电子电路基础

第九讲～谐振电路和滤波器

谐振电路和滤波器

- **5.1 *RLC*串联谐振电路**
- 5.1.1 *RLC*串联谐振电路的组成和参数特性（包括谐振条件、谐振时的电压、功率和能量、品质因数）
- 5.1.2 *RLC*串联谐振电路的频率响应
- **5.2 *RLC*并联谐振电路**
- 5.2.1 *RLC*并联谐振电路的组成和参数特性（包括谐振条件、谐振时的功率和能量、品质因数）
- 5.2.2 *RLC*并联谐振电路的频率响应
- **5.3 基本滤波器**
- 5.3.1 滤波器的分类及其频率响应（低通、高通、带通、带阻和全通）
- 5.3.2 无源滤波器简介（低通、高通、带通和带阻）

Frequency Response
Chapter 14

14.1 Introduction

14.2 Transfer Function

14.3 Series Resonance

14.4 Parallel Resonance

14.5 Passive Filters

频率响应

什么是电路的频率响应？

> The **frequency response** of a circuit is the variation in its behavior with change in signal frequency.

电路的**频率响应**：当正弦激励信号的**振幅保持不变**，而**频率变化**时，分析**电路特性**随频率变化的情况。尤其是分析**电路传递函数**的**幅度**（增益）和**相位**随频率变化的情况。

- 振幅 vs 频率：幅频特性
- 相位 vs 频率：相频特性

14.2 Transfer Function 传递函数

The **transfer function** $\mathbf{H}(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\omega)$ (an element voltage or current) to a phasor input $\mathbf{X}(\omega)$ (source voltage or current).

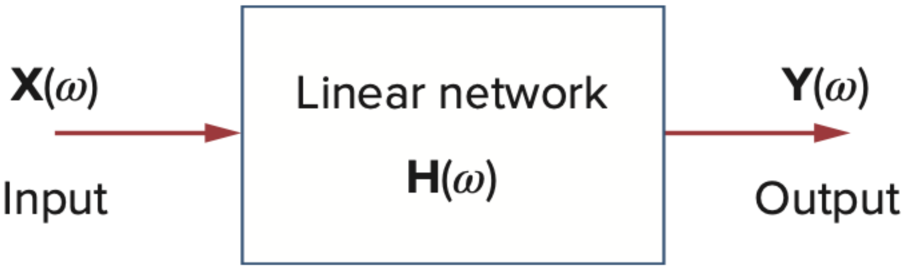


Figure 14.1
A block diagram representation of a linear network.

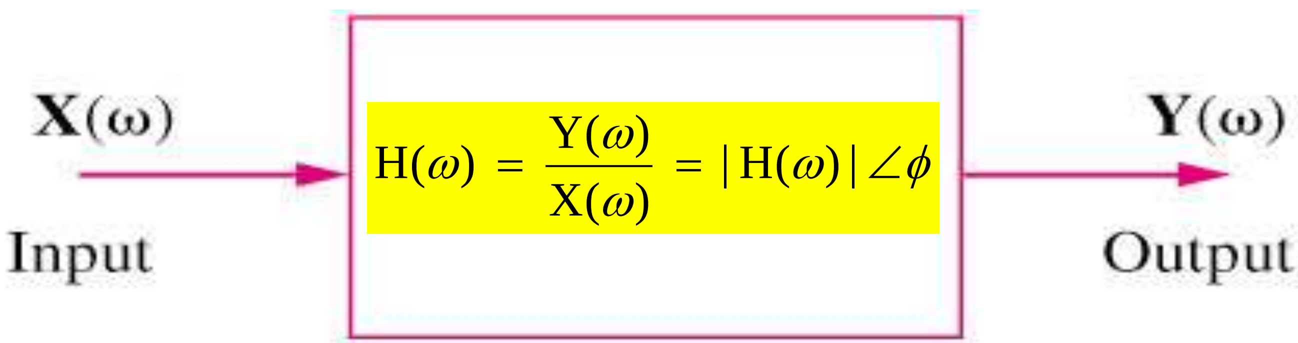
$$\boxed{\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}}$$

14.2 Transfer Function (2)

- 四种可能的传递函数:

$$H(\omega) = \text{Voltage gain} = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = \text{Transfer Impedance} = \frac{V_o(\omega)}{I_i(\omega)}$$



$$H(\omega) = \text{Current gain} = \frac{I_o(\omega)}{I_i(\omega)}$$

$$H(\omega) = \text{Transfer Admittance} = \frac{I_o(\omega)}{V_i(\omega)}$$

14.2 Transfer Function (3)

Example 14.1

For the *RC* circuit in Fig. 14.2(a), obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and its frequency response. Let $v_s = V_m \cos \omega t$.

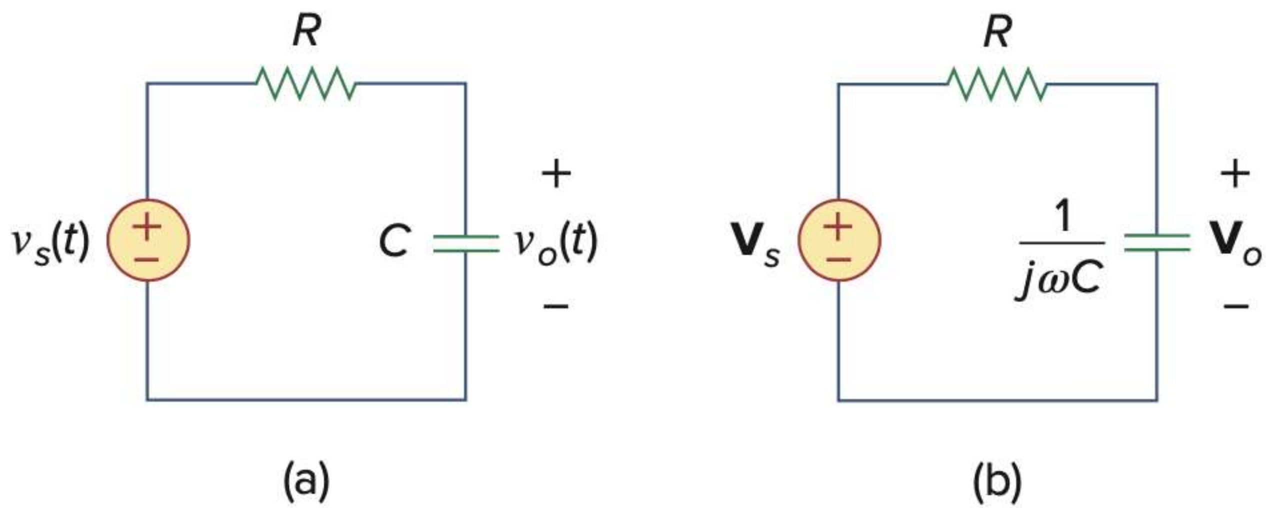


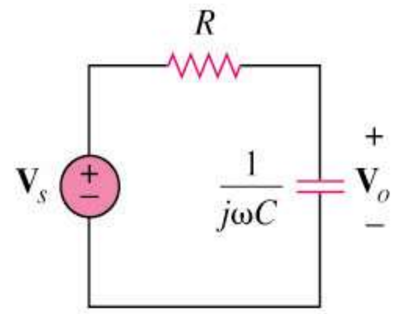
Figure 14.2
For Example 14.1: (a) time-domain *RC* circuit, (b) frequency-domain *RC* circuit.

14.2 Transfer Function (4)

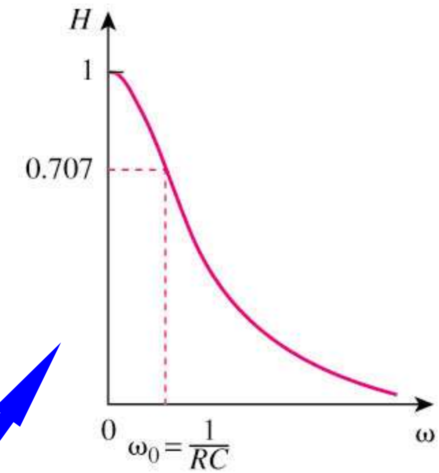
Solution:

The transfer function is

$$H(\omega)=\frac{V_o}{V_s}=\frac{\frac{1}{j\omega C}}{R+1/j\omega C}=\frac{1}{1+j\omega RC}=\frac{1-j\omega RC}{1+(\omega RC)^2}$$



(b)

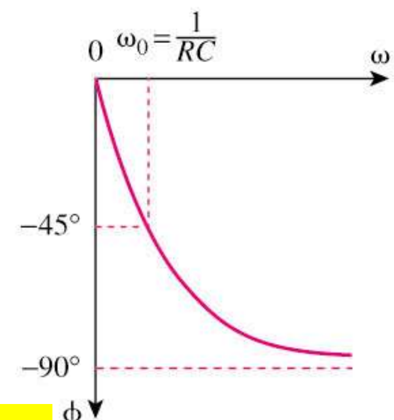


(a)

The **magnitude** is $$|H(\omega)|=\frac{1}{\sqrt{1+(\omega/\omega_o)^2}}$$

The **phase** is $$\phi=-\tan^{-1}\frac{\omega}{\omega_o}$$

$$\omega_o=1/RC$$



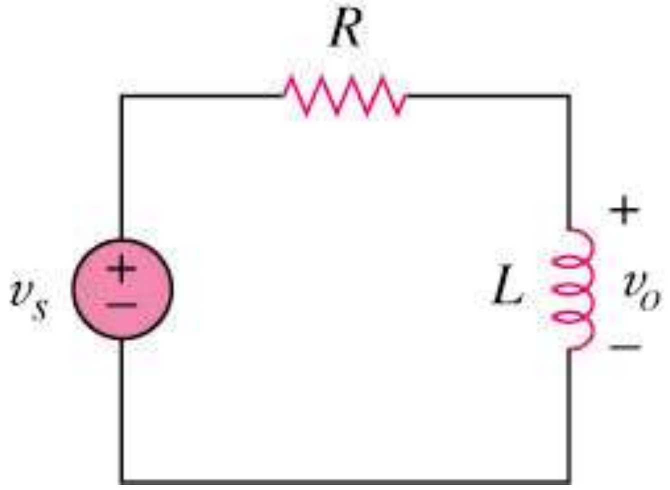
(b)

Low Pass Filter

14.2 Transfer Function (5)

Example 2

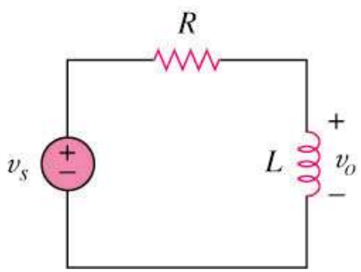
Obtain the transfer function $\mathbf{V}_o / \mathbf{V}_s$ of the *RL* circuit shown below, assuming $v_s = V_m\cos(\omega t)$. Sketch its frequency response.



14.2 Transfer Function (6)

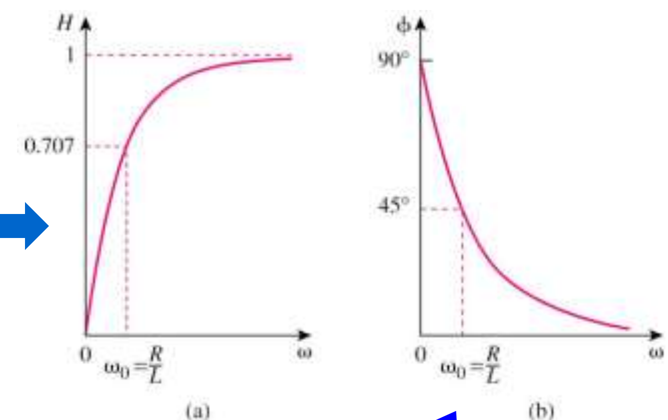
Solution:

The transfer function is $$\mathrm{H}(\omega)=\frac{\mathrm{V_o}}{\mathrm{V_s}}=\frac{\mathrm{j}\omega\mathrm{L}}{\mathrm{R}+\mathrm{j}\omega\mathrm{L}}=\frac{1}{1+\dfrac{\mathrm{R}}{\mathrm{j}\omega\mathrm{L}}}=\frac{1+jR/(\omega L)}{1+R^2/(\omega L)^2}$$



High Pass Filter

The magnitude is $$\mathrm{H}(\omega)=\frac{1}{\sqrt{1+(\frac{\omega_o}{\omega})^2}}$$



The phase is $$\phi=\angle 90^\circ-\tan^{-1}\frac{\omega}{\omega_o}$$

$$\omega_o=\mathrm{R/L}$$

零极点分析

$$\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)}$$

分子为零 → 传递函数的零点

分母为零 → 传递函数的极点

A **zero,** as a *root* of the numerator polynomial, is a value that results in a zero value of the function. A **pole,** as a *root* of the denominator polynomial, is a value for which the function is infinite.

A zero may also be regarded as the value of $s = j\omega$ that makes $\mathbf{H}(s)$ zero, and a pole as the value of $s = j\omega$ that makes $\mathbf{H}(s)$ infinite.

零极点的分析也往往在S域进行

Example 14.2

For the circuit in Fig. 14.6, calculate the g ain $\mathbf{I}_o(\omega)/\mathbf{I}_i(\omega)$ and its poles and zeros.

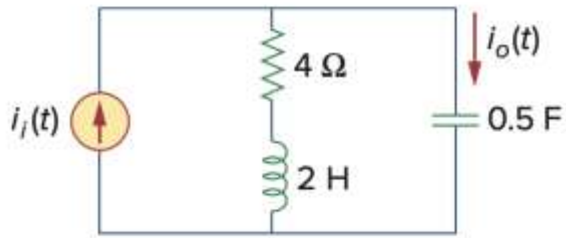


Figure 14.6
For Example 14.2.

Solution:

By current division,

$$\mathbf{I}_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + 1/j0.5\omega}\mathbf{I}_i(\omega)$$

or

$$\frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s+2)}{s^2 + 2s + 1}, \qquad s = j\omega$$

The zeros are at

$$s(s+2) = 0 \qquad \Rightarrow \qquad z_1 = 0,\ z_2 = -2$$

The poles are at

$$s^2 + 2s + 1 = (s+1)^2 = 0$$

Thus, there is a repeated pole (or double pole) at $p = -1$.

dB

- 对数运算法则

$$1.\ \log P_1P_2 = \log P_1 + \log P_2$$
$$2.\ \log P_1/P_2 = \log P_1 - \log P_2$$
$$3.\ \log P^n = n \log P$$
$$4.\ \log 1 = 0$$

- 将两个功率的比值用对数表示 → 单位为dB

$$\boxed{G_{\mathrm{dB}} = 10 \log_{10} \frac{P_2}{P_1}}$$

$$G_{\mathrm{dB}} = 10 \log_{10} 2 \simeq 3\ \mathrm{dB}$$
$$G_{\mathrm{dB}} = 10 \log_{10} 0.5 \simeq -3\ \mathrm{dB}$$

  - 功率与电压平方关联，所以若表示电压的比值：

$$\boxed{G_{\mathrm{dB}} = 20 \log_{10} \frac{V_2}{V_1}}$$

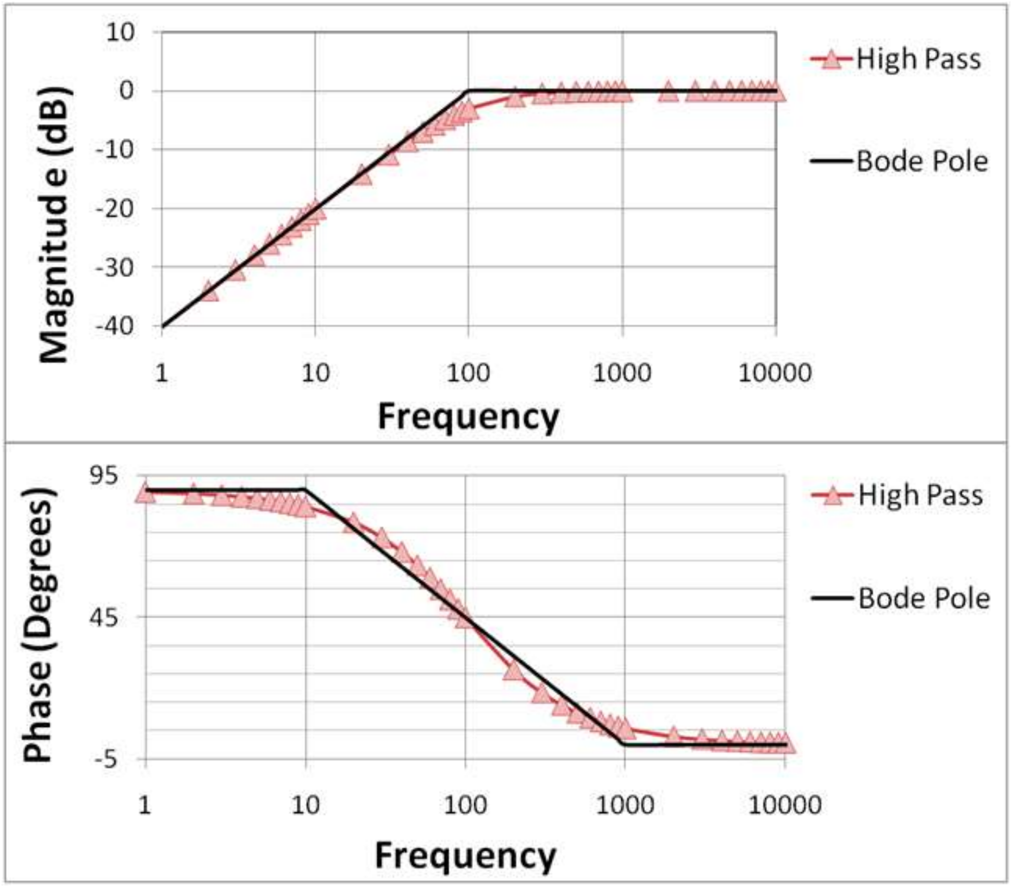
dB

Three things are important to note from Eqs. (14.5), (14.10), and (14.11):

1. That 10 $\log_{10}$ is used for power, while 20 $\log_{10}$ is used for voltage or current, because of the square relationship between them ($P = V^2/R = I^2R$).
2. That the dB value is a logarithmic measurement of the *ratio* of one variable to another *of the same type*. Therefore, it applies in expressing the transfer function H in Eqs. (14.2a) and (14.2b), which are dimensionless quantities, but not in expressing H in Eqs. (14.2c) and (14.2d).
3. It is important to note that we only use voltage and current magnitudes in Eqs. (14.10) and (14.11). Negative signs and angles will be handled independently as we will see in Section 14.4.

