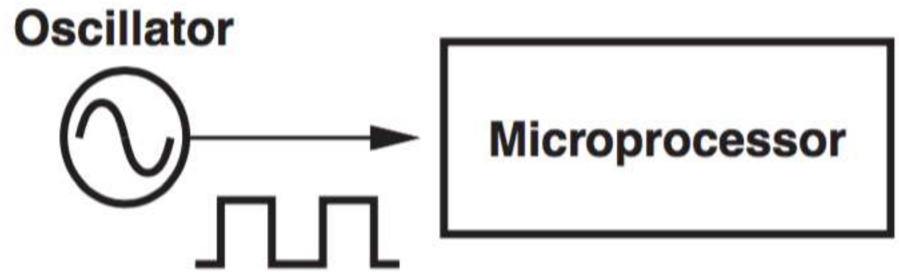
Lecture 28 正反馈和振荡电路

12.4 正反馈和振荡电路

- 12.4.1 振荡器的组成和原理
- 12.4.2 振荡器的平衡与起振条件
- 12.4.3 *LC*和*RC*振荡器（概念、原理）
- 12.4.4 石英晶体振荡器（概念、原理）

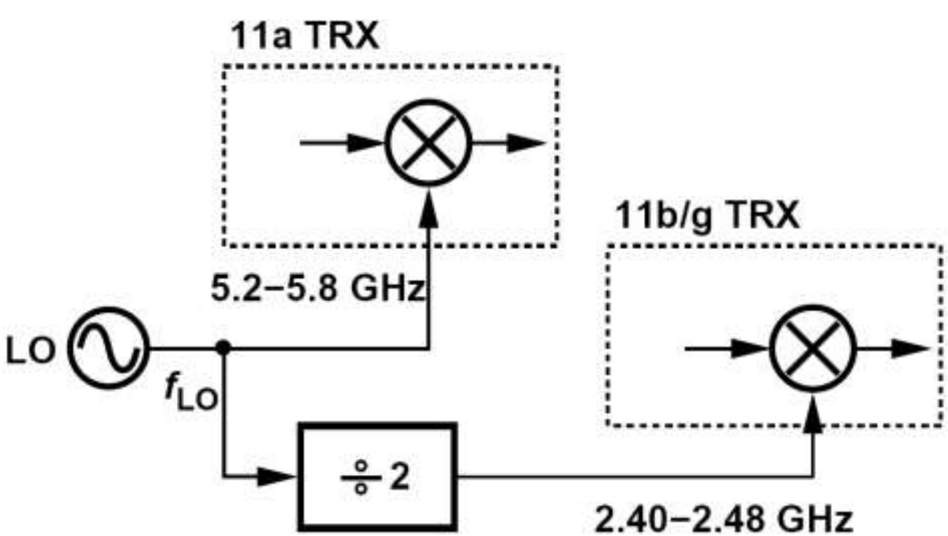
振荡器有什么用？

应用1: 计算机CPU的主频



High-speed oscillator driving a microprocessor.

应用2: 无线通信的载频

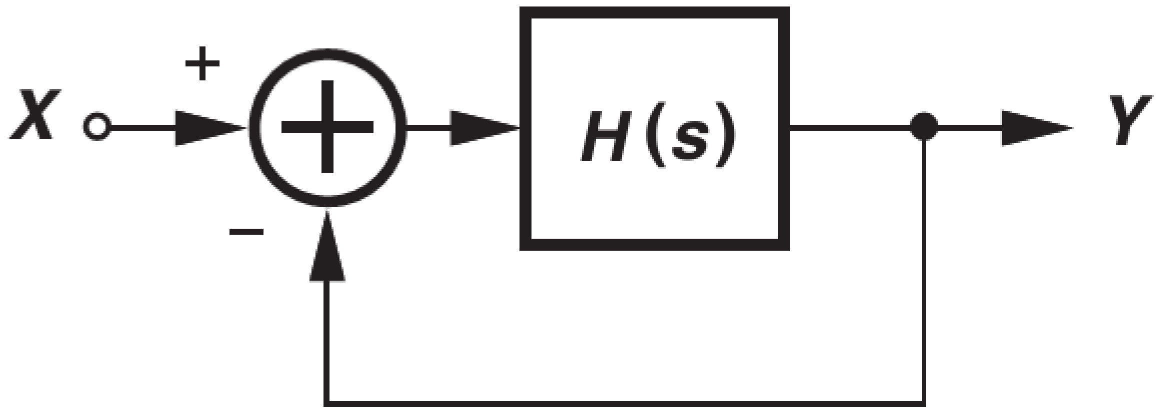


应用3: 石英表的时钟基准

Did you know?

The most commonplace use of oscillators is in (electronic) watches. A crystal oscillator (studied later in this chapter) runs at a precise frequency of 32,768 (= 2^{15}) Hz. This frequency is then divided down by means of a 15-bit counter to generate a 1-Hz square waveform, providing the "time base." This waveform shows the seconds on the watch. It is also divided by 60 and another 60 to count the minutes and the hours, respectively. A great challenge in early electronic watches was to design these counters for a very low power consumption so that the watch battery would last a few months.

负反馈电路回顾



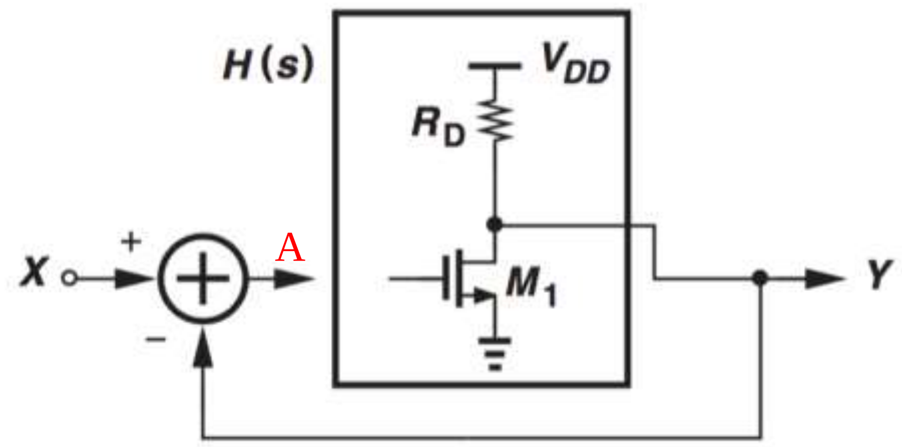
$$\frac{Y}{X}(s)=\frac{H(s)}{1+H(s)}$$

- **Closed-loop transfer function goes to infinity at frequency ω_1 if $H(s=j\omega_1)=-1$, or, equivalently, $|H(j\omega_1)|=1$ and $\angle H(j\omega_1)=180°$.**
- 注意：此处为按负反馈的定义“输入相减”，分析振荡电路时，也常常定义成“输入相加”，则相位平衡条件就变成**360°。但总相移都是360°！**

起振过程

$$\frac{Y}{X}(s)=\frac{H(s)}{1+H(s)}$$

当H(s)= -1时，
1.微小输入X → 无穷大的输出
（负反馈放大器的不稳定性）
2.零输入 → 有限幅度的输出
（振荡器）



Feedback loop containing a common-source stage.

- **思考1：最初的信号从哪里来的？** 电路中的噪声（如A点）
- **思考2：输出幅度真的可以趋向无穷大吗？**

  不会，随着输出幅度的增加，电路的非线性效应会使增益H(s)降低，最终实现一个稳定幅度的输出。如输出摆幅增加 → M1进入非饱和区 → g_m减小

- **振荡器的瞬态过程：**接通电源瞬间起，输出幅度逐渐从0增加到稳定值，这个过程称为“起振”。起振过程中，环路增益大于1，输出幅度逐渐增加；稳定后，环路增益＝1

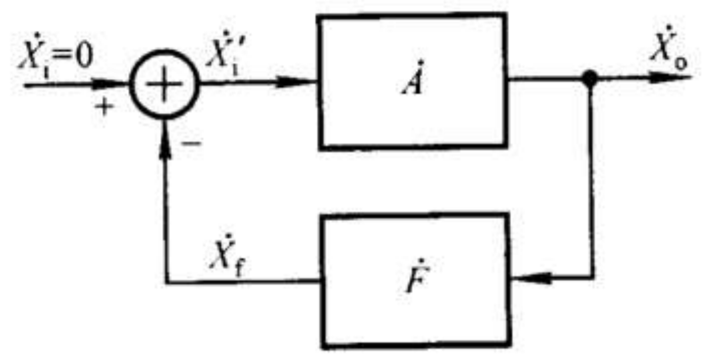


图 6.6.1　负反馈放大电路的自激振荡

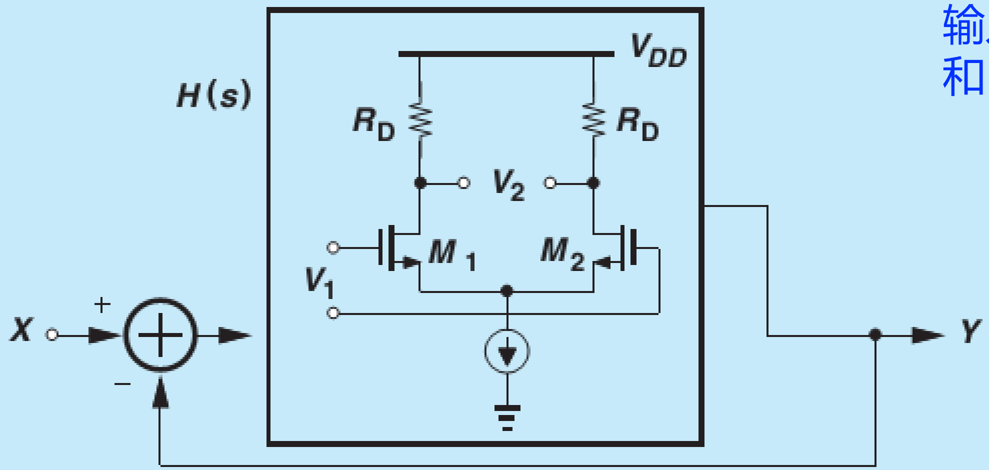
$$\begin{cases} |\dot{A}\dot{F}| = 1 & (6.6.3a) \\ \varphi_A + \varphi_F = (2n+1)\pi \quad (n\text{ 为整数}) & (6.6.3b) \end{cases}$$

上式称为自激振荡的平衡条件，式(6.6.3a)为幅值平衡条件，式(6.6.3b)为相位平衡条件，简称幅值条件和相位条件。只有同时满足上述两个条件，电路才会产生自激振荡。在起振过程中，$|\dot{X}_o|$有一个从小到大的过程，故起振条件为

$$|\dot{A}\dot{F}| > 1 \tag{6.6.4}$$

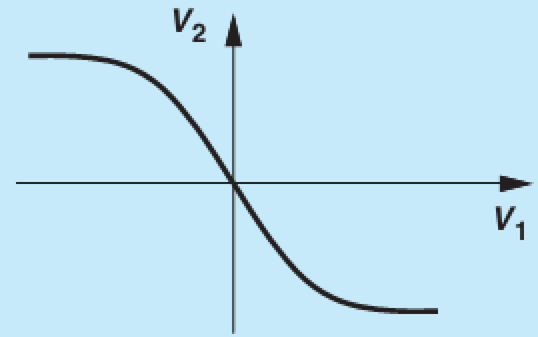
Example

An oscillator employs a differential pair. Explain what limits the output amplitude.



