电子电路基础

第三讲：电路分析的基本方法和定理~part2

电路分析的基本方法和定理

- 2.1 电阻电路的一般分析方法
  - 2.1.1 电阻的串联和并联
  - 2.1.2 电阻的混联和Y-Δ等效变换
- 2.2 电容和电感的串联和并联
- 2.3 电路定理
  - 2.3.1 节点、支路、回路和网孔基本概念
  - 2.3.2 网孔电流法和节点电压法（包括不含受控源和含受控源的电路的分析）
  - 2.3.3 叠加定理和替代定理
  - 2.3.4 戴维南定理和诺顿定理
  - 2.3.5 最大功率传递定理
- 2.4 电路等效和输入电阻

Circuit Theorems - Chapter 4

4.1 Motivation

4.2 Linearity Property 线性性质

4.3 Superposition 叠加性

4.4 Source Transformation 电源变换

4.5 Thevenin’s Theorem 戴维南定理

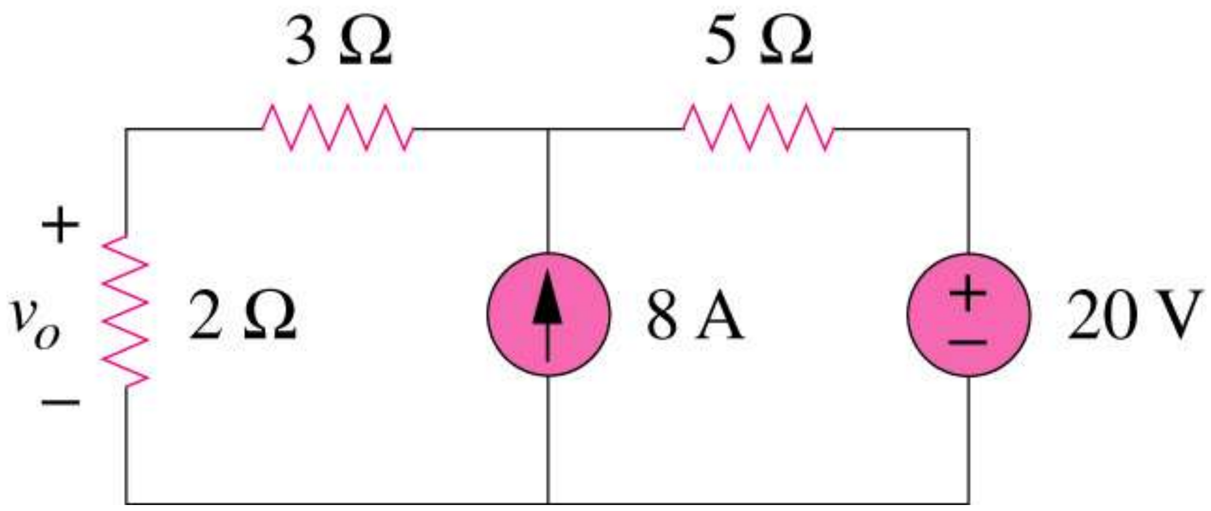
4.6 Norton’s Theorem 诺顿定理

4.7 Maximum Power Transfer 最大功率传输

补充：替代定理

4.1 Motivation (1)

If you are given the following circuit, are there any other alternative(s) to determine the voltage across 2Ω resistor?



What are they? And how? ①节点电压法；②网孔电流法

4.2 线性性质

It is the property of an element describing a linear relationship between cause and effect.

A linear circuit is one whose output is linearly related (or directly proportional) to its input. 【输入和输出呈线性关系！】

Homogeneity (scaling) property
【①齐次性：输入放大k倍→输出亦放大k倍】

线性性质：即同时满足①齐次性；②可加性

$$v = iR \longrightarrow \quad kv = (ki)R$$

Additive property
【②可加性：输入相加→输出相加】

$$v_1 = i_1R \text{ and } v_2 = i_2R$$

$$\longrightarrow \quad v = (i_1 + i_2)R = v_1 + v_2$$

线性性质

- **电阻是线性元件**，因为其电压-电流关系满足①齐次性；②可加性；
- **线性电路：**仅包含线性元件的电路；
- Q: 功率是线性的吗？

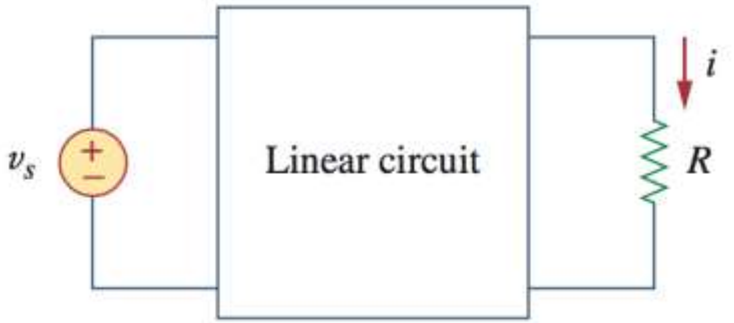


Figure 4.1
A linear circuit with input v_s and output i.

For example, when current i_1 flows through resistor R, the power is $p_1 = Ri_1^2$, and when current i_2 flows through R, the power is $p_2 = Ri_2^2$. If current $i_1 + i_2$ flows through R, the power absorbed is $p_3 = R(i_1 + i_2)^2 = Ri_1^2 + Ri_2^2 + 2Ri_1i_2 \neq p_1 + p_2$. Thus, the power relation is nonlinear.

Example 4.1

For the circuit in Fig. 4.2, find I_o when $v_s = 12$ V and $v_s = 24$ V.

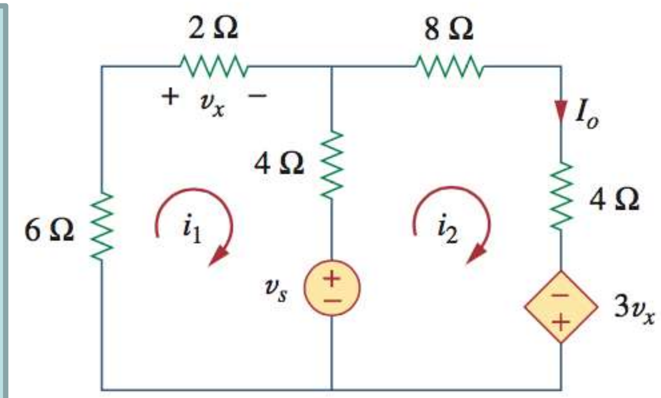


Figure 4.2
For Example 4.1.

验证线性性质

Example 4.2

线性性质的应用

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

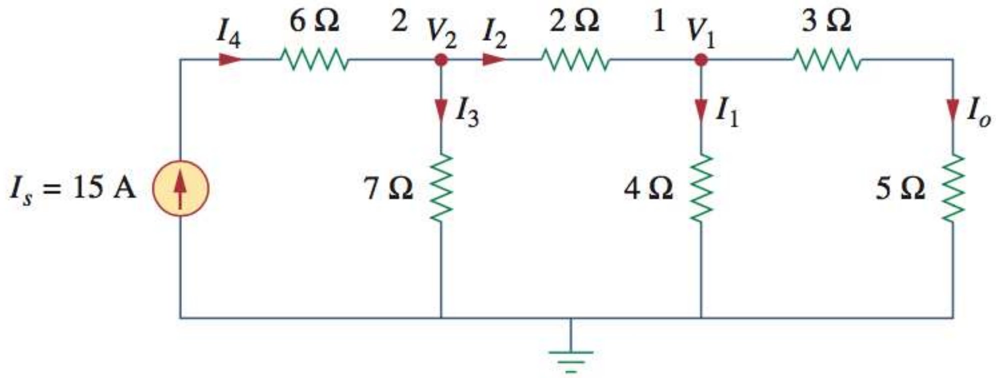


Figure 4.4
For Example 4.2.

I_s是“因”，I_o是“果”，“因”如果按比例缩放，“果”也会按比例缩放，反之亦然。

我们先假设I_o为1A，反推出相应的I_s，然后根据得到的I_s与实际I_s（15A）的比例关系，相应地缩放之前假设的I_o（1A），得到实际的I_o

4.3 Superposition Theorem 叠加定理

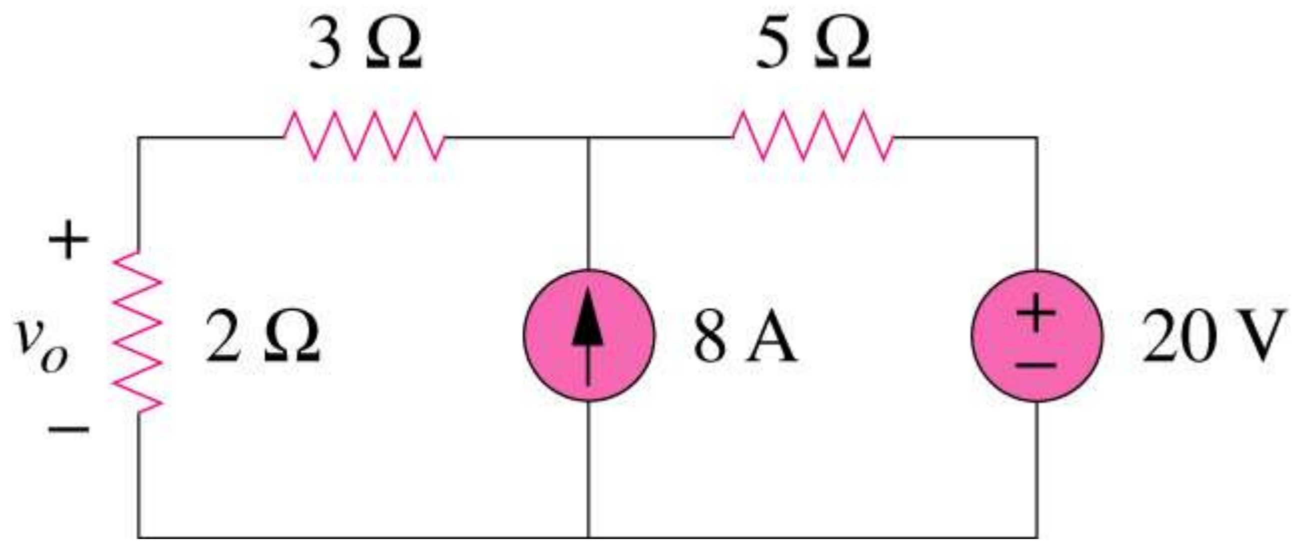
It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

- 分别计算每个独立源的贡献(考虑一个独立源时，其他独立源均设为零/turn off)，再线性叠加
- 适用于有多个独立源的线形电路

4.3 Superposition Theorem

We consider the effects of 8A and 20V **one by one**, then add the two effects together for final v_o.



4.3 Superposition Theorem

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

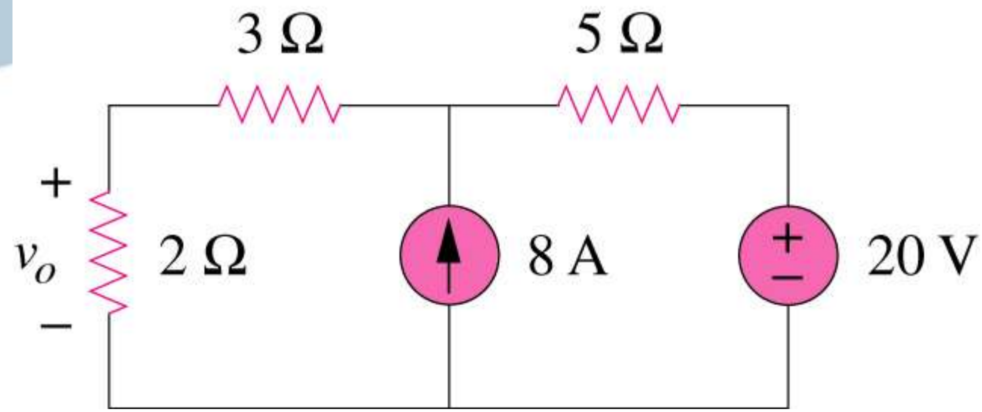
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

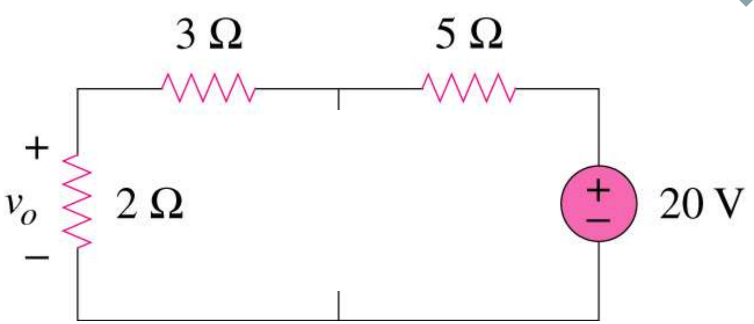
4.3 Superposition Theorem

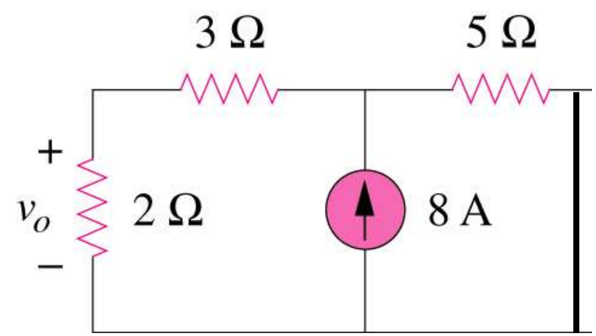
Two things have to be keep in mind:

1. When we say turn off all other independent sources: 【独立电压源短路、独立电流源开路】
   - Independent voltage sources are replaced by **0 V** (short circuit) and
   - Independent current sources are replaced by **0 A** (open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables. 【受控源不变】



