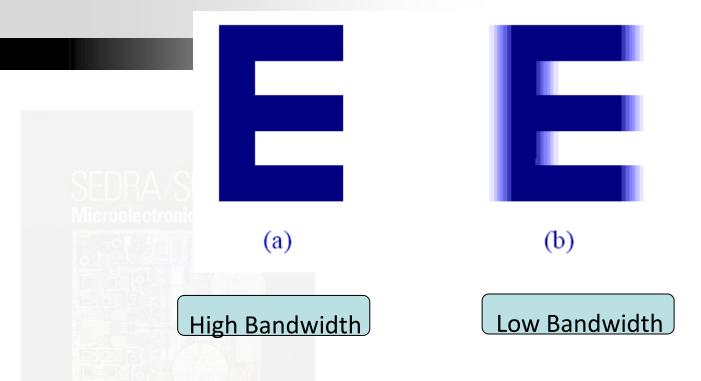
举例:人类的声音信号Ⅱ

从"嘴巴"到"空气"到"录音机"的路径



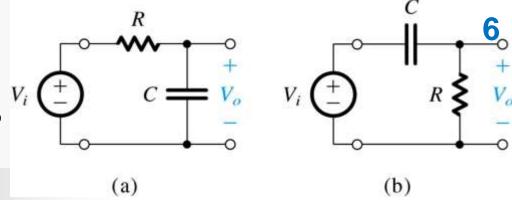
▶ 因为信号路径不一样(不同路径具有不同的频率特性),所以最终得到的信号也不一样。有没有听自己的录音觉得很奇怪的经历?

举例:视频信号



■ 若传输视频信号的路径带宽不够大 → 输出的信号会变 模糊 (信号不能快速从0变到1,或从1变到0)

1.6.4. Single Time-Constant Networks



- single time-constant (STC) network is composed of (or may be reduced to) one reactive component and one resistance.
 - (a) low pass filter attenuates output at high frequencies, allow low frequencies to pass
 - (b) high pass filter attenuates output at low frequencies, allow high frequencies to pass
- time constant (τ) describes the length of time required for a network transient to settle from step change $(\tau = L / R = RC)$

S (复频率)域分析,极 点、零点和波特图

- 分析放大器的频率响应时,主要工作之一是分析放大器的电压 增益在s域的表达式;
 - 电容 → 导纳为 sC; 电感 → 阻抗为 sL;
 - 分析出电压增益(电压传输函数)的表达式 $T(s) \equiv V_o(s)/V_i(s)$
 - F.1 Find the voltage transfer function $T(s) \equiv V_o(s)/V_i(s)$ for the STC network shown in Fig. EF.1.

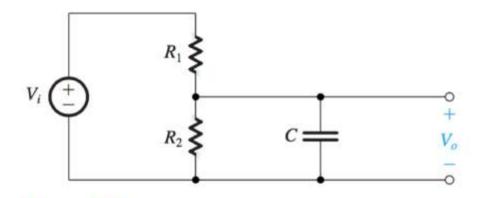


Figure EF.1

Ans.
$$T(s) = \frac{1/CR_1}{s + 1/C(R_1//R_2)}$$

S域分析,极点、零点 和波特图

- s域 → 物理频率域: replacing s by ja.
- $T(s) \to T(j\omega)$
 - T(jω) 的幅度: 幅频特性;
 - *T(jω*) 的角度(相位): 相频特性;
 - 波特图,即画出放大器的上述幅频特性和相频特性
- 在许多情况下,也可以不转变到物理频率域,而直接在s域中进行分析 $T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{s^n + b_{n-1} s^{n-1} + \dots + b_0}$
 - 其中系数a、b是实数;分子的阶数m≤分母的阶数n;n称为传输函数的阶数;
 - 对于一个稳定电路(自己不产生信号),分母多项式的所有根都具有负的实部

■ 一阶传输函数(n=1)
$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

■ 低通STC:
$$T(s) = \frac{a_0}{s + \omega_0}$$

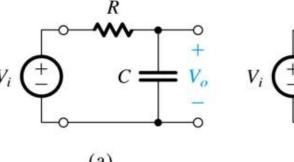
分子无一次项

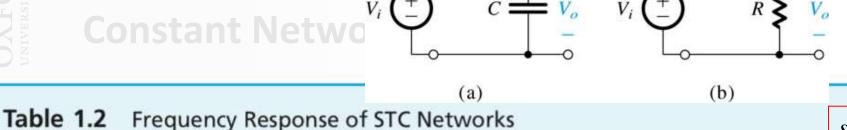
In this case the dc gain is a_0/ω_0 , and ω_0 is the corner or 3-dB frequency