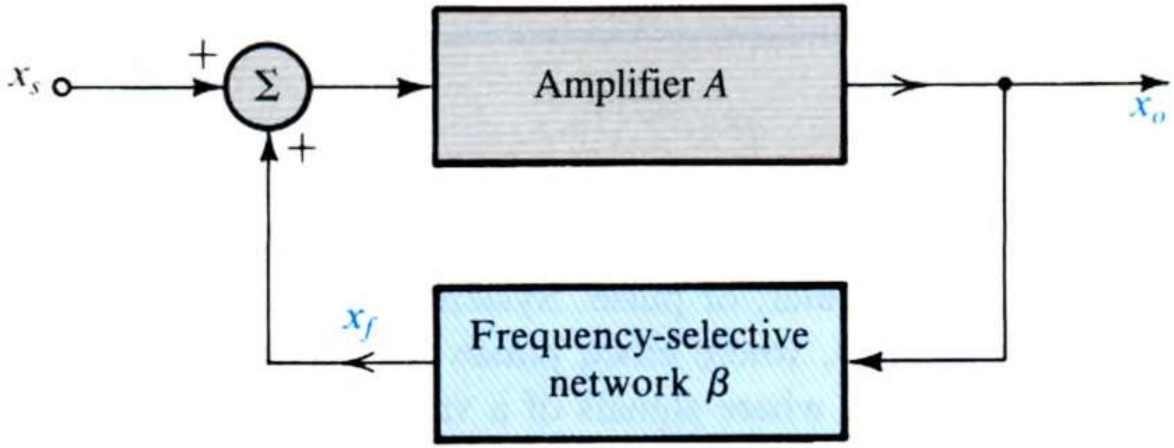
(a)

输入信号增加到一定程度，输出会饱和，增益下降



(b)

- **The gain of the differential pair drops and so does the loop gain as the input swing grows.**
- **The oscillation amplitude reaches its maximum when the tail current is steered completely to either side, i.e. swing from $-I_{SS}R_D$ to $I_{SS}R_D$.**



$$A_f(s) = \frac{A(s)}{1 - A(s)\beta(s)}$$

Figure 15.1 The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network. In an actual oscillator circuit, no input signal will be present; here we are showing an input signal x_s to help explain the principle of operation.

- 振荡器设计时，反馈信号常用“+”
- 常用反馈网络实现频率选择功能
- 振荡器设计常常分两步：
  1）频率分析，获得谐振频率ω_0
  2）设计非线性机制控制输出幅度

反馈电路分析方法一：

1. Break the feedback loop, 获得 loop gain $A(s)\beta(s)$
2. 令 $A(j\omega)\beta(j\omega)$相位等于零，获得振荡频率 ω_0
3. 根据 $|A(j\omega_0)\beta(j\omega_0)| \geq 1$，分析起振条件

Example 15.1

Figure 15.2(a) shows a sinusoidal oscillator formed by placing a second-order LCR bandpass filter [see Fig. 14.19(c)] in the feedback path of a positive-gain amplifier. Find the frequency of oscillation ω_0, and the condition for oscillations to start. Assume an ideal op amp.

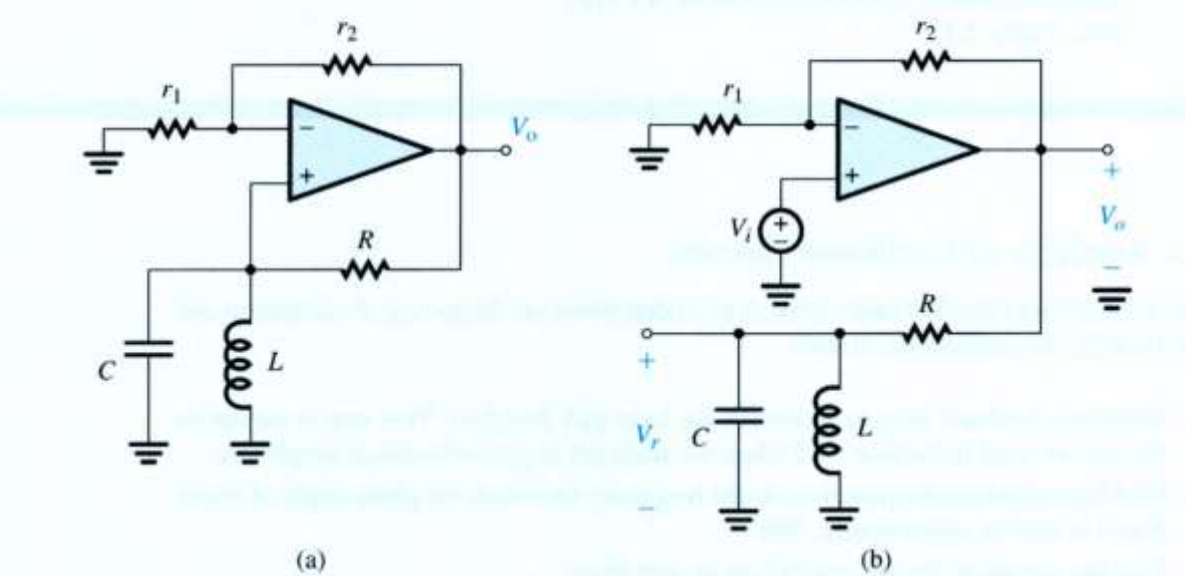


Figure 15.2 **(a)** An oscillator formed by connecting a positive-gain amplifier in a feedback loop with a bandpass RLC circuit. **(b)** Breaking the feedback loop at the input of the op amp to determine $A(s) \equiv V_o(s)/V_i(s)$ and $\beta(s) \equiv V_r(s)/V_o(s)$, and hence the loop gain $A(s)\beta(s)$.

$$A(j\omega)\beta(j\omega) = \frac{j\frac{\omega}{CR}\left(1+\frac{r_2}{r_1}\right)}{\left(-\omega^2+\frac{1}{LC}\right)+j\frac{\omega}{CR}}$$

相位为零 $\Rightarrow$ $\omega_0 = 1/\sqrt{LC}$ $\Rightarrow$

$$|A(j\omega_0)\beta(j\omega_0)| = 1+\frac{r_2}{r_1}$$

$$r_2/r_1 \geq 0$$

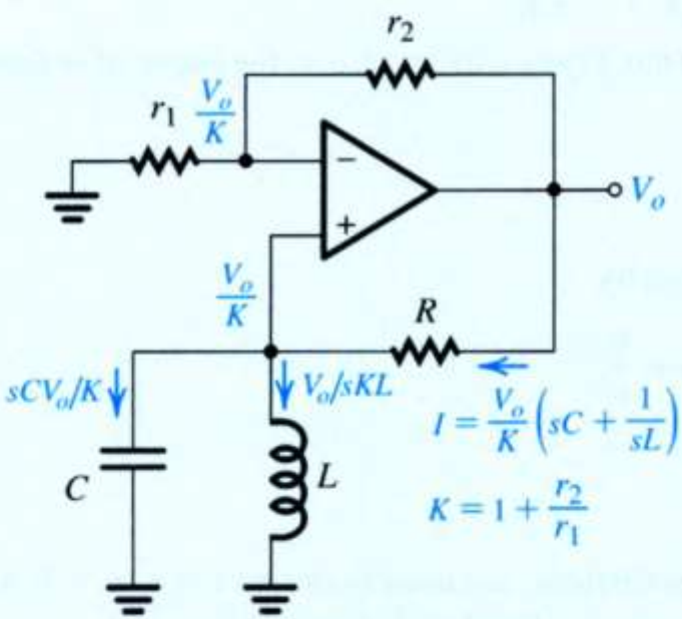
反馈电路分析方法二：

不需要 break the feedback loop

1. 假设电路在频率 ω_0 振荡
2. 分析电路，获得约束方程 $D(j\omega_0) = 0$
3. 求解方程，获得谐振频率 ω_0 和起振条件

Example 15.2

Use the alternative analysis method described above to find the frequency of oscillation ω_0 and the condition for sustained oscillation of the circuit in Fig. 15.2(a).



$$V_o = IR + \frac{V_o}{K} = \frac{V_o}{K}R\left(sC + \frac{1}{sL}\right) + \frac{V_o}{K}$$

Figure 15.3 Analysis of the oscillator circuit of Fig. 15.2(a).

$$\left(-\omega^2 + \frac{1}{LC}\right) - j\omega\frac{1}{CR}\frac{r_2}{r_1} = 0$$

$$R_e[D(j\omega)] = -\omega^2 + \frac{1}{LC}$$

$$\mathrm{Im}[D(j\omega)] = -\frac{\omega}{CR}\frac{r_2}{r_1}$$

$$R_e = 0 \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\mathrm{Im} = 0 \Rightarrow \frac{r_2}{r_1} = 0 \Rightarrow r_2 = 0$$

分析谐振频率比较简单

To ensure that oscillations start, we usually design for

$$r_2 > 0$$

非线性机制控制输出幅度

- Barkhausen criterion描述了振荡稳定时需满足的条件;
  - $A(j\omega_0)\beta(j\omega_0)=1$，即在谐振频率 ω_0 时， loop gain 的相位为零，且幅度为1

- 此外，我们需要引入非线性机制来控制振荡器的输出幅度在合理的值
  - 起振时 $|A(j\omega_0)\beta(j\omega_0)|>1$
  - 稳定时 $|A(j\omega_0)\beta(j\omega_0)|=1$，输出电压/电流幅度在某一合理值
  - 若因某种原因导致瞬间 $|A(j\omega_0)\beta(j\omega_0)|<1$，非线性机制需要将其调回到 1

非线性机制控制输出幅度

- 利用二极管实现非线性控制
  - 流过二极管的电流增加时，其小信号电阻减小（Why？）
  - 所以合理地放置二极管，可以使得输出信号增加时增益减小

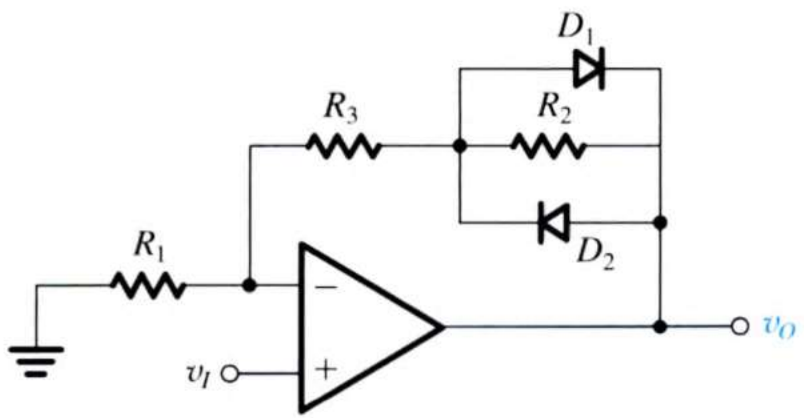


Figure 15.4 By placing two diodes in opposite directions across part of the feedback resistance, the amplifier gain v_O/v_I is made to decrease as the amplitude of v_O increases.

- 当输出信号幅度比较小时，二极管截止，反馈电阻是 R_2+R_3
- 当输出信号幅度增加时，二极管导通，反馈电阻是 $R_3+R_2 \| r_D$，增益降低；输出越大，反馈电阻越低，增益越低；
- 若稳定后某种原因导致增益瞬间降低，反馈电阻会变大，把增益回调

非线性机制控制输出幅度

- 利用 limiter 实现非线性控制
  - 方波的基频分量是正弦波；同频率，幅度为$(\pi/4)V$，傅里叶展开
  - 谐振频率 ω_0 由频率选择电路LC决定；

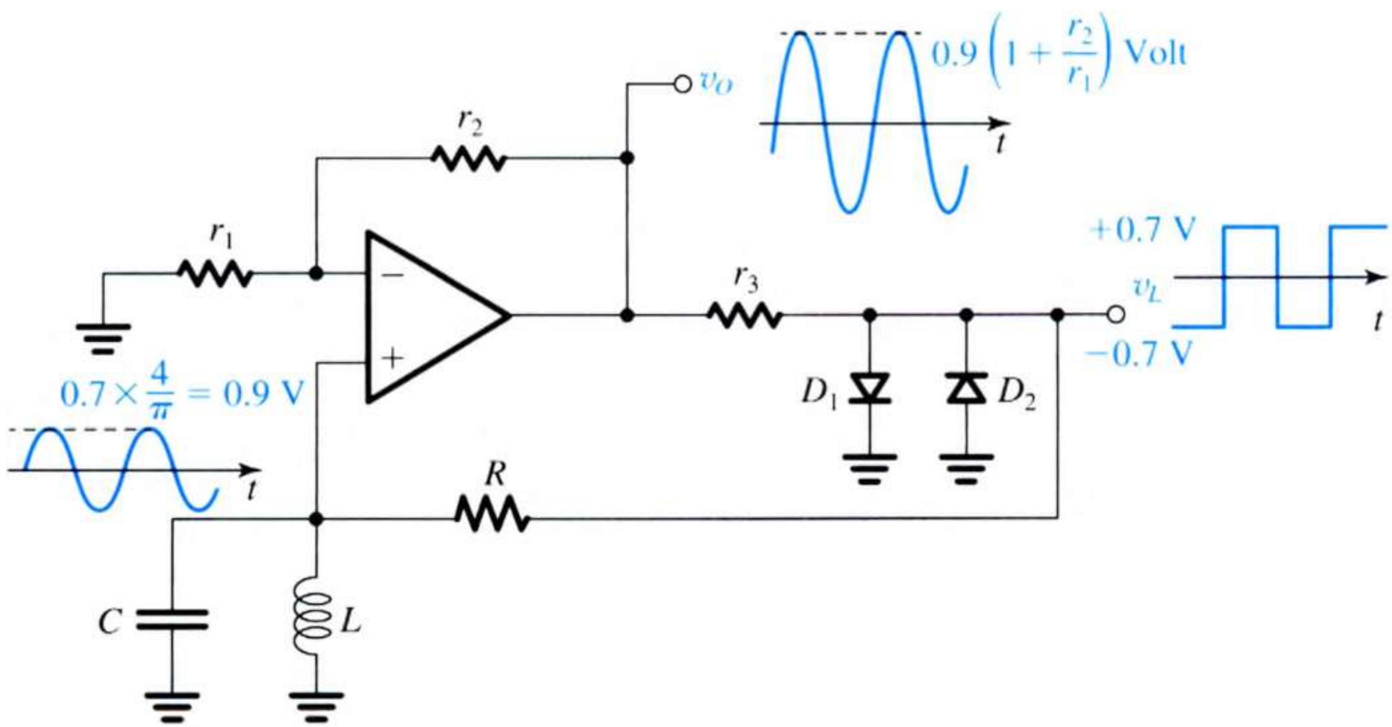


Figure 15.5 The oscillator circuit of Fig. 15.2 with a limiter inserted in the feedback loop for amplitude control.