Bode Plots 波特图

- 电路的频率响应
  - 传递函数幅度 vs 频率
  - 传递函数相位 vs 频率
- 波特图
  - 描述频率响应的业界标准
  - 频率用对数坐标
  - 幅度用dB
  - 相位用角度



Bode plots are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

波特图

- 传递函数：响应（电压或电流）÷ 激励（电压或电流）

$$\boxed{H_{\mathrm{dB}} = 20 \log_{10} H}$$

系数是20

Magnitude H	$20 \log_{10} H$ (dB)
0.001	−60
0.01	−40
0.1	−20
0.5	−6
$1/\sqrt{2}$	−3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

常见的传递函数（如电压增益）
幅度所对应的dB值

波特图

- 分析电路，写出传递函数，转换为以下的**标准形式**

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots} \quad (14.15)$$

$\mathbf{H}(\omega)$ may include up to seven types of different factors that can appear in various combinations in a transfer function. These are:

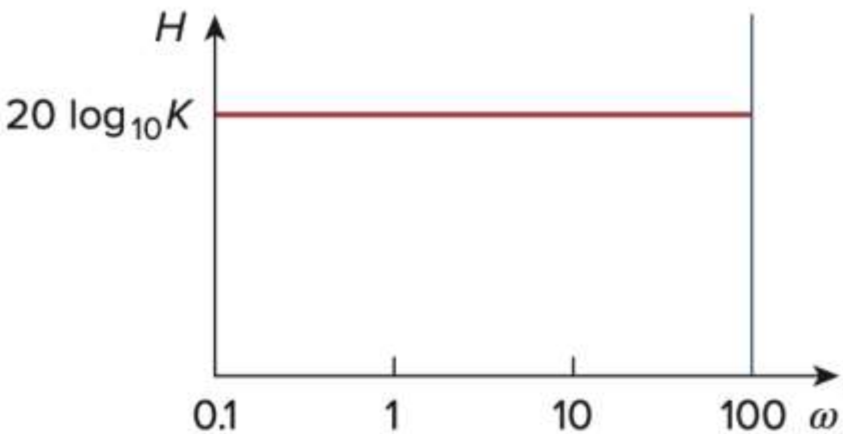
1. A gain K
2. A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin $\omega = 0$
3. A simple pole $1/(1 + j\omega/p_1)$ or zero $(1 + j\omega/z_1)$ $j\omega = -p_1$ *or* $-z_1$
4. A quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ or zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

幅度采用dB，增加了数学运算的方便性，在前计算机时代，大幅增加了工程计算便利性

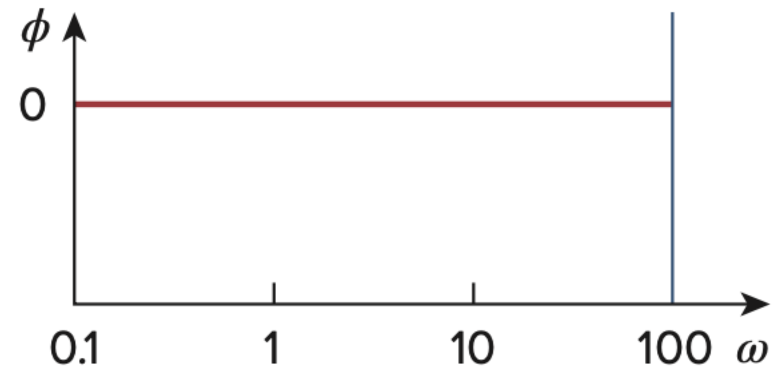
- H(ω)的幅度（dB）：各项代数和（乘除在对数坐标下转化为加减）
- H(ω)的相位（dB）：各项代数和

波特图

- ①常数项 $\mathbf{H}(\omega)=\dfrac{K(j\omega)^{\pm 1}(1+j\omega/z_1)[1+j2\zeta_1\omega/\omega_k+(j\omega/\omega_k)^2]\cdots}{(1+j\omega/p_1)[1+j2\zeta_2\omega/\omega_n+(j\omega/\omega_n)^2]\cdots}$



(a)



(b)

波特图

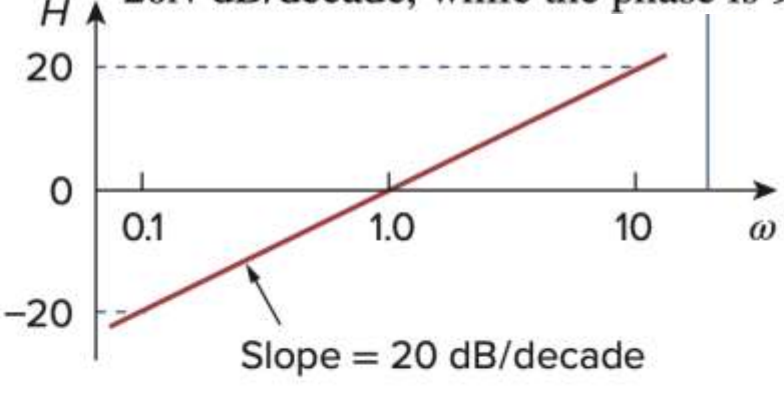
- ②③原点处的零极点 $\mathbf{H}(\omega) = \dfrac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$

Pole/zero at the origin: For the zero $(j\omega)$ at the origin, the magnitude is $20 \log_{10} \omega$ and the phase is 90°. These are plotted in Fig. 14.10, where we notice that the slope of the magnitude plot is 20 dB/decade, while the phase is constant with frequency. **Q1：极点呢？**

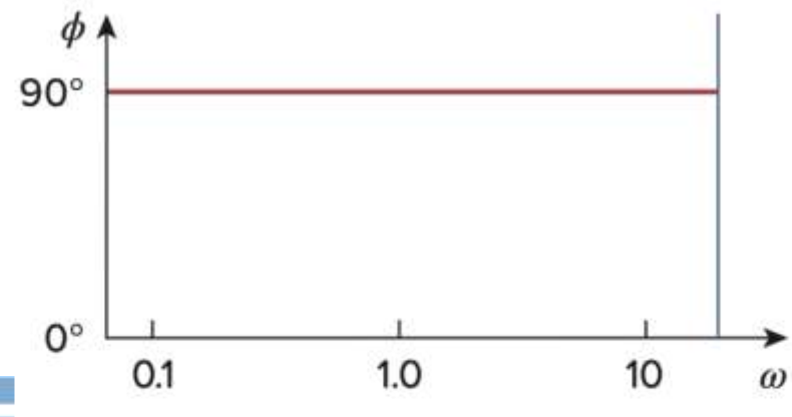
−20 dB/decade while the phase is −90°

Q2：原点处的N重零极点呢？

for $(j\omega)^N$, where N is an integer, the magnitude plot will have a slope of $20N$ dB/decade, while the phase is $90N$ degrees.



(a)



(b)

- ④⑤一阶零极点

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

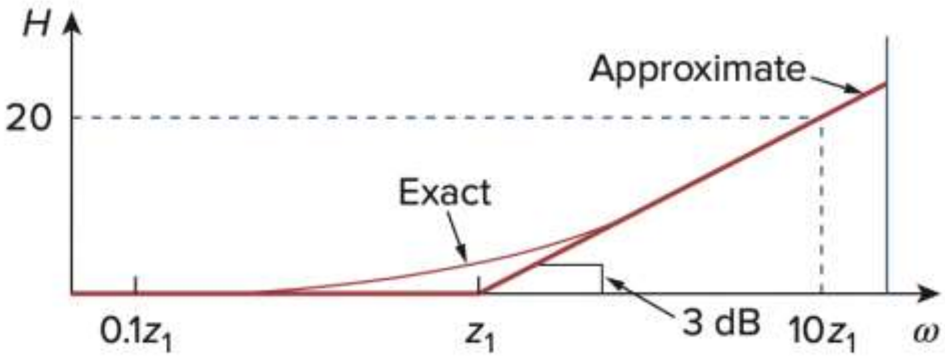
Simple pole/zero: For the simple zero $(1 + j\omega/z_1)$, the magnitude is $20\log_{10}|1 + j\omega/z_1|$ and the phase is $\tan^{-1}\omega/z_1$. We notice that

$$H_{dB} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Rightarrow \quad 20\log_{10}1 = 0 \quad \text{as} \quad \omega \to 0 \tag{14.16}$$

* 勾勒出近似曲线（**培养直觉**）
* 精确曲线用数学软件可以画出

$$H_{dB} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Rightarrow \quad 20\log_{10}\frac{\omega}{z_1} \quad \text{as} \quad \omega \to \infty \tag{14.17}$$

showing that we can approximate the magnitude as zero (a straight line with zero slope) for small v alues of ω and by a straight line with slope 20 dB/decade for large values of ω. The frequency $\omega = z_1$ where the two asymptotic lines meet is called the *corner frequency* or *break frequency*.



(a)

一阶零极点项 $(1 + j\omega/z_1)$ $(1 + j\omega/p_1)$
对应着一个corner frequency，
有时也简单地把这个corner frequency
称为零极点频率

corner frequency 处有3 dB误差

- ④⑤一阶零极点

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

The phase $\tan^{-1}(\omega/z_1)$ can be expressed as

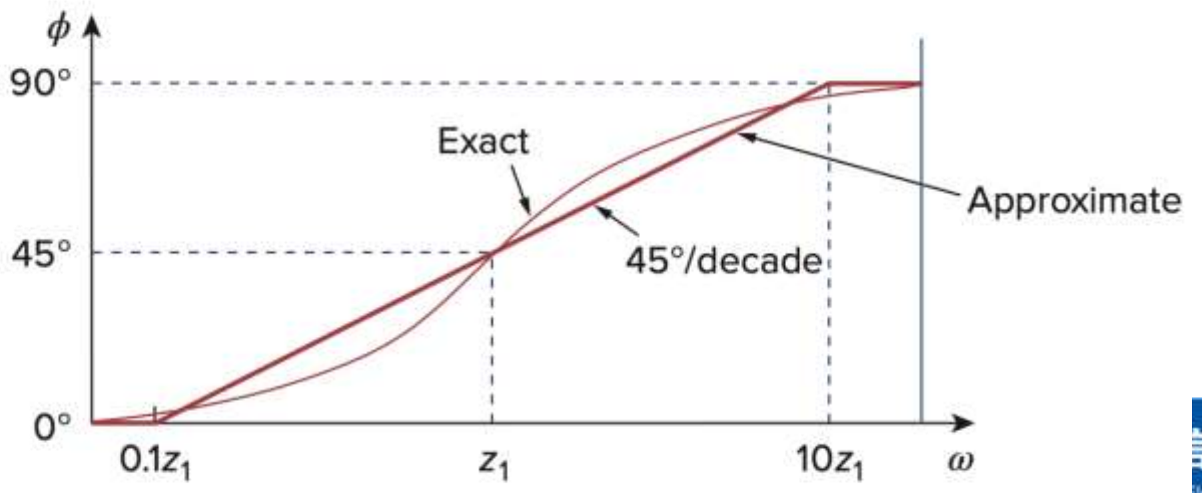
工程上差10倍即可略去

$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \to \infty \end{cases} \qquad \textbf{(14.18)}$$

As a straight-line approximation, we let $\phi \simeq 0$ for $\omega \leq z_1/10$, $\phi \simeq 45^\circ$ for $\omega = z_1$, and $\phi \simeq 90^\circ$ for $\omega \geq 10z_1$. As shown in Fig. 14.11(b) along with the actual plot, the straight-line plot has a slope of 45° per decade.

Q：一阶极点呢？

Fig. 14.11 except that the corner frequency is at $\omega = p_1$, the magnitude has a slope of −20 dB/decade, and the phase has a slope of −45° per decade.



- ⑥⑦二阶零极点

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

Quadratic pole/zer o: The magnitude of the quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ is $-20\log_{10}|1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2|$ and the phase is $-\tan^{-1}(2\zeta_2\omega/\omega_n)/(1 - \omega^2/\omega_n^2)$. But

$$H_{\mathrm{dB}} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \quad \Rightarrow \quad 0$$

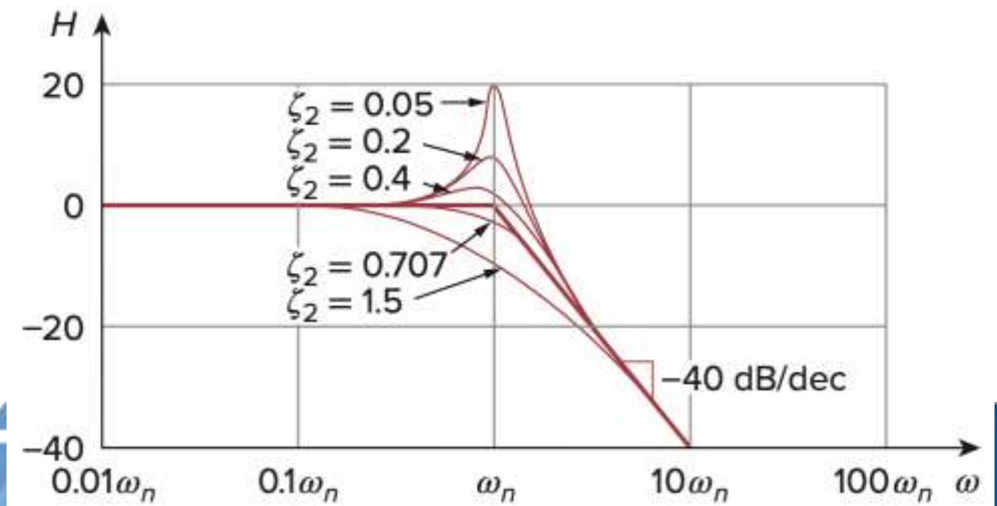
$$\text{as} \quad \omega \to 0$$

$$H_{\mathrm{dB}} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right| \quad \Rightarrow \quad -40\log_{10}\frac{\omega}{\omega_n}$$

$$\text{as} \quad \omega \to \infty$$

Thus, the amplitude plot consists of tw o straight asymptotic lines: one with zero slope for $\omega < \omega_n$ and the other with slope −40 dB/decade for $\omega > \omega_n$, with ω_n as the corner frequenc y. Figure 14.12(a) sho ws

corner frequency 处有误差，误差值取决于一次项系数



- ⑥⑦二阶零极点

$$\mathbf{H}(\omega)=\frac{K(j\omega)^{\pm1}(1+j\omega/z_1)[1+j2\zeta_1\omega/\omega_k+(j\omega/\omega_k)^2]\cdots}{(1+j\omega/p_1)[1+j2\zeta_2\omega/\omega_n+(j\omega/\omega_n)^2]\cdots}$$

The phase can be expressed as

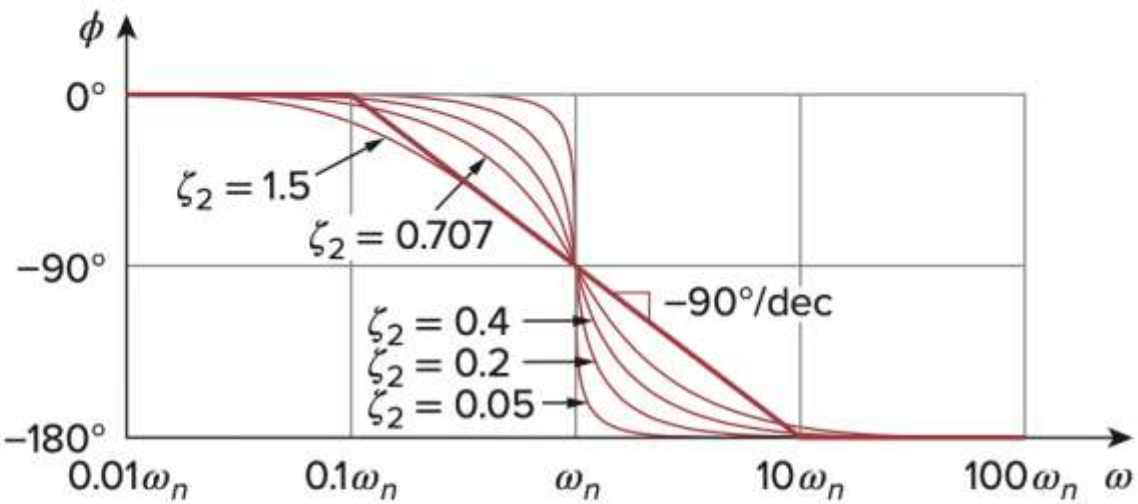
$$\phi=-\tan^{-1}\frac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2}=\begin{cases}0, & \omega=0\\ -90^\circ, & \omega=\omega_n\\ -180^\circ, & \omega\to\infty\end{cases} \qquad (14.21)$$

The phase plot is a straight line with a slope of −90° per decade starting at $\omega_n/10$ and ending at $10\omega_n$, as shown in Fig. 14.12(b). We see again that

Q1：二阶零点呢？ 40 dB/decade while the phase plot has a slope of 90° per decade.