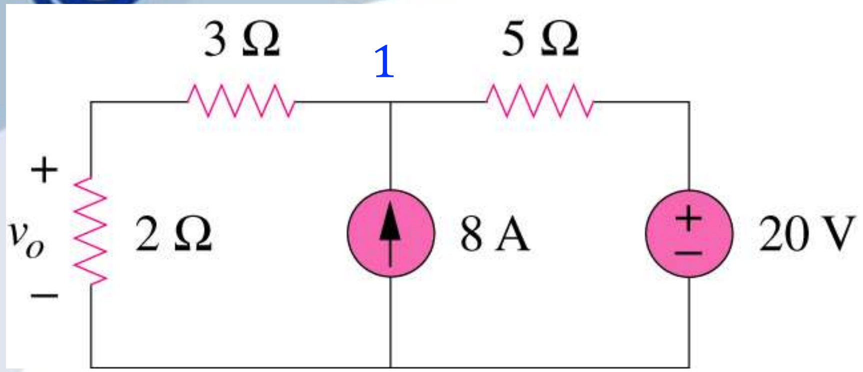




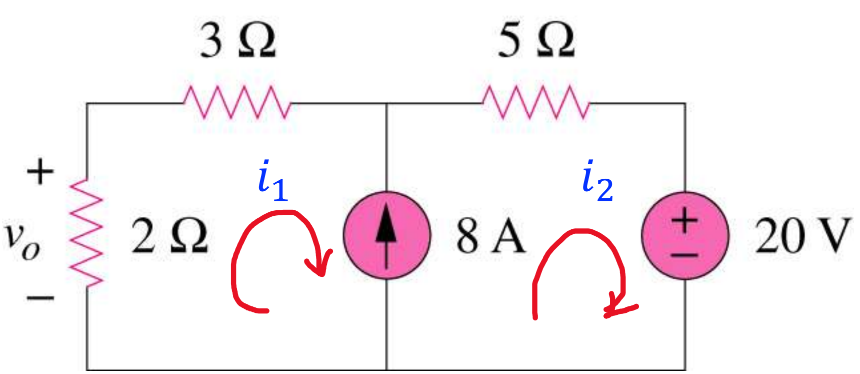
$$v_o = \frac{20}{10} \times 2 = 4\ \mathrm{V}$$

$$v_o = 4 \times 2 = 8\ \mathrm{V}$$

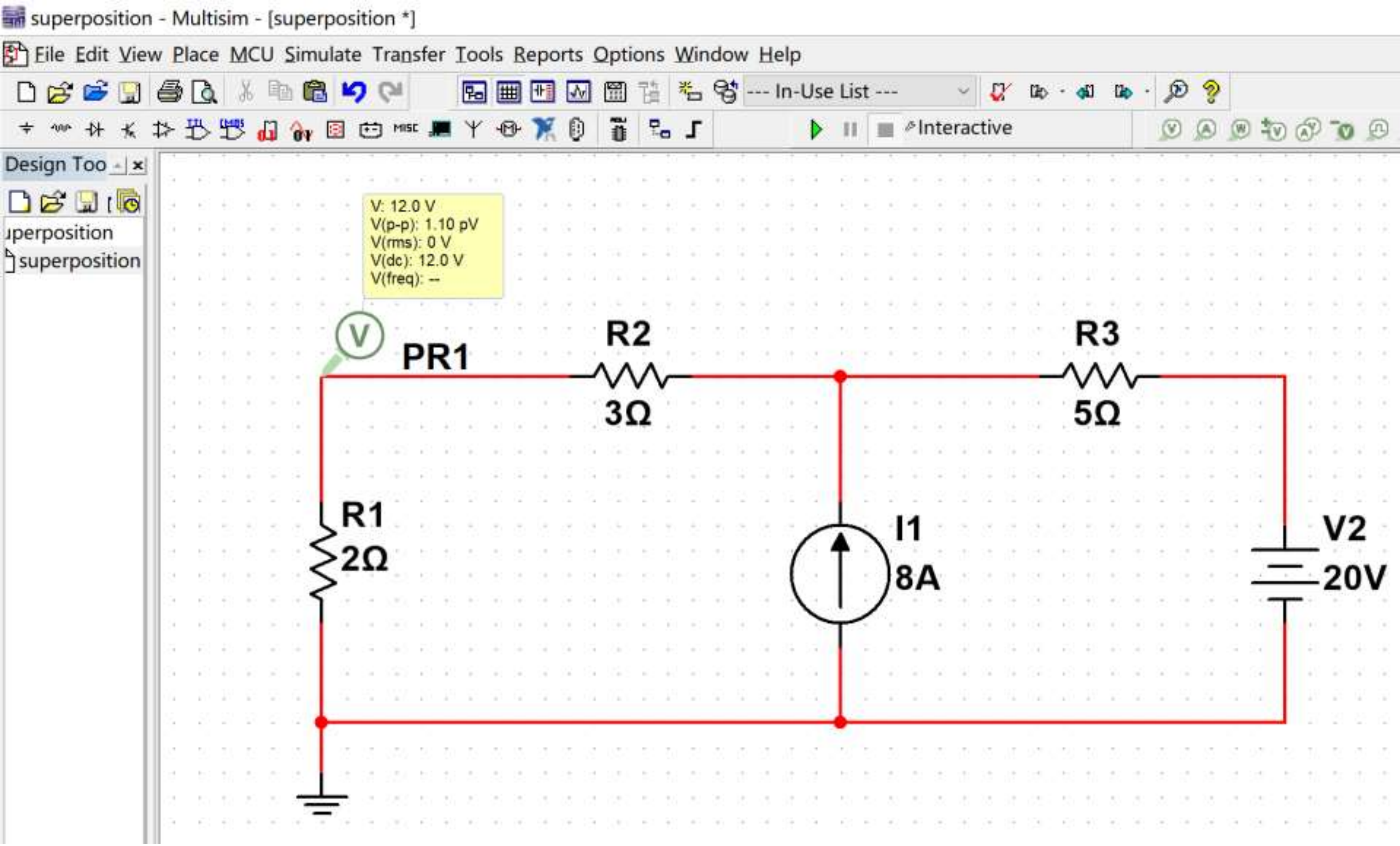
$$v_o = 12\ \mathrm{V}$$



节点电压法:



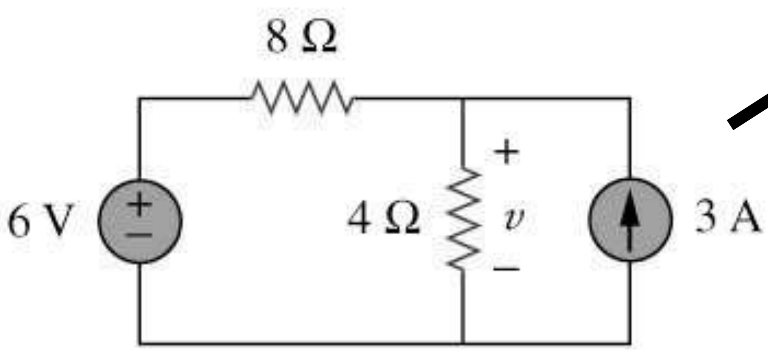
网孔电流法:


superposition - Multisim - [superposition *]
File Edit View Place MCU Simulate Transfer Tools Reports Options Window Help
--- In-Use List ---
Interactive
Design Too
uperposition
superposition
V: 12.0 V
V(p-p): 1.10 pV
V(rms): 0 V
V(dc): 12.0 V
V(freq): --
PR1
R2
3Ω
R3
5Ω
R1
2Ω
I1
8A
V2
20V

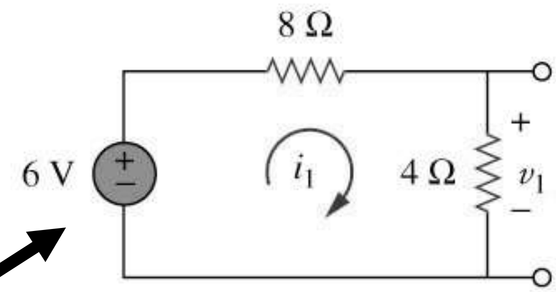
4.3 Superposition Theorem

<u>Example 4.3</u> ①不含受控源

Use the superposition theorem to find v in the circuit shown below.



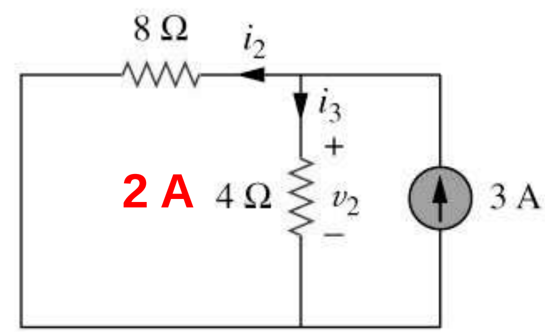
只考虑电压源：
3A is turned off by open-circuit



(a)

2 V

只考虑电流源：
6V is turned off by short-circuit



2 A

(b)

$2\ \text{V} + 2\ \text{A} \times 4\ \Omega = 10\ \text{V}$

Example 4.4

Find i_o in the circuit of Fig. 4.9 using superposition.

②含受控源

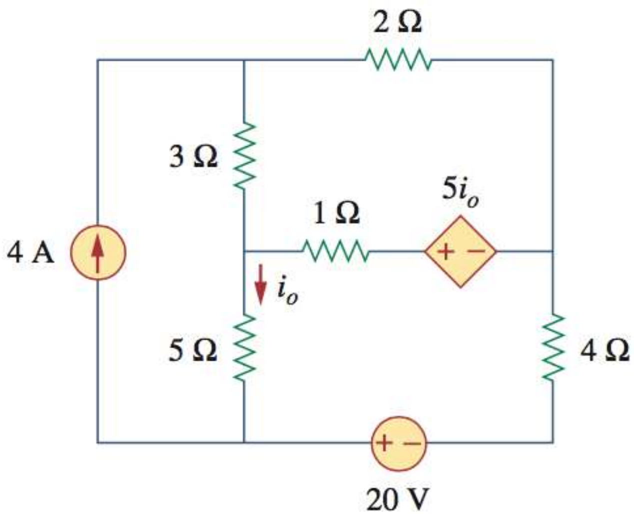


Figure 4.9
For Example 4.4.

$$\boxed{i_o = i'_o + i''_o}$$

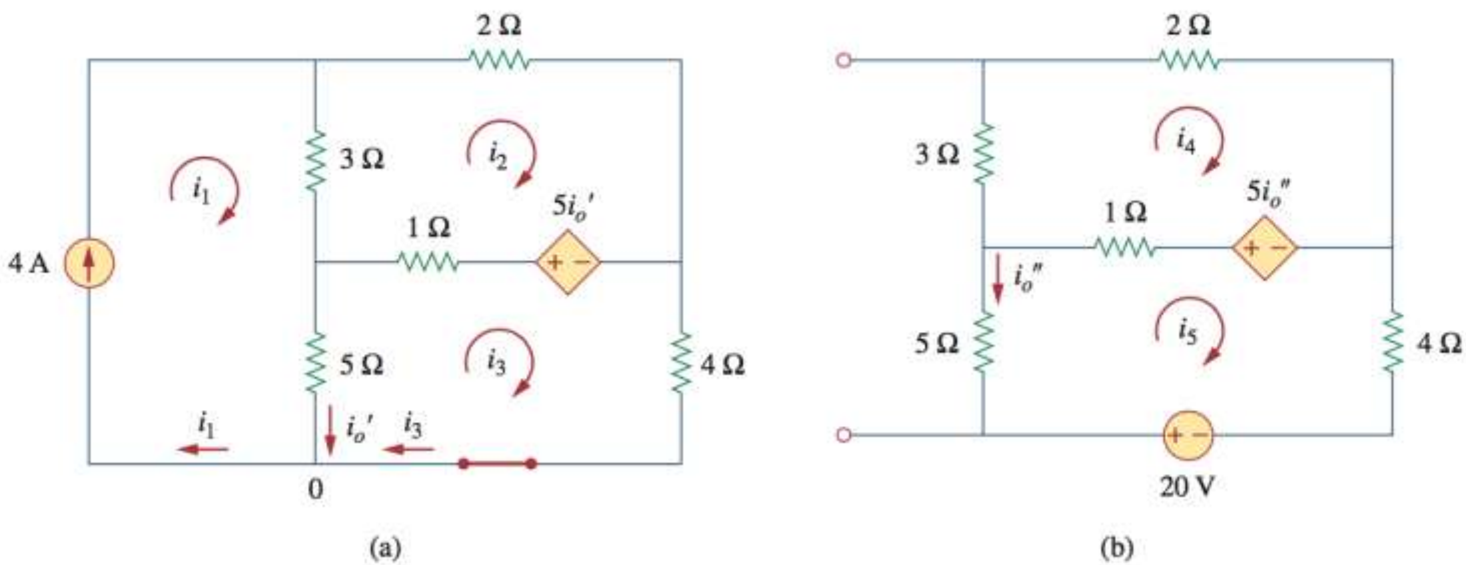


Figure 4.10
For Example 4.4: Applying superposition to (a) obtain i'_o, (b) obtain i''_o.

Example 4.5

For the circuit in Fig. 4.12, use the superposition theorem to find i.

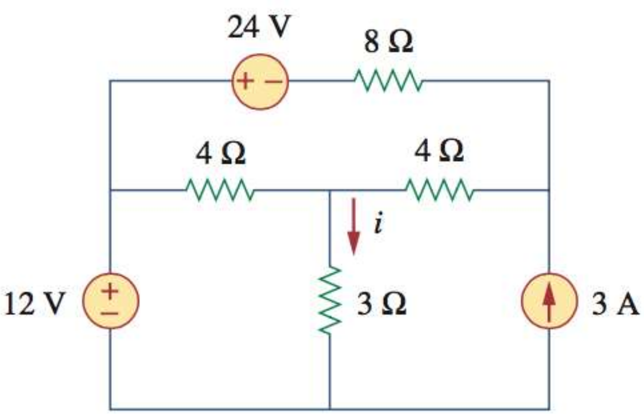
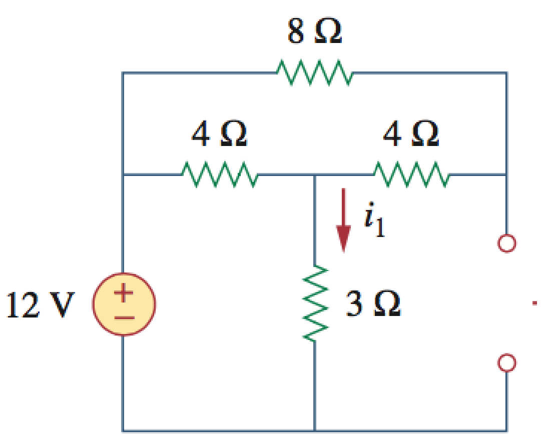


Figure 4.12
For Example 4.5.