Summary of Oscillator topologies and applications

Oscillator Topology	Ring Oscillator	Cross-Coupled Oscillator (LC Oscillators)	Colpitts Oscillator (LC Oscillators)	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

- **Oscillators can be realized as either integrated or discrete circuits. The topologies are quite different in the two cases but still rely on Barkhausen's criteria （起振稳定后，即振荡器达到平衡后，需满足的条件）.**

Startup Condition 起振条件

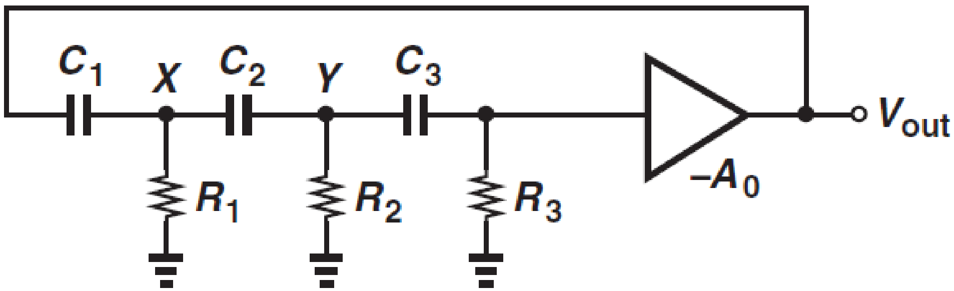
- **Startup condition: a unity loop gain at the desired oscillation frequency, ω_0.**
- **The loop gain is usually quite larger than unity to leave margin for process, temperature or supply voltage variation.**
- **Design specifications: oscillation frequency, output amplitude, power consumption, complexity and noise.**

- 起振时，环路增益幅度大于**1**（在希望产生振荡的频率ω_0处）；
- 在振荡频率ω_0处，环路总相移（加上负反馈的**180**度）刚好等于**360**度（相位平衡条件决定了振荡器的振荡频率）
- 稳定后，环路增益幅度会因电路中的非线性效应降低并维持在**1**；
- 振荡器的设计指标：振荡频率、输出电压幅度、功耗、复杂性、噪声等

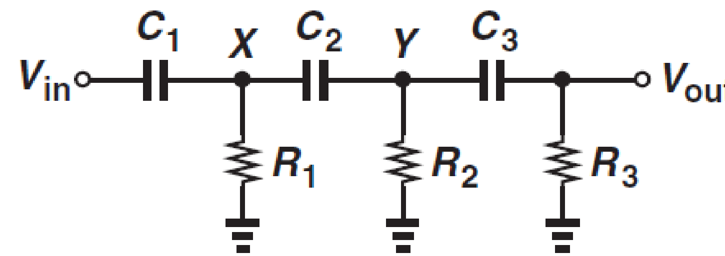
RC振荡器

Oscillator Topology	Ring Oscillator	LC Oscillators Cross-Coupled Oscillator	LC Oscillators Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

Phase Shift Oscillator （RC移相振荡器）



(a)



(b)

$$\frac{V_{out}}{V_{in}} = \frac{(RCs)^3}{(RCs+1)^3}$$

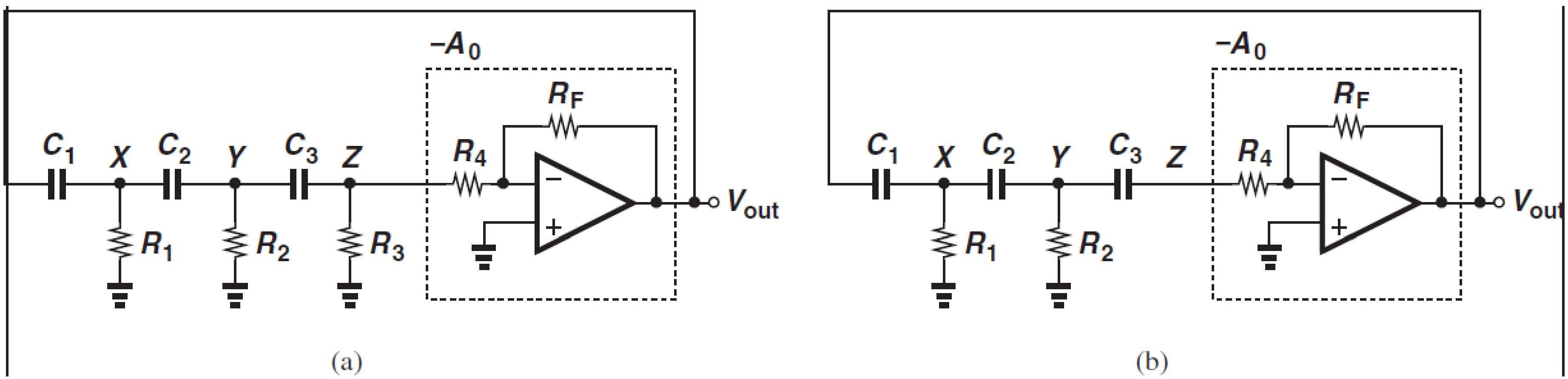
$$\omega_1 = \frac{1}{\sqrt{3}RC}$$

$$\frac{ARC\omega_1}{\sqrt{R^2C^2\omega_1^2+1}} = 1$$

- 可以通过RC网络移相。Three RC sections can provide 180° phase shift at oscillation frequency.
- The signal attenuation of the passive stages must be compensated by the amplifier to fulfill the startup condition.
- Occasionally used in discrete design.

Example

Design the phase shift oscillator using an op amp.



- **The op amp is configured as an inverting amplifier.**
- **Due to R_4 equivalently shunting R_3 , we must choose $R_3||R_4 = R_2 = R_1 = R$.**
- **Alternatively, we may simply eliminate R_3 and set R_4 to be equal to R .**

非线性控制输出幅度

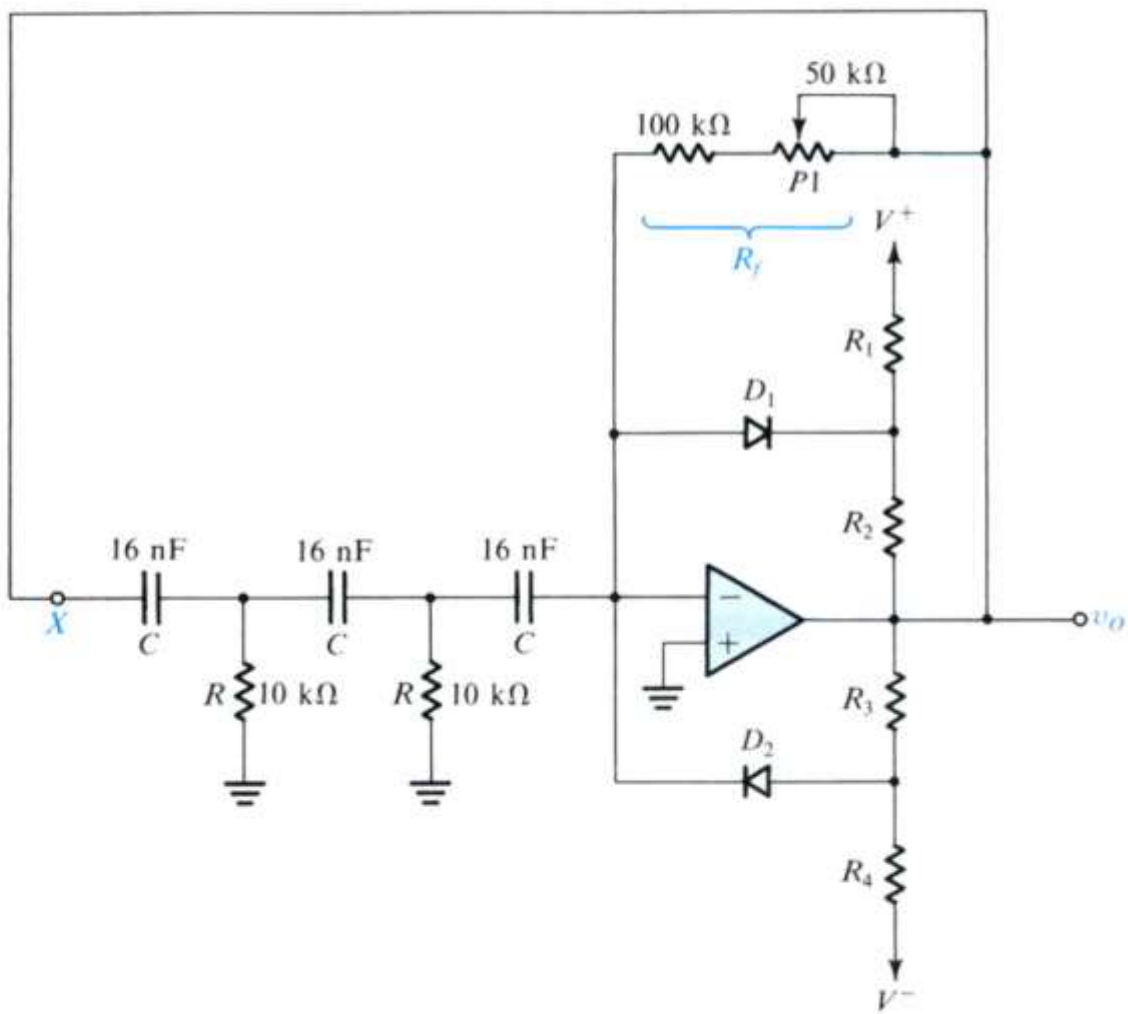
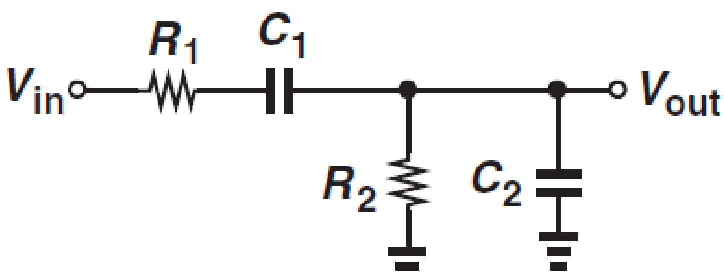


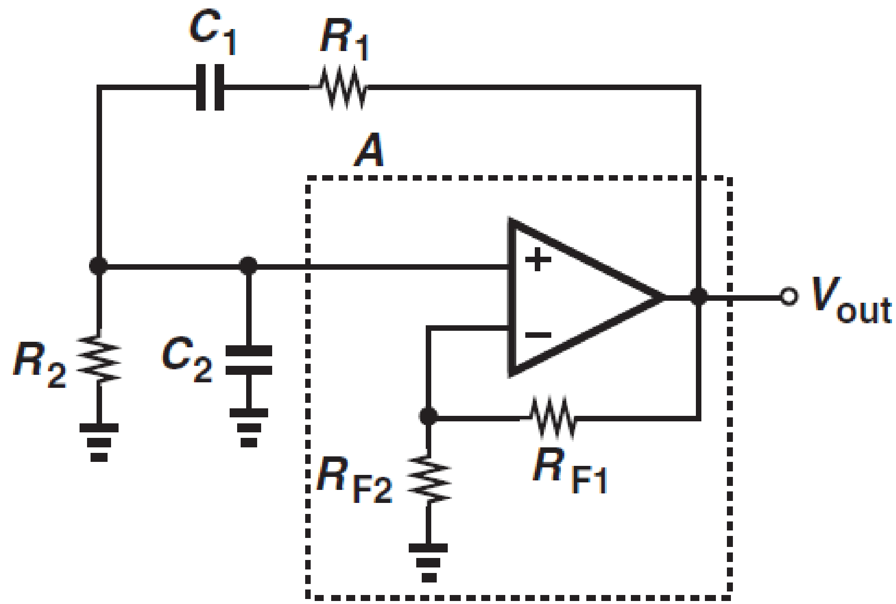
Figure 15.10 A practical phase-shift oscillator with a limiter for amplitude stabilization.