Q2：双重零极点呢？ the double pole $(1+j\omega/\omega_n)^{-2}$ equals the quadratic pole $1/[1+j2\zeta_2\omega/\omega_n+(j\omega/\omega_n)^2]$ when $\zeta_2=1$. Thus, the quadratic pole can be treated as a double pole as far as straight-line approximation is concerned.

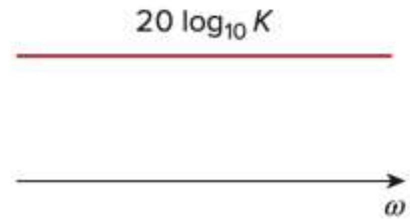
二阶零极点的分析，可简单地用双重零极点的分析来近似

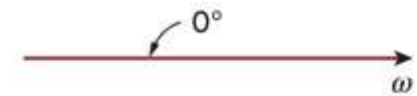


Summary of Bode straight-line magnitude and phase plots.

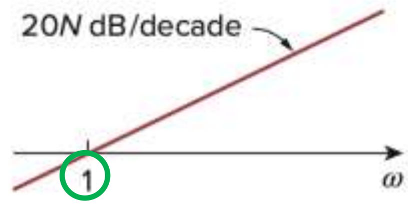
Factor | **Magnitude** | **Phase**

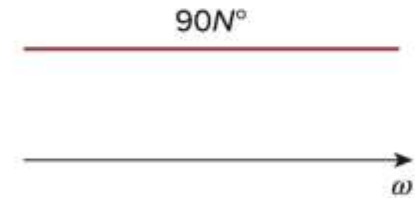
K



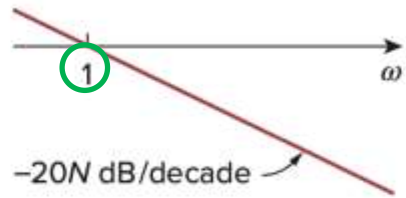


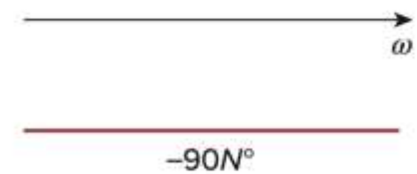
$(j\omega)^N$



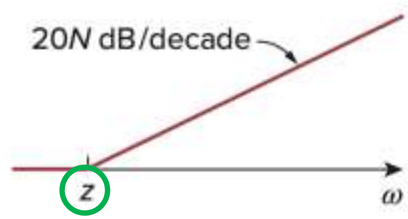


$\dfrac{1}{(j\omega)^N}$

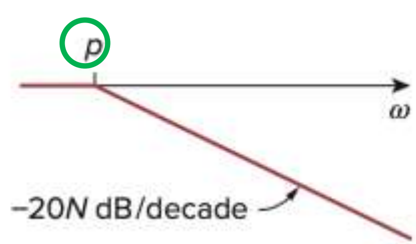


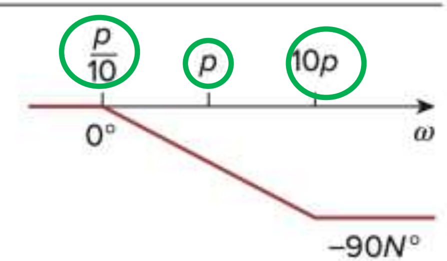


$$\left(1+\frac{j\omega}{z}\right)^N$$

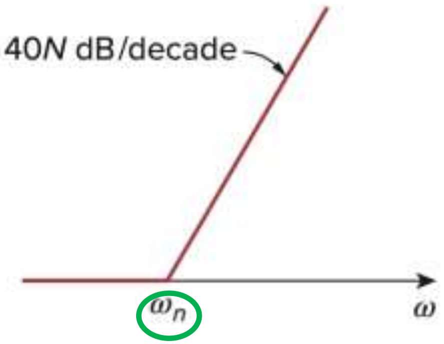


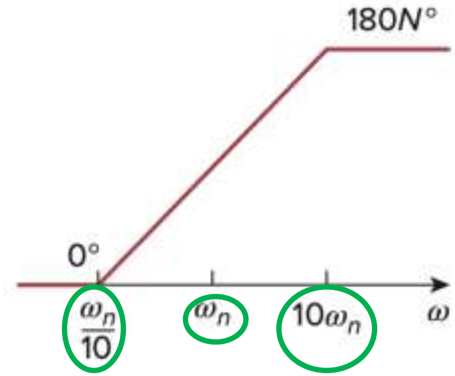
$\dfrac{1}{(1+j\omega/p)^N}$



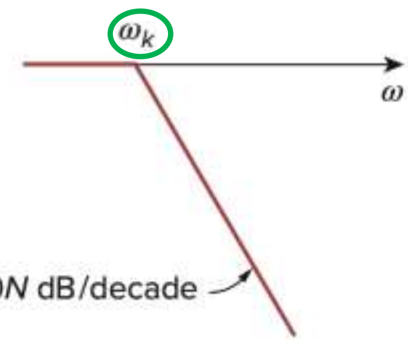


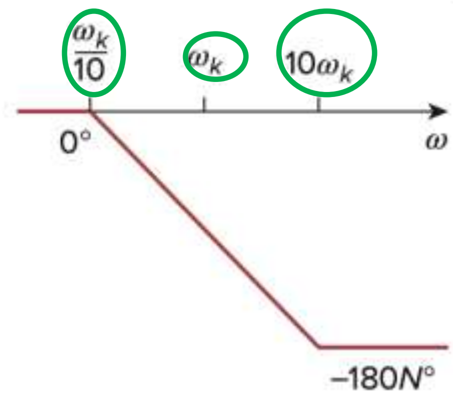
$$\left[1+\frac{2j\omega\zeta}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]^N$$





$$\frac{1}{[1+2j\omega\zeta/\omega_k+(j\omega/\omega_k)^2]^N}$$





- 构建波特图的方法
  - 方法1: 按上述规则，做各项的代数和
  - 方法2: 先分析常数项和原点处的零极点，然后，随着频率的增加，**每碰到一个零点**，幅度的斜率**增加20dB/十倍频**，相位**增加90**度（从0.1到10倍频率）；**每碰到一个极点**，幅度的斜率**减小20dB/十倍频**，相位**减小90度**（从0.1到10倍频率）

Example 14.3

Construct the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{200j\omega}{(j\omega+2)(j\omega+10)}$$

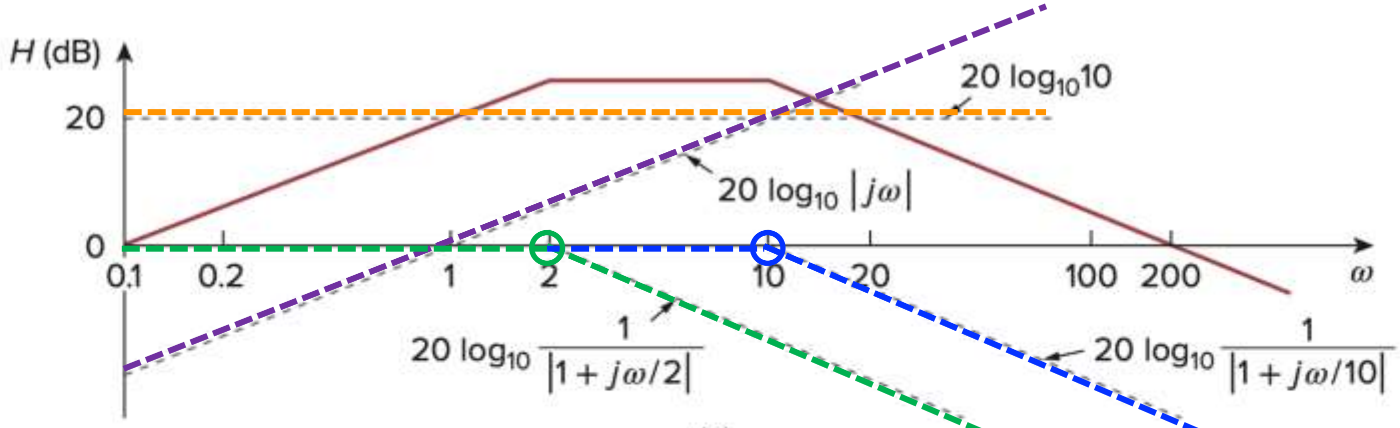
方法1:

①转换为标准形式: $$\mathbf{H}(\omega) = \frac{10j\omega}{(1+j\omega/2)(1+j\omega/10)}$$

先画常数或零点，或全部各自画出

②分析常数项和零极点：增益10；原点处零点ω=0；一阶极点ω =2， ω =10

③画出各部分的波特图，再做代数和



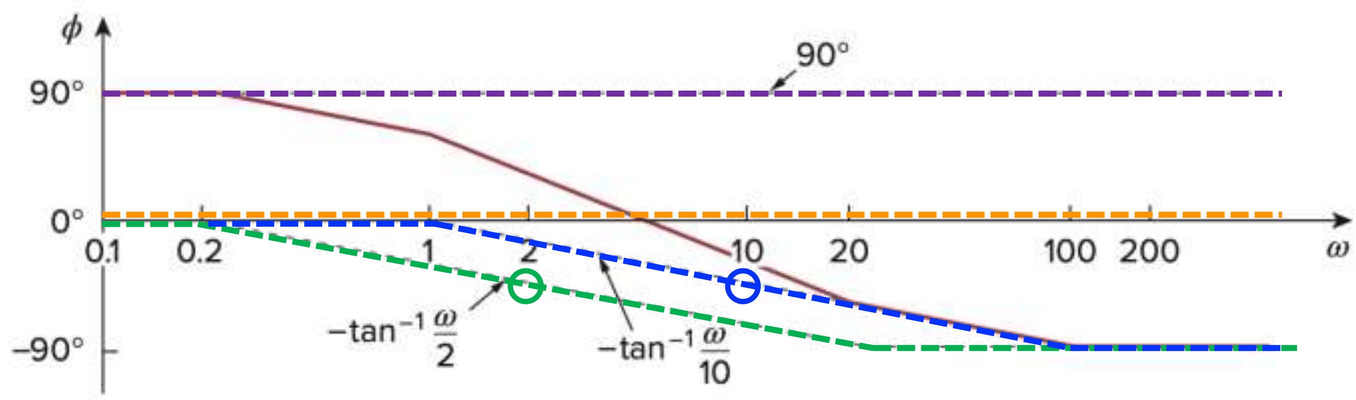
(a)

方法2:

①先分析常数项和原点处的零极点

②按低频到高频的顺序，逐渐分析各个零极点的影响

* 若零极点之间频率间隔太近，相位变化起止点会有重叠



(b)

Example 14.6

Given the Bode plot in Fig. 14.19, obtain the transfer function $\mathbf{H}(\omega)$.

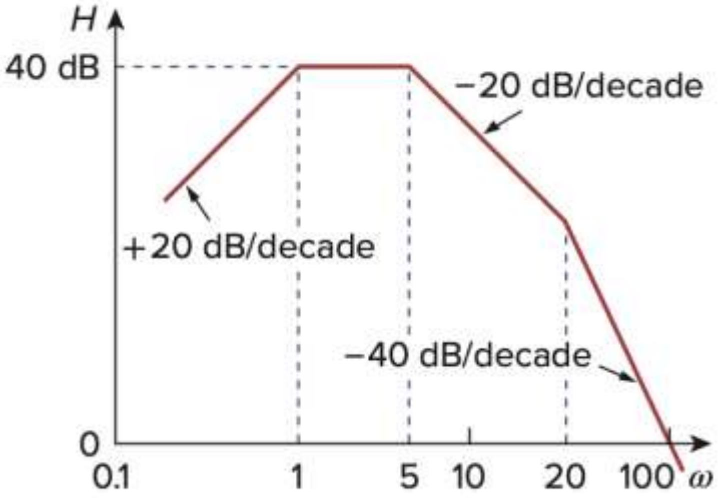


Figure 14.19
For Example 14.6.

①低频的“20dB/十倍频”斜率是原点处的零点引起的，其与横轴的交点为ω=1， 因此ω=1处实际的 40dB幅度是常数项导致的

$$40 = 20\log_{10} K \quad \Rightarrow \quad K = 10^2 = 100$$

② ω=1处斜率下降“20dB/十倍频”，所以ω=1是一个极点

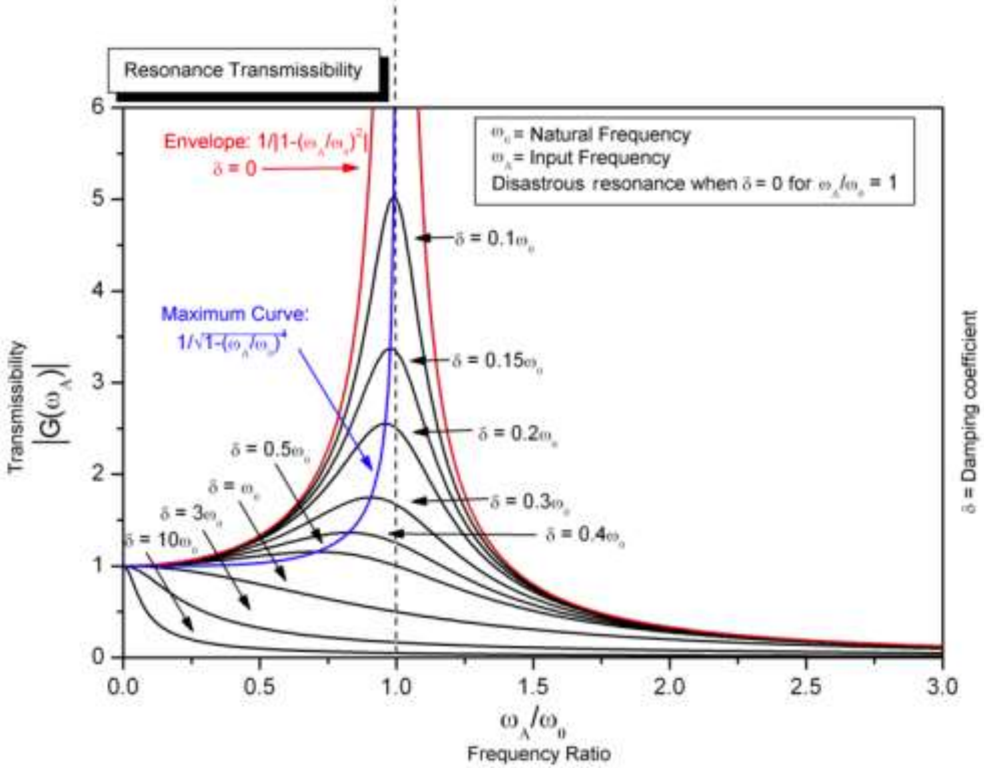
③ ω=5处斜率再下降“20dB/十倍频”，所以ω=5也是一个极点

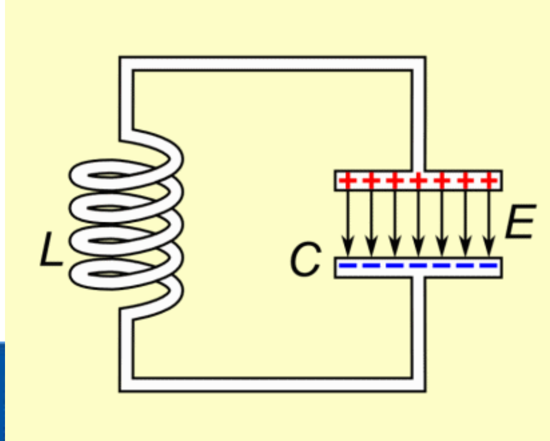
④ 同理，ω=20也是一个极点

$$\mathbf{H}(\omega) = \frac{100\, j\omega}{(1 + j\omega/1)(1 + j\omega/5)(1 + j\omega/20)}$$

谐振电路

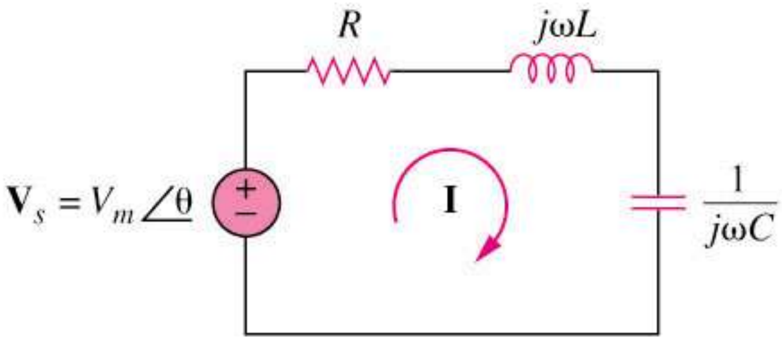
- 谐振时频率响应曲线会出现一个尖峰（sharp peak）
- 电路（或系统）如果有一对共轭极点，则会产生谐振
- 若电路中至少有一个L和一个C，则会产生谐振
- 谐振是系统中不同能量之间的来回互换
- 谐振电路是滤波器的基础





14.3 Series Resonance 串联谐振

Resonance is a condition in an *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.