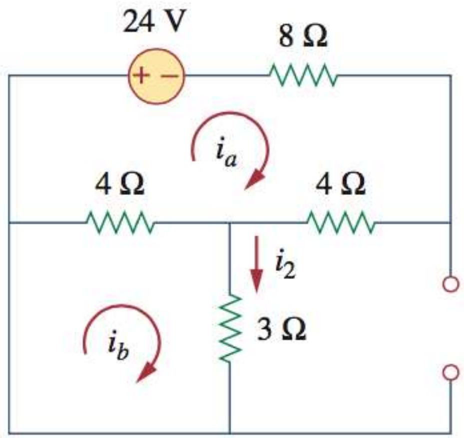
③含多个独立源（3个及以上）

$$\boxed{i = i_1 + i_2 + i_3}$$

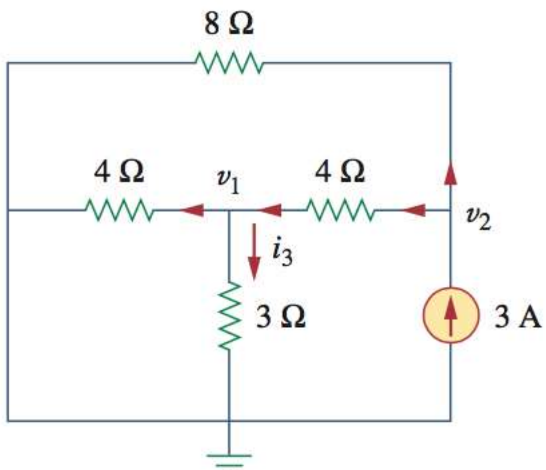
注：根据线性叠加原理，可以
① 每次考虑一个独立源，计算三次；
② 一次考虑两个独立源，另一次考虑剩下的一个独立源，计算两次



(a)



(b)



(c)

端口（port）的概念

- 电路或网络的一个端口（a port），是指它向外引出的一对端子（a pair of terminals），这对端子可以与外部电源或其他电路相联结。
- 对一个端口来说，从它的一个端子流入的电流一定等于从另一个端子流出的电流。
- 这种具有向外引出一对端子（两对端子）的电路或网络称为一端口网络（二端口网络）

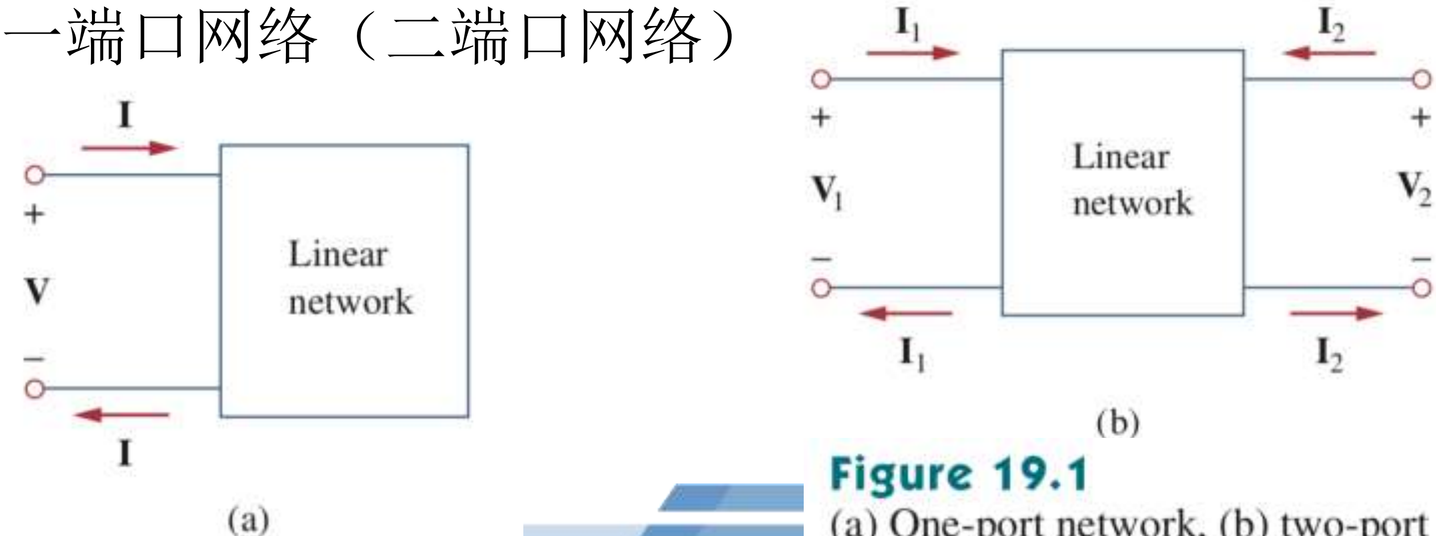
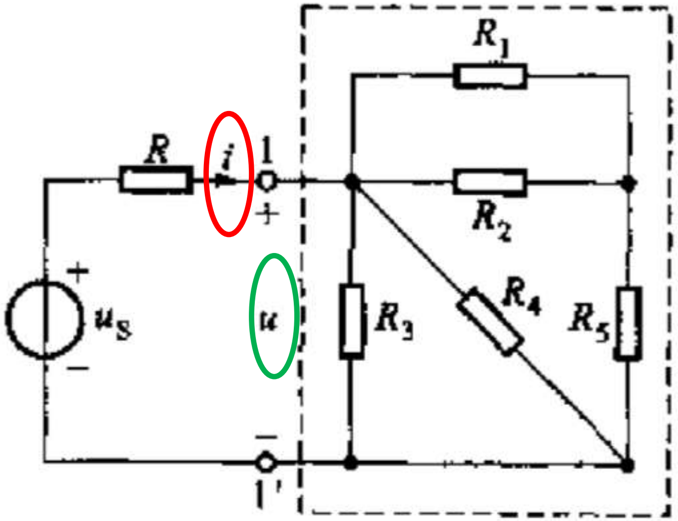


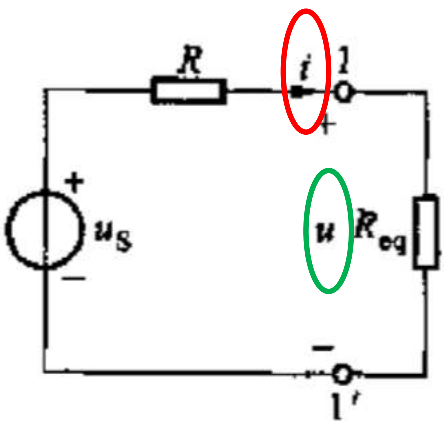
Figure 19.1
(a) One-port network, (b) two-port network.

电路等效

- 【概念】：当电路中的某一部分用其**等效电路**代替后，未被代替部分的电压和电流均应保持不变；
- 【方法】：**接口**处（端口）电压、电流保持不变；



(a)



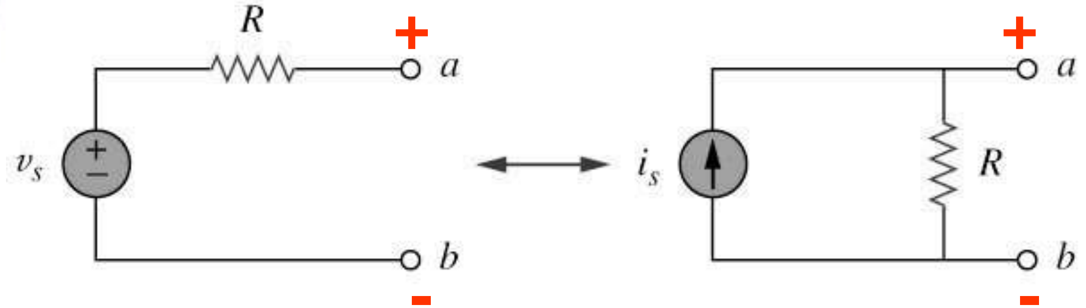
(b)

4.4 Source Transformation 电源变换

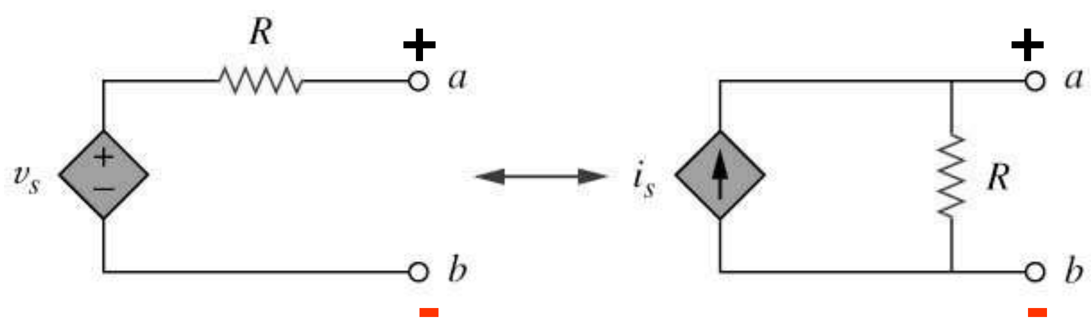
- An equivalent circuit is one whose *v*-*i* characteristics are identical with the original circuit. 【等效的原则：*v*-*i* 特性保持不变，即不能对其余电路产生影响】
- It is the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa. 【电压源串联电阻 ←→电流源并联电阻】

4.4 Source Transformation

$R \neq 0$ $R \neq \infty$



(a) Independent source transform



(b) Dependent source transform

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

① 不适用于无损电源；
② 受控源和独立源一样处理

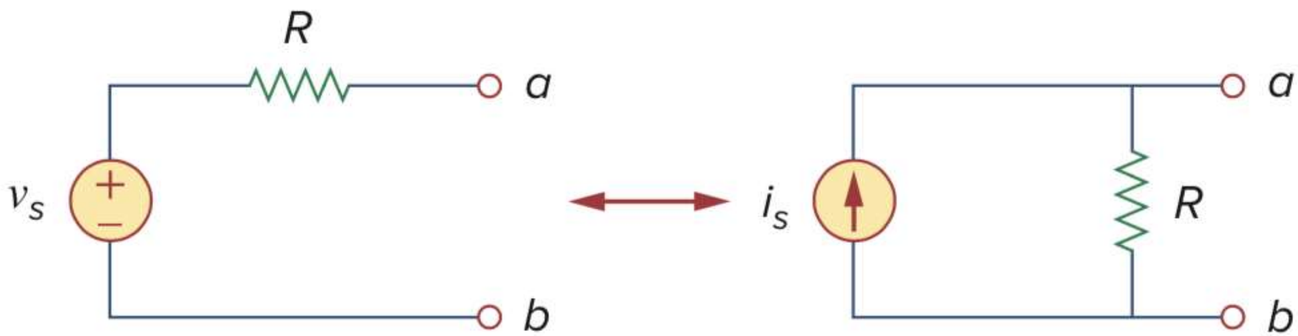


Figure 4.15
Transformation of independent sources.

$$v_s = i_s R \qquad \text{or} \qquad i_s = \frac{v_s}{R}$$

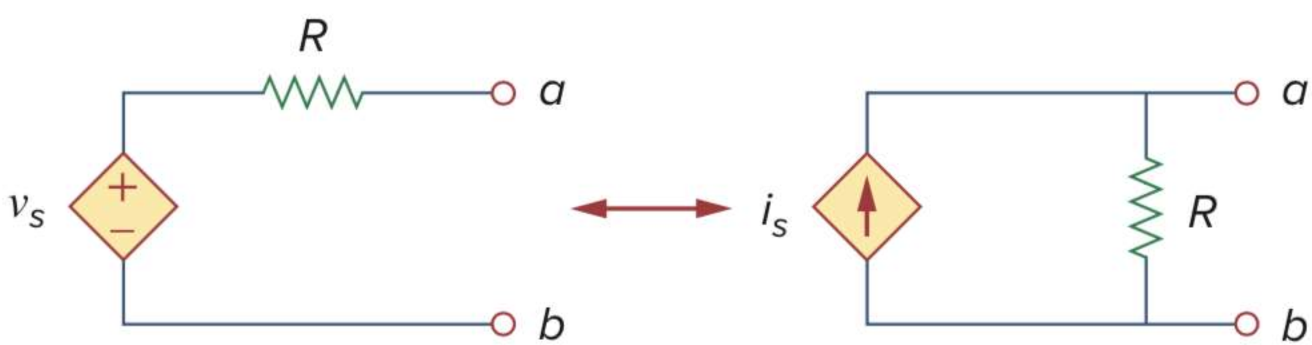


Figure 4.16
Transformation of dependent sources.

1）“电源变换”，受控源也同样适用
2）只适合于“一端口网络”

Example 4.6

Use source transformation to find v_o in the circuit of Fig. 4.17.

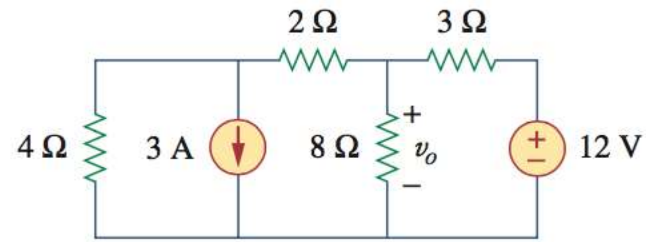


Figure 4.17
For Example 4.6.

①不含受控源

Solution:
We first transform the current and voltage sources to obtain the circuit in Fig. 4.18(a). Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us Fig. 4.18(b). We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. 4.18(c).

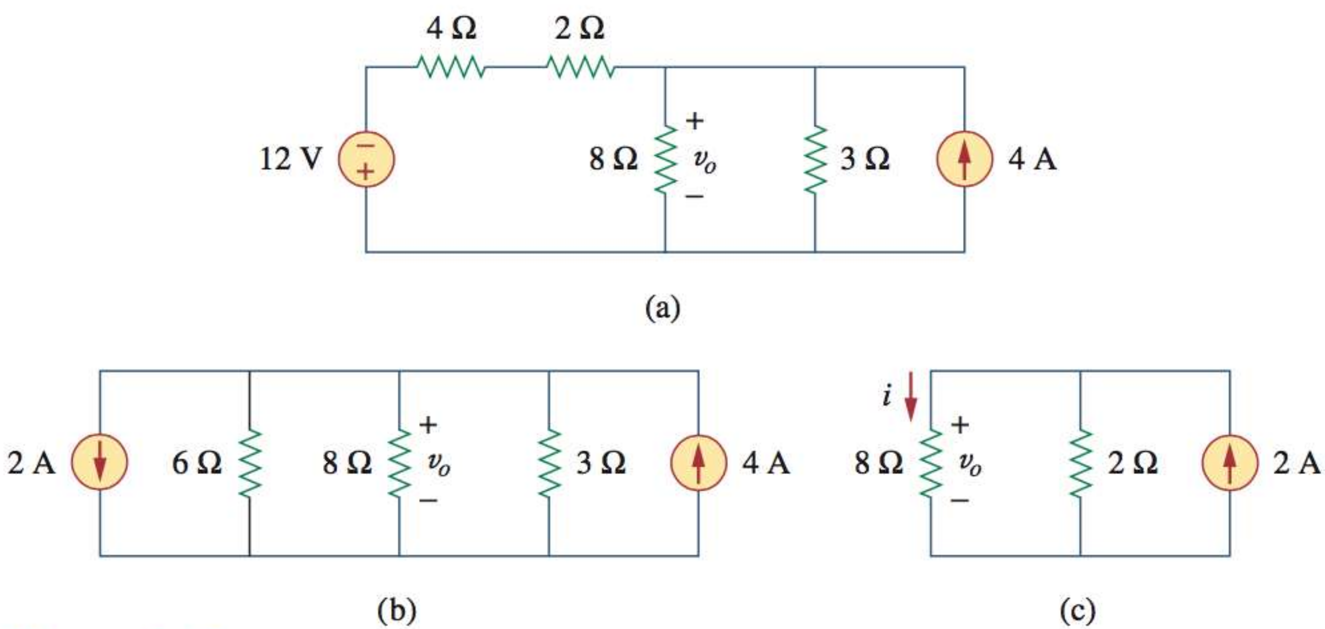


Figure 4.18
For Example 4.6.