RC振荡器

Oscillator Topology	Ring Oscillator	Cross-Coupled Oscillator	Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
		LC Oscillators				
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

Wien-Bridge Oscillator （文氏振荡器）





If $R_1 = R_2 = R$ and $C_1 = C_2 = C$.

$$\frac{V_{out}}{V_{in}}(s) = \frac{\dfrac{R}{RCs+1}}{\dfrac{R}{RCs+1} + \dfrac{1}{Cs} + R} = \frac{RCs}{R^2C^2s^2 + 3RCs + 1}$$

The phase thus emerges as

$$\angle \frac{V_{out}}{V_{in}}(s = j\omega) = \frac{\pi}{2} - \tan^{-1}\frac{3RC\omega}{1 - R^2C^2\omega^2}$$

零相移 $\Rightarrow$ $\boxed{\omega_1 = \frac{1}{RC}}$

$$\left|\frac{ARCj\omega}{1 - R^2C^2\omega^2 + 3jRC\omega}\right| = 1$$

$$\Downarrow$$

$$\boxed{A = 3}$$

That is, we choose $R_{F1} \geq 2R_{F2}$

- Passive feedback network provides zero phase shift.
- The amplifier is non-inverting. 反馈网络零相移＋放大器正增益＝360度相移

非线性机制控制输出幅度

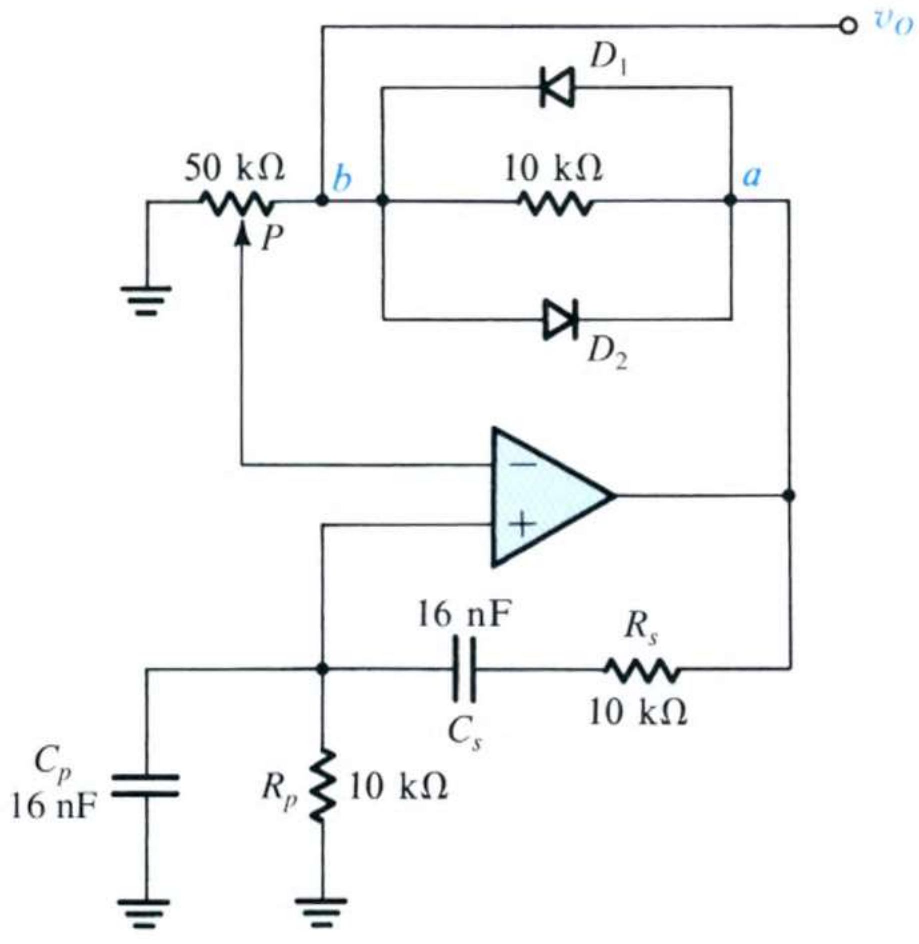


Figure 15.8 A Wien-bridge oscillator with an alternative method for amplitude stabilization.

输出幅度由电位器P控制

科技创业的典范

Did you know?

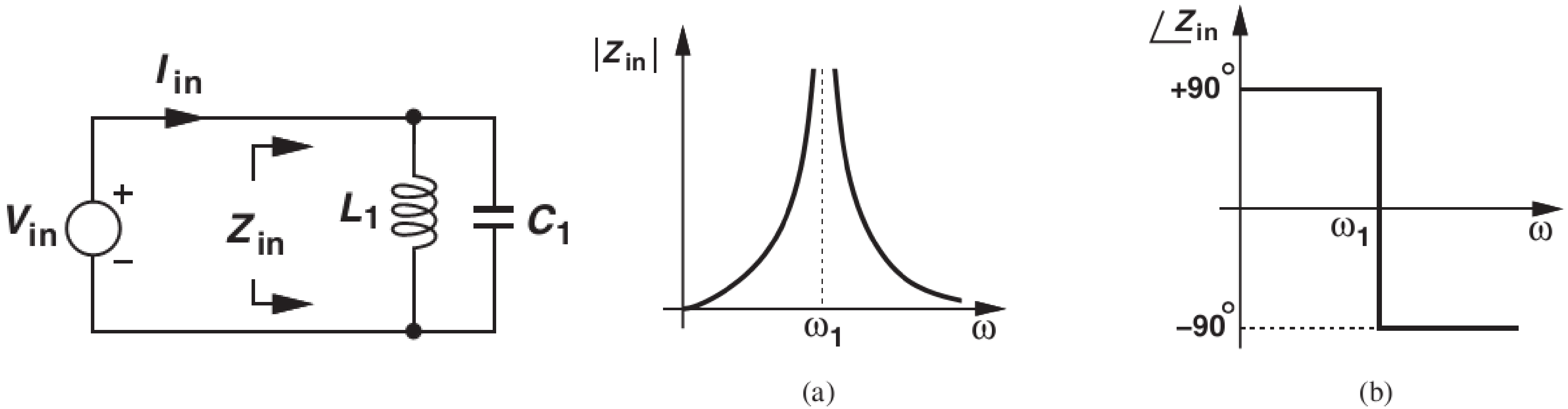
In 1939, two young Stanford graduates named William Hewlett and David Packard used the Wien-bridge oscillator to design a sound generator for the soundtrack of the Disney movie *Fantasia*. History has it that Hewlett and Packard borrowed $500 from their advisor, Fredrick Terman, to construct and sell eight of these generators to Disney. Thus began the company known today as HP. For the first several decades, HP designed and manufactured only test equipment, e.g., oscilloscopes, signal generators, power supplies, etc.

LC振荡器

Oscillator Topology	Ring Oscillator	Cross-Coupled Oscillator	Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
	LC Oscillators					
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

- RC振荡器频率适用频率较低，Q值也较低

LC oscillators: Ideal Parallel LC Tanks

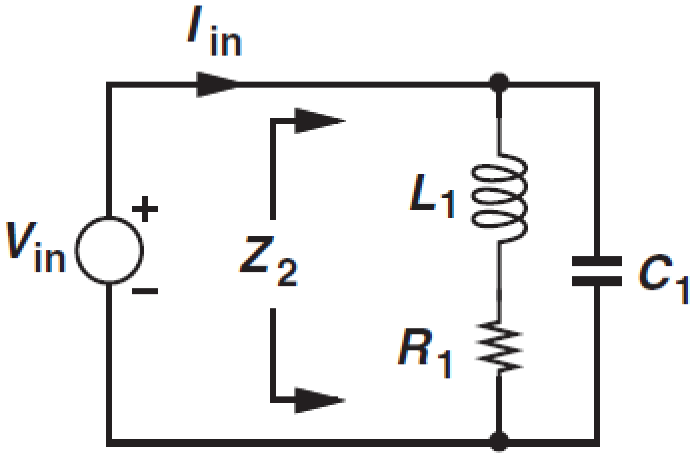


$$Z_{in}(j\omega) = \frac{jL_1\omega}{1 - L_1C_1\omega^2}$$

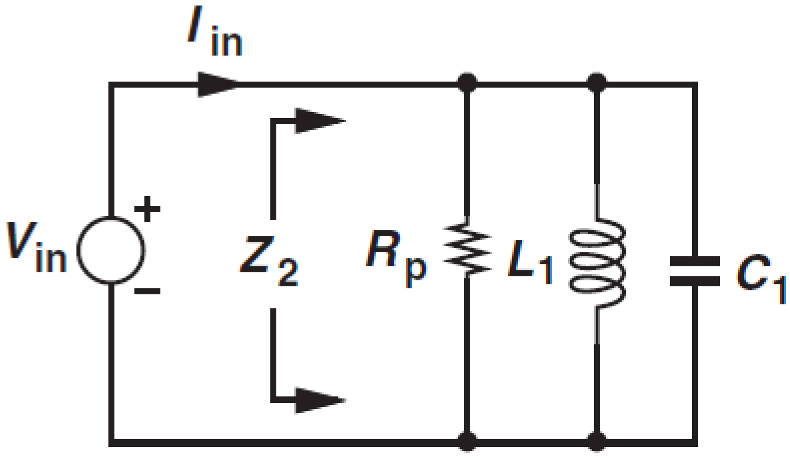
$$\omega_1 = 1/\sqrt{L_1C_1}$$

- **The impedance goes to infinity at ω_1, i.e. LC tank resonates.并联LC，谐振时阻抗无穷大**
- **The tank has an inductive behavior for $\omega < \omega_1$ and a capacitive behavior for $\omega > \omega_1$. 频率小于谐振频率时，总体呈现感性；频率大于谐振频率时，总体呈现容性**

Lossy LC Tank (1)



(a)

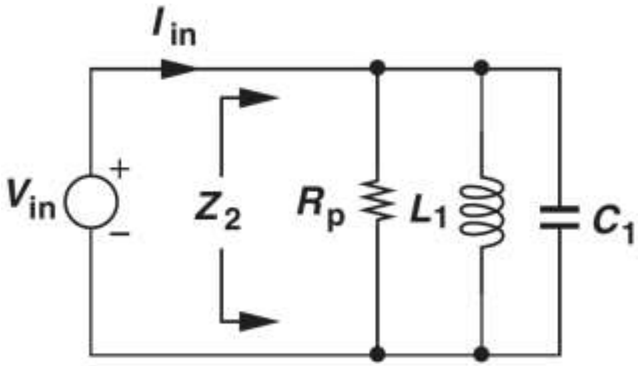


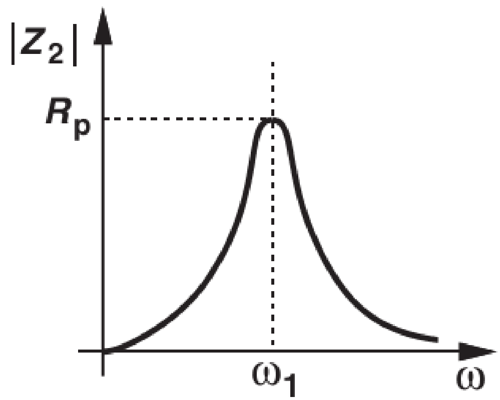
(b)

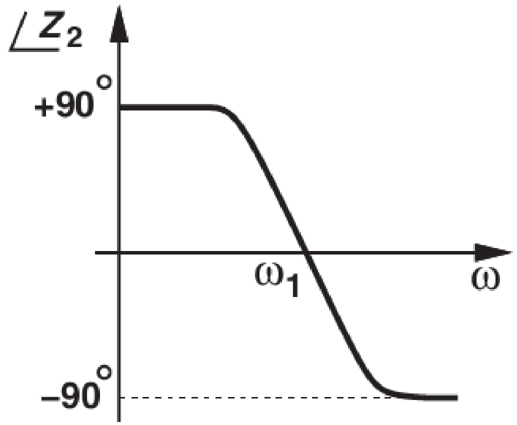
$$R_p = \frac{L_1^2 \omega^2}{R_1}$$

- In practice, the impedance of LC tank does not goes to infinity at the resonance frequency due to finite resistance of the inductor. 实际电路中，电感是有损耗的，用串联R_1表示
- Circuit (a) and (b) are only equivalent for a narrow range around the resonance frequency. 只在谐振频率附近等效