■ 高通STC:
$$T(s) = \frac{a_1 s}{s + \omega_0}$$

的模为DC时的√2 倍

(回顾) 1.6.4. Single Tin Constant Netwo

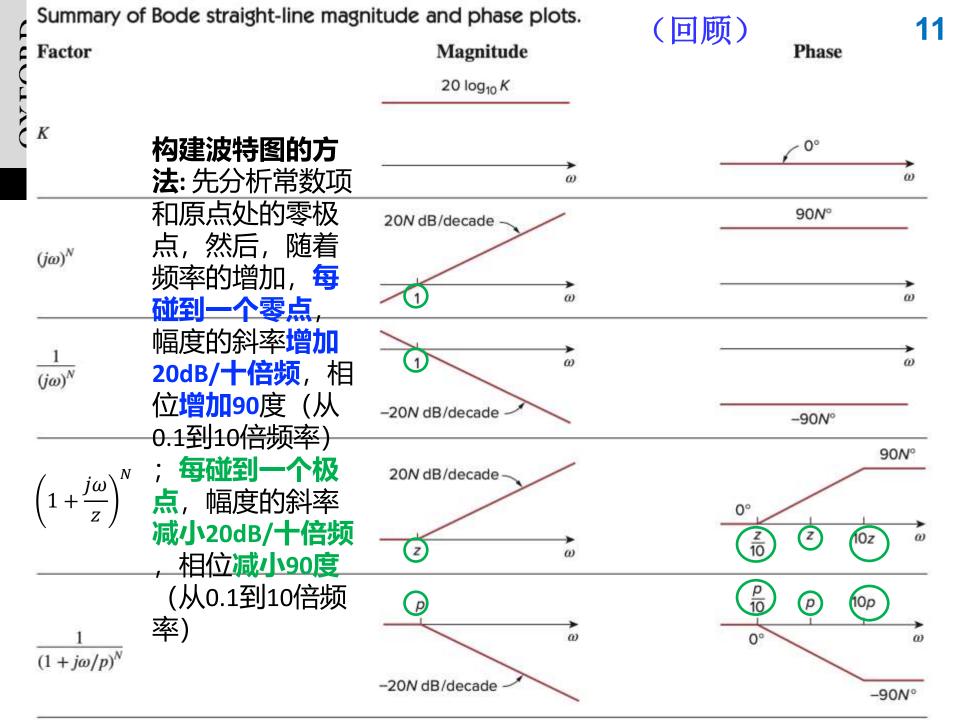


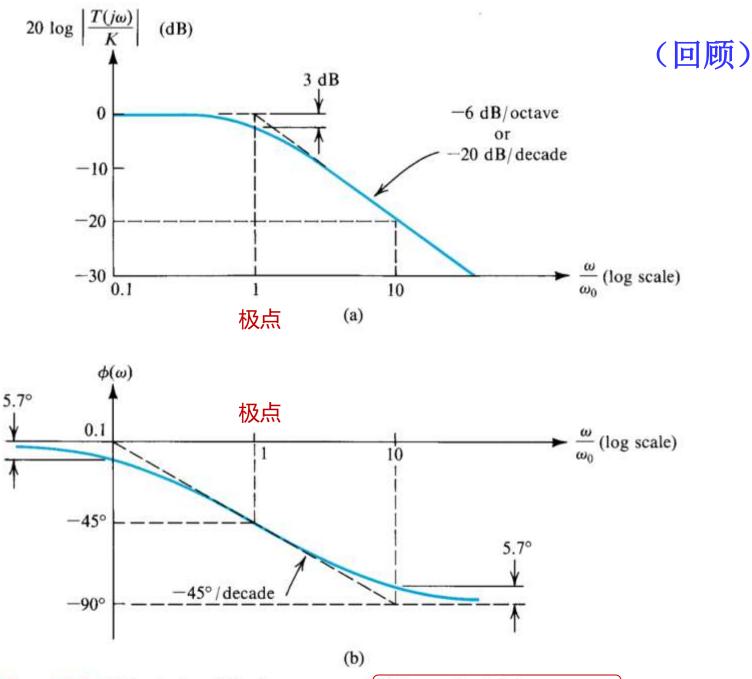


$S = J\omega$		
	低通STC Low-Pass (LP)	高通STCHigh-Pass (HP)
Transfer Function $T(s)$ 传递函数 标准形式	$\frac{K}{1 + (s/\omega_0)} \frac{1}{j\omega C}$	$\frac{Ks}{s+\omega_0} \qquad \frac{R}{R+\frac{1}{s+\omega_0}}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1+j(\omega/\omega_0)} = R + \frac{1}{j\omega C}$	$\frac{K}{1 - j(\underline{\omega_0/\omega})} j\omega C$
Magnitude Response $ T(j\omega) $	$\frac{ K }{\sqrt{1+(\omega/\omega_0)^2}}$	$\frac{ K }{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv \text{time constant}$	

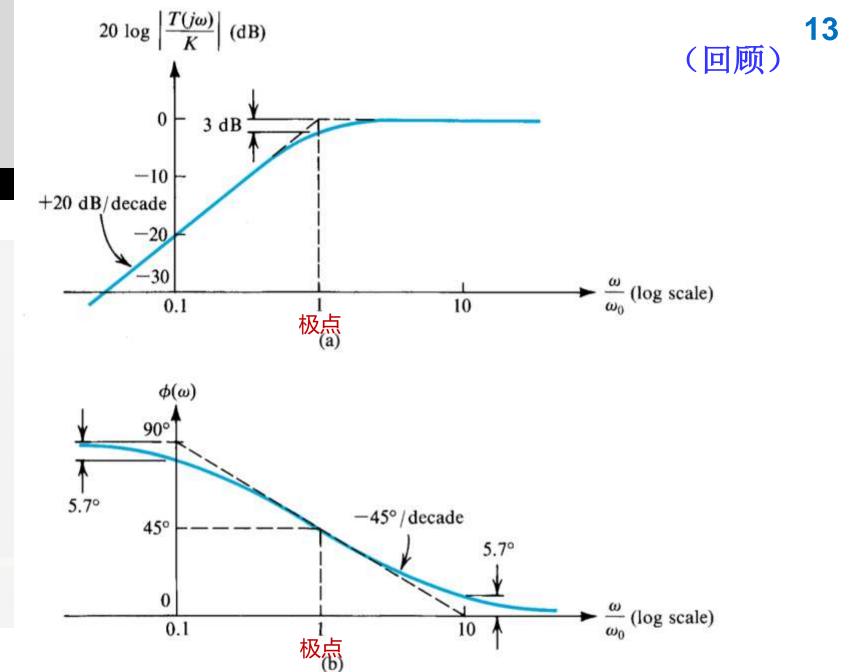
in Fig. 1.23 **Bode Plots**

 $\tau = CR \text{ or } L/R$ in Fig. 1.24





Microelectronic Figure 1.23 (a) Magnitude and (b) phase response of STC networks of the low-pass type.



Microelectronic Figure 1.24 (a) Magnitude and (b) phase response of STC networks of the high-pass type.

分量分立电路(由"分立/离散"元件组成的电路)的频率响应曲线 14

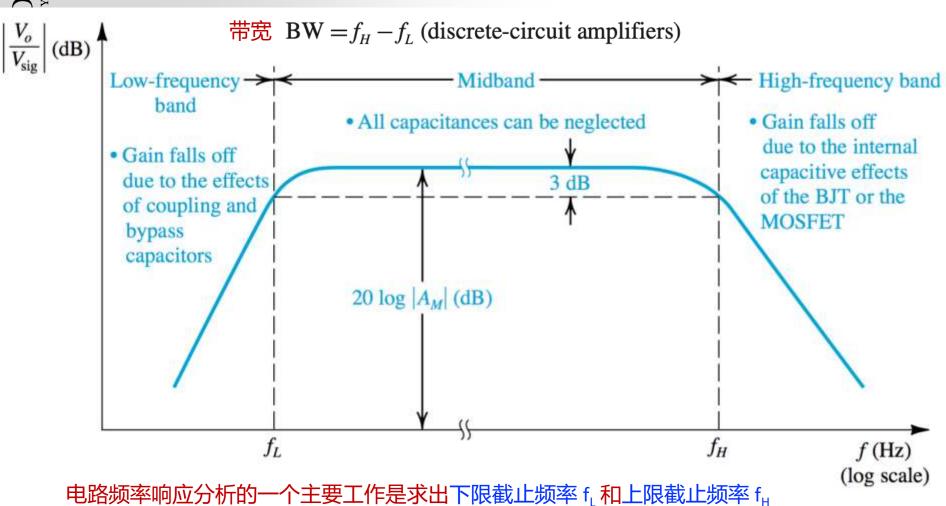


Figure 9.1 Sketch of the magnitude of the gain of a discrete-circuit BJT or MOS amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