Example 4.7

Find v_x in Fig. 4.20 using source transformation.

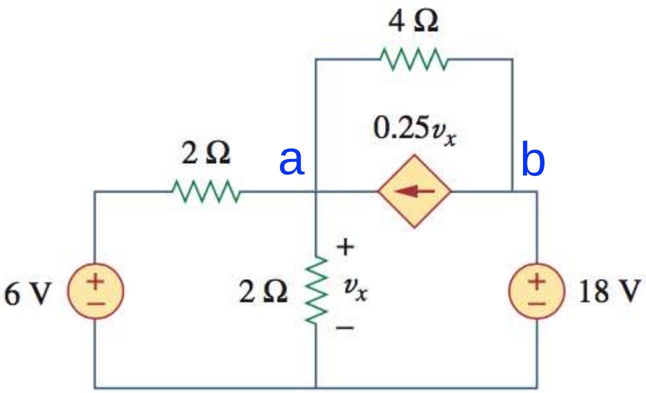


Figure 4.20
For Example 4.7.

②含受控源

a、b构成一端口

Solution:

The circuit in Fig. 4.20 involves a voltage-controlled dependent current source. We transform this dependent current source as well as the 6-V independent voltage source as shown in Fig. 4.21(a). The 18-V voltage source is not transformed because it is not connected in series with any resistor. The two 2-Ω resistors in parallel combine to give a 1-Ω resistor, which is in parallel with the 3-A current source. The current source is transformed to a voltage source as shown in Fig. 4.21(b). Notice that the terminals for v_x are intact. Applying KVL around the loop in Fig. 4.21(b) gives

$$-3 + 5i + v_x + 18 = 0 \tag{4.7.1}$$

$$v_x = 3 - i$$

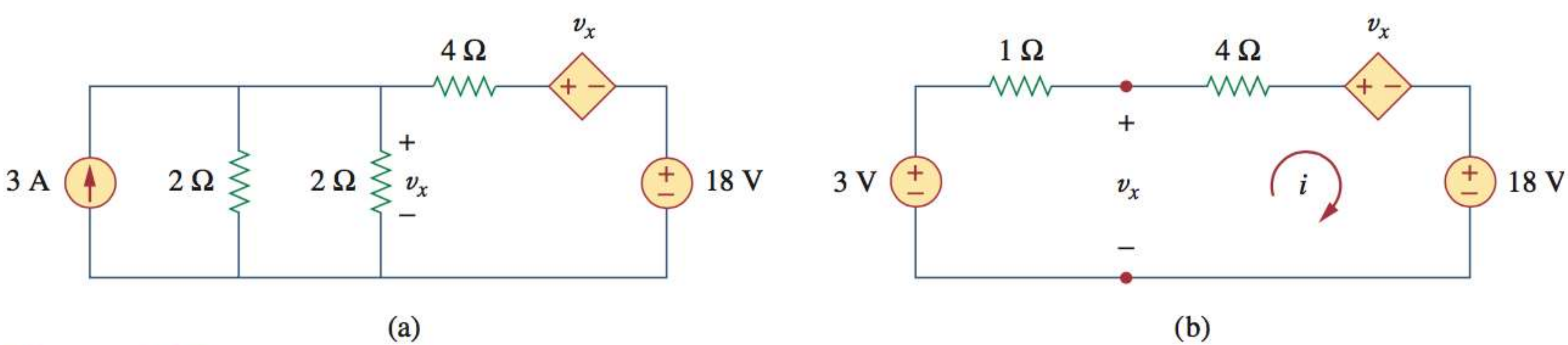
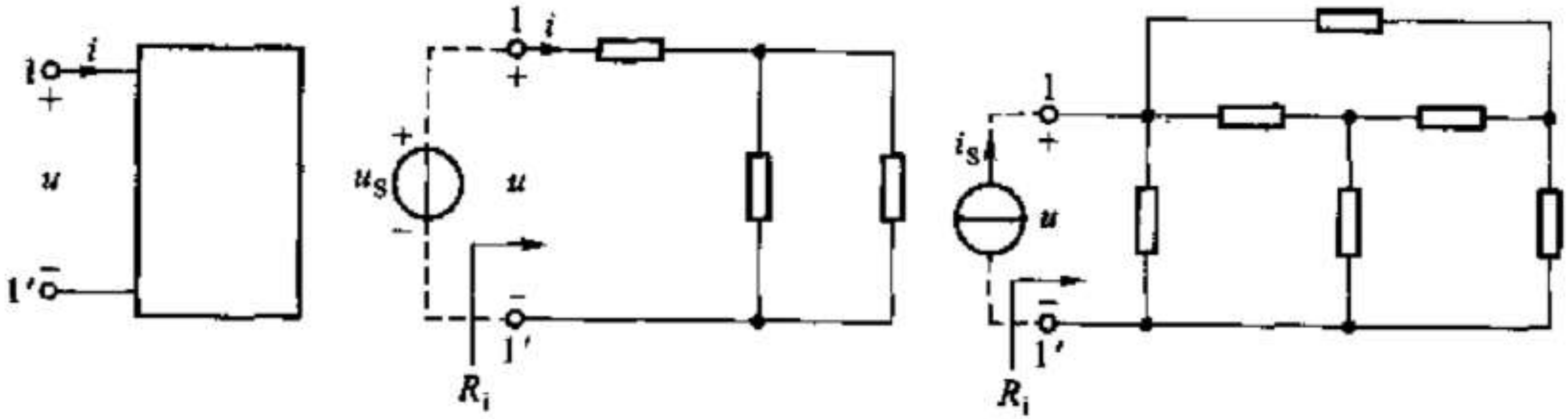


Figure 4.21
For Example 4.7: Applying source transformation to the circuit in Fig. 4.20.

输入电阻 / 等效电阻

- **针对一端口网络：**
  - 如果一端口网络内部**仅含电阻**。则应用电阻的串并联、Δ – Y 变换等方法，可以求得它的等效电阻，该等效电阻就为输入电阻；
  - 如果一端口网络内部除电阻外，还**含受控源**，但**不含任何独立电源**。不论内部如何复杂，端口电压和端口电流成正比，其比例就是输入电阻R_i【计算方法】：施加电压求电流 / 施加电流求电压
  - 如果一端口网络内如含独立电源，则先将其**turn off**（电压源 → 0V → 短路；电流源 → 0A → 开路），再计算；



Turn off 独立源，若仅含电阻，则化简；若还含受控源，则施加激励求响应

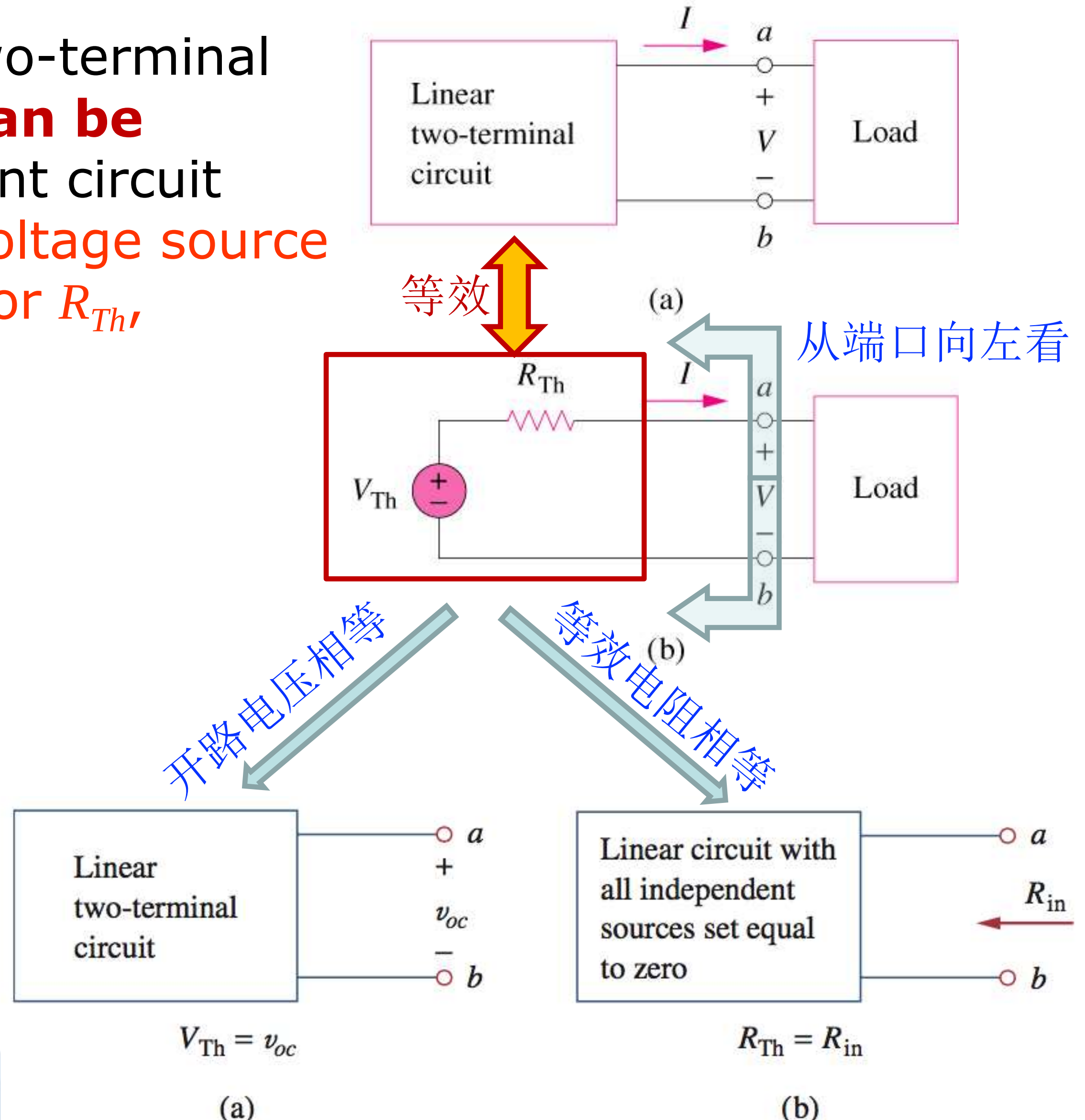
图 2 – 17 一端口的输入电阻

4.5 Thevenin's Theorem 戴维南定理

It states that a **linear** two-terminal (一端口) circuit (Fig. a) **can be replaced** by an equivalent circuit (Fig. b) consisting of a voltage source V_{Th} in series with a resistor R_{Th},

where

- V_{Th} is the open-circuit voltage at the terminals. 【开路电压】
- R_{Th} is the input or equivalent resistance at the terminals when the **independent** sources are **turned off**. 【输入电阻/等效电阻】



Example 4.8

Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6, 16,$ and $36\ \Omega$.

①不含受控源

① 对【32V和4ohm】 用“source transformation”化简电路

② 若对【12ohm和2A】 用“source transformation”，c、b构成一端口

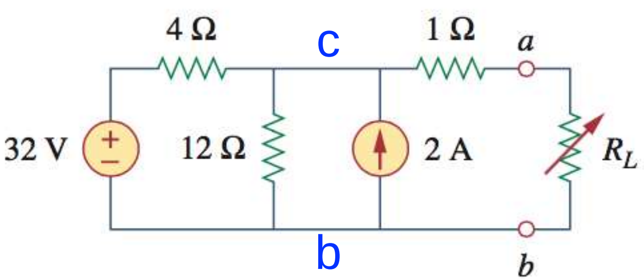


Figure 4.27
For Example 4.8.

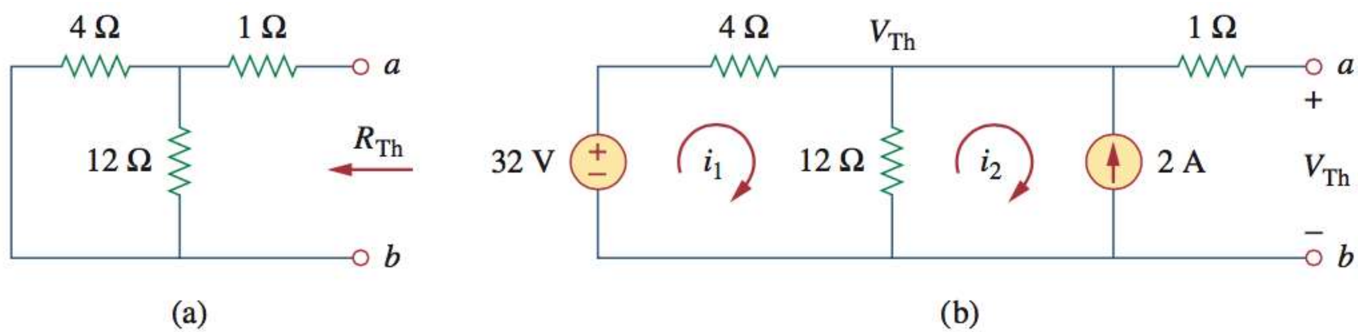


Figure 4.28
For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

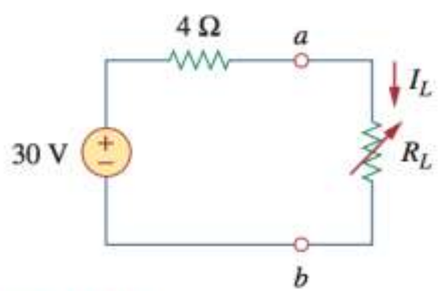


Figure 4.29
The Thevenin equivalent circuit for Example 4.8.

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\ \text{A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\ \text{V}$$

Example 4.9

Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals a-b.

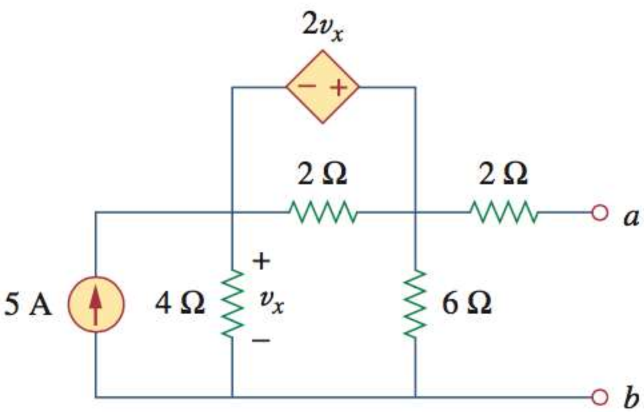


Figure 4.31
For Example 4.9. ②含受控电压源

Solution:

This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source v_o connected to the terminals as indicated in Fig. 4.32(a). We may set $v_o = 1$ V to ease calculation, since the circuit is linear. Our goal is to find the current i_o through the terminals, and then obtain $R_{Th} = 1/i_o$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage v_o, and obtain $R_{Th} = v_o/1$.)