Lossy LC Tank (2)





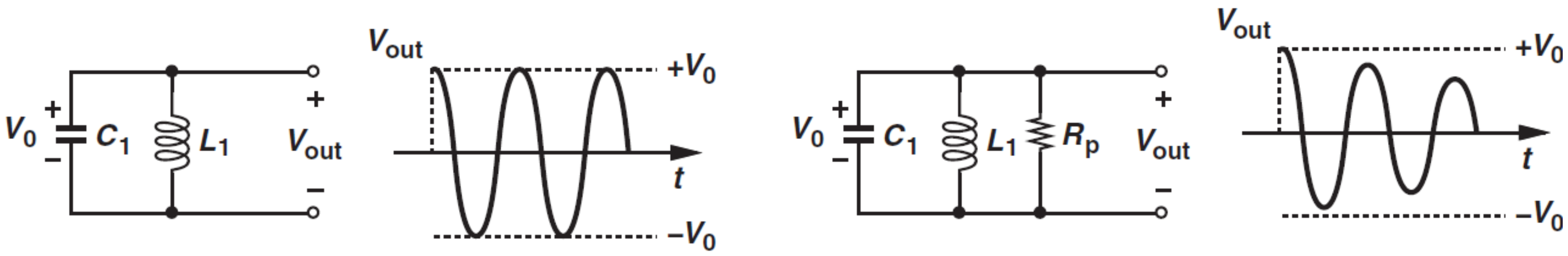


$$Z_2(j\omega) = \frac{jR_pL_1\omega}{R_p(1-L_1C_1\omega^2)+jL_1\omega}$$

- **In the analysis of LC oscillators, we prefer to model the loss of the tank by a parallel resistance, R_p .**
- **Z_2 reduces to a single resistance, R_p , at ω_1 .**
- **At very low frequency, $Z_2 \approx jL_1\omega$; at very high frequency, $Z_2 \approx 1/(jC_1\omega)$.**
- 注意：损耗导致相位的渐变

Example

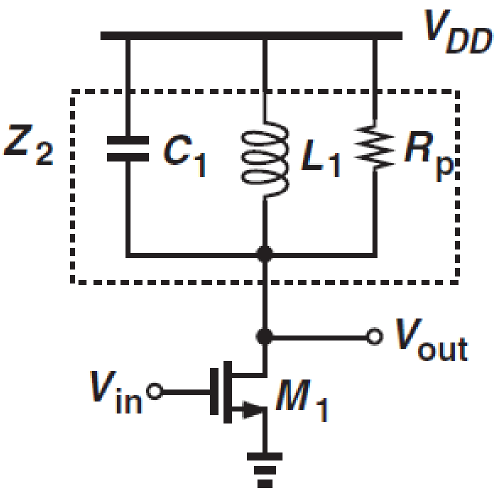
Suppose we apply an initial voltage of V_0 across the capacitor in an isolated parallel tank. Study the behavior of the circuit in the time domain if the tank is ideal or lossy.



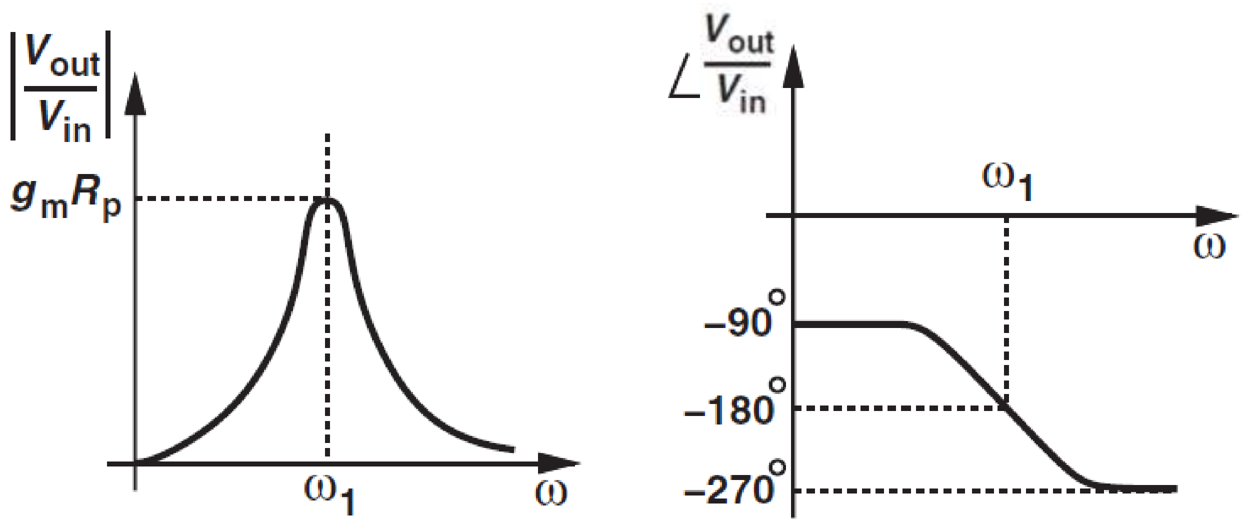
(a) (b)

- **For ideal tank, the transfer of energy between C_1 and L_1 repeats and the tank oscillates indefinitely.**
- **For lossy tank, the current flowing through R_p dissipates energy and thus the tank loses some energy each cycle, producing a decaying oscillatory output.**

Single CS Stage with a Tank Load



(a)



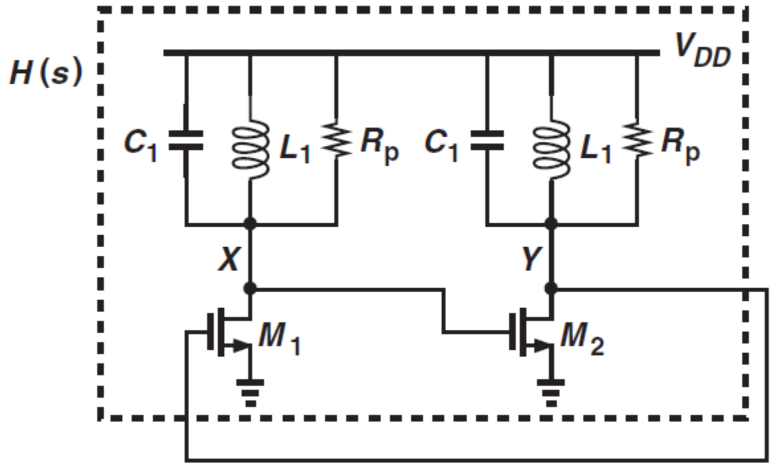
(b)

$$\frac{V_{out}}{V_{in}} = -g_m Z_2(s)$$

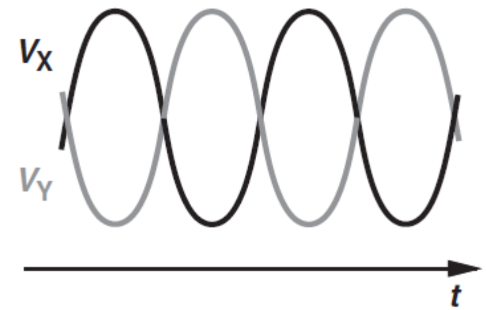
$$Z_2(j\omega) = \frac{jR_p L_1 \omega}{R_p(1 - L_1 C_1 \omega^2) + jL_1 \omega}$$

- The gain reaches a maximum of at resonance and approaches zero at very low or very high frequencies.
- The phase shift at resonance frequency is equal to 180°.（将 LC tank 的相位曲线下移180度）

Two LC-load CS Stages in a Loop



(a)



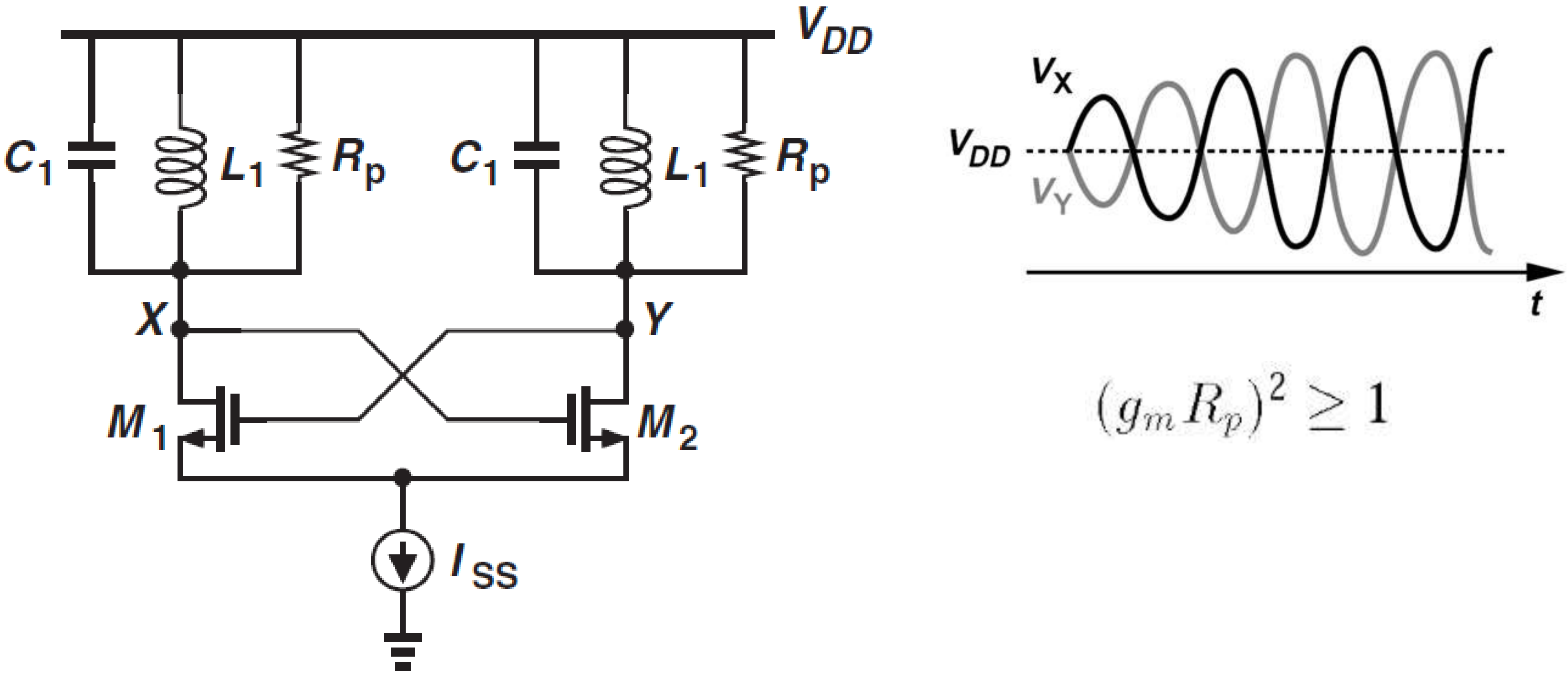
(b)

$$(g_m R_p)^2 \geq 1$$

$$\omega_1 = 1/\sqrt{L_1 C_1}$$

- Each stage provides 180° at ω_1 to achieve the total phase shift of 360°.
- Differential signals at nodes X and Y.
- However, the bias current of the transistors is poorly defined. 偏置对温度、工艺敏感（对V_T值敏感）