对于IC而言,无低频衰减,因为IC无耦合电容和旁路电容

集成电路 (IC) 的频率响应曲线

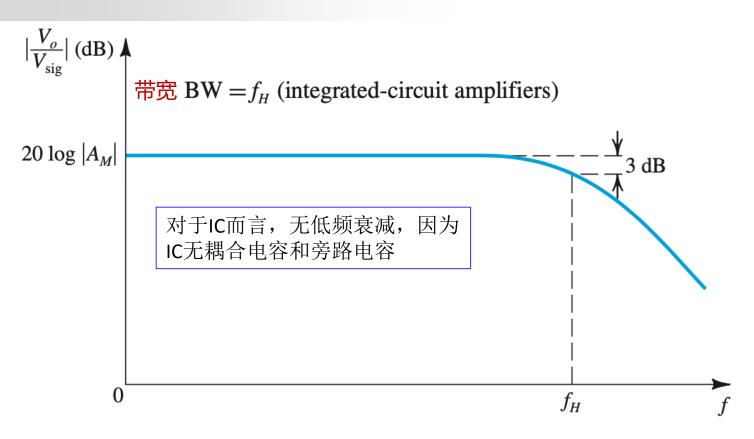
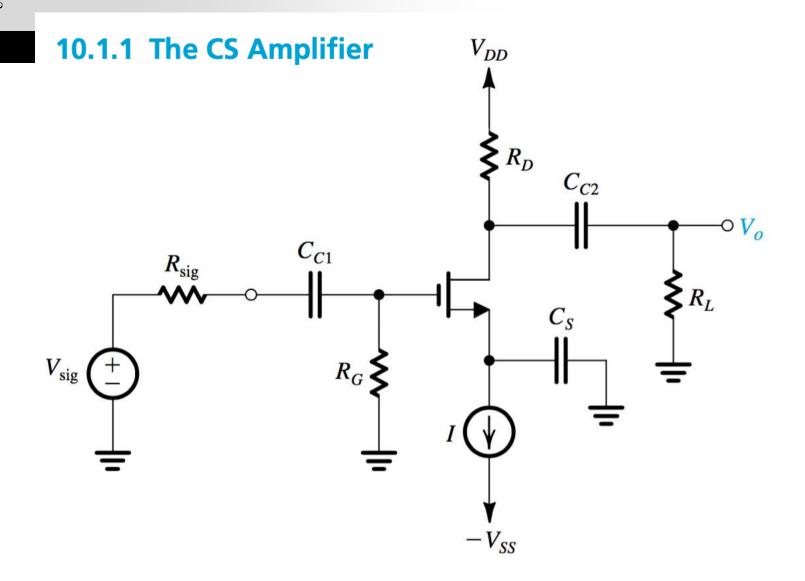
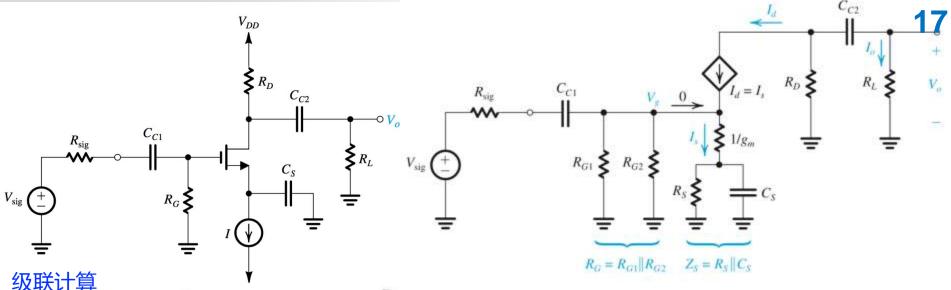


Figure 10.2 Frequency response of a direct-coupled (dc) amplifier. Observe that the gain does *not* fall off at low frequencies, and the midband gain A_M extends down to zero frequency.

A figure of merit for the amplifier is its **gain-bandwidth product**, $GB = |A_M| BW$

放大器电路低频响应





$$V_g = V_{\text{sig}} \frac{R_G}{R_G + \frac{1}{sC_{C1}} + R_{\text{sig}}} = V_{\text{sig}} \frac{R_G}{R_G + R_{\text{sig}}} \frac{s}{s + \frac{1}{C_{C1}(R_G + R_{\text{sig}})}} \Longrightarrow \omega_{P1} = \omega_0 = \frac{1}{C_{C1}(R_G + R_{\text{sig}})}$$

$$g_m$$
 $s + \frac{1}{C_S}$ $\omega_z = \frac{1}{C_S R_S}$ $\omega_{P2} \gg \omega_z$ 写出电压增益的传递函数,分析零极点 $I_o = -I_d \frac{R_D}{R_D + \frac{1}{sC_{C2}} + R_L}$ $V_o = I_o R_L = -I_d \frac{R_D R_L}{R_D + R_L}$ $s + \frac{1}{C_{C2}(R_D + R_L)}$ $\omega_{P3} = \frac{1}{C_{C2}(R_D + R_L)}$ ③ C_{C2} 引起一个极点

$$\frac{V_o}{V_{\text{sig}}} = -\frac{R_G}{R_C + R_{\text{sig}}} g_m(R_D \| R_L) \left(\frac{s}{s + \omega_{\text{Pl}}} \right) \left(\frac{s + \omega_Z}{s + \omega_{\text{Pl}}} \right) \left(\frac{s}{s + \omega_{\text{Pl}}} \right)$$
三个电容总共引起三个极点

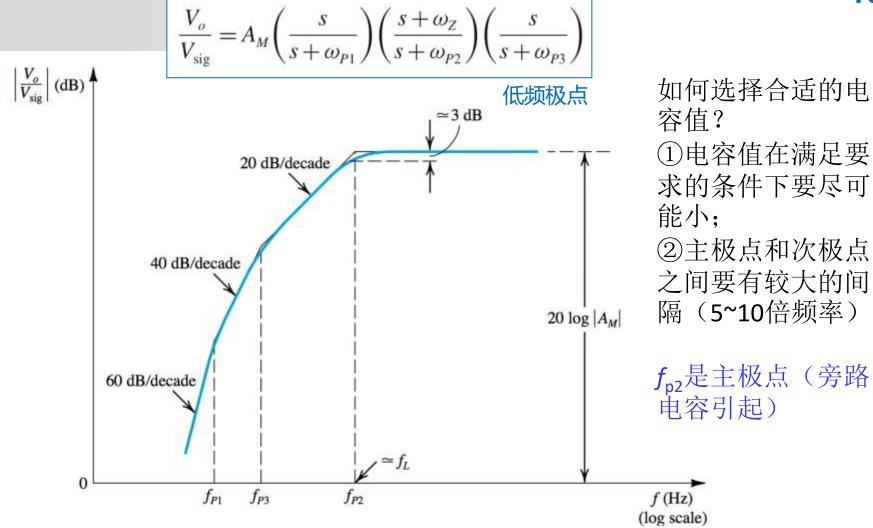


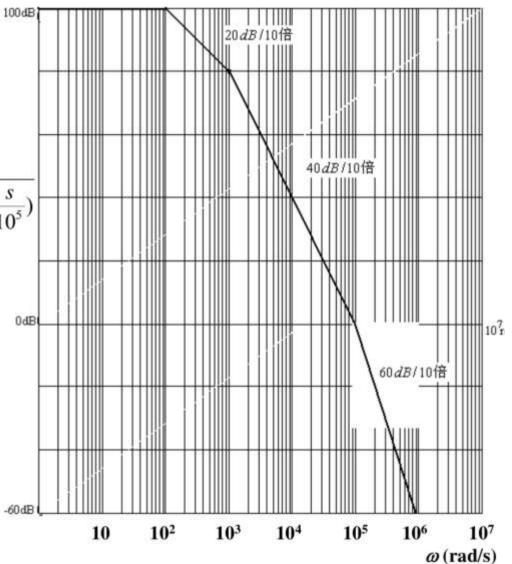
Figure 9.3 Sketch of the low-frequency magnitude response of a CS amplifier for which the three pole frequencies are sufficiently separated for their effects to appear distinct.