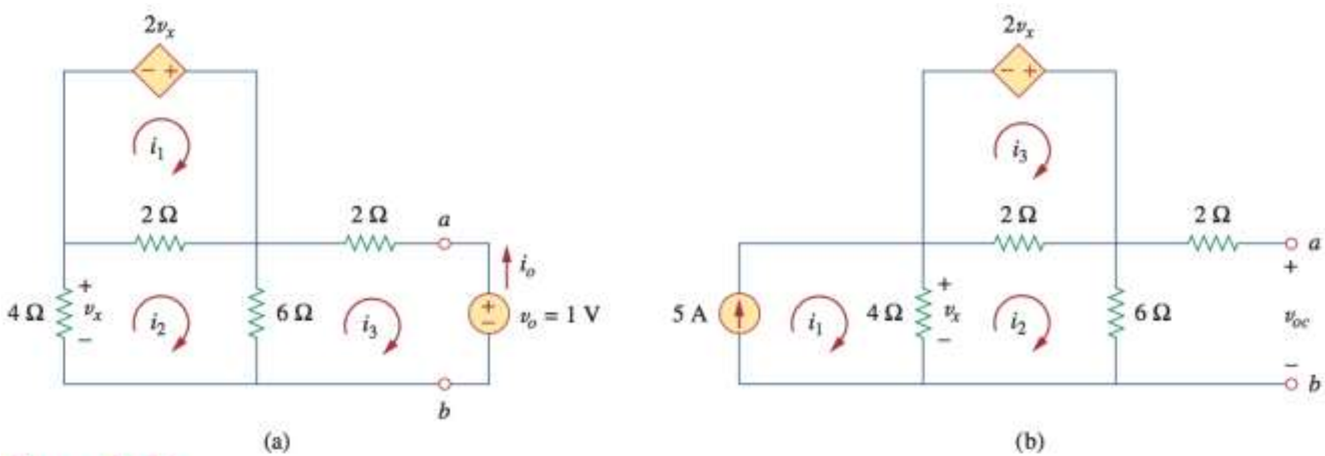


Figure 4.32
Finding R_{Th} and V_{Th} for Example 4.9.

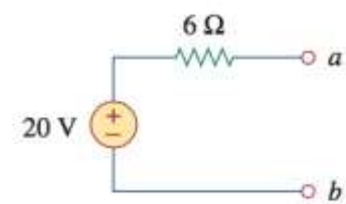
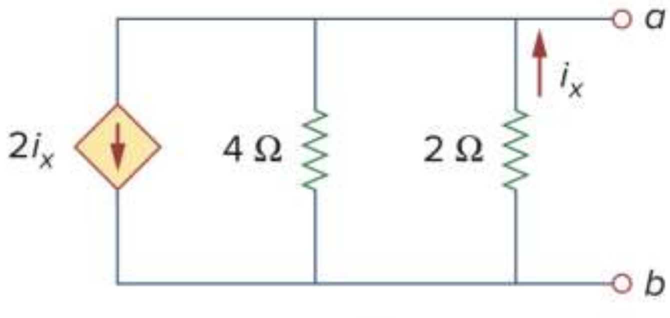


Figure 4.33
The Thevenin equivalent of the circuit in Fig. 4.31.

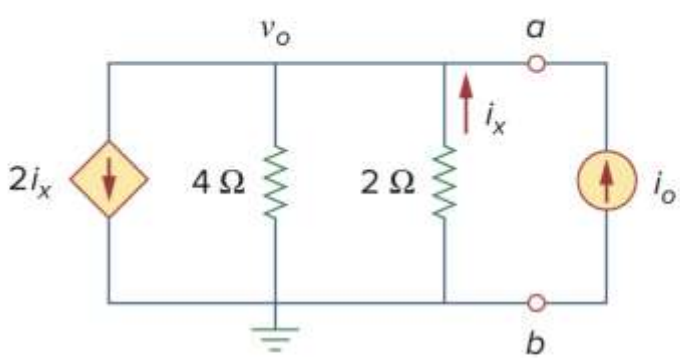
Example 4.10

Determine the Thevenin equi valent of the circuit in Fig. 4.35(a) at terminals *a*-*b*.



(a)

③含受控电流源

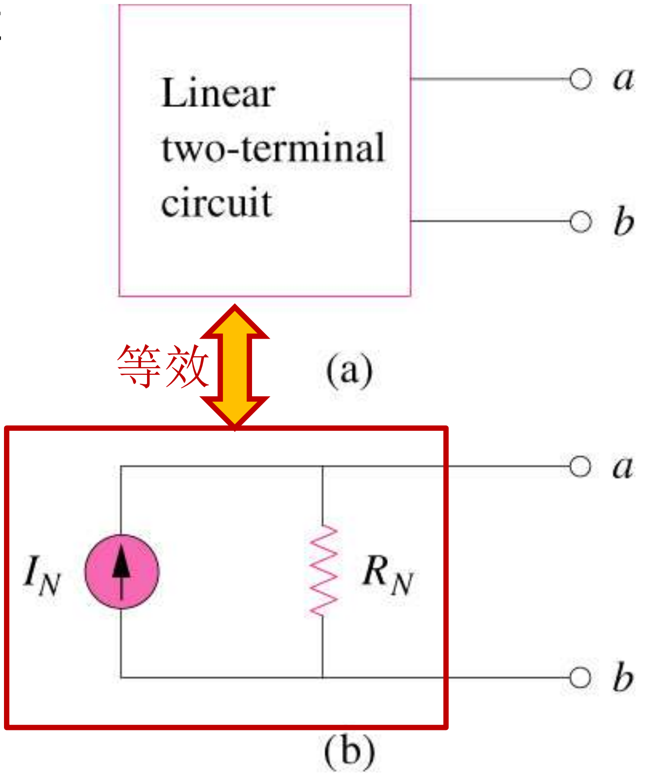


(b)

$$R_{Th} = -4\,\Omega.$$

4.6 Norton’s Theorem 诺顿定理

It states that a **linear** two-terminal （一端口） circuit **can be replaced** by an equivalent circuit of a current source I_N in parallel with a resistor R_N,

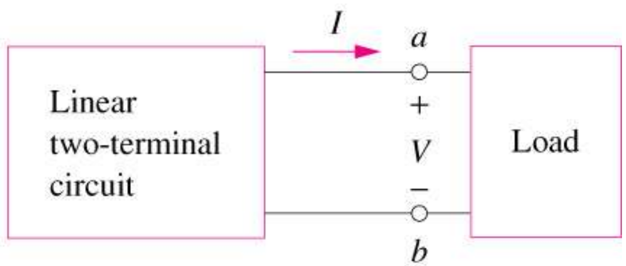


Where

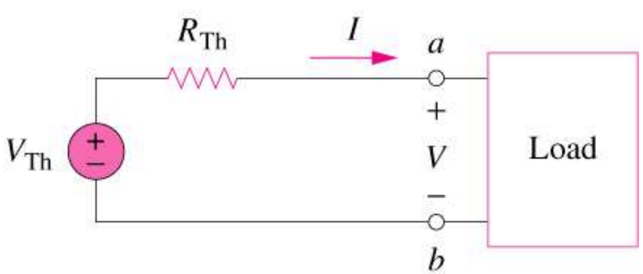
- I_N is the **short circuit current** through the terminals. 【短路电流】

- R_N is the input or equivalent resistance at the terminals when **the independent** sources are **turned off**. 【输入电阻/等效电阻】

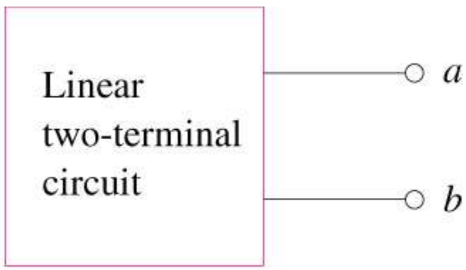
The Thevenin's and Norton equivalent circuits are related by a source transformation. 【戴维南定理和诺顿定理的关系~电源转换】



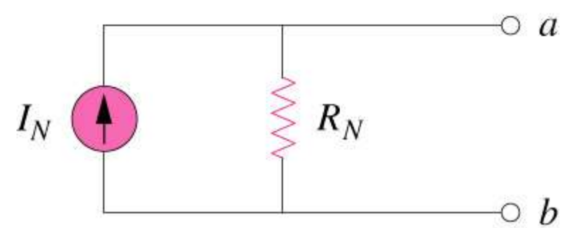
(a)



(b)



(a)



(b)

$$R_N = R_{Th}$$

$$I_N = \frac{V_{Th}}{R_{Th}}$$

Example 4.11

Find the Norton equivalent circuit of the circuit in Fig. 4.39 at terminals *a*-*b*.

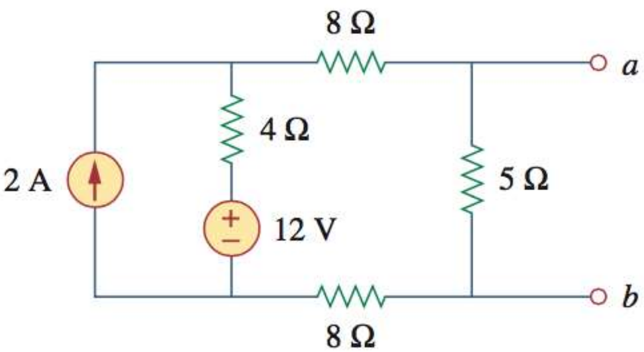
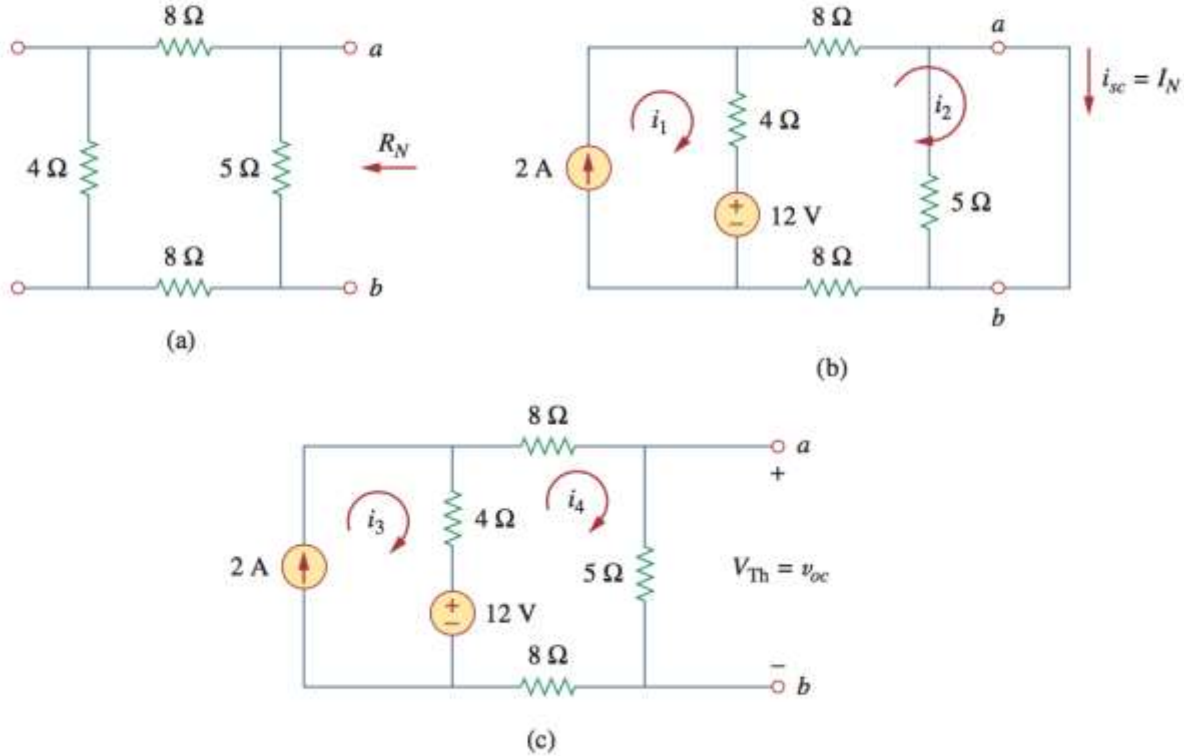


Figure 4.39
For Example 4.11.

①不含受控源

先用source transformation化简



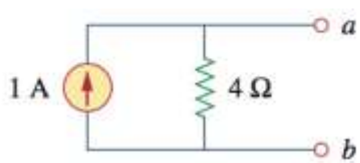


Figure 4.41
Norton equivalent of the circuit in Fig. 4.39.

Figure 4.40
For Example 4.11; finding: (a) R_N, (b) $I_N = i_{sc}$, (c) $V_{Th} = v_{oc}$.

$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

$$v_{oc} = V_{Th} = 5i_4 = 4\text{ V}$$

Example 4.12

Using Norton's theorem, find R_N and I_N of the circuit in Fig. 4.43 at terminals *a*-*b*.