Cross-Coupled Oscillator

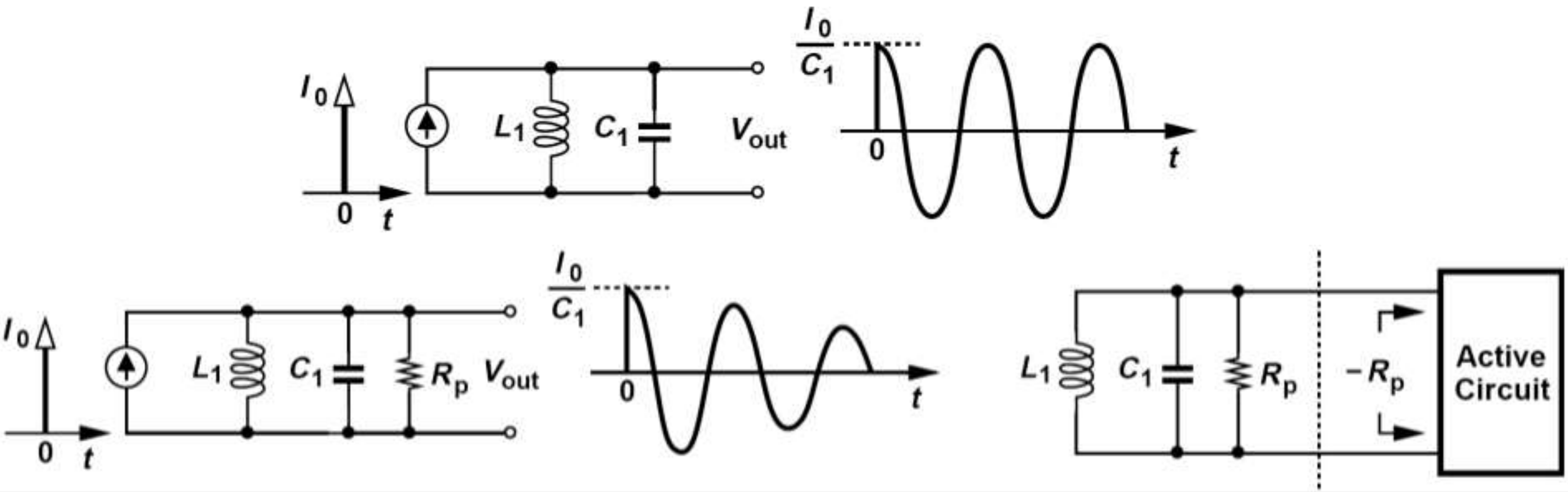


$$(g_m R_p)^2 \geq 1$$

- **A tail current source is added to set bias condition for the transistors.**
- **Most popular and robust LC oscillator used in integrated circuits.**
- **M_1的G与M_2的D相连；M_2的G与M_1的D相连；**
- **A unique attribute of inductive loads is that they can provide peak voltages above the supply**
- **优点：适用于射频（65nm可实现300GHz）；噪声小**

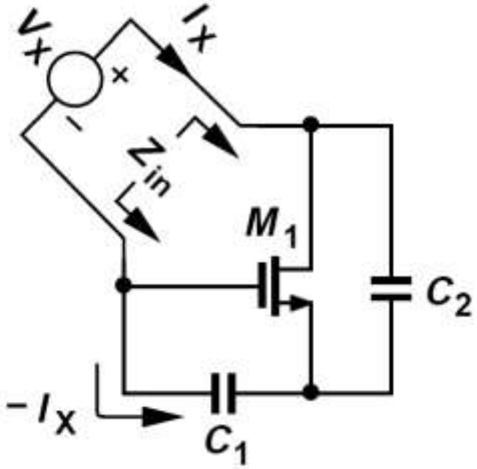
振荡器分析的另一角度：One-Port View of Oscillators

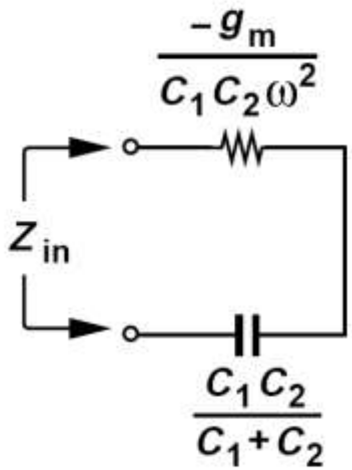
- **An alternative perspective views oscillators as two one-port components, namely, a lossy resonator and an active circuit that cancels the loss.**



- **If an active circuit replenishes the energy lost in each period, then the oscillation can be sustained.** 有源电路提供一个负电阻，用以消除R_p
- **In fact, we predict that an active circuit exhibiting an input resistance of -R_p can be attached across the tank to cancel the effect of R_p.**

How Can a Circuit Present a Negative Input Resistance?





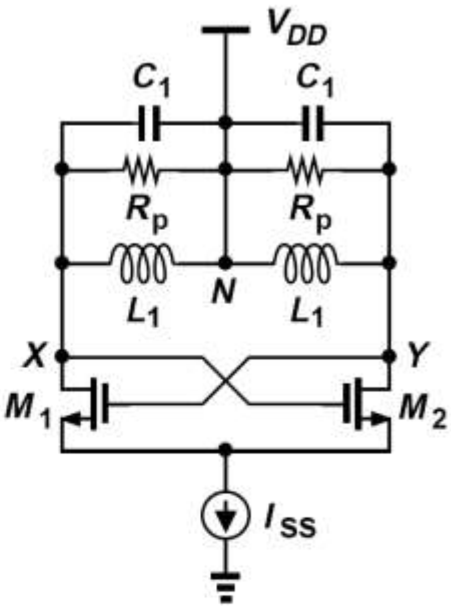
$$-\frac{I_X}{C_1 s} + V_X = \left(I_X + I_X\frac{g_m}{C_1 s}\right)\frac{1}{C_2 s}$$

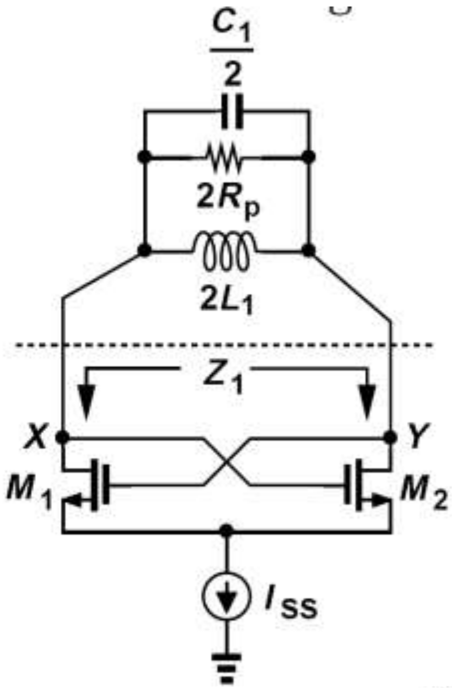
$$\frac{V_X}{I_X}(s) = \frac{1}{C_1 s} + \frac{1}{C_2 s} + \frac{g_m}{C_1 C_2 s^2}$$

$$\frac{V_X}{I_X}(j\omega) = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1 C_2 \omega^2}$$

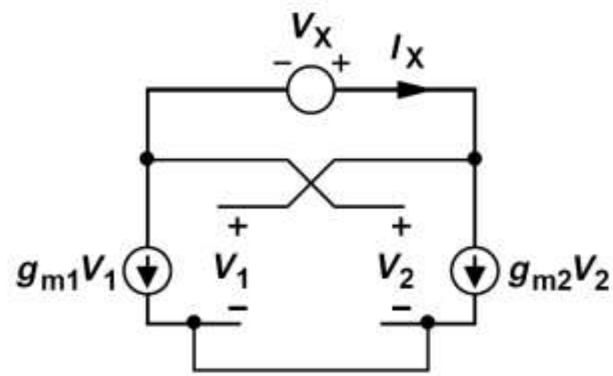
- The negative resistance varies with frequency.

One-Port View of Cross-Coupled Oscillator





不考虑 r_o



$$I_X = -g_{m1}V_1 = g_{m2}V_2 \quad \Rightarrow \quad \frac{V_X}{I_X} = -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right)$$

For $g_{m1} = g_{m2} = g_m$ $\quad \dfrac{V_X}{I_X} = -\dfrac{2}{g_m}$

For oscillation to occur, the negative resistance must cancel the loss of the tank:

产生的负阻要能够足以抵消
并联谐振电路的阻抗

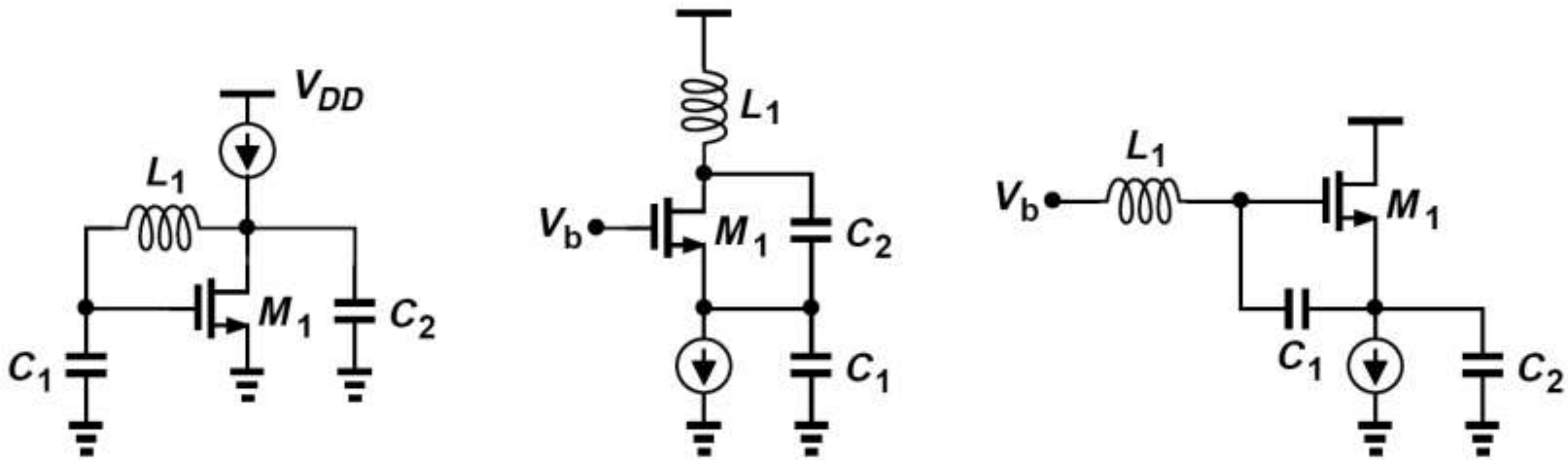
$$\frac{2}{g_m} \leq 2R_p \quad \Rightarrow \quad g_m R_p \geq 1$$

LC振荡器

Oscillator Topology	Ring Oscillator	LC Oscillators: Cross-Coupled Oscillator	LC Oscillators: Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

Three-Point Oscillators 三点式振荡器（三端振荡器）

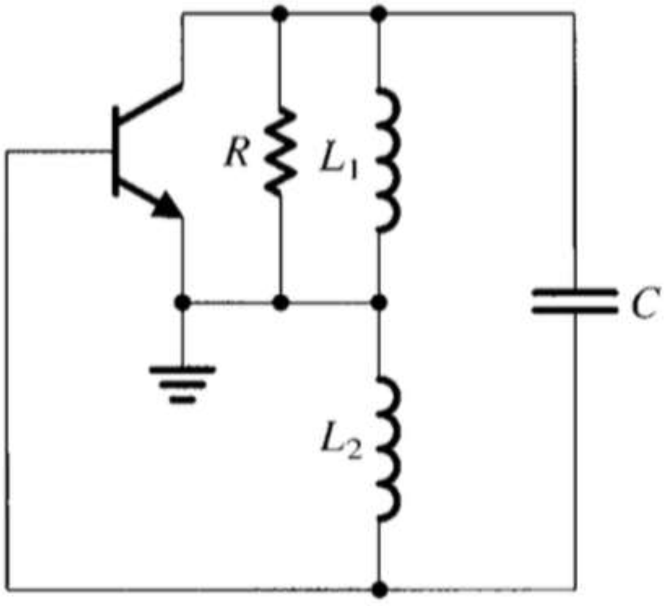
三点式振荡器是指晶体管的三个电极分别与LC谐振回路的三个端点连接组成的一种振荡器。为满足振荡条件，BE（GS）与CE（DS）必须是同性质的电抗元件，而CB（DG）必须为另一性质的电抗元件。（可从负阻的观点来理解）



If $C_1 = C_2$, the transistor must provide sufficient transconductance to satisfy

- **The circuits above may fail to oscillate if the inductor *Q* is not very high.**