$$Z = R + j(\omega L - \frac{1}{\omega C})$$

谐振频率

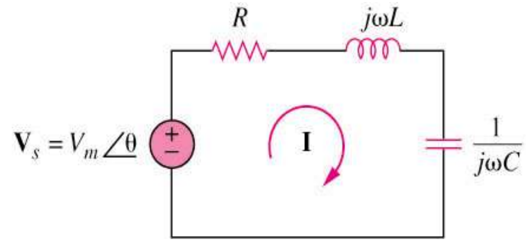
Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s} \qquad or$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

谐振条件：虚部 = 0

14.3 Series Resonance (2)



$$Z = R + j(\omega L - \frac{1}{\omega C})$$

串联谐振时的特性:

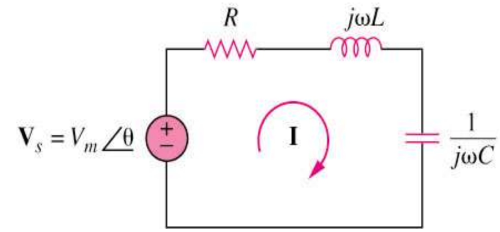
- The impedance is **purely resistive**, Z = R;
- The supply voltage $\mathbf{V}_s$ and the current $\mathbf{I}$ are **in phase**;
- The magnitude of the transfer function H(ω) = Z(ω) is minimum; 串联谐振时，阻抗幅度最小 → 电流最大
- The inductor voltage and capacitor voltage can be much more than the source voltage. 串联谐振时，电容/电感上的电压幅度会大于所施加的电压源

谐振电路里，Q一般都大于10，
也可达到几千甚至几万

$$|\mathbf{V}_L| = \frac{V_m}{R}\omega_0 L = QV_m$$

$$|\mathbf{V}_C| = \frac{V_m}{R}\frac{1}{\omega_0 C} = QV_m$$

14.3 Series Resonance (3)

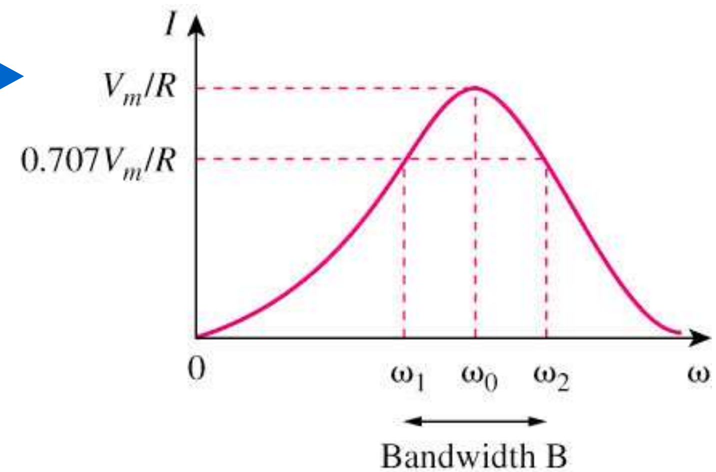


$$Z = R + j(\omega L - \frac{1}{\omega C})$$

Bandwidth（带宽） B

The **frequency response** of the resonance circuit current is

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$



The average power absorbed by the RLC circuit is

$$P(\omega) = \frac{1}{2} I^2 R$$

The highest power dissipated occurs at resonance:

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$

谐振时响应：电流特性
（施加信号为电压时）

浙江大学 信息与电子工程学院
College of Information Science & Electronic Engineering, Zhejiang University

14 3 Series Resonance (4)

Half-power frequencies ω_1 and ω_2 are frequencies at which the dissipated power is half the maximum value:

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$$

The half-power frequencies can be obtained by setting $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

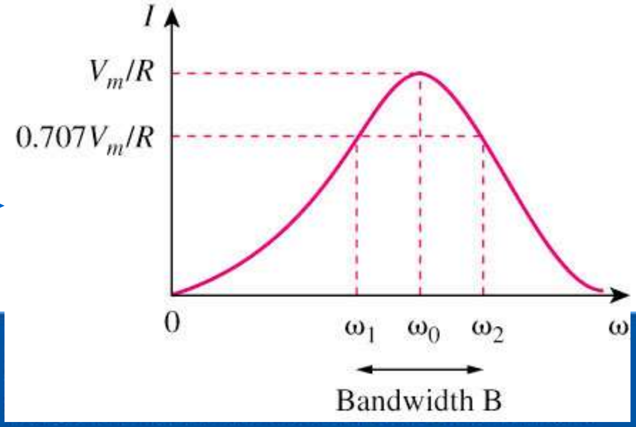
$$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$

带宽的定义：两个半功率频率点之间的频率差

Bandwidth B

$$B = \omega_2 - \omega_1$$



14.3 Series Resonance (5)

Quality factor,

品质因数

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R}$$

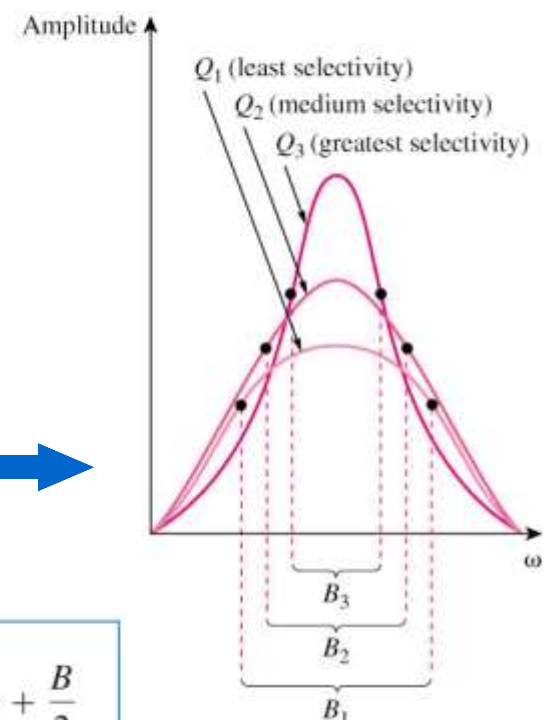
R 越小，*Q* 越大

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}}$$

$$\boxed{B = \frac{R}{L} = \frac{\omega_0}{Q}}$$

R 越小，*B* 越小

- 曲线越窄 （带宽越小）→ **Q** 值越大；
- **Q** 值的测量方法：中心频率 ÷ 带宽



$$\omega_o = \sqrt{\omega_1 \omega_2}$$

If Q > 10

实际工程近似

$$\boxed{\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}}$$

In the circuit of Fig. 14.24, $R = 2\ \Omega$, $L = 1$ mH, and $C = 0.4\ \mu$F. (a) Find the resonant frequency and the half-power frequencies. (b) Calculate the quality factor and bandwidth. (c) Determine the amplitude of the current at ω_0, ω_1, and ω_2.

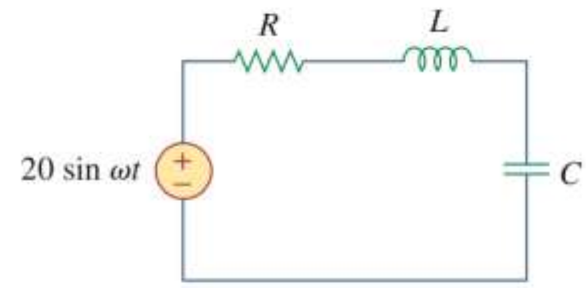


Figure 14.24
For Example 14.7.

Solution:

(a) The resonant frequency is

① $$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/s}$$

② $$\boxed{Q = \frac{\omega_0 L}{R}} = \frac{50 \times 10^3 \times 10^{-3}}{2} = 25$$

From Q, we find

③ $$\boxed{B = \frac{\omega_0}{Q}} = \frac{50 \times 10^3}{25} = 2 \text{ krad/s}$$

Since $Q > 10$, this is a high-Q circuit and we can obtain the half-power frequencies as

④ $$\boxed{\omega_1 = \omega_0 - \frac{B}{2}} = 50 - 1 = 49 \text{ krad/s}$$

⑤ $$\boxed{\omega_2 = \omega_0 + \frac{B}{2}} = 50 + 1 = 51 \text{ krad/s}$$

串联谐振电路的5个基本参数

思考：若不知RLC具体值，但可以测量出电流响应曲线（频率可直接读取），如何决定Q、B、R、L、C？

(c) At $\omega = \omega_0$,

$$I = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$$

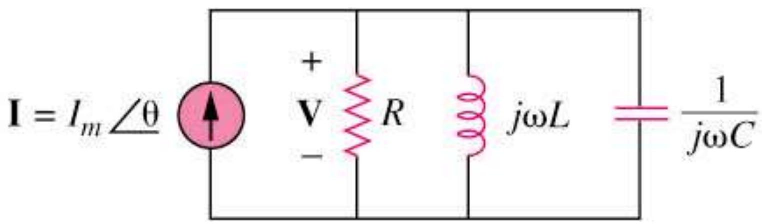
At $\omega = \omega_1, \omega_2$,

$$I = \frac{V_m}{\sqrt{2}R} = \frac{10}{\sqrt{2}} = 7.071 \text{ A}$$

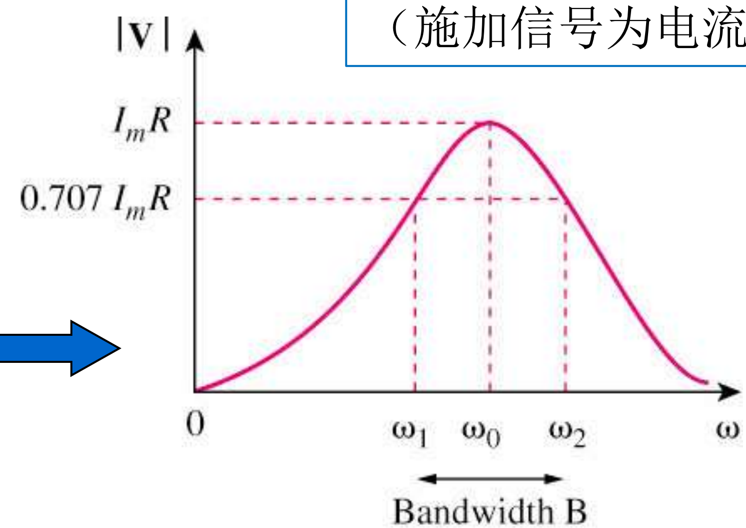
14.4 Parallel Resonance并联谐振

It occurs when imaginary part of Y is zero

谐振时响应：电压特性（施加信号为电流时）



$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$



Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{rad/s} \quad \text{or} \quad f_o = \frac{1}{2\pi\sqrt{LC}} Hz$$

$$|\mathbf{I}_L| = \frac{I_m R}{\omega_0 L} = Q I_m$$

$$|\mathbf{I}_C| = \omega_0 C I_m R = Q I_m$$

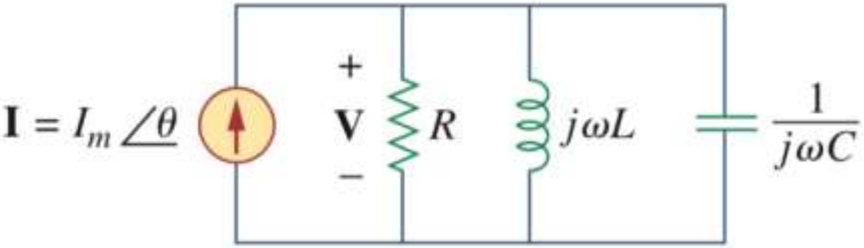


Figure 14.25
The parallel resonant circuit.

实际用不到

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

是串联谐振Q公式的倒数

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

并联时R越大损耗越小

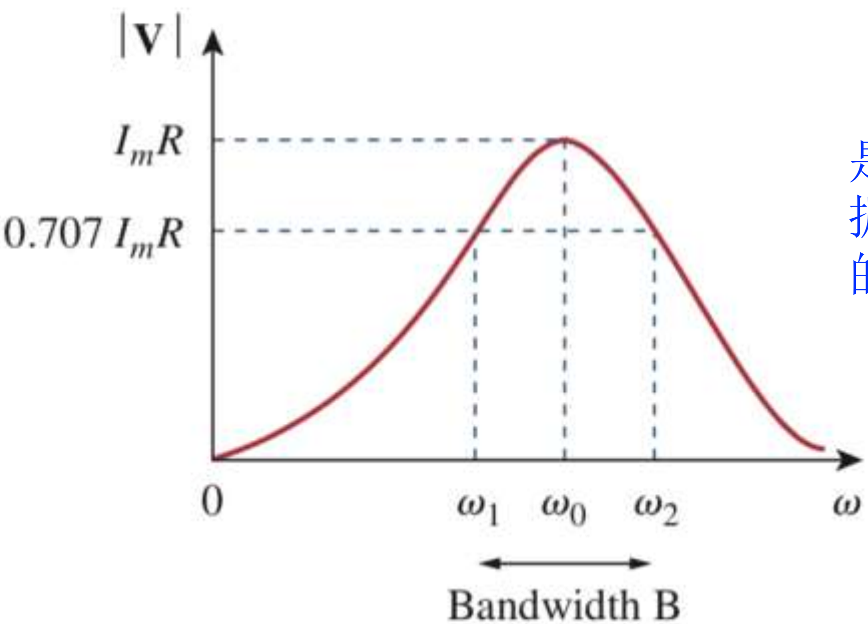


Figure 14.26
The current amplitude versus frequency for the series resonant circuit of Fig. 14.25.

Again, for high-Q circuits ($Q \geq 10$)

实际工程近似用：

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

14.4 串并联比较

Summary of series and parallel resonance circuits:

characteristic	*Series circuit*	*Parallel circuit*
① ω_o	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
② Q	$\frac{\omega_o L}{R}$ or $\frac{1}{\omega_o RC}$	$\frac{R}{\omega_o L}$ or $\omega_o RC$
③ B	$\frac{\omega_o}{Q}$	$\frac{\omega_o}{Q}$
$\omega_1,\ \omega_2$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$
$Q \geq 10$, $\omega_1,\ \omega_2$ ④ ⑤	$\omega_o \pm \frac{B}{2}$	$\omega_o \pm \frac{B}{2}$

谐振电路的5个基本参数

Example 14.8

In the parallel *RLC* circuit of Fig. 14.27, let $R = 8\ \text{k}\Omega$, $L = 0.2$ mH, and $C = 8\ \mu$F. (a) Calculate ω_0, Q, and B. (b) Find ω_1 and ω_2. (c) Determine the power dissipated at ω_0, ω_1, and ω_2.

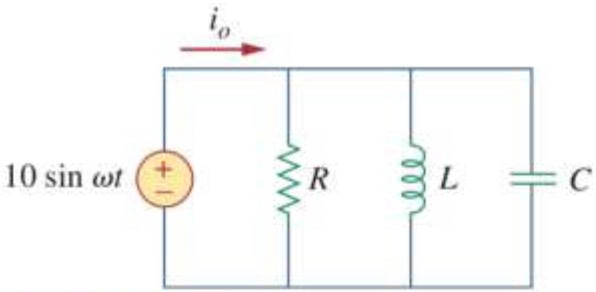


Figure 14.27
For Example 14.8.

Solution:

(a)

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 8 \times 10^{-6}}} = \frac{10^5}{4} = 25\ \text{krad/s}$$

$$\boxed{Q = \frac{R}{\omega_0 L}} = \frac{8 \times 10^3}{25 \times 10^3 \times 0.2 \times 10^{-3}} = 1{,}600$$

$$\boxed{B = \frac{\omega_0}{Q}} = 15.625\ \text{rad/s}$$

$$\boxed{\omega_1 = \omega_0 - \frac{B}{2}} = 25{,}000 - 7.812 = 24{,}992\ \text{rad/s}$$

$$\boxed{\omega_2 = \omega_0 + \frac{B}{2}} = 25{,}000 + 7.812 = 25{,}008\ \text{rad/s}$$

(c) At $\omega = \omega_0$, $\mathbf{Y} = 1/R$ or $\mathbf{Z} = R = 8\ \text{k}\Omega$. Then

$$\mathbf{I}_o = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\angle -90^\circ}{8{,}000} = 1.25\angle -90^\circ\ \text{mA}$$

Since the entire current flows through R at resonance, the average power dissipated at $\omega = \omega_0$ is

$$P = \frac{1}{2}|\mathbf{I}_o|^2 R = \frac{1}{2}(1.25 \times 10^{-3})^2(8 \times 10^3) = 6.25\ \text{mW}$$

At $\omega = \omega_1, \omega_2$,

$$P = \frac{V_m^2}{4R} = 3.125\ \text{mW}$$

Example 14.9

Determine the resonant frequency of the circuit in Fig. 14.28.

Solution:

The input admittance is

$$\mathbf{Y} = j\omega 0.1 + \frac{1}{10} + \frac{1}{2 + j\omega 2} = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$ and

$$\omega_0 0.1 - \frac{2\omega_0}{4 + 4\omega_0^2} = 0 \quad \Rightarrow \quad \omega_0 = 2 \text{ rad/s}$$

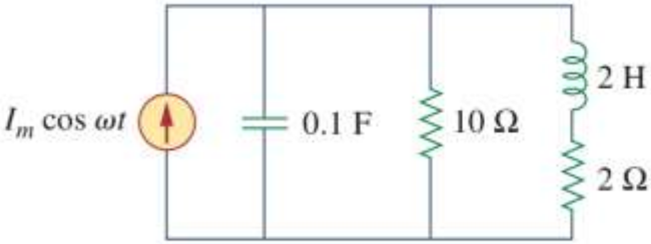


Figure 14.28
For Example 14.9.