②含受控源

计算短路电流时，i_x可先计算得出

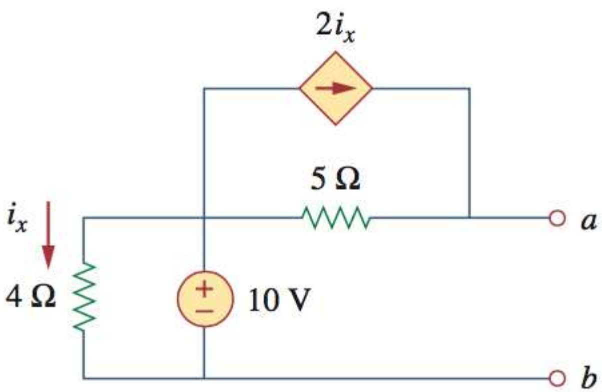


Figure 4.43
For Example 4.12.

$$R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\ \Omega$$

At node *a*, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\text{ A}$$

Thus,

$$I_N = 7\text{ A}$$

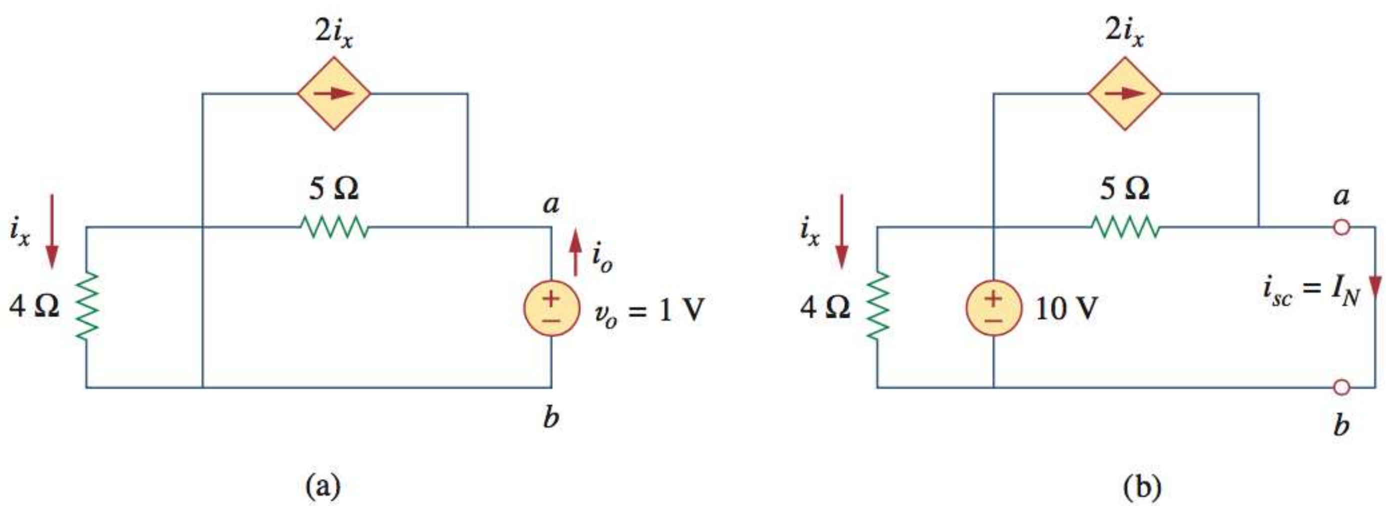
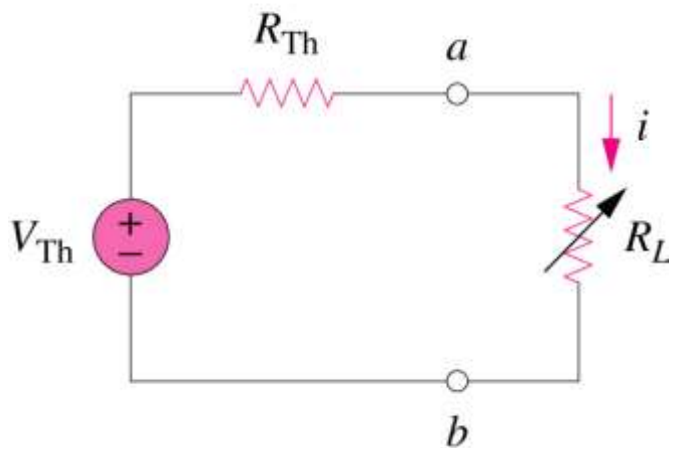


Figure 4.44
For Example 4.12: (a) finding R_N, (b) finding I_N.

4.7 Maximum Power Transfer 最大功率传输

If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:

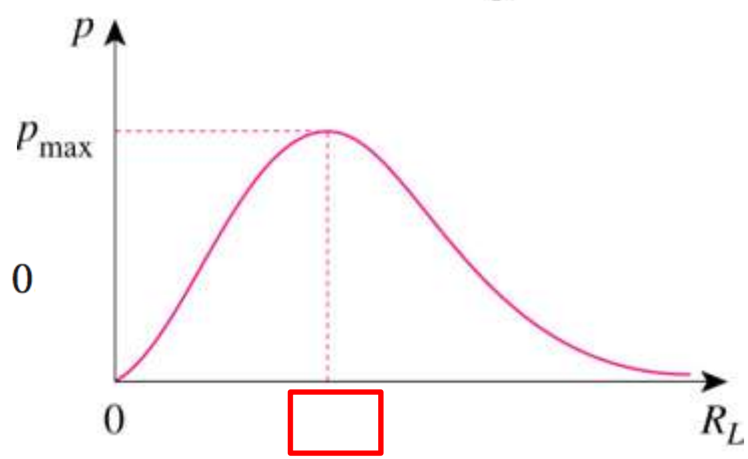


$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

- 如何求P的最大值？变量为 R_L

$$\frac{dp}{dR_L} = V_{Th}^2\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$

$$= V_{Th}^2\left[\frac{(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^3}\right] = 0 \qquad d^2p/dR_L^2 < 0$$



The power transfer profile with different R_L

$$\boxed{R_L = R_{Th}}$$

$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

The source and load are said to be *matched* when $R_L = R_{Th}$.

Example 4.13

Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

最大功率传输：
先计算戴维南等效电路

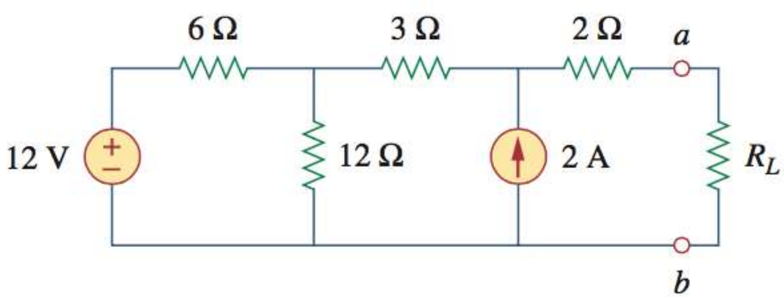


Figure 4.50
For Example 4.13.

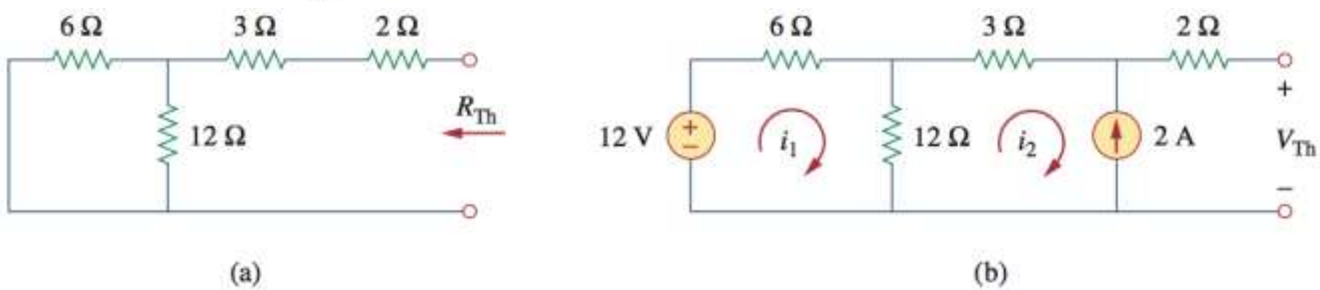


Figure 4.51
For Example 4.13: (a) finding R_{Th}, (b) finding V_{Th}.

$$R_{Th} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\text{ V}$$

For maximum power transfer,

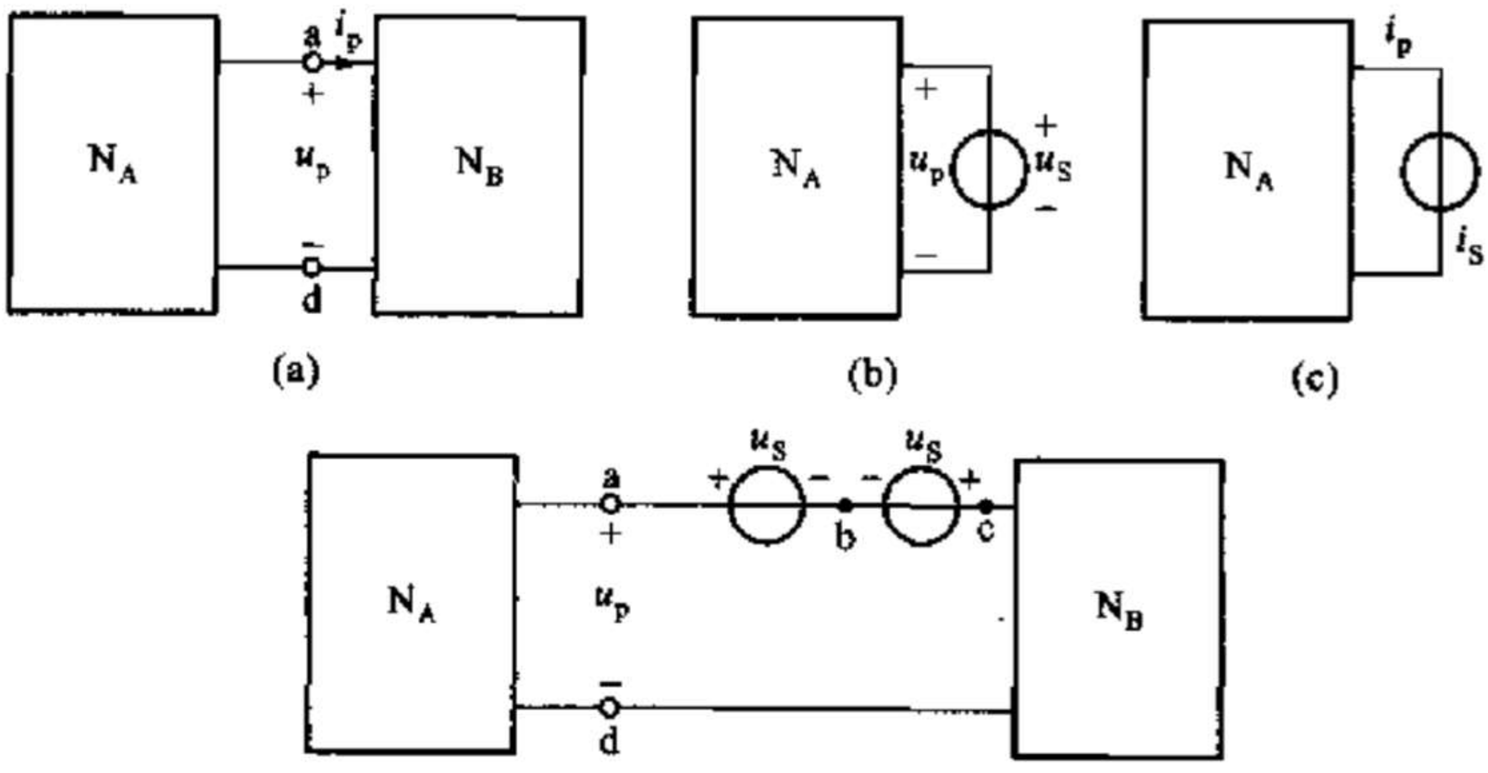
$$R_L = R_{Th} = 9\ \Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}$$

替代定理

在电路中，已知 N_A、N_B 两个一端口网络连接的电压为 u_p 和 i_p，那么就可以用一个$u_s = u_p$的电压源，或者一个 $i_s = i_p$ 的电流源来替代其中的一个网络



应用——实际电源

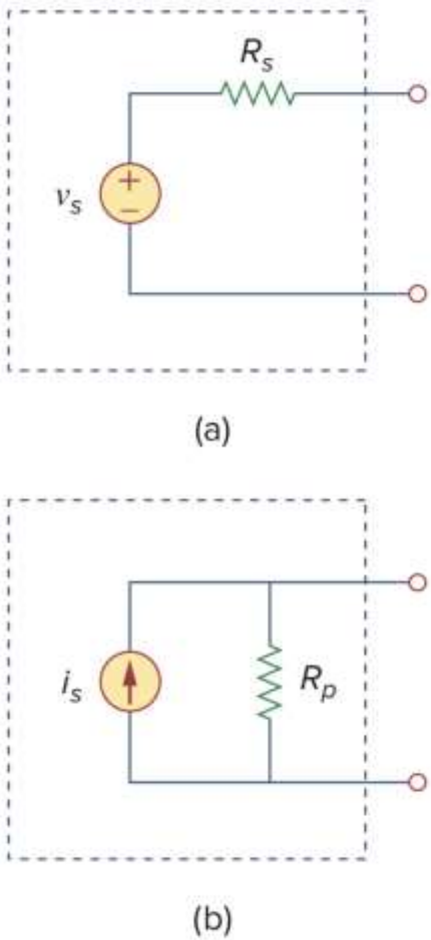


Figure 4.58
(a) Practical voltage source, (b) practical current source.

$$v_L = \frac{R_L}{R_s + R_L} v_s$$

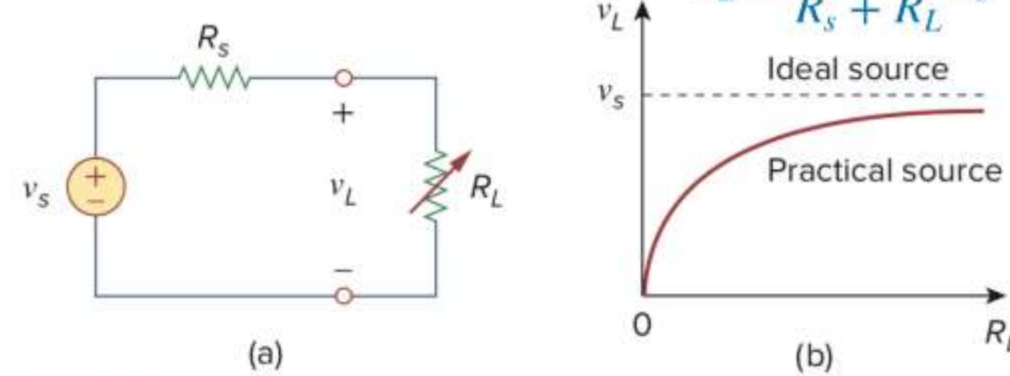
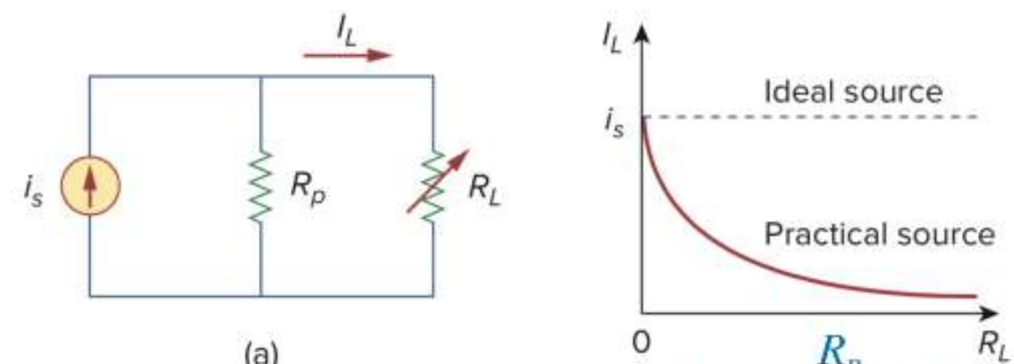