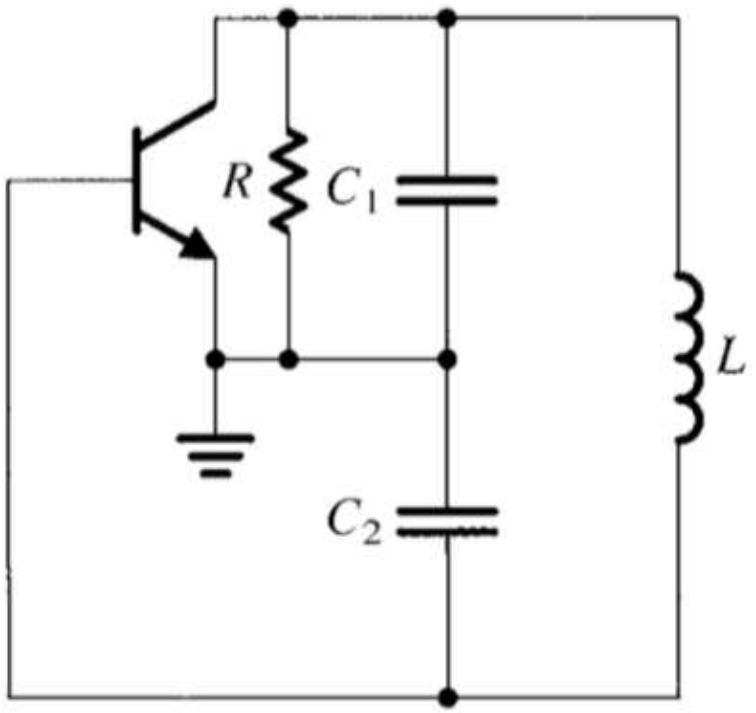
电感三点式振荡器（Hartley）



谐振频率： $\omega_0 = 1/\sqrt{(L_1 + L_2)C}$

起振条件： $g_m R > \dfrac{L_1}{L_2}$

电容三点式振荡器（Colpitts）



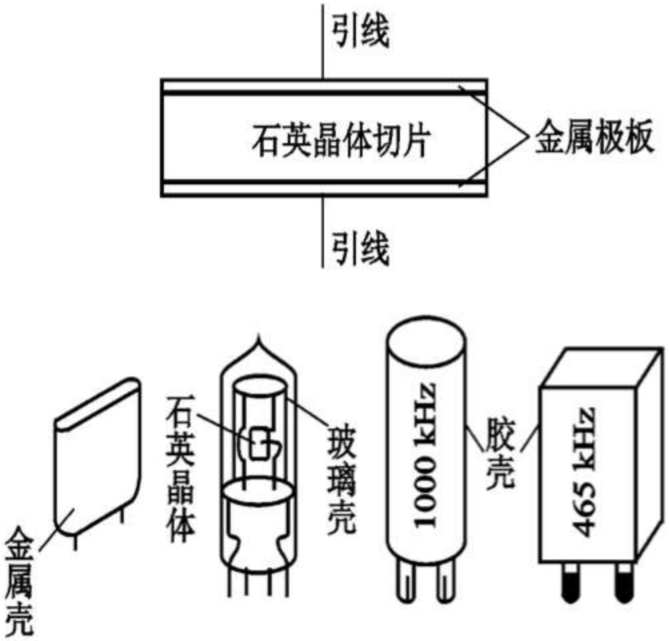
谐振频率： $\omega_0 = 1 \Big/ \sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}$

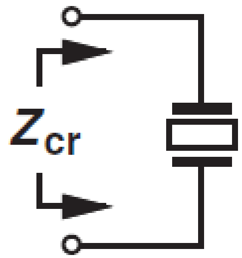
起振条件： $g_m R > \frac{C_2}{C_1}$

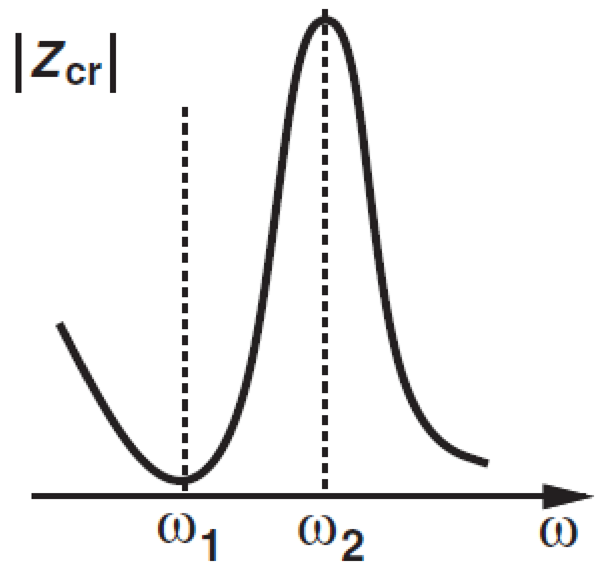
石英晶体振荡器

Oscillator Topology	Ring Oscillator	LC Oscillators: Cross-Coupled Oscillator	LC Oscillators: Colpitts Oscillator	Phase Shift Oscillator	Wien-Bridge Oscillator	Crystal Oscillator
Implementation	Integrated	Integrated	Discrete or Integrated	Discrete	Discrete	Discrete or Integrated
Typical Frequency Range	Up to Several Gigahertz	Up to Hundreds of Gigahertz	Up to Tens of Gigahertz	Up to a Few Megahertz	Up to a Few Megahertz	Up to About 100 MHz
Application	Microprocessors and Memories	Wireless Transceivers	Stand-Alone oscillators	Prototype Design	Prototype Design	Precise Reference

Crystal Oscillator: Crystal Model (1)

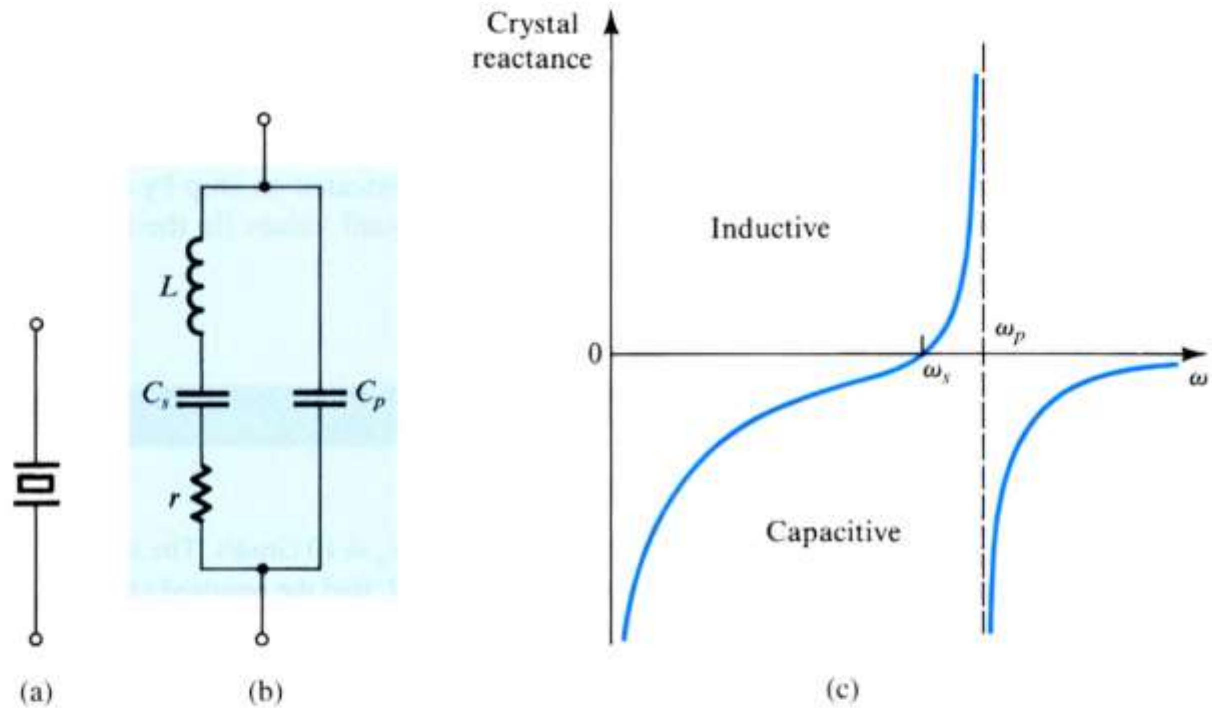






- 石英晶体振荡器，因压电效应而具有有图所示的阻抗特性
- **Attractive as frequency reference: (1) vibration frequency extremely stable; (2) easy to be cut to produce a precise frequency; (3) very low loss.**
- **The impedance falls to nearly zero at ω_1 (ω_s) and rises to a very high value at ω_2 (ω_p) .**

Crystal Model (2)



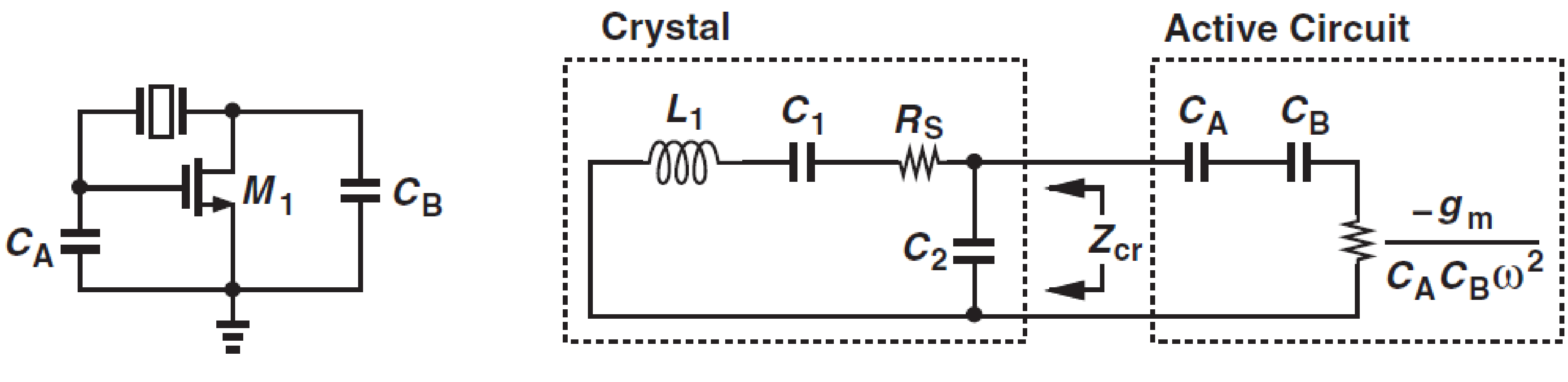
$$\omega_s = 1/\sqrt{LC_s}$$

$$\omega_p = 1\Bigg/\sqrt{L\left(\frac{C_sC_p}{C_s+C_p}\right)}$$

Figure 15.17 A piezoelectric crystal. **(a)** Circuit symbol. **(b)** Equivalent circuit. **(c)** Crystal reactance versus frequency [note that, neglecting the small resistance r, $Z_{crystal} = jX(\omega)$].