当电路不是简单的RLC并联或RLC串联时：
写出传递函数分析谐振频率

14.5 Passive Filters (1)

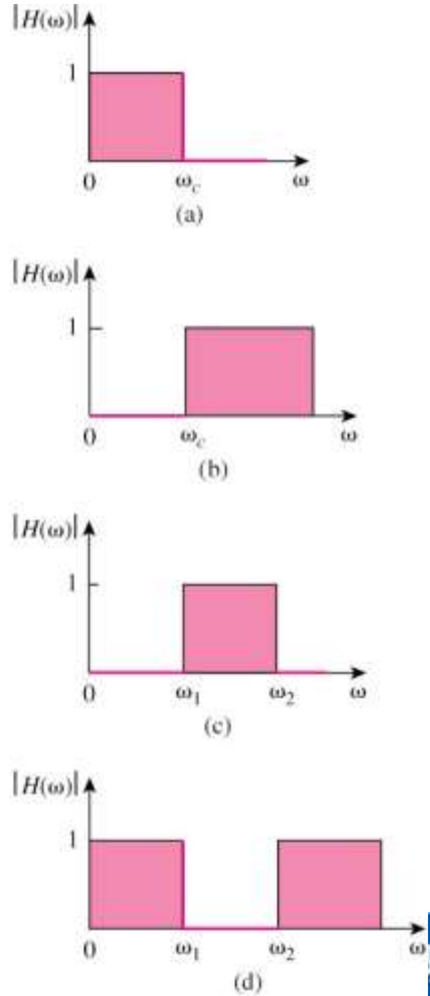
- A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

- Passive filter consists of only passive element R, L and C.

- There are four types of filters.

理想滤波器的传递函数

Low Pass 低通滤波器
High Pass 高通滤波器
Band Pass 带通滤波器
Band Stop 带阻滤波器



低通滤波器典型案例

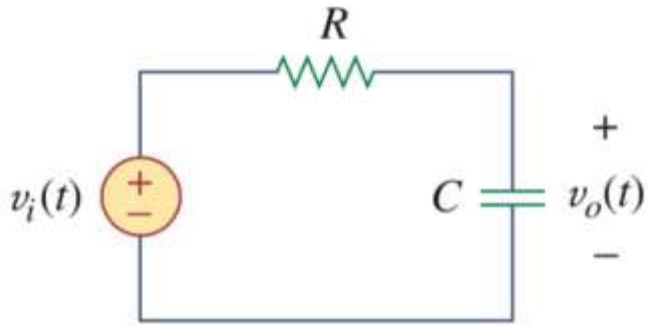


Figure 14.31
A lowpass filter.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

A **lowpass filter** is designed to pass only frequencies from dc up to the cutoff frequency ω_c.

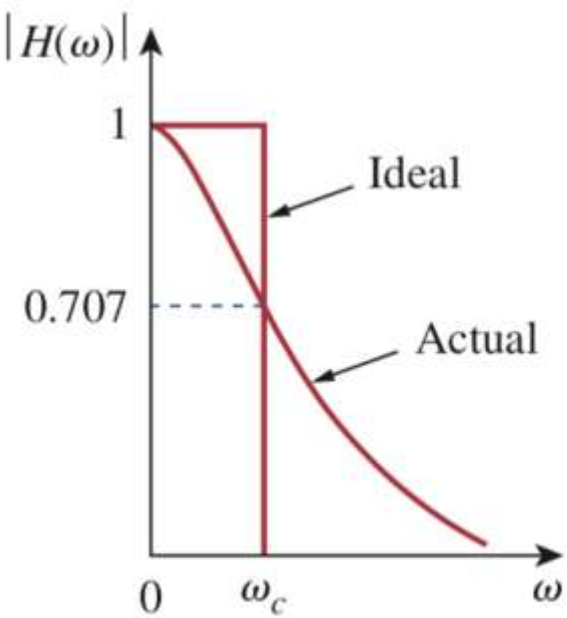


Figure 14.32
Ideal and actual frequency response of a lowpass filter.

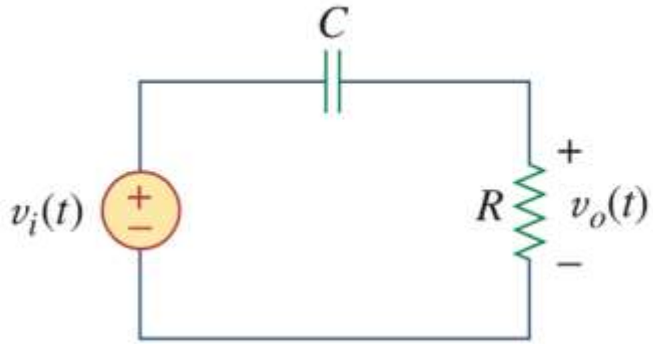
半功率处的频率称为“**截止频率**”(cutoff frequency)

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{RC}$$

判断滤波特性的方法：special cases

高通滤波器典型案例



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

Figure 14.33
A highpass filter.

A **highpass filter** is designed to pass all frequencies above its cutoff frequency ω_c.

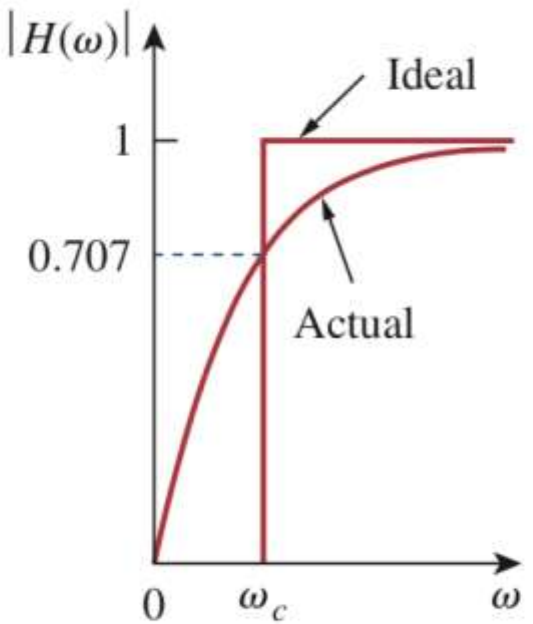


Figure 14.34
Ideal and actual frequency response of a highpass filter.

半功率处的频率称为“截止频率”(cutoff frequency)

$$\omega_c = \frac{1}{RC}$$

带通滤波器典型案例

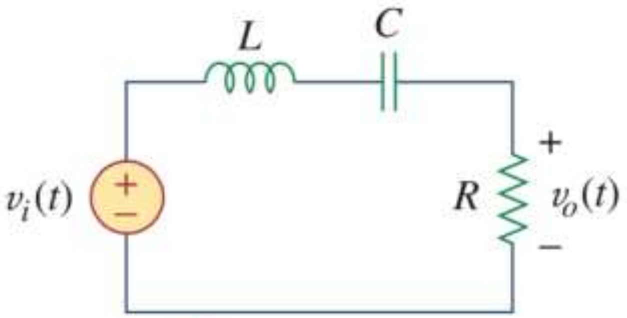


Figure 14.35
A bandpass filter.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

A **bandpass filter** is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

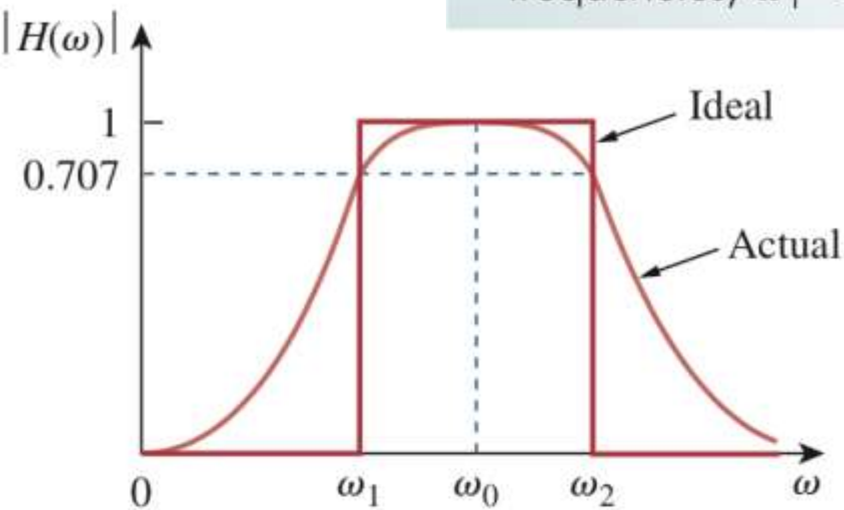


Figure 14.36
Ideal and actual frequency response of a bandpass filter.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

带阻滤波器典型案例

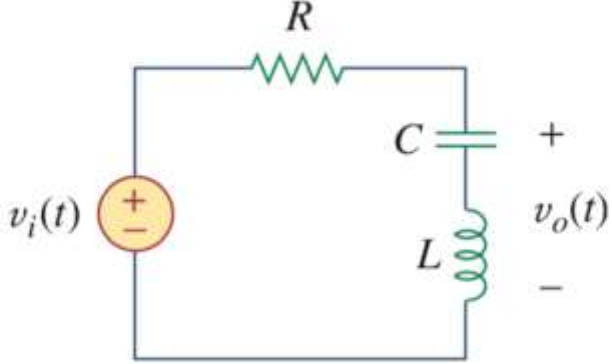


Figure 14.37
A bandstop filter.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

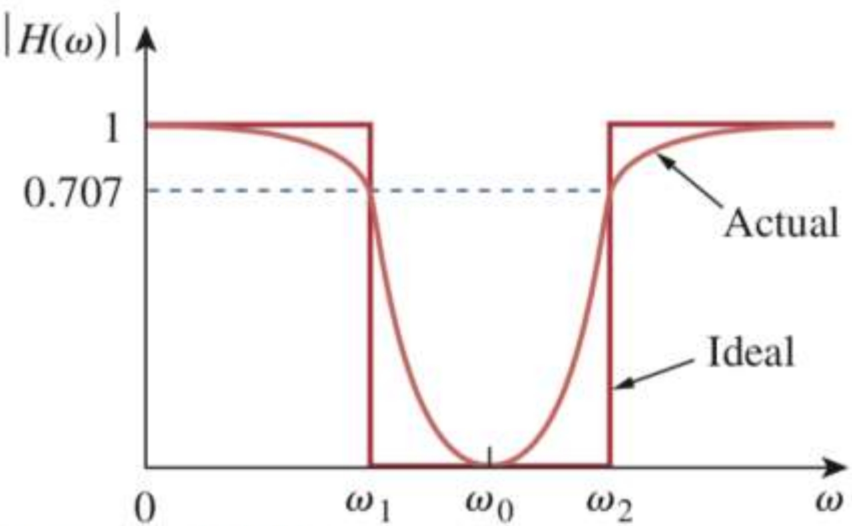


Figure 14.38
Ideal and actual frequency response of a bandstop filter.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Example 14.10

Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2\ \text{k}\Omega$, $L = 2$ H, and $C = 2\ \mu$F.

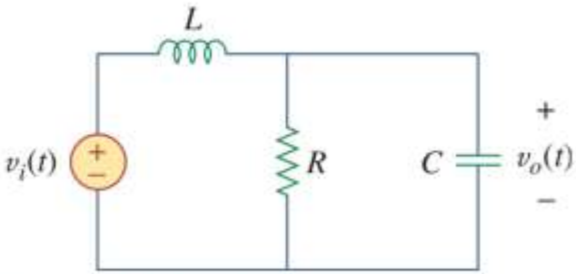


Figure 14.39
For Example 14.10.

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \qquad s = j\omega$$

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$. $\Longrightarrow$ 猜测低通滤波器

传递函数的幅度：$H = \dfrac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}}$

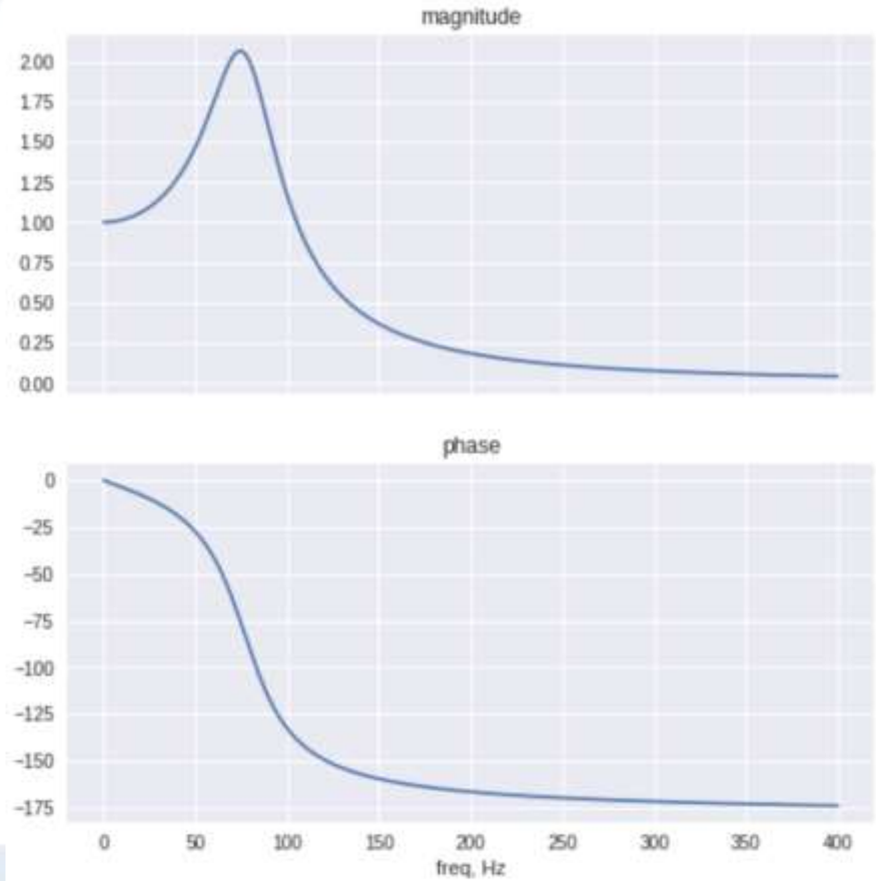
$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

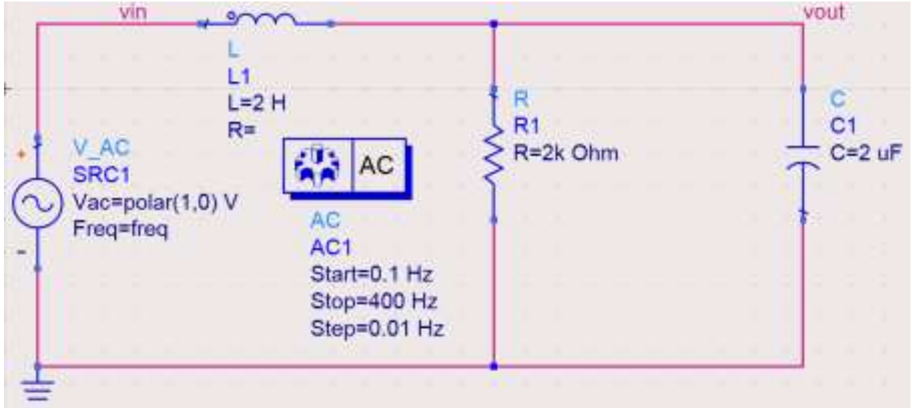
$$\omega_c = 0.742\ \text{krad/s} = 742\ \text{rad/s}$$

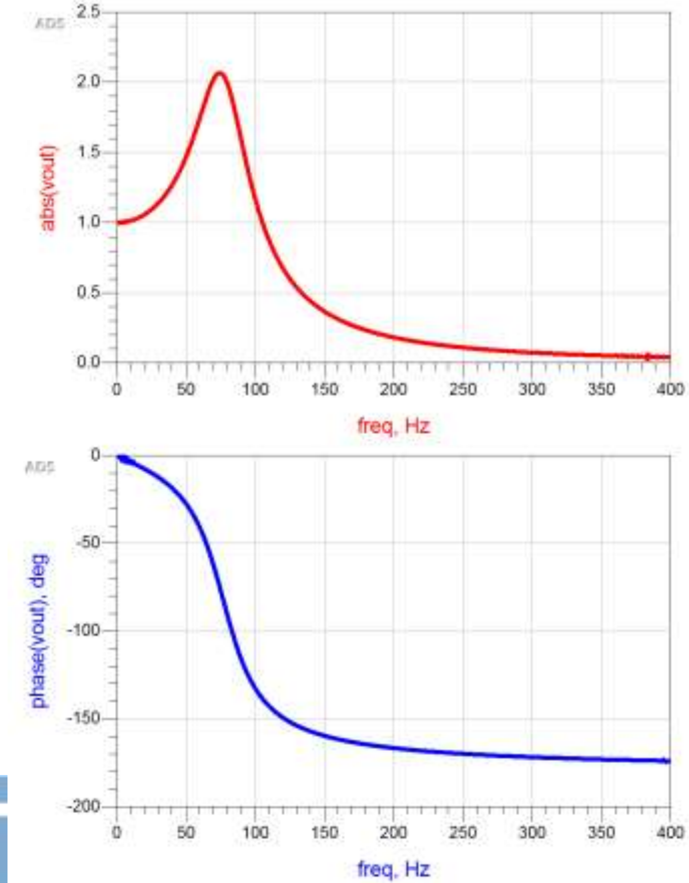
数学仿真

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$



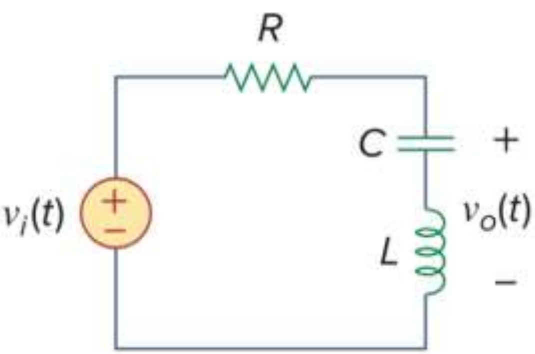
电路仿真





Example 14.11

If the band-stop filter in Fig. 14.37 is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C. Take $R = 150\ \Omega$ and the bandwidth as 100 Hz.