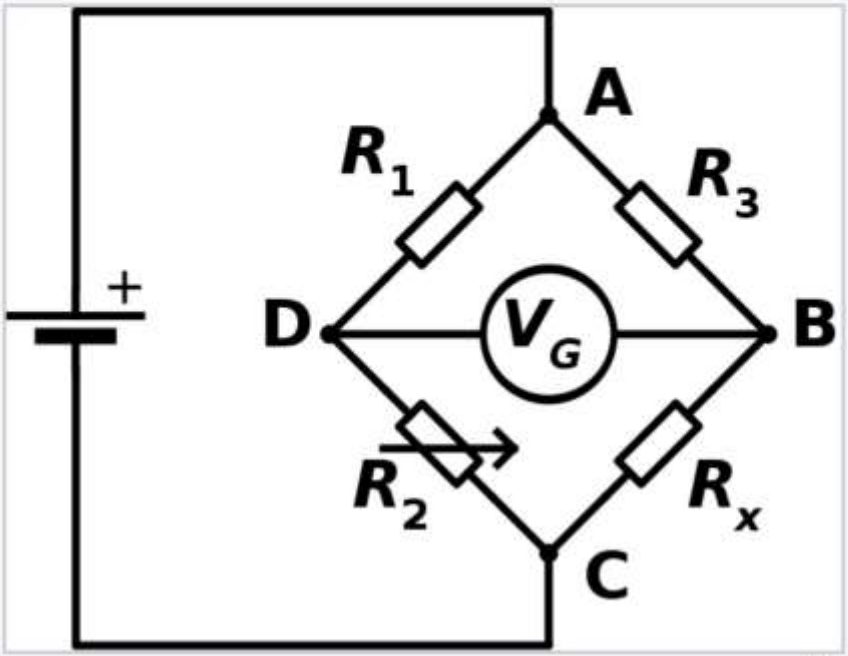
Figure 4.59
(a) Practical voltage source connected to a load R_L, (b) load voltage decreases as R_L decreases.



$$i_L = \frac{R_p}{R_p + R_L} i_s$$

- 电压源希望内阻R_s越小越好
- 电流源希望内阻R_p越大越好
- Q：如何测量v_s、R_s？调节R_L使$v_L = 0.5\ v_{oc}$

应用——电阻测量（惠斯通电桥）



Wheatstone bridge circuit diagram. The unknown resistance R_x is to be measured; resistances R_1, R_2 and R_3 are known and R_2 is adjustable. If the measured voltage V_G is 0, then $R_2/R_1 = R_x/R_3$.

- R_x 是待测电阻；R_1、R_3已知；R_2可调
- 调节R_2，使得电压表V_G=0时，电桥平衡

$$\boxed{R_x = \frac{R_3}{R_1} R_2}$$

Example 4.18

The circuit in Fig. 4.64 represents an unbalanced bridge. If the galvanometer has a resistance of 40 Ω, find the current through the galvanometer.

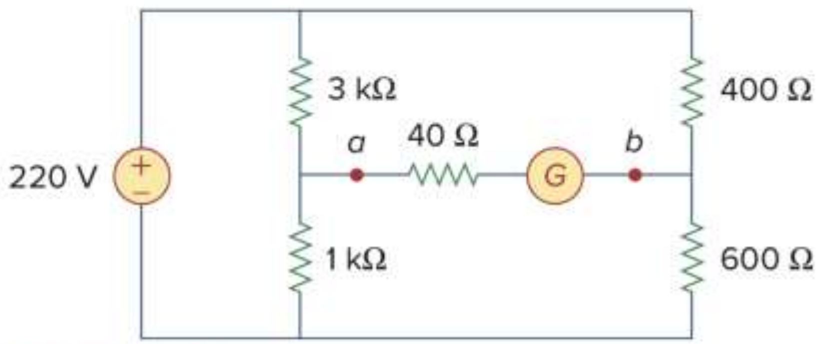


Figure 4.64
Unbalanced bridge of Example 4.18.

当电桥不平衡时，用戴维南定理等效a-b端口的网络

应用——基于惠斯通电桥的温度传感器

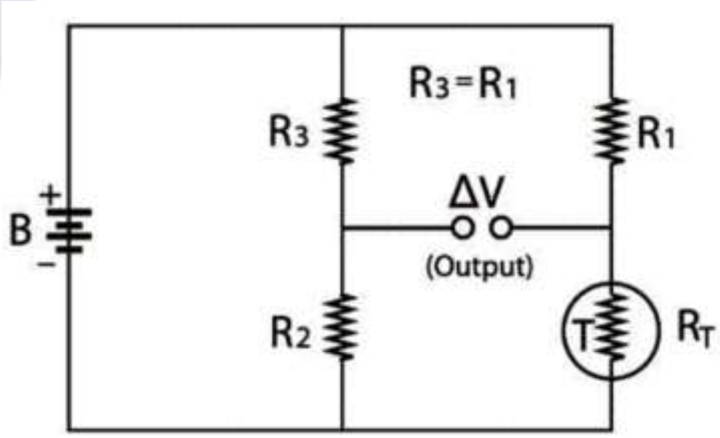


Figure 2. Temperature Measurement Using Thermistors

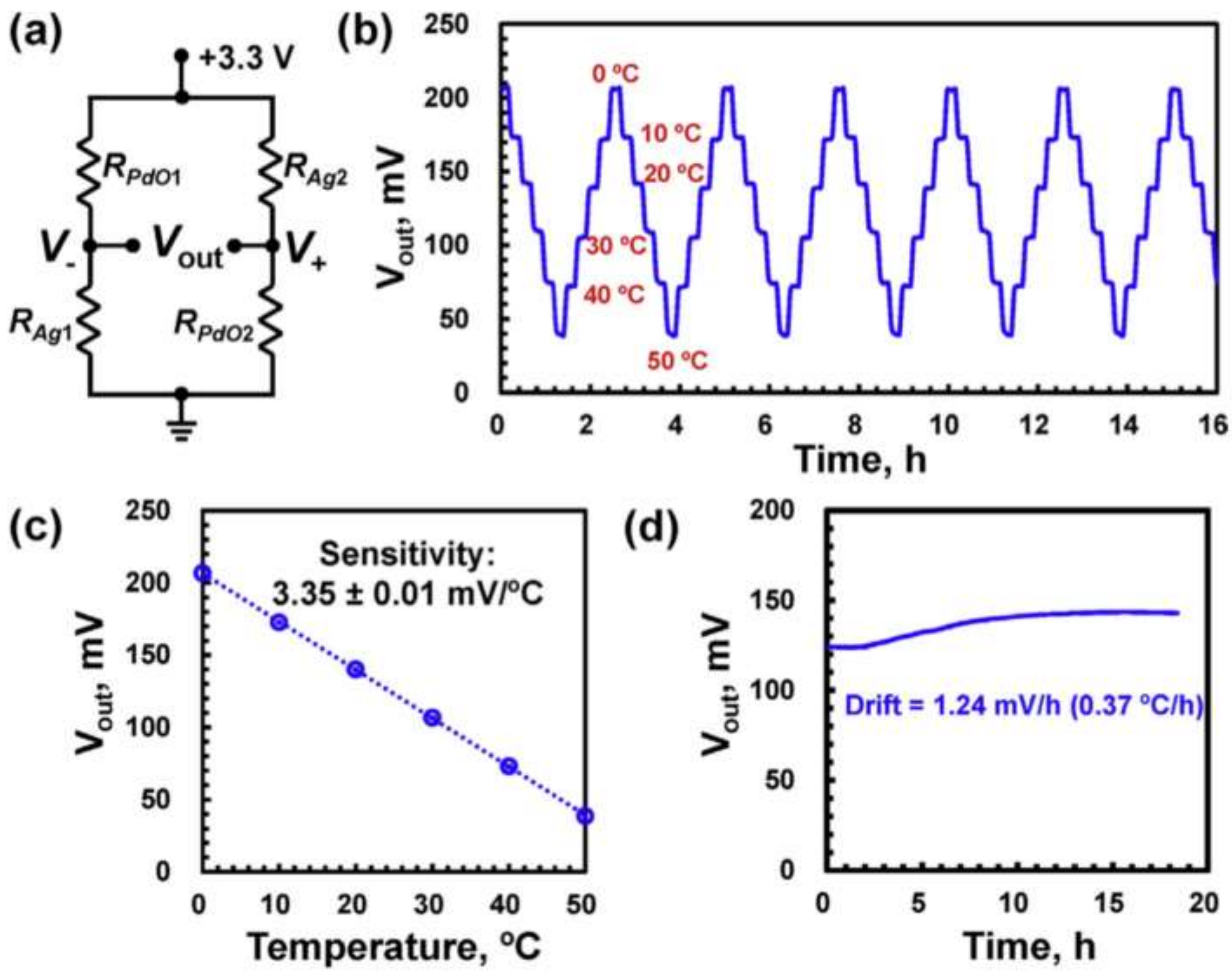
- 基于非平衡的惠斯通电桥
- R_T是温度敏感的电阻，一般随温度增加，阻值减小（NTC电阻，negative temperature coefficient）
- 通过测量 delta V，得出温度信息T

$$\Delta V = E\left(\frac{R_2}{R_2 + R_3} - \frac{R_T}{R_1 + R_T}\right)$$

Ag：正温度系数材料

PdO：负温度系数材料

Y. Qin et al. / Sensors and Actuators B 255 (2018) 781–790



Capacitors and Inductors
Chapter 6

电容、电感，及它们的串并联

6.1 Capacitors 电容

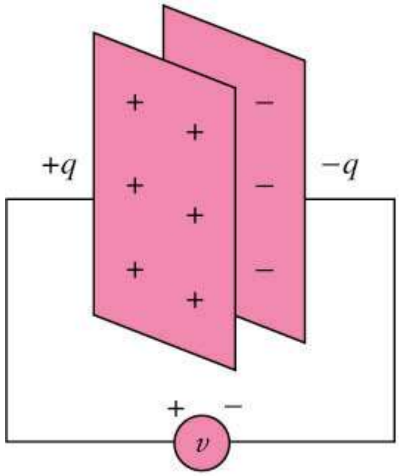
6.2 Series and Parallel Capacitors

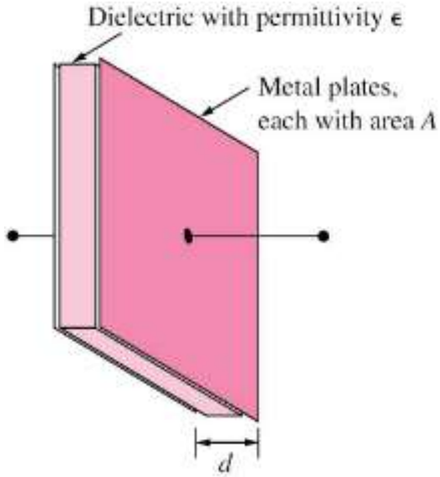
6.3 Inductors 电感

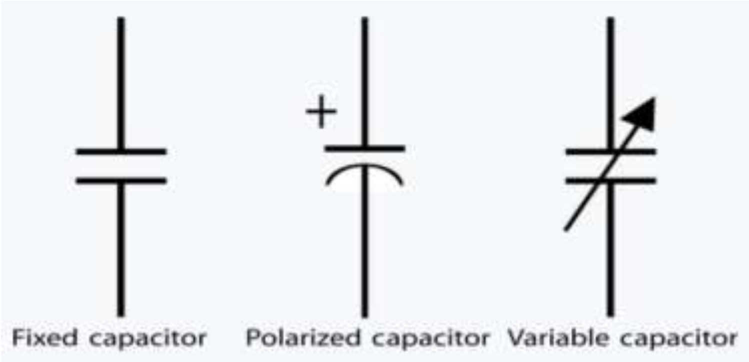
6.4 Series and Parallel Inductors

6.1 Capacitors 电容

- A capacitor is a **passive** (无源) element designed to **store energy** in its **electric field.** 储存电场能量



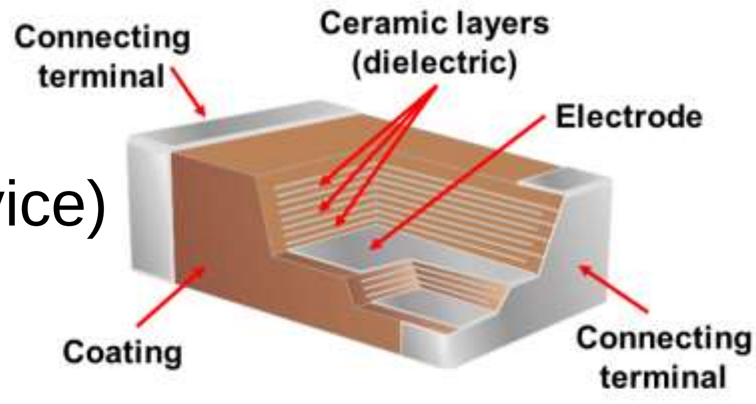


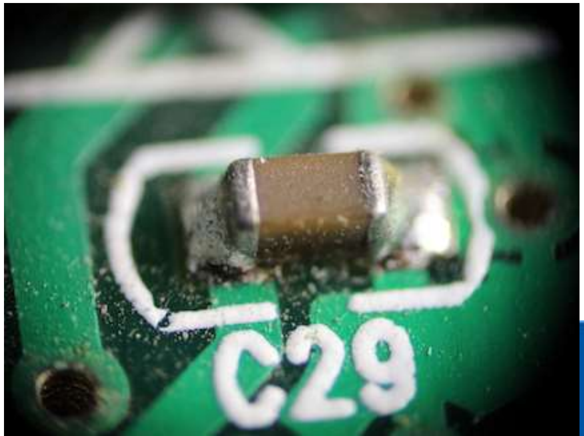


- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

Through-hole
直插式电容

SMD
(surface mount device)
贴片电容

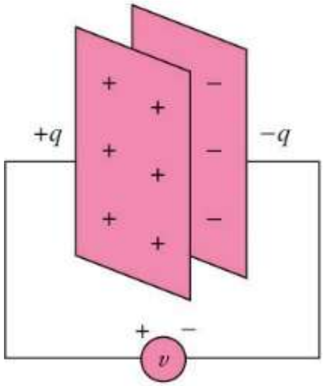




6.1 Capacitors (2)

电容的定义：导板上的电荷和导板之间电势差的比值

- **Capacitance** C is the ratio of the charge q on one plate of a capacitor to the voltage difference v between the two plates, measured in farads (F).



$$\boxed{q = C\,v}$$

平板上的电荷量　　电压

and

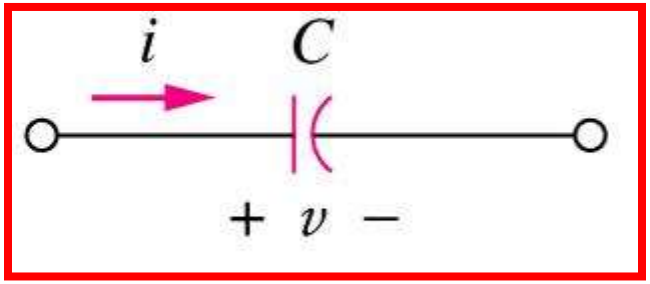
$$C = \frac{\varepsilon A}{d}$$

介电常数　平板面积　平板间距离

- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

6.1 Capacitors (3)

- If i is flowing into the $+v$ terminal of C
  - 电流从+端流入→ 充电Charging
  - 电流从+端流出→ 放电Discharging

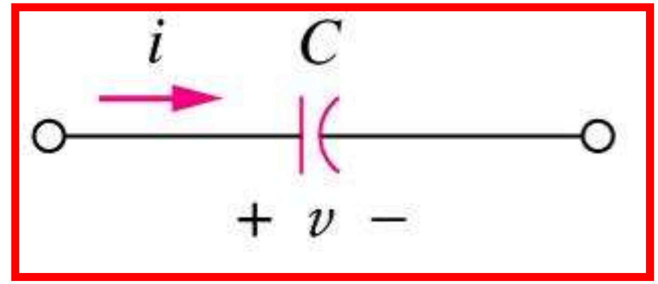


- 电容也是基本的电路元件，我们需要熟悉流过它的电流和它两端电压之间的关系，就如熟悉电阻一样。The current-voltage relationship of capacitor according to above convention is ：电容的“电压-电流”关系

$$\left.\begin{array}{l} i = \dfrac{dq}{dt} \\ q = Cv \end{array}\right\} \Rightarrow \boxed{i = C\frac{dv}{dt}} \qquad \boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)}$$

6.1 Capacitors (4)

- The energy, w, stored in the capacitor is (电容存储的电能):



$$p = vi = Cv\frac{dv}{dt}$$

$$w = C\int_{-\infty}^{t} v\frac{dv}{d\tau}\,d\tau = \frac{1}{2}Cv^2\Big|_{v(-\infty)}^{v(t)}$$

$$\boxed{w = \frac{1}{2}Cv^2}$$

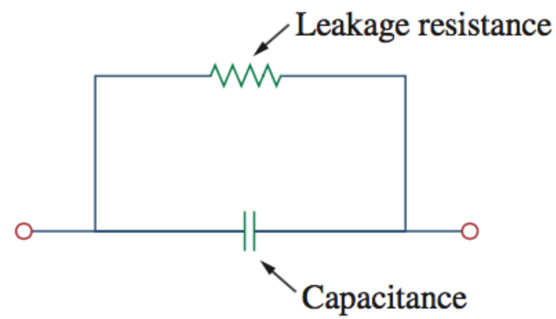
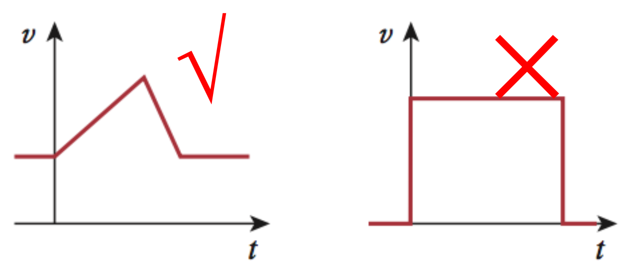


Figure 6.8
Circuit model of a nonideal capacitor.

- 电容的两个基本属性
  - an **open circuit** to dc (dv/dt = 0). 直流开路
  - its voltage **cannot change abruptly**. 电容两端电压不能突变, Why?

$$i = C\frac{dv}{dt}，\ v突变，则i为\infty$$



Example 6.1

$$\boxed{q = C\,v}$$

$$\boxed{w = \frac{1}{2} C\,v^2}$$

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

Example 6.2

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:
By definition, the current is

$$\begin{aligned} i(t) &= C\frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt}(10 \cos 6000t) \\ &= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A} \end{aligned}$$

Example 6.3

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

Since $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$

$$= \frac{3 \times 10^3}{-3000} e^{-3000t} \bigg|_0^t = (1 - e^{-3000t}) \text{ V}$$

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,d\,t + v(t_0)}$$

Example 6.4

Determine the current through a 200- μF capacitor whose voltage is shown in Fig. 6.9.

Solution:

The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\, dv/dt$ and $C = 200\ \mu$F, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus, the current waveform is as shown in Fig. 6.10.

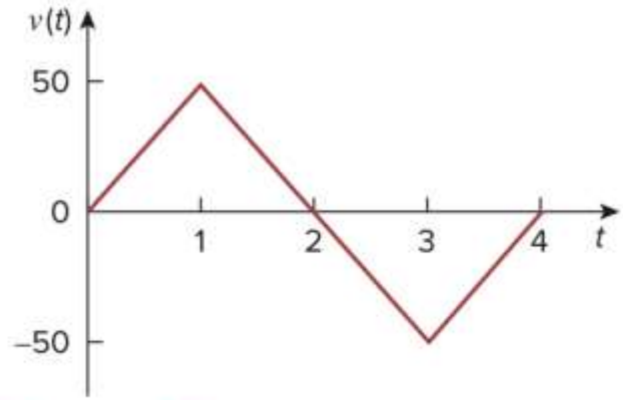


Figure 6.9
For Example 6.4.

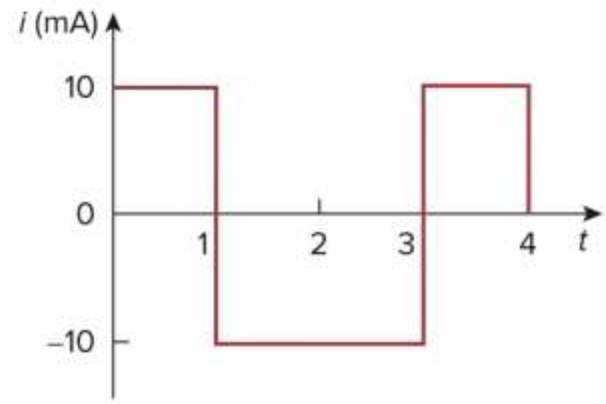


Figure 6.10
For Example 6.4.

Example 6.5

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.

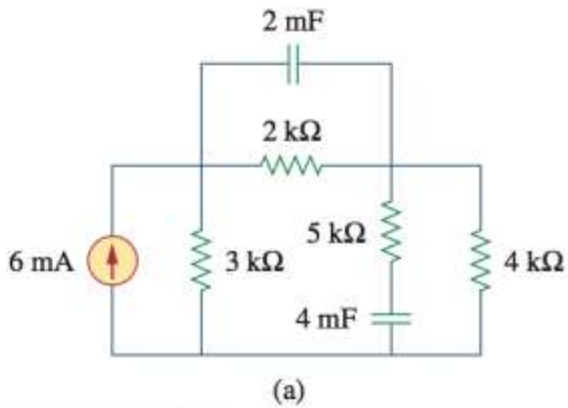
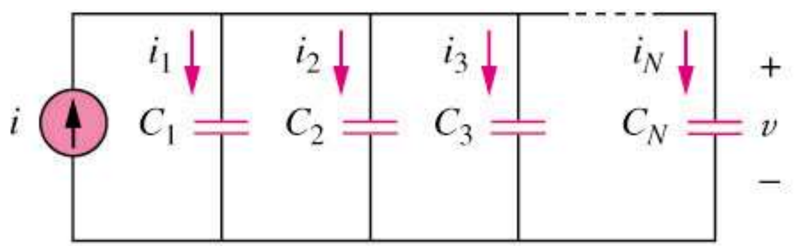


Figure 6.12
For Example 6.5.

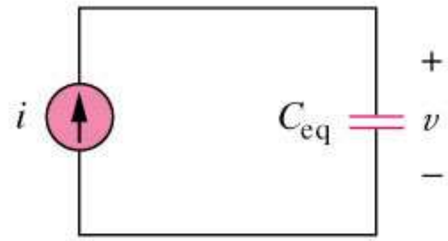
电容：直流开路

6.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



(a)



(b)

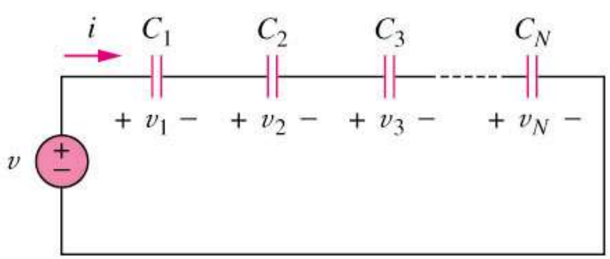
$$\boxed{i = C\frac{dv}{dt}}$$

$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

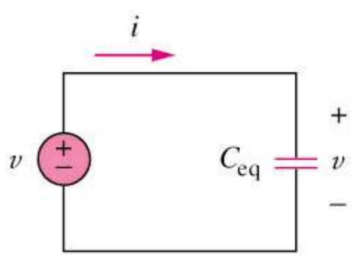
电容并联：
电压相等，电流相加 → C相加

6.2 Series and Parallel Capacitors (2)

- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)



(b)

$$v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

若N=2：

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

电容串联：

电流相等，电压相加 →1/C相加 → 类似于电阻并联

Example 6.6

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.

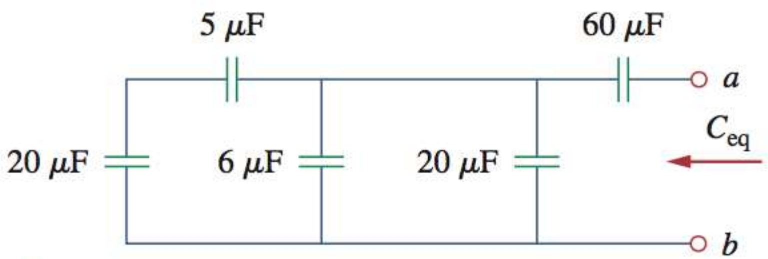


Figure 6.16
For Example 6.6.

Example 6.7

For the circuit in Fig. 6.18, find the voltage across each capacitor.

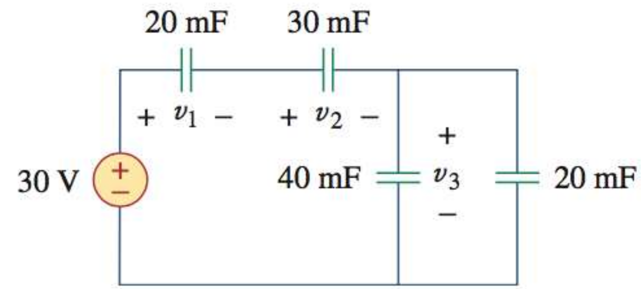


Figure 6.18
For Example 6.7.

Solution:
We first find the equivalent capacitance C_{eq}, shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get 40 + 20 = 60 mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

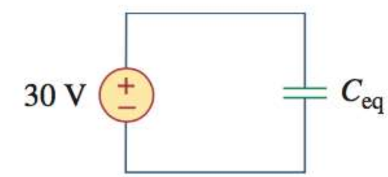


Figure 6.19
Equivalent circuit for Fig. 6.18.

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \qquad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2, we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is 40 + 20 = 60 mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

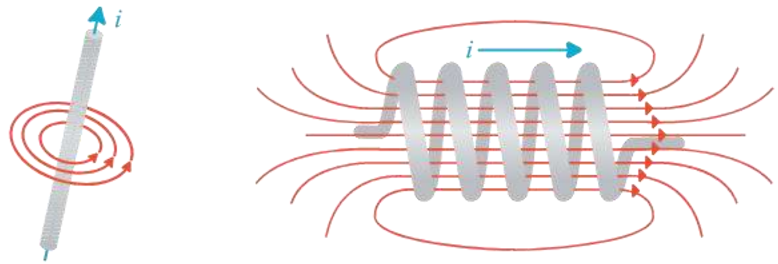
$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

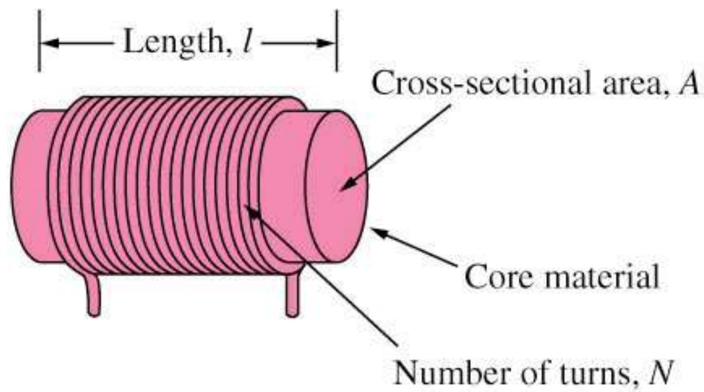
串联电容的q相等

Q=CV, V之比等于1/C之比

6.3 Inductors 电感

- An inductor is a **passive** element designed to store energy in its magnetic field. 储存磁能
- An inductor consists of a coil of conducting wire.





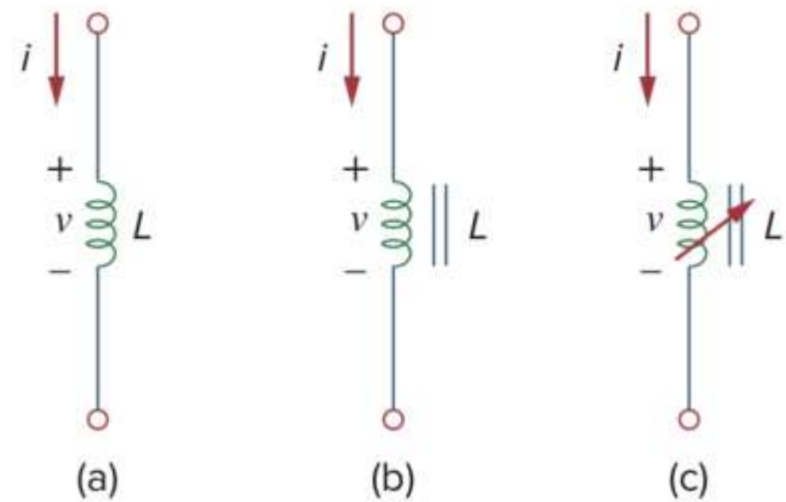
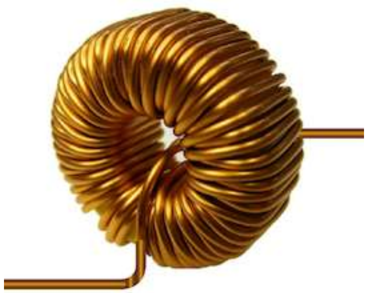