当电路存在主极点时,即某一个极点频率远高于其他极点时,该主极点频率就是电路的下限截止频率 f_i

传递函数←→波特图

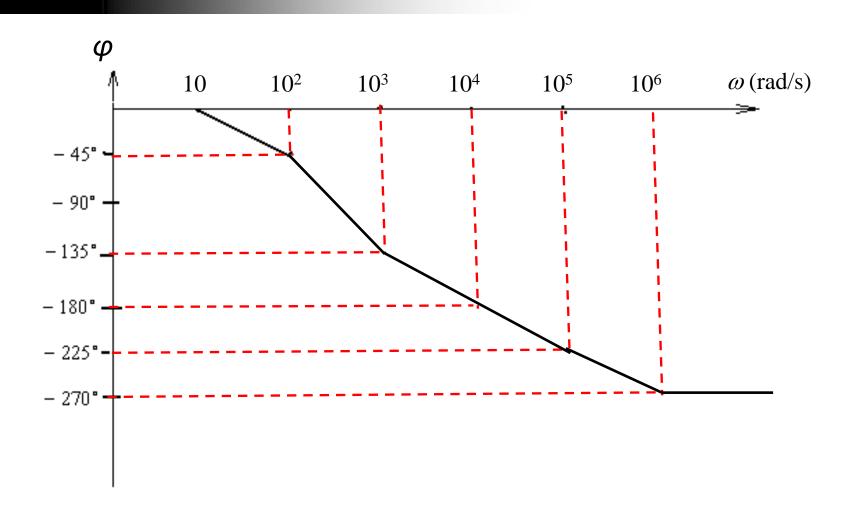
高频极点(低通滤波结构) **幅度波特图**

 $T(s) = \frac{10^{5}}{(1 + \frac{s}{10^{2}})(1 + \frac{s}{10^{3}})(1 + \frac{s}{10^{5}})}$



$$T(s) = \frac{10^5}{(1 + \frac{s}{10^2})(1 + \frac{s}{10^3})(1 + \frac{s}{10^5})}$$

高频极点



We wish to select appropriate values for the coupling capacitors C_{C1} and C_{C2} and the bypass capacitor C_S for a CS amplifier for which $R_G = 4.7$ M Ω , $R_D = R_L = 15$ k Ω , $R_{\rm sig} = 100$ k Ω , $R_S = 10$ k Ω , and $g_m = 1$ mA/V. We are required to have f_L at 100 Hz and the nearest break frequency at least a decade lower.

Solution

We select C_s so that

$$f_{p_2} = \frac{g_m + \frac{1}{R_s}}{2\pi C_s} = f_L$$
 ①主极点由旁路电容引起

Thus,

$$C_s = \frac{1.1 \times 10^{-3}}{2\pi \times 100} = 1.75 \,\mu\text{F}$$

For $f_{p_1} = f_{p_3} = 10$ Hz, we obtain

$$10 = \frac{1}{2\pi C_{c1}(0.1 + 4.7) \times 10^6}$$
 ②其他两个极点频率远低于主极点

which yields

$$C_{C1} = 3.3 \text{ nF}$$

and

$$10 = \frac{1}{2\pi C_{C2}(15+15) \times 10^3}$$

which results in

$$C_{C2} = 0.53 \, \mu \text{F}$$

Finally, we calculate the frequency of the zero f_z as

$$f_Z = \frac{1}{2\pi C_S R_S} = \frac{1}{2\pi \times 1.75 \times 10^{-6} \times 10 \times 10^3}$$

= 9.1 Hz

下限截止频率 f_L 的通 用近似计算方法

当电路不存在主极点时,可用"短路时间常数法"估算电路的下限截止频率f_l

10.1.2 The Method of Short-Circuit Time Constants

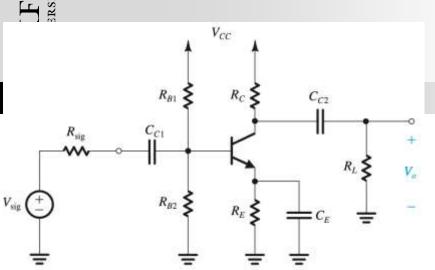
- 1. Set the input signal $V_{\text{sig}} = 0$. ①输入信号置零
- Consider the capacitors one at a time. That is, while considering capacitor C_i, set all the other capacitors to infinite values (i.e., replace them with short circuits—hence the name of the method).
 ②一次考虑一个电容,分析时将其他电容短路(理想化)
- 3. For each capacitor C_i , find the total resistance R_i seen by C_i . This can be determined either by inspection or by replacing C_i with a voltage source V_x and finding the current I_x drawn from V_x ; $R_i \equiv V_x/I_x$. ③求电容两端的等效电阻,并得到相应的时间常数RC
- 4. Calculate the 3-dB frequency f_L using

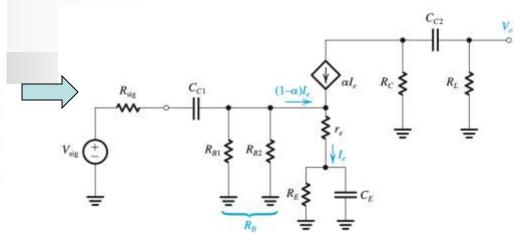
 (4) 求和计算下限截止频率 f_L

$$\omega_L \simeq \sum_{i=1}^n \frac{1}{C_i R_i} \tag{10.13}$$

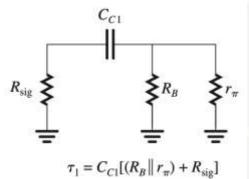
where n is the total number of capacitors.

10.1.3 The CE Amplifier





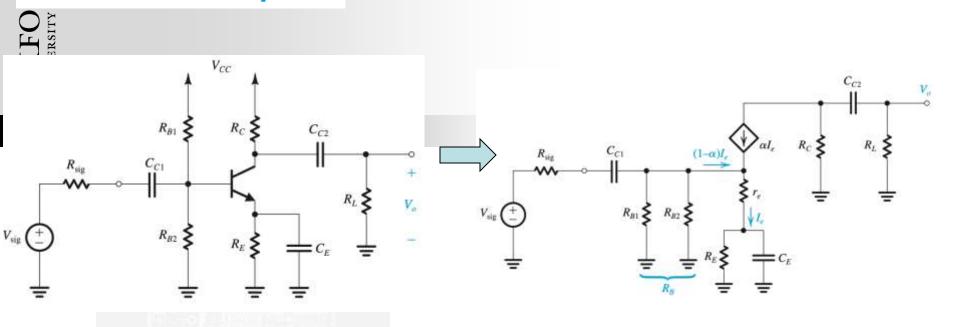
1)考虑C_{C1}



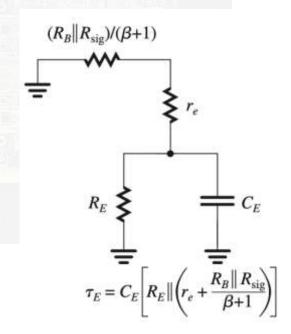
$$R_{C1} = (R_B \parallel r_\pi) + R_{\text{sig}}$$