Solution:

We use the formulas for a series resonant circuit in Section 14.5.

$$\boxed{B = 2\pi(100)} = 200\pi \text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387 \text{ H}$$

Rejection of the 200-Hz sinusoid means that f_0 is 200 Hz, so that ω_0 in Fig. 14.38 is

$$\boxed{\omega_0 = 2\pi f_0 = 2\pi(200)} = 400\pi$$

Given that $\boxed{\omega_0 = 1/\sqrt{LC}}$,

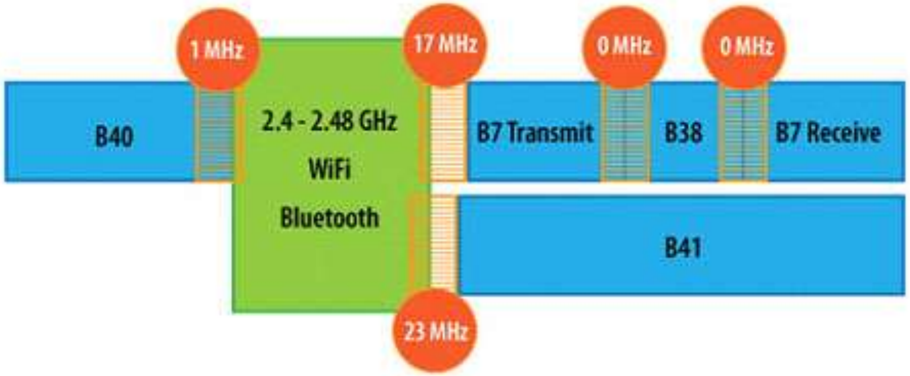
$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.653\ \mu\text{F}$$

$$\boxed{B = \frac{\omega_0}{Q}} \Rightarrow \quad Q = 2$$

$$\boxed{Q = \frac{\omega_0 L}{R}} \Rightarrow \quad L = 300\pi/400$$

滤波器的在移动终端中的应用

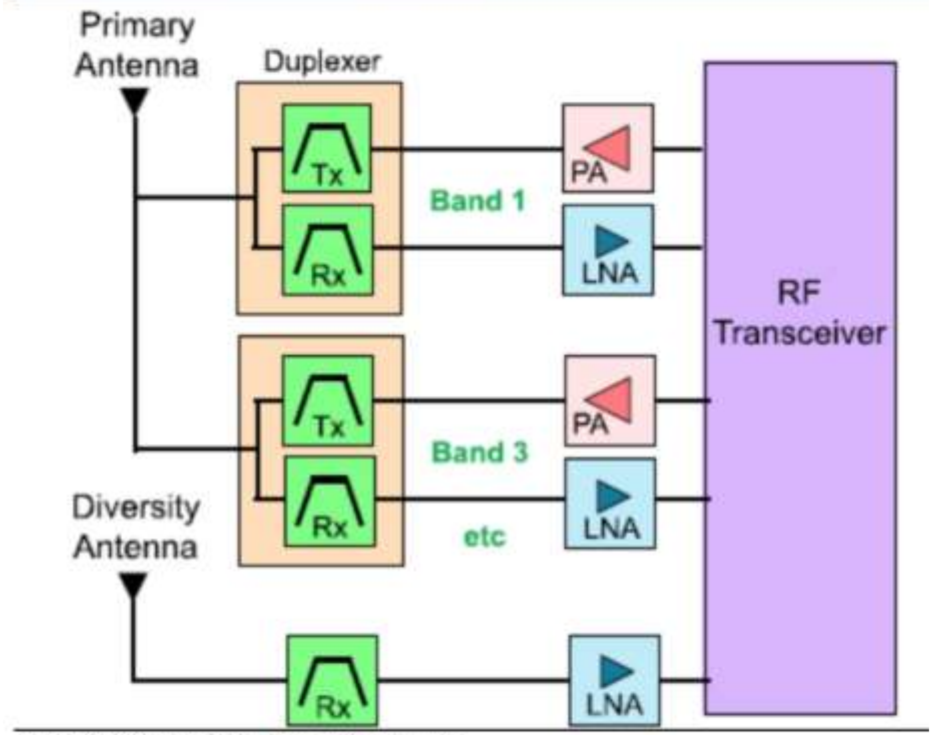
2.3-2.7 GHz Ecosystem Coexistence Requirements



Minimal guard bands require FBAR Filter performance to solve difficult coexistence problems

FBAR Filter Advantages

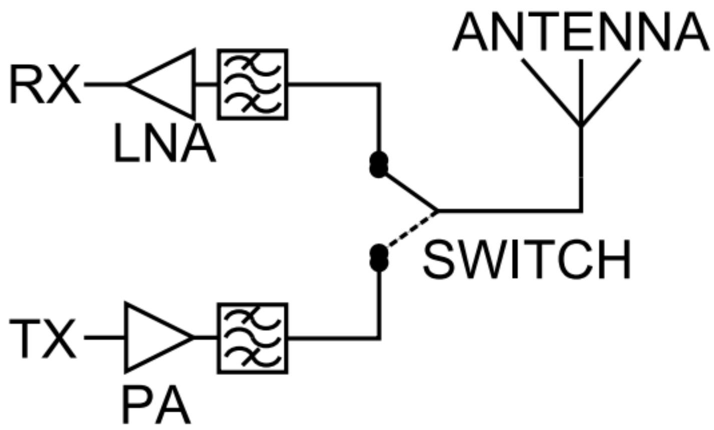
Figure 11. Typical LTE RF Subsystem



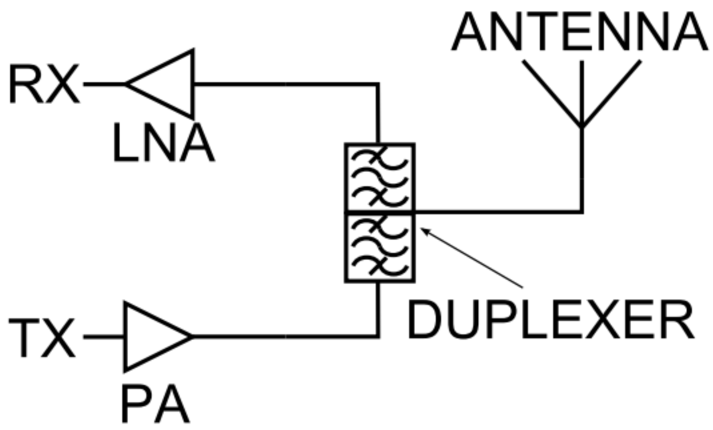
Sources: Company Reports and Joseph Gunnar

频谱资源越来越紧张 → 高性能滤波器需求

时分复用系统



频分复用系统



滤波器的主要参数

1) 中心频率（Center Frequency）f_0。

2) 3dB 带宽（Band-Width）BW_{3dB}。也称通频带。

3) 带内波动（Ripple）。应尽量小，以减少频率失真。

4) 选择性（Selectivity）。也称带外衰减，描述滤波器对频带外信号的衰减程度，带外衰减越大，选择性越好。

5) 插入损耗（Insertion Loss）。插入损耗定义为通频带内滤波器的输入功率和输出功率之比：

$$IL = \frac{P_{in}}{P_{out}} \tag{1.1}$$

一般用 dB 表示，无源滤波器都有一定的损耗，损耗越小越好。

6) 零点（Zeros）和极点（Poles）。即滤波器传输系数表达式的零点和极点，在零点处，传输系数为 0，在极点处，传输系数最大。

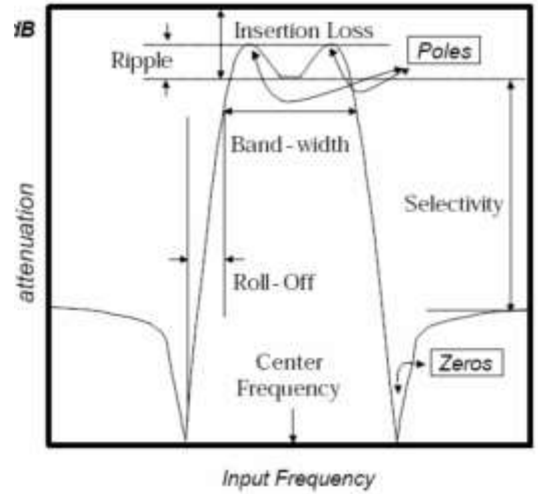
7) 滚降（Roll-Off）：滤波器传输系数从滤波器通频带边缘到邻近零点的下滑程度，要求越陡越好。

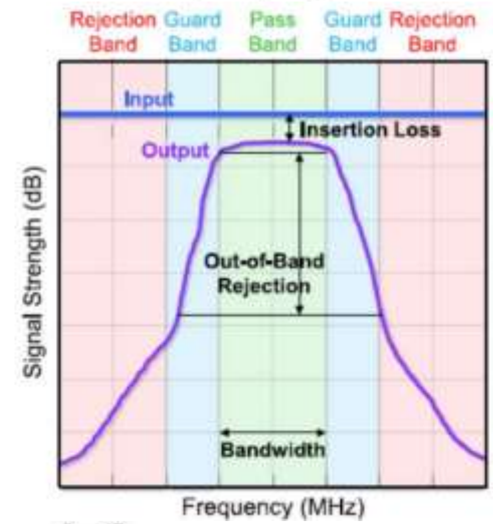
8) 品质因数 Q。Q 值定义为储能和损耗之比，其值也可以用中心频率 f_0 和带宽 BW_{3dB} 之比来定量表示：

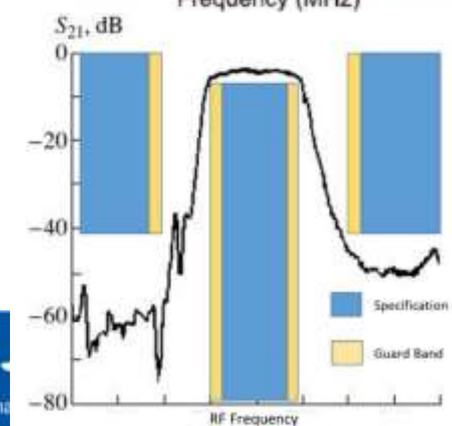
$$Q = \frac{f_0}{BW_{3dB}} \tag{1.2}$$

9) 输入输出阻抗。滤波器的性能指标都是在其输入输出端均匹配时测得的，在应用时，必须知道其输入输出阻抗，并很好的匹配，才能使滤波器发挥其最佳性能。

10) 相频特性。滤波器传输系数相位随频率变化的曲线，要求其接近线性。

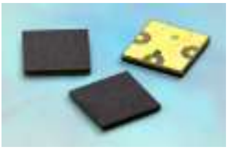




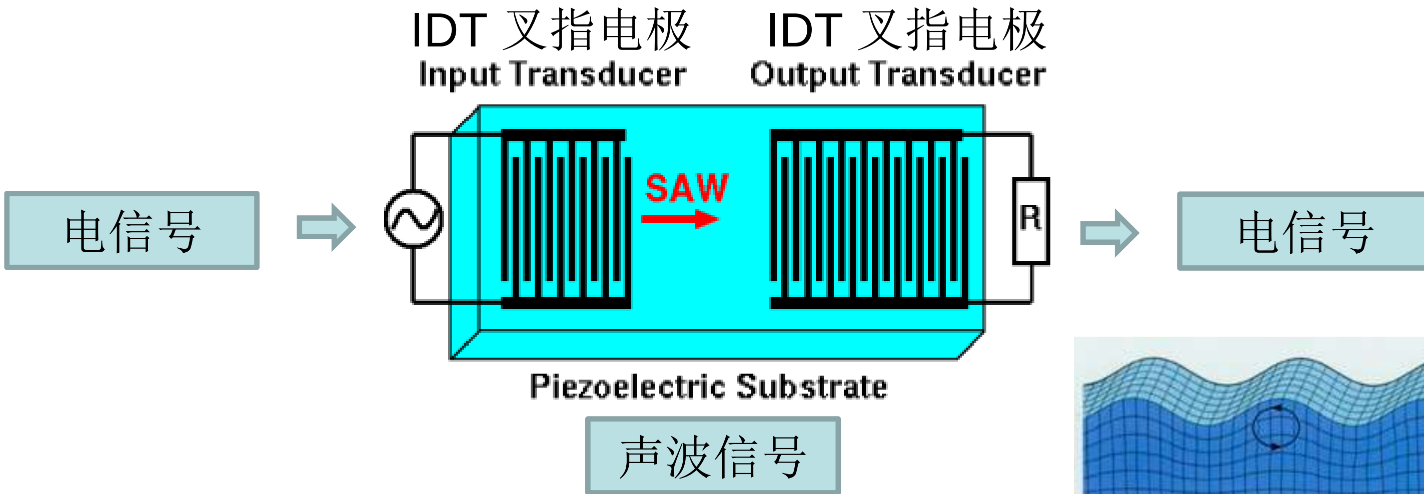


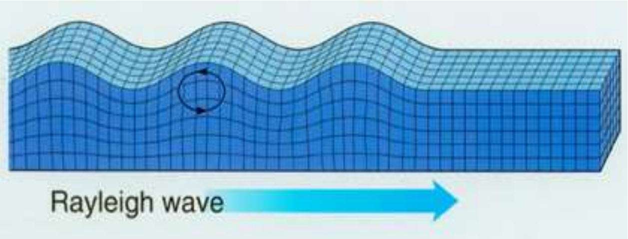
移动终端滤波器方案

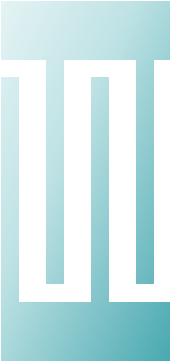
➢ **目前滤波器解决方案：介质滤波器、声表面波（SAW）、FBAR**

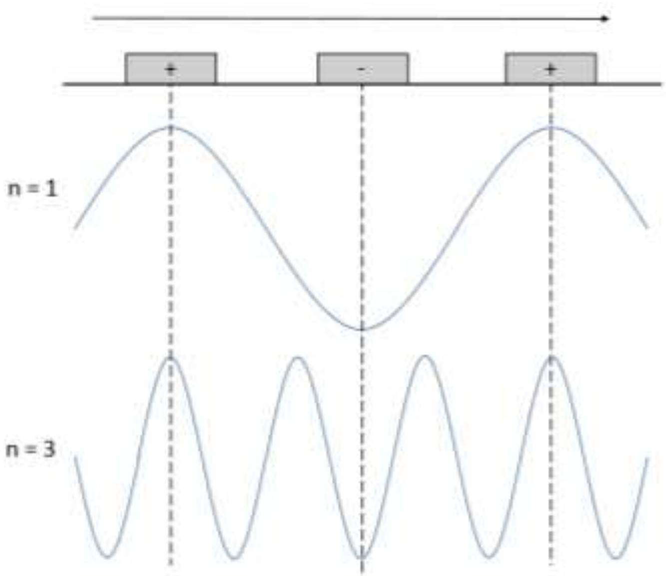
	介质滤波器 	SAW 	FBAR 
工作频率	1MHz-10GHz	30MHz-2GHz	500MHz-10GHz
插入损耗	1-2 dB	2.5-4 dB	1-1.5 dB
带外抑制	<40 dB	<45 dB	<50 dB
温度系数	-10 ~ +10 ppm/^{o}C	-35 ~ -95 ppm/^{o}C	-25 ~ -30 ppm/^{o}C
Q	300-700	200-400	700-1000
功率容量	>>1W	<1W	>1W
尺寸	5-10mm^2	2-8mm^2	0.1*0.1mm^2
制备工艺	成熟	成熟	工艺要求高
可集成性	不能	不能	可以

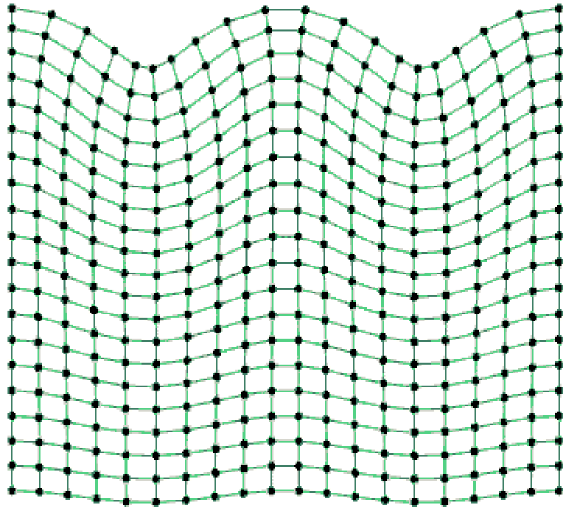
SAW原理


IDT 叉指电极
Input Transducer
IDT 叉指电极
Output Transducer
电信号
SAW
R
电信号
Piezoelectric Substrate
声波信号