- **At ω_s the device experiences series resonance（串联谐振）, while at ω_p it experiences parallel resonance（并联谐振）.**
- **In practice, ω_s and ω_p are very close which means $C_p \gg C_s$.**

Crystal Oscillator



(a) (b)

$$L_1 C_1 \omega^2 - 1 \le g_m R_S \frac{C_1 C_2}{C_A C_B} \; (Parallel\, resonance)$$

- **Attach a crystal to a negative-resistance circuit to form an oscillator.** 具体推导不做要求，但要理解概念
- **C_A and C_B are chosen 10 to 20 times smaller than C_2 to minimize their effect on the oscillation frequency and to make negative resistance strong enough to cancel the loss.**

More on crystal oscillator

- 根据石英晶振在振荡器中的作用原理，晶体振荡器可分成两类。一类是将其作为等效电感元件用在三点式电路中， 工作在感性区， 称为并联型晶体振荡器； 另一类是将其作为一个短路元件串接于正反馈支路上，工作在它的串联谐振频率上，称为串联型晶体振荡器。

$$\omega_0 \simeq 1/\sqrt{LC_s} = \omega_s$$

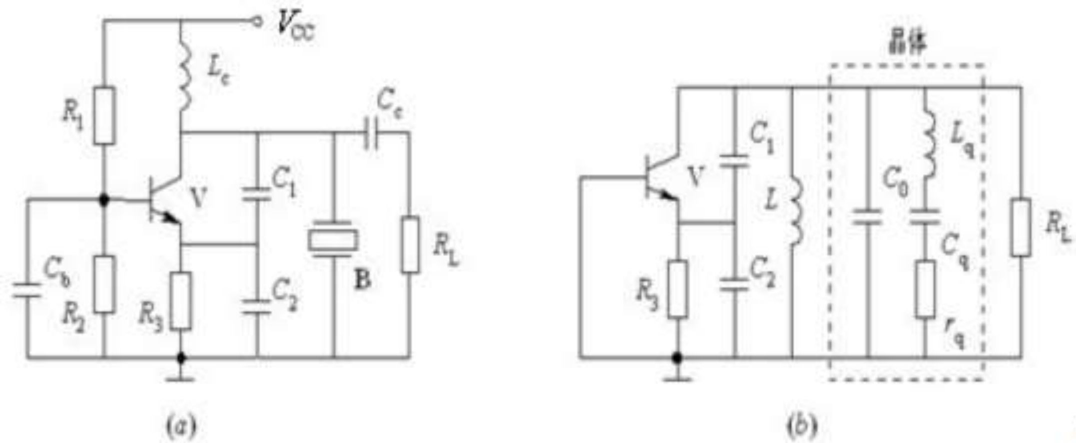


图 7.20　并联型晶体振荡电路及其等效电路

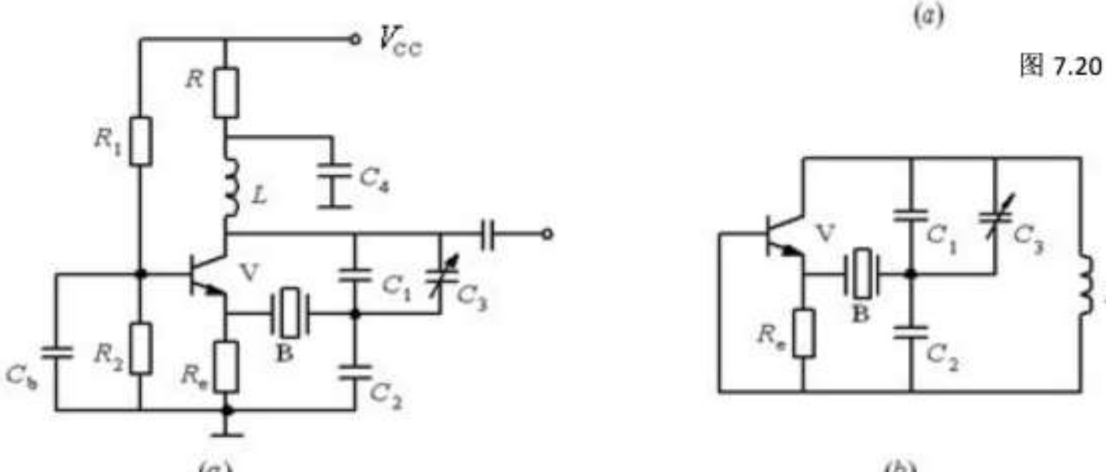
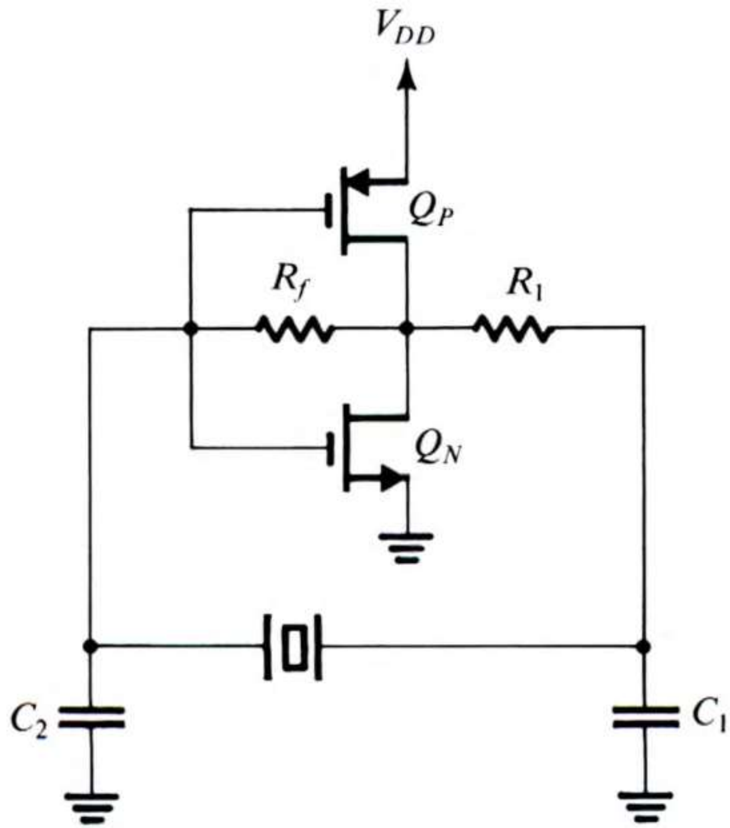


图 7.22　串联型晶体振荡电路及其等效电路



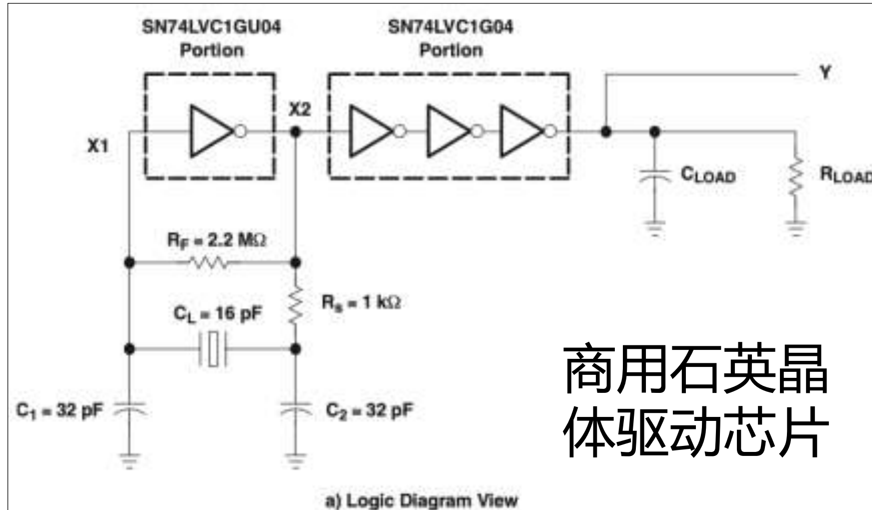


Figure 5. Oscillator Circuit

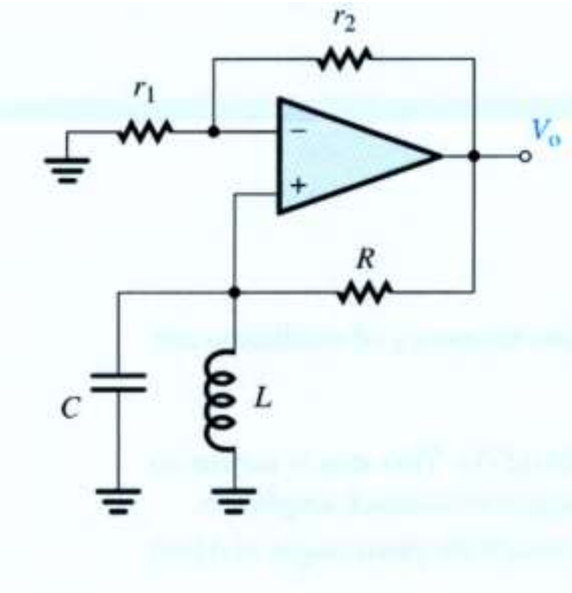
商用石英晶体驱动芯片

Figure 15.18 A Pierce crystal oscillator using a CMOS inverter as an amplifier.

- ***R*$_f$ 用来确定CMOS反相器的直流工作点；**
- ***R*$_1$和*C*$_1$构成低通滤波器，滤除石英晶体的高次谐波；**
- ***C*$_1$、*C*$_2$串联，再与石英晶体的*C*$_p$并联；**

作业

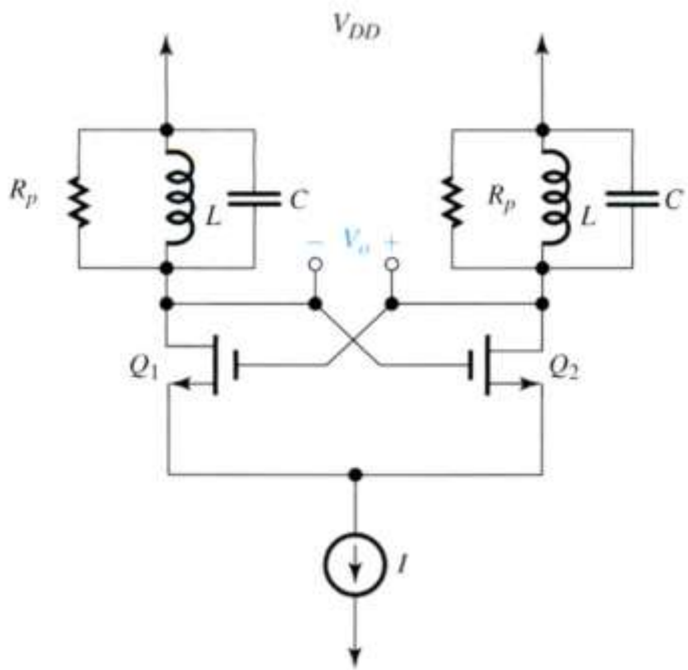
15.2 For the oscillator circuit in Fig. 15.2(a), let $r_1 = 10\ \text{k}\Omega$, $r_2 = 100\ \Omega$, $R = 10\ \text{k}\Omega$, $C = 10$ nF, and $L = 0.1$ mH. Find the frequency of oscillation ω_0. Is the condition of oscillation satisfied?
Ans. 10^6 rad/s; yes, since $r_2 > 0$.



(a)

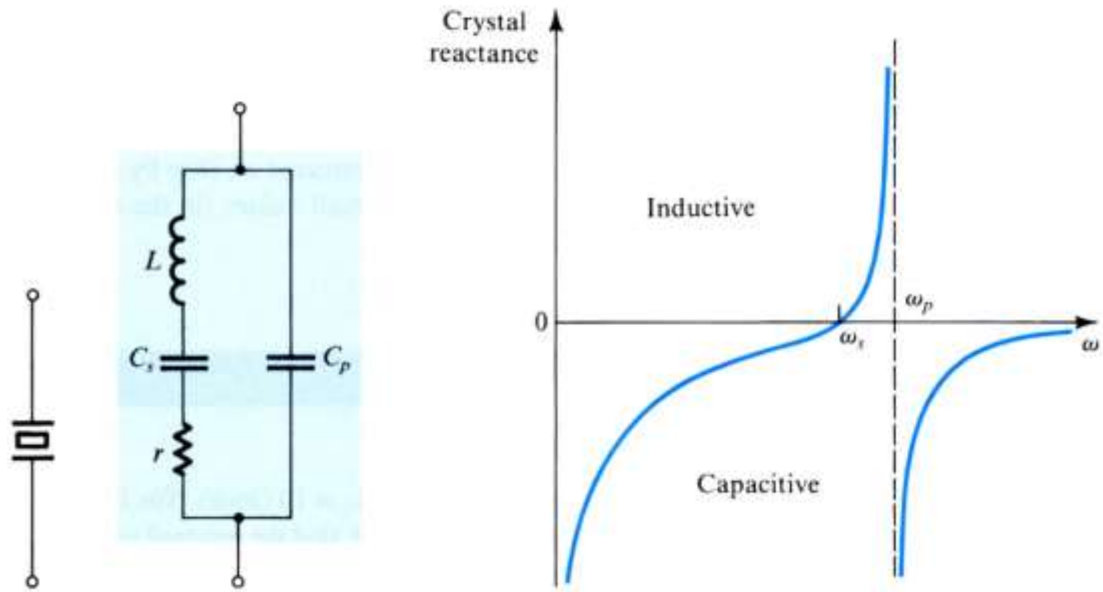
D15.12 Design the cross-coupled oscillator to operate at $\omega_0 = 10$ Grad/s. The IC inductors available have $L = 10$ nH and $Q = 10$. If the transistor $r_o = 10$ kΩ, find the required value of C and the minimum required value of g_m at which Q_1 and Q_2 are to be operated.

Ans. 1 pF; 1.1 mA/V



(a)

15.13 A 2-MHz quartz crystal is specified to have $L = 0.52$ H, $C_s = 0.012$ pF, $C_p = 4$ pF, and $r = 120\ \Omega$. Find f_s, f_p, and Q.
Ans. 2.015 MHz; 2.018.MHz; 55,000



(a) (b) (c)