$$\tau_{C1} = C_{C1}R_{C1}$$

10.1.3 The CE Amplifier



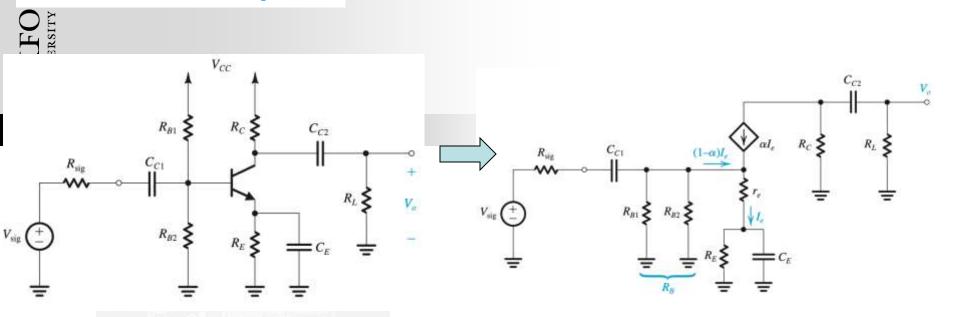
②考虑C_E



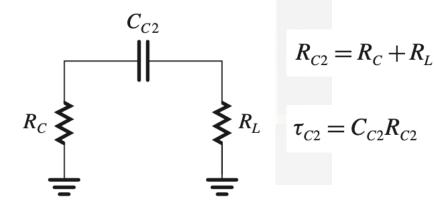
$$R_{CE} = R_E \| \left[r_e + \frac{R_B \| R_{\text{sig}}}{\beta + 1} \right]$$

$$\tau_{CE} = C_E R_{CE}$$

10.1.3 The CE Amplifier



③考虑C_{C2}



 $\tau_2 = C_{C2}(R_C + R_L)$

④求和,得到下限截止频率 f_

$$f_L = \frac{\omega_L}{2\pi} = \frac{1}{2\pi} \left[\frac{1}{C_{C1}R_{C1}} + \frac{1}{C_ER_E} + \frac{1}{C_{C2}R_{C2}} \right]$$

We wish to select appropriate values for C_{C1} , C_{C2} , and C_E for the common-emitter amplifier, which has $R_B = 100 \text{ k}\Omega$, $R_C = 8 \text{ k}\Omega$, $R_L = 5 \text{ k}\Omega$, $R_{\text{sig}} = 5 \text{ k}\Omega$, $R_E = 5 \text{ k}\Omega$, $\beta = 100$, $g_m = 40 \text{ mA/V}$, and $r_{\pi} = 2.5 \text{ k}\Omega$. It is required to have $f_L = 100 \text{ Hz}$.

Solution

We first determine the resistances seen by the three capacitors C_{C1} , C_{E} , and C_{C2} as follows:

$$R_{C1} = (R_B \| r_\pi) + R_{\text{sig}}$$

$$= (100 \| 2.5) + 5 = 7.44 \text{ k}\Omega$$

$$R_{CE} = R_E \| \left[r_e + \frac{R_B \| R_{\text{sig}}}{\beta + 1} \right]$$

$$= 5 \| \left(0.025 + \frac{100 \| 5}{101} \right) = 0.071 \text{ k}\Omega$$

$$R_{C2} = R_C + R_L = 8 + 5 = 13 \text{ k}\Omega$$

Now, selecting C_E so that it contributes 80% of the value of ω_L gives

$$\frac{1}{C_E \times 71} = 0.8 \times 2\pi \times 100$$
$$C_E = 28 \,\mu\text{F}$$

Next, if C_{C1} is to contribute 10% of f_L ,

$$\frac{1}{C_{C1} \times 7.44 \times 10^3} = 0.1 \times 2\pi \times 100$$

$$C_{C1} = 2.1 \,\mu\text{F}$$

Similarly, if C_{C2} is to contribute 10% of f_L , its value should be selected as follows:

$$\frac{1}{C_{C2} \times 13 \times 10^3} = 0.1 \times 2\pi \times 100$$
$$C_{C2} = 1.2 \,\mu\text{F}$$

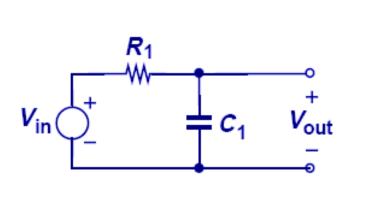
In practice, we would select the nearest standard values for the three capacitors while ensuring that $f_L \le 100$ Hz. Finally, the frequency of the zero introduced by C_E can be found,

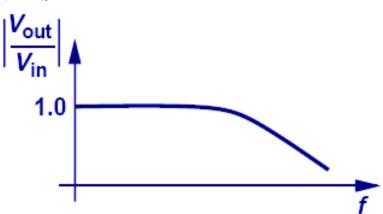
$$f_Z = \frac{1}{2\pi C_E R_E} = \frac{1}{2\pi \times 28 \times 10^{-6} \times 5 \times 10^3} = 1.1 \text{ Hz}$$

which is very far from f_L and thus has an insignificant effect.

放大器电路高频响应

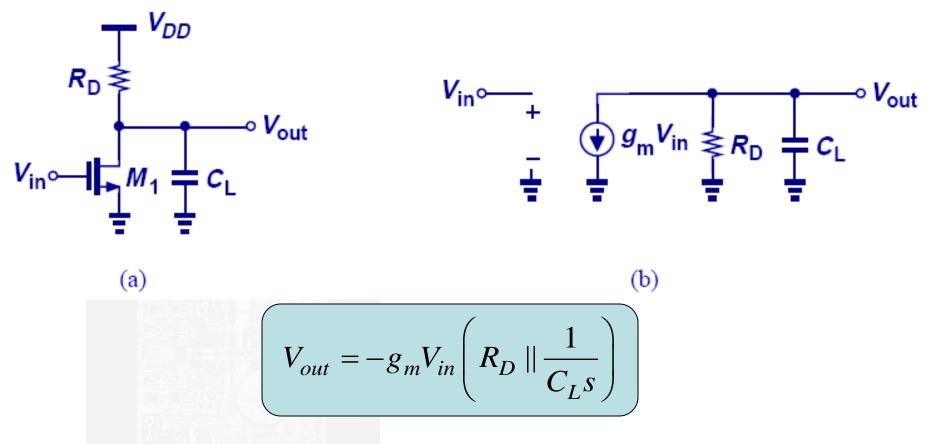
增益下降: 简单的低通滤波器





■ 频率增加 \rightarrow C_1 的阻抗下降 \rightarrow V_{out} 下降

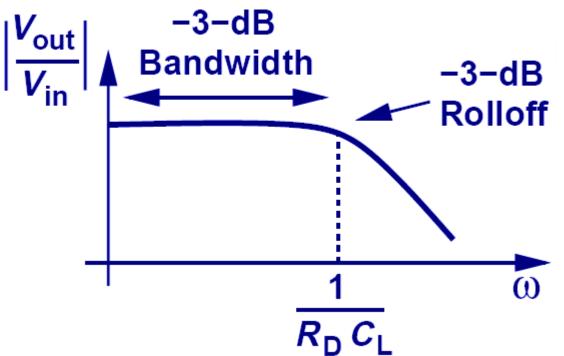
增益下降: Common Source



■ 频率增加 → 电容负载 C_L的阻抗降低 → "输出端到地的总阻抗"降低 → 放大器增益下降;



CS 结构的频率响应(IC)

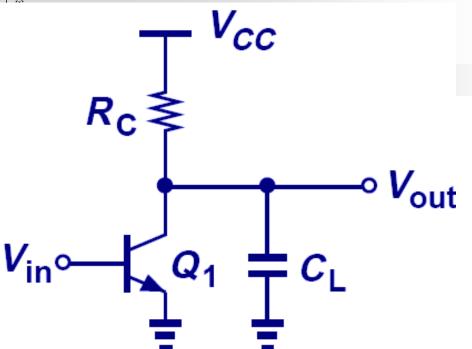


$$V_{out} = -g_m V_{in} \left(R_D \parallel \frac{1}{C_L s} \right)$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

- 低频时, 电容开路, 增益是平坦的;
- 频率增加时,电容阻抗逐渐下降,并趋于短路 → 增益 下降;
- 当 ω=1/(R_DC_L), 增益下降 3dB.

举例: Figure of Merit

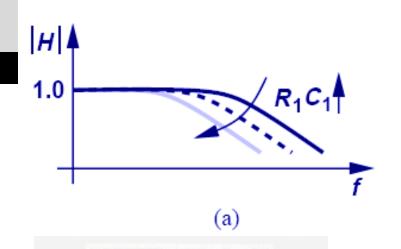


$$F.O.M. = \frac{1}{V_T V_{CC} C_L}$$

$$\frac{\text{Gain} \times \text{Bandwidth}}{\text{Power Consumption}} = \frac{\frac{I_C}{V_T} R_C \times \frac{1}{R_C C_L}}{I_C \cdot V_{CC}}$$
$$= \frac{1}{V_T \cdot V_{CC}} \frac{1}{C_L}.$$

- 一般情况下,我们希望放大器的"增益"要大、"带宽"要大、"功耗"要小,因此,将"增益"ד带宽"÷"功耗"定义成电路优值(Figure of Merit, FOM),FOM越大,放大器的性能越好;
- 对于CS(CE)结构,低温、低电压、低负载电容有利于提高电路FOM

Example: Relationship between Frequency Response and Step Response



$$V_{\text{out}}$$
 t
(b)

$$\left|H\left(s=j\omega\right)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0 \left(1 - \exp \frac{-t}{R_1 C_1} \right) u(t)$$

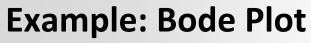
频域传输函数

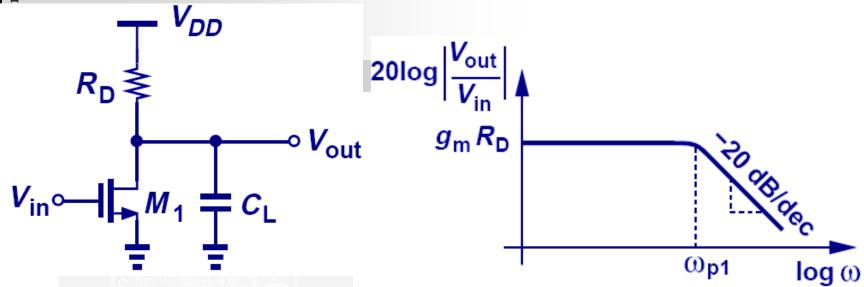
时域阶跃响应

Arr R_1C_1 增加 \rightarrow 带宽下降(频域) \rightarrow 阶跃响应变慢(时域)

$$H(s) = A_0 \frac{\left(1 + \frac{s}{\omega_{z1}}\right)\left(1 + \frac{s}{\omega_{z2}}\right)\cdots}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)\cdots}$$

- 频率从低到高,碰到零点 ω_{zi} ,幅度的斜率上升20 dB/dec
- 频率从低到高,碰到极点 ω_{pi} ,幅度的斜率下降20 dB/dec



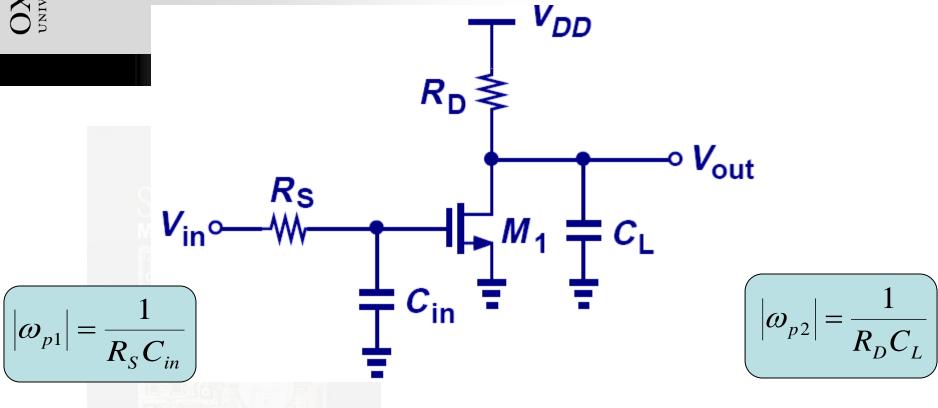


■ 该电路只有一个极点 $1/(R_DC_L)$,没有零点,因此经过 ω_{p1} 后,波特图的斜率从 0 下降到 -20dB/dec.

电路"结点"对高频 "极点"的贡献

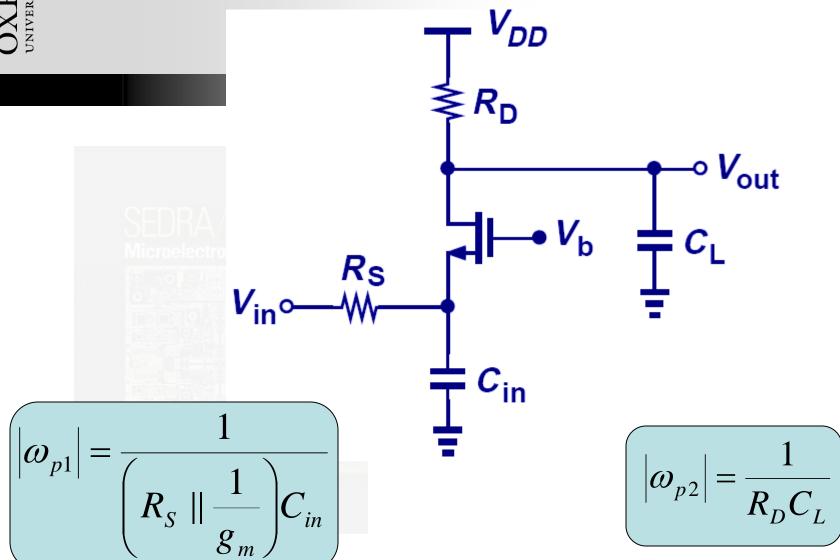
- 如果信号通路上的"结点"j有一个对地的小信号电阻 R_j ,同时又有一个对地的小信号电容 C_j ,那么,该结点将对传输函数贡献一个"高频极点"(R_iC_i)-1
- 该方法直观性较好,但精确性稍差,在电路的初步分析中较为有用,**考试时不要用!**

Pole Identification Example I

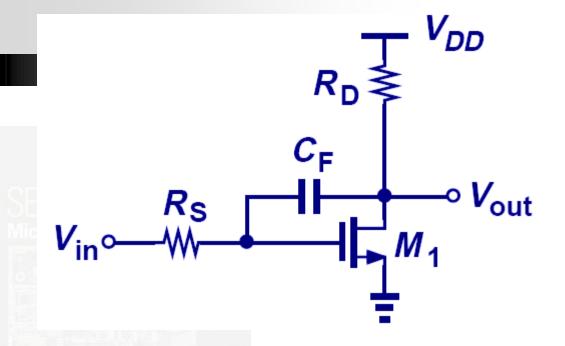


$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{\left(1 + \omega^2 / \omega_{p1}^2\right) \left(1 + \omega^2 / \omega_{p2}^2\right)}}$$

Pole Identification Example II



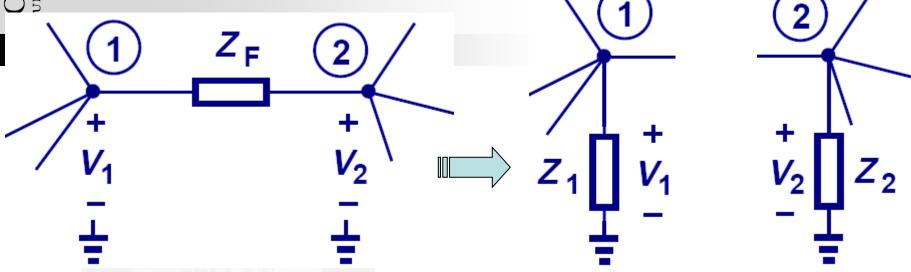
浮接电容怎么处理?



- 上述极点的原则是计算信号通路结点上对地的有效小信号电阻与电容;
- 但上图所示 C_F 两端均不接地,如何处理?

Miller's Theorem

密勒定理



$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

■ 如果结点①到结点②的增益为 A_v ,那么浮动阻抗 Z_F 可以等效成两个到地的阻抗 Z_1 和 Z_2 .

密勒定理的推导

$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1} \tag{11.19}$$

$$\frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}. (11.20)$$

Denoting the voltage gain from node 1 to node 2 by A_v , we obtain

$$Z_1 = Z_F \frac{V_1}{V_1 - V_2} \tag{11.21}$$

$$=\frac{Z_F}{1-A_v}\tag{11.22}$$

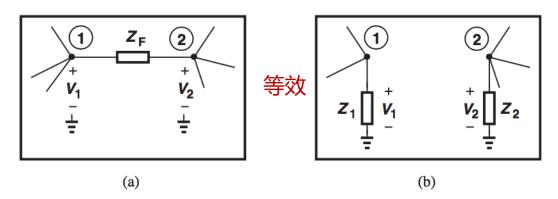
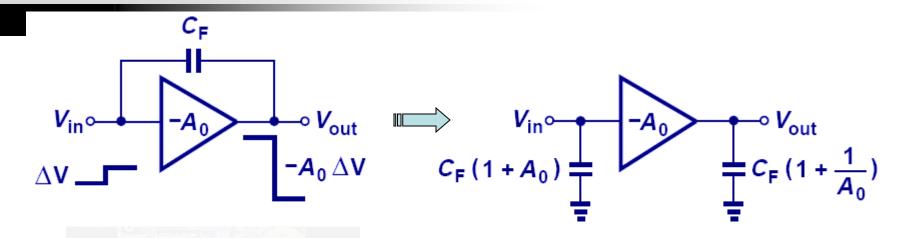


Figure 11.13 (a) General circuit including a floating impedance, (b) equivalent of (a) as obtained from Miller's theorem.

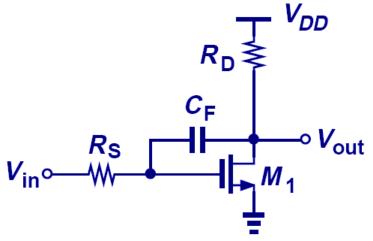
Miller Multiplication

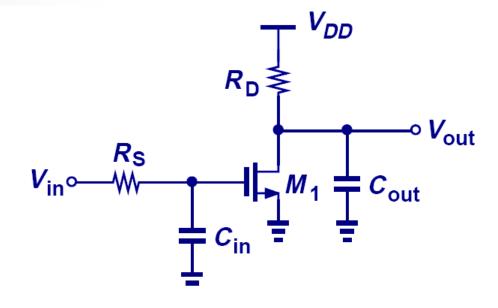


- 使用Miller定理,我们可以方便地将跨接在输入与输出 之间的浮动电容等效成两个分别到地的电容;
- 若放大器的增益为 $-A_0$ ($A_0>1$),那么等效到输入端的电容值为 C_F ($1+A_0$),相比原先的 C_F 放大到了($1+A_0$)倍,这一现象称之为 Miller multiplication.



Example: Miller Theorem





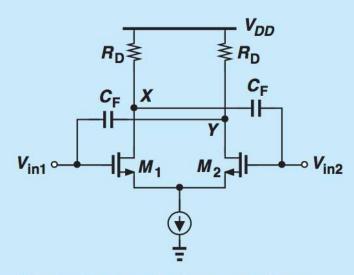
$$\omega_{in} = \frac{1}{R_S (1 + g_m R_D) C_F}$$

$$\omega_{out} = \frac{1}{R_D \left(1 + \frac{1}{g_m R_D}\right) C_F}$$

如果结点①到结点②之间的增益为"正"呢?

Did you know?

The Miller effect was discovered by John Miller in 1919. In his original paper, Miller observes that "the apparent input capacity can become a number of times greater than the actual capacities between the tube electrodes." (Back then, capacitance and capacitor were called "capacity" and "condensor," respectively.) A curious effect with respect to Miller multiplication is that, if the amplifier gain is positive and greater than unity, then we obtain a negative input capacitance, $(1-A_v)C_F$. Does this happen? Yes, indeed. Shown below is a circuit realizing a negative capacitance. Since the gain from V_{in1} to V_Y (and from V_{in2} to V_X) is positive, C_F can be multiplied by a negative number. This technique is used in many high-speed circuits to partially cancel the effect of undesired (positive) capacitances.



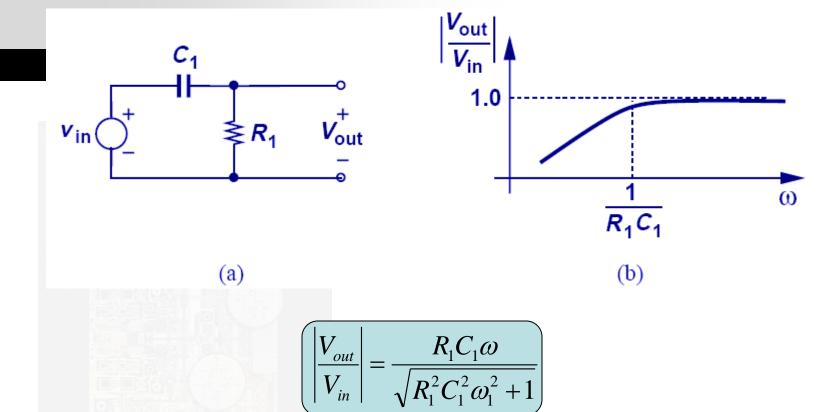
Circuit with negative input capacitance.