Rayleigh wave




+
-
+
n = 1
n = 3



FBAR原理

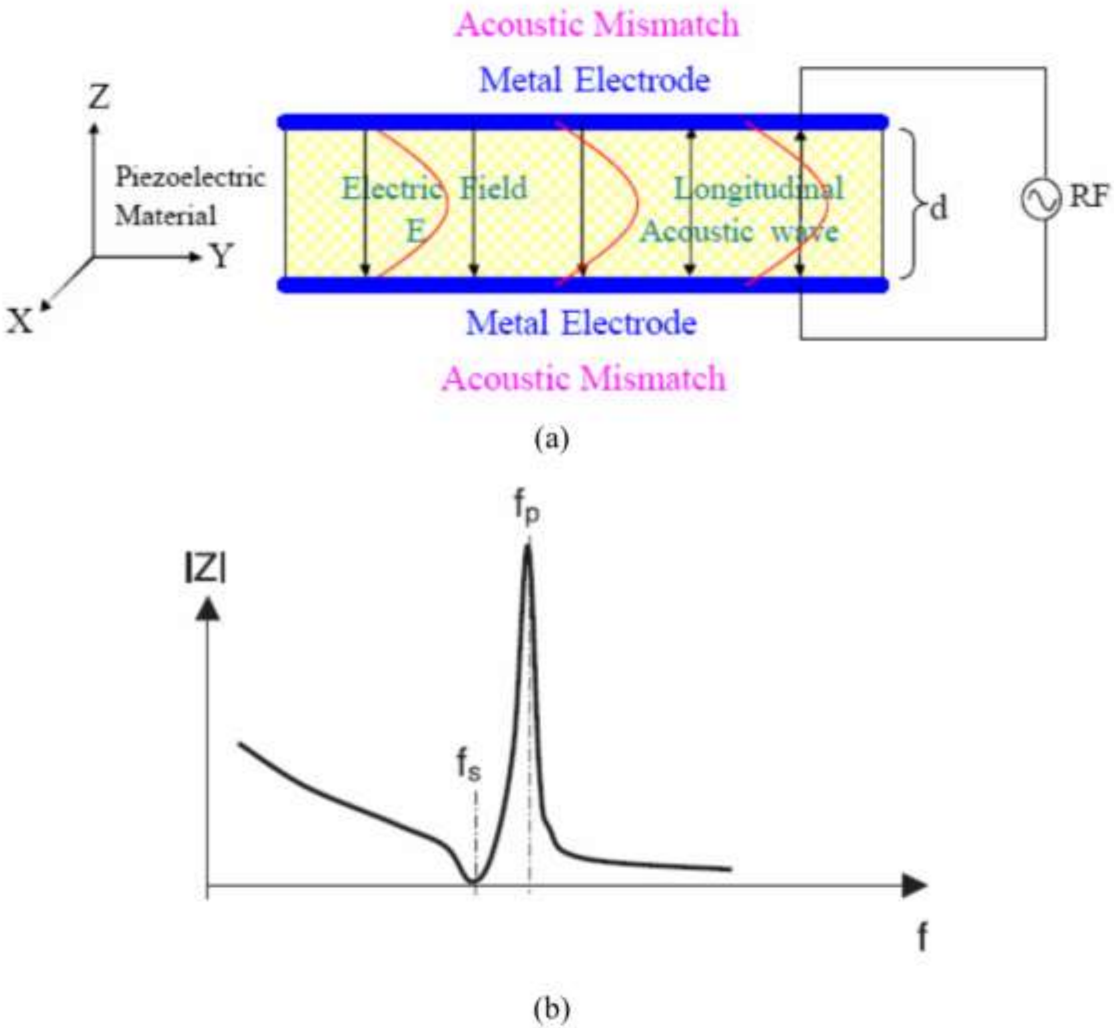
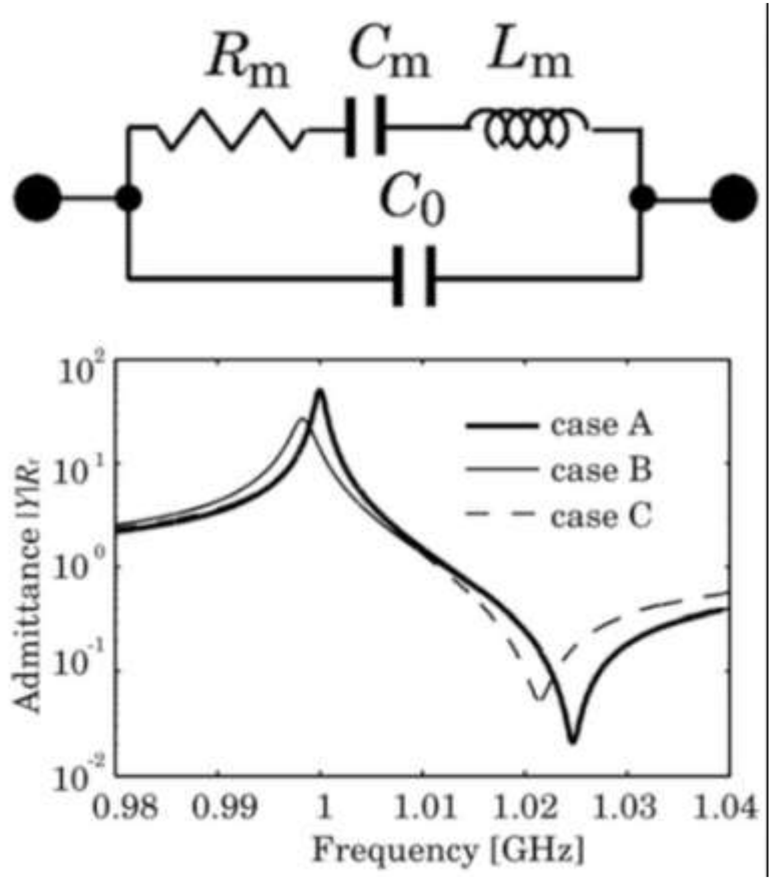


图 1. 7 FBAR 工作原理：(a)声波激励；(b)电学阻抗特性[23]

SAW / FBAR 谐振器的等效电路



FBAR滤波器设计方法

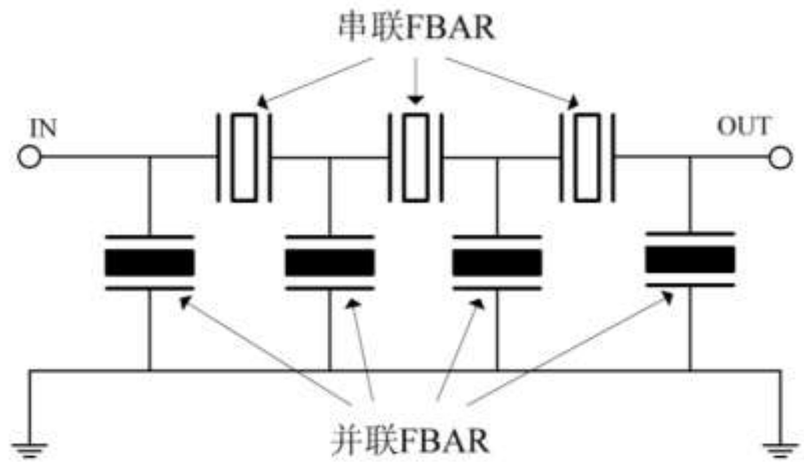


图 6.1 阶梯型 FBAR 滤波器拓扑结构

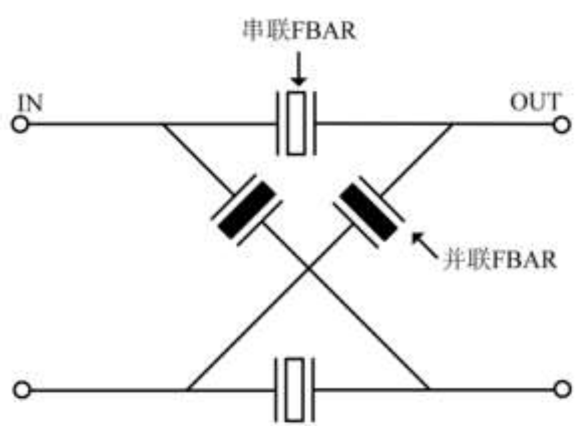
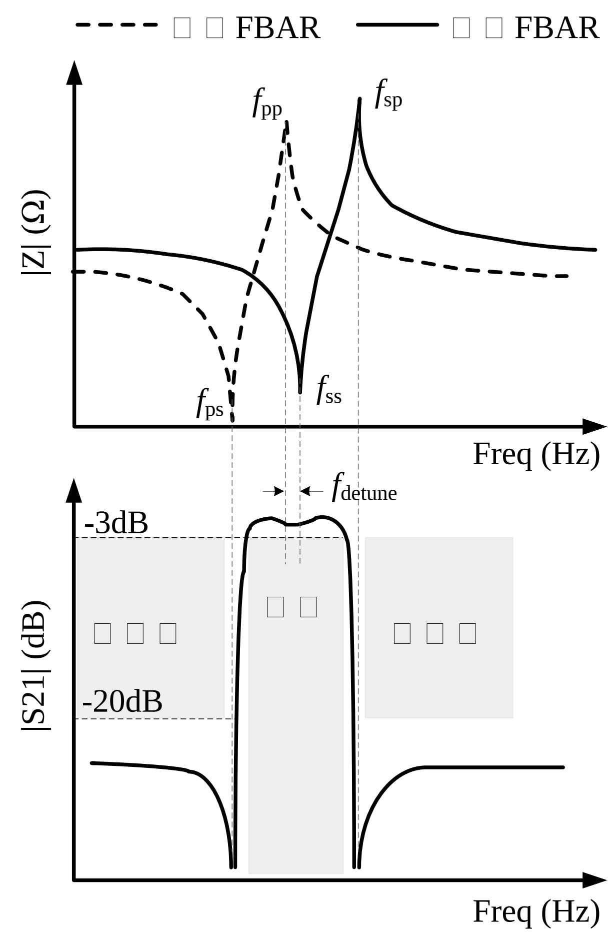