等效的负电容可以抵消部分与之并联的寄生电容

$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

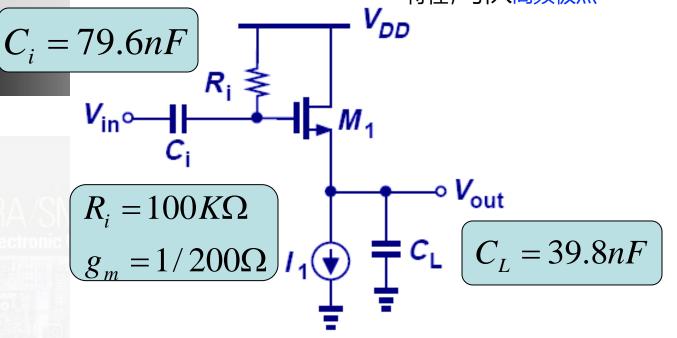
高通滤波器响应



- \triangleright 该传输函数有一个零点($\mathbf{0}$)和一个极点($\mathbf{1/R_1C_1}$);
- ▶ 直流时, C1开路, 传输函数为零; 过了零点后, 波特图的斜率增加20dB/dec; 过了极点后, 波特图的斜率减小20dB/dec;

Example: Audio Amplifier

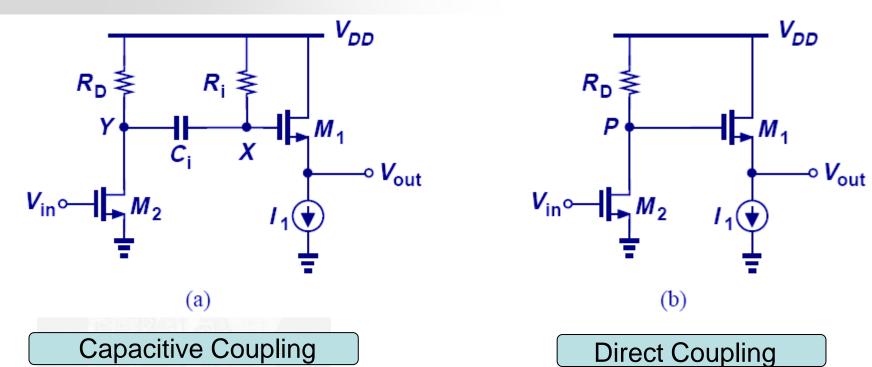
- 信号通路上<mark>串接</mark>的电容,具有<mark>高通</mark> 特性,引入低频极点
- 信号通路上到地的电容,具有低通 特性,引入高频极点



- ➤ 为了成功地传输声音信号 (20 Hz-20 KHz), 需要合适的输入输出 电容值;
- C_i是信号通路上的串接电容,所以其极点决定了下限截止频率 20Hz (1/R_iC_i=2pi*20Hz → C_i=79.6nF);
- \mathbf{C}_{L} 是信号通路上到地的电容,所以其极点决定了上限截止频率 **20kHz** $\omega_{p,out} = \frac{g_m}{C}$

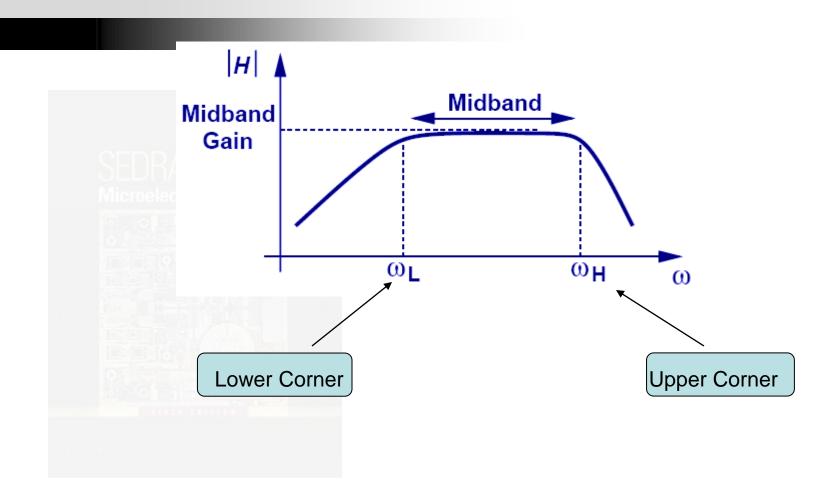
$$=2\pi\times(20\,\mathrm{kHz}),$$

电容耦合 vs. 直接耦合



- 电容耦合使 AC 信号能从 Y 到达 X, 但直流信号不能通过;
- 电容耦合使的M2和M1的直流偏置相互不影响,直接耦合不能实现这一功能

典型的频率响应

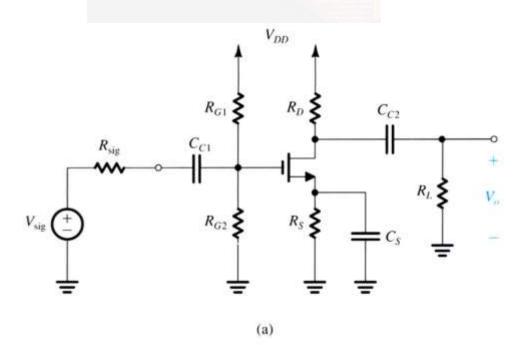


作业

$$R_G = R_{G1} \parallel R_{G2}$$

10.26 A CS amplifier has $C_{C1} = C_S = C_{C2} = 1 \,\mu\text{F}$, $R_G = 10 \,\text{M}\Omega$, $R_{\text{sig}} = 100 \,\text{k}\Omega$, $g_m = 2 \,\text{mA/V}$, $R_D = R_L = R_S = 10 \,\text{k}\Omega$. Find A_M , f_{P1} , f_{P2} , f_{P3} , f_Z , and f_L .

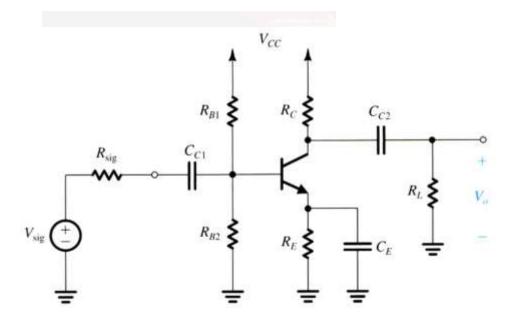
Ans. -9.9 V/V; 0.016 Hz; 334.2 Hz; 8 Hz; 15.91 Hz; 334.2 Hz



$$R_B = R_{B1} || R_{B2}$$

A common-emitter amplifier has $C_{C1} = C_E = C_{C2} = 1 \,\mu\text{F}$, $R_B = 100 \,\text{k}\Omega$, $R_{\text{sig}} = 5 \,\text{k}\Omega$, $g_m = 40 \,\text{mA/V}$, $r_\pi = 2.5 \,\text{k}\Omega$, $R_E = 5 \,\text{k}\Omega$, $R_C = 8 \,\text{k}\Omega$, and $R_L = 5 \,\text{k}\Omega$. Find the value of the time constant associated with each capacitor, and hence estimate the value of f_L . Also compute the frequency of the transmission zero introduced by C_E and comment on its effect on f_L .

Ans. $\tau_{C1} = 7.44$ ms; $\tau_{CE} = 0.071$ ms; $\tau_{C2} = 13$ ms; $f_L = 2.28$ kHz; $f_Z = 31.8$ Hz, which is much smaller than f_L and thus has a negligible effect on f_L





Example 10.4

Figure 10.15(a) shows an ideal voltage amplifier with a gain of -100 V/V and an impedance Z connected between its output and input terminals. Find the Miller equivalent circuit when Z is (a) a 1-M Ω resistance and (b) a 1-pF capacitance. In each case, use the equivalent circuit to determine V_o/V_{sig} .