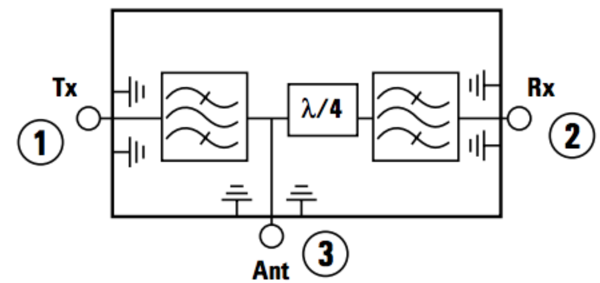
图 6.4 网格型 FBAR 滤波器拓扑结构

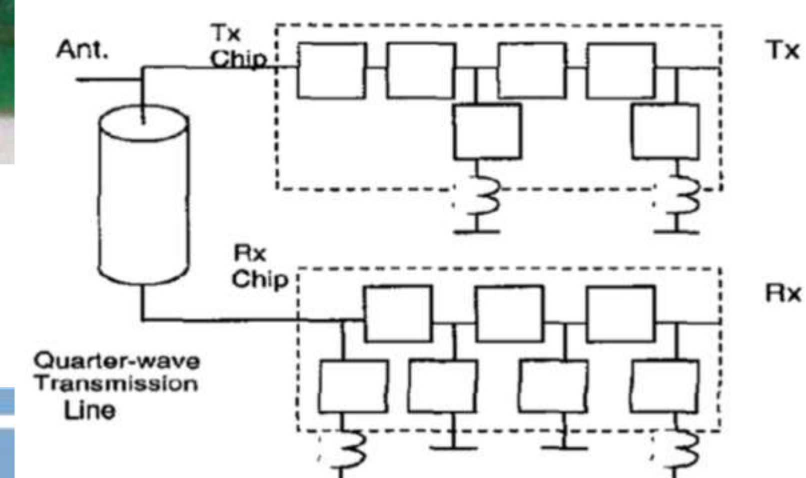


FBAR主要产品及剖析

第一款产业化的FBAR产品：
Avago的针对US PCS1900的第一代FBAR双工器HPMD-7904

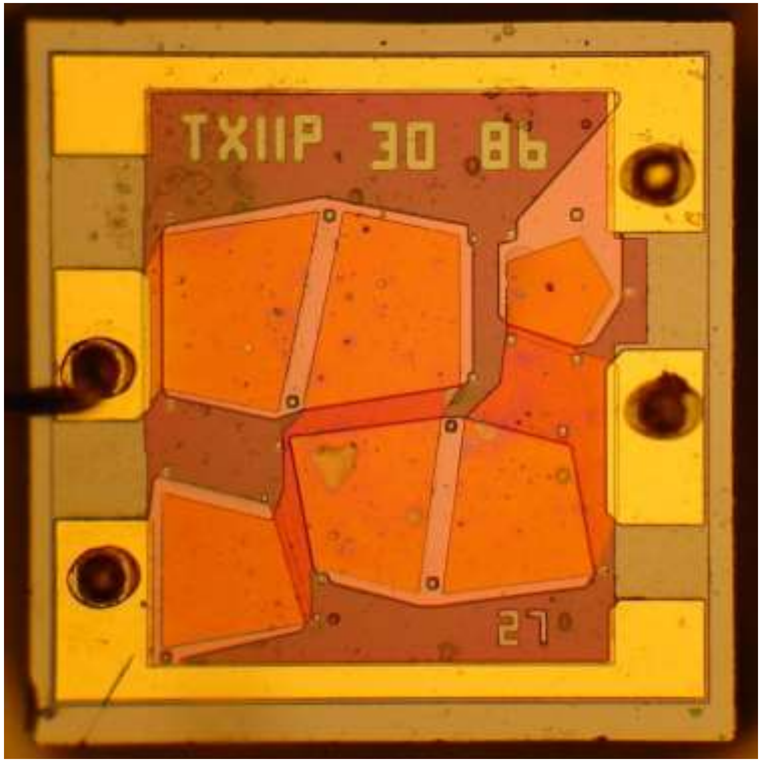
- 样品整体情况

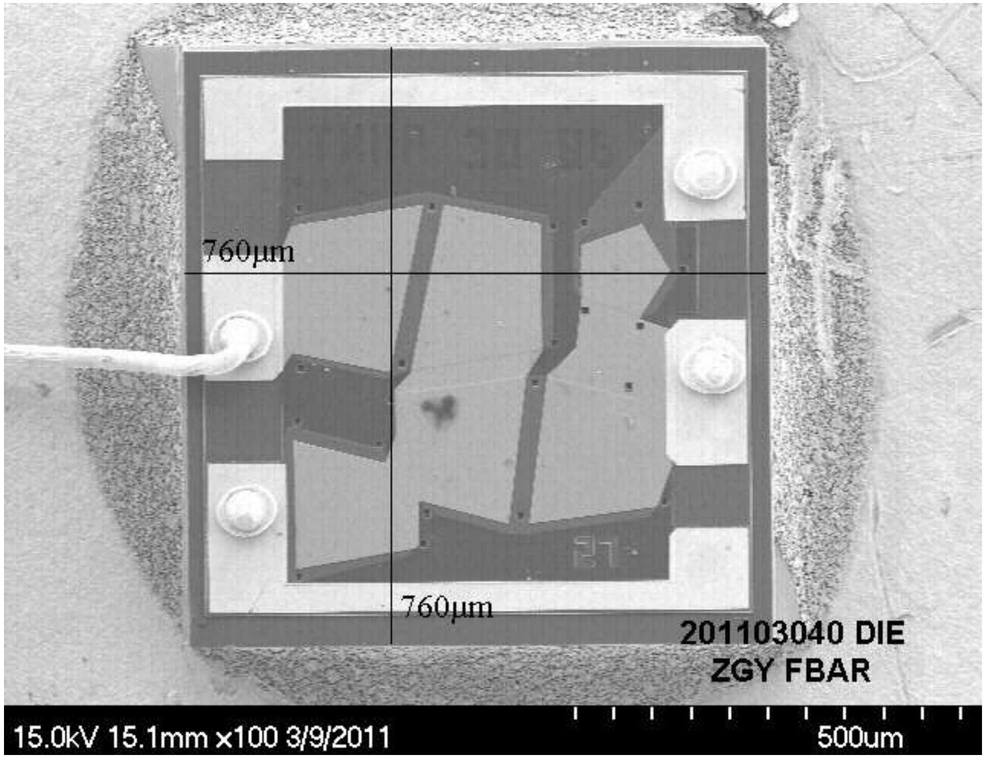




FBAR主要产品及剖析

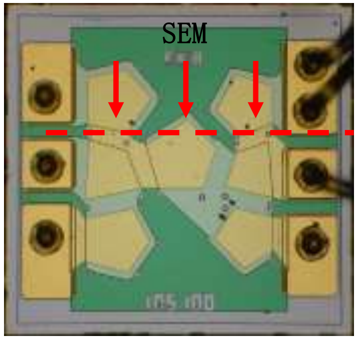
- TX部分结构



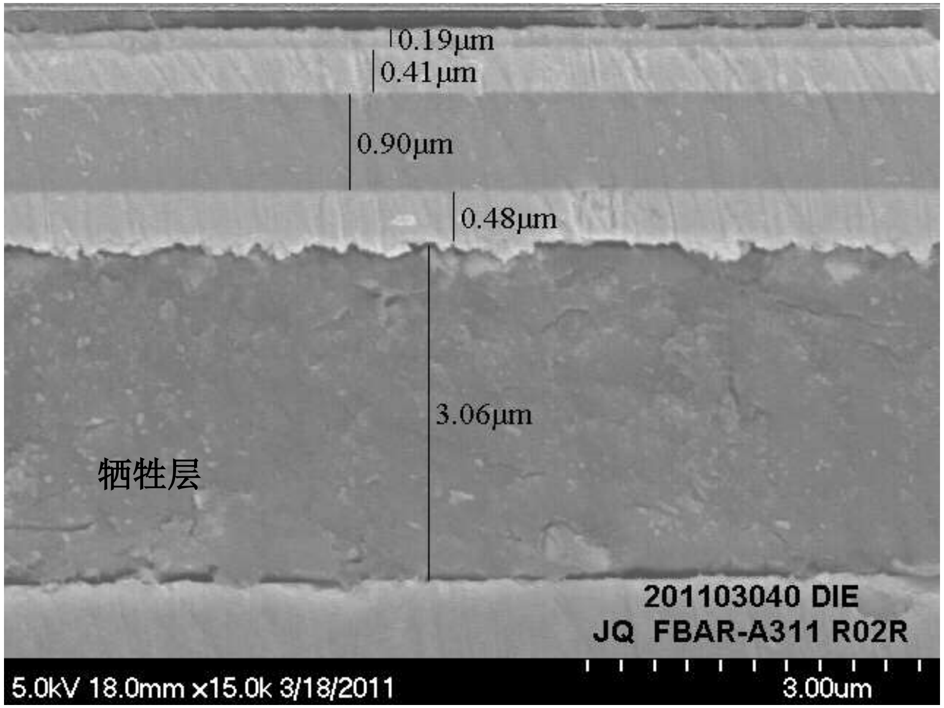


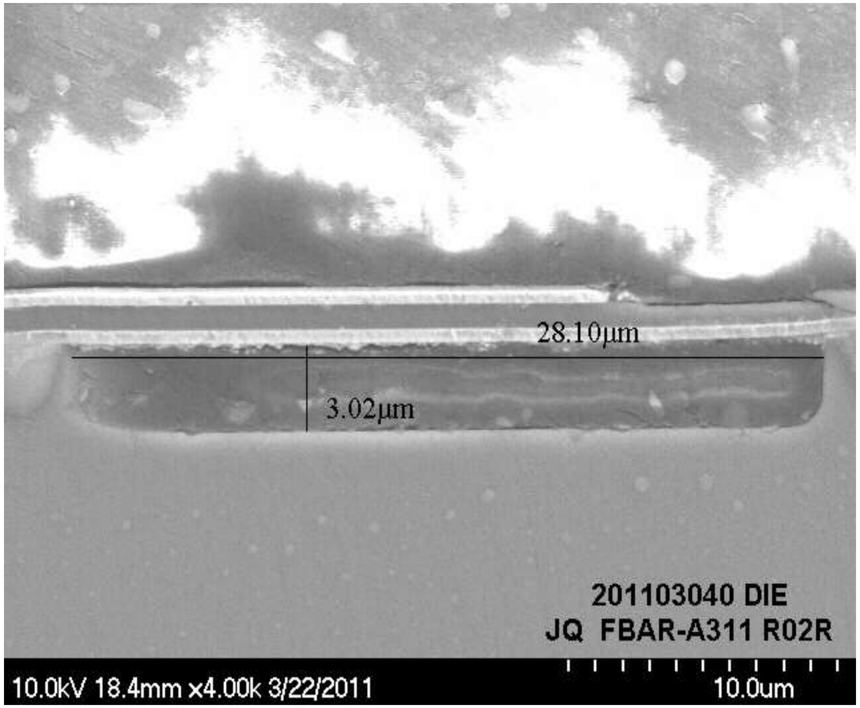
芯片大小为760um*760um

FBAR主要产品及剖析

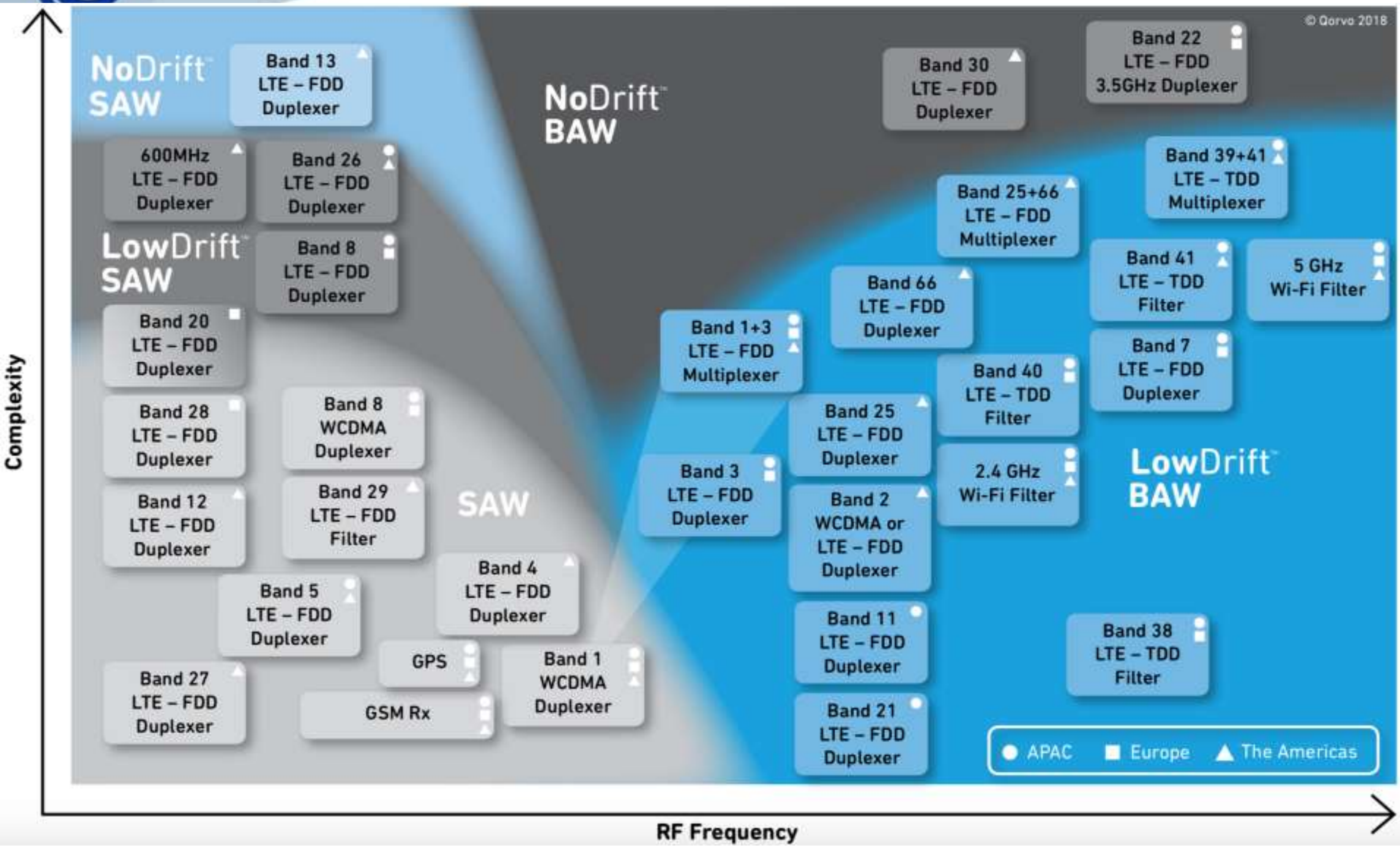


- 剖面分析结果





SEM显示牺牲层上存在四层结构，牺牲层厚度为3um。



© Qorvo 2018
NoDrift SAW
NoDrift BAW
LowDrift SAW
SAW
LowDrift BAW
Band 13 LTE – FDD Duplexer
600MHz LTE – FDD Duplexer
Band 26 LTE – FDD Duplexer
Band 8 LTE – FDD Duplexer
Band 20 LTE – FDD Duplexer
Band 28 LTE – FDD Duplexer
Band 8 WCDMA Duplexer
Band 12 LTE – FDD Duplexer
Band 29 LTE – FDD Filter
Band 5 LTE – FDD Duplexer
Band 4 LTE – FDD Duplexer
Band 27 LTE – FDD Duplexer
GPS
GSM Rx
Band 1 WCDMA Duplexer
Band 30 LTE – FDD Duplexer
Band 22 LTE – FDD 3.5GHz Duplexer
Band 39+41 LTE – TDD Multiplexer
Band 25+66 LTE – FDD Multiplexer
Band 41 LTE – TDD Filter
5 GHz Wi-Fi Filter
Band 66 LTE – FDD Duplexer
Band 1+3 LTE – FDD Multiplexer
Band 7 LTE – FDD Duplexer
Band 40 LTE – TDD Filter
Band 25 LTE – FDD Duplexer
Band 3 LTE – FDD Duplexer
2.4 GHz Wi-Fi Filter
Band 2 WCDMA or LTE – FDD Duplexer
Band 11 LTE – FDD Duplexer
Band 38 LTE – TDD Filter
Band 21 LTE – FDD Duplexer
APAC
Europe
The Americas
Complexity
RF Frequency

小结

- 频率响应
- dB的概念 $\boxed{G_{dB} = 10\log_{10}\frac{P_2}{P_1}}$ $\boxed{G_{dB} = 20\log_{10}\frac{V_2}{V_1}}$
- 会根据波特图写出传递函数；也会根据传递函数画出波特图
- 谐振电路的五个基本参数，注意串并联的Q值公式互为倒数

串联

① $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$ ② $\boxed{Q = \frac{\omega_0 L}{R}}$ ③ $\boxed{B = \frac{\omega_0}{Q}}$ ④ $\boxed{\omega_1 = \omega_0 - \frac{B}{2}}$ ⑤ $\boxed{\omega_2 = \omega_0 + \frac{B}{2}}$

- 滤波器的类型，以及 special case 判断滤波器类型的方法

作业

Practice Problem 14.6

Obtain the transfer function H(ω) corresponding to the Bode plot in Fig. 14.20.

Answer: $\mathbf{H}(\omega) = \dfrac{2{,}000{,}000(s+5)}{(s+10)(s+100)^2}$.

用双重极点表示

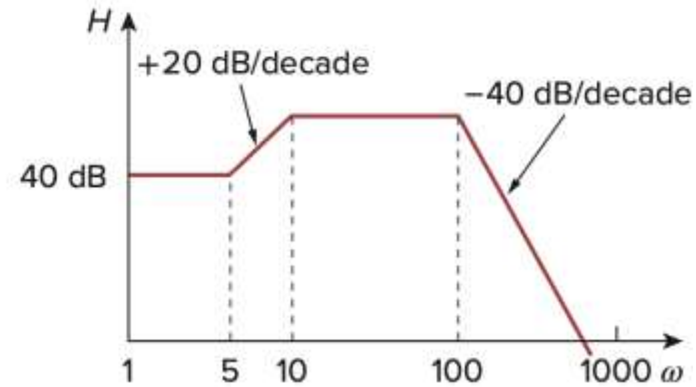


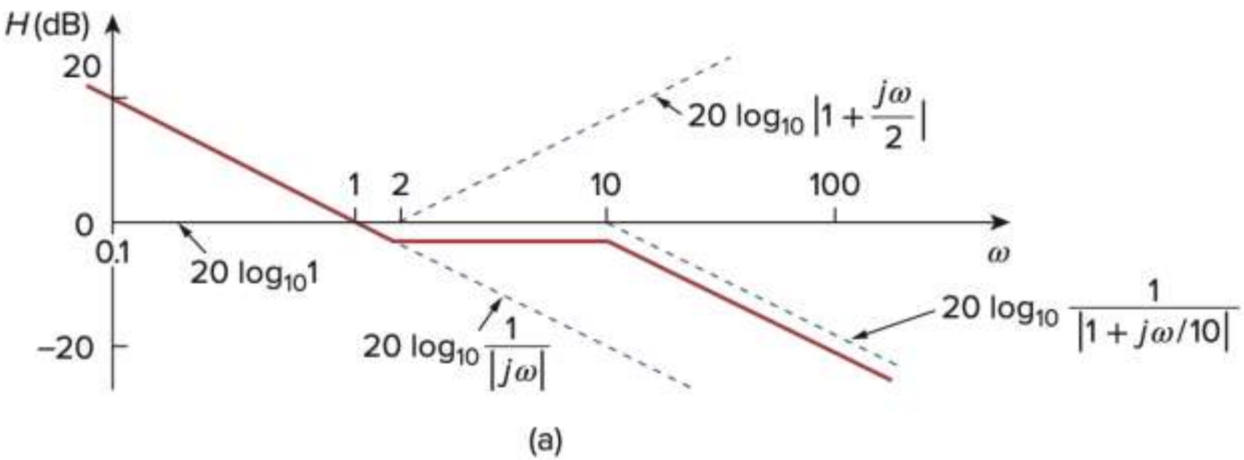
Figure 14.20
For Practice Prob. 14.6.

Draw the Bode plots for the transfer function

Practice Problem 14.3

$$\mathbf{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$

Answer: See Fig. 14.14.



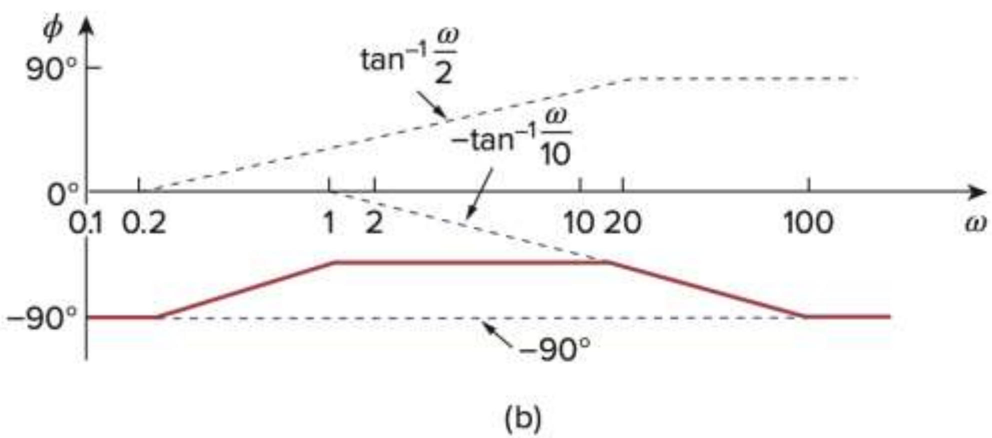
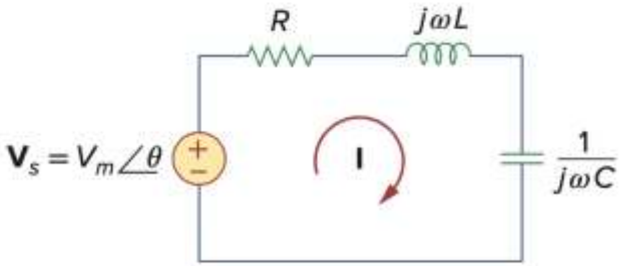


Figure 14.14
For Practice Prob. 14.3: (a) magnitude plot, (b) phase plot.

Practice Problem 14.7



A series-connected circuit has $R = 4\ \Omega$ and $L = 25$ mH. (a) Calculate the value of C that will produce a quality factor of 50. (b) Find ω_1, ω_2, and B. (c) Determine the average power dissipated at $\omega = \omega_0, \omega_1, \omega_2$. Take $V_m = 100$ V.

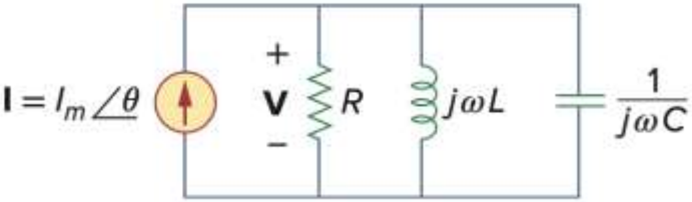
串联谐振

Answer: (a) 0.625 μF, (b) 7920 rad/s, 8080 rad/s, 160 rad/s, (c) 1.25 kW, 0.625 kW, 0.625 kW.

Practice Problem 14.8

A parallel resonant circuit has $R = 100\ \mathrm{k\Omega}$, $L = 50\ \mathrm{mH}$, and $C = 2\ \mathrm{nF}$. Calculate ω_0, ω_1, ω_2, Q, and B.

Answer: 100 krad/s, 97.5 krad/s,102.5 krad/s, 20, 5 krad/s.



并联谐振

Practice Problem 14.9

Calculate the resonant frequency of the circuit in Fig. 14.29.

Answer: 173.21 rad/s.

非简单串并联情况计算谐振频率

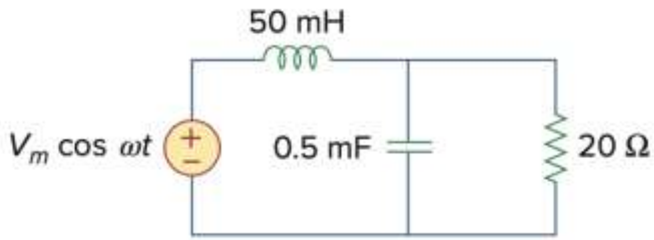


Figure 14.29
For Practice Prob. 14.9

Practice Problem 14.11

Design a band-pass filter of the form in Fig. 14.35 with a lower cutoff frequency of 20.1 kHz and an upper cutoff frequency of 20.3 kHz. Take $R = 30\ \text{k}\Omega$. Calculate L, C, and Q.

Answer: 23.87 H, 2.6 pF, 101.

带通滤波器

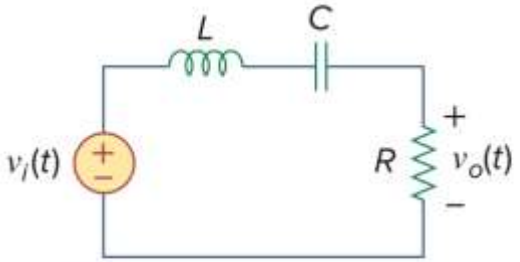


Figure 14.35
A band-pass filter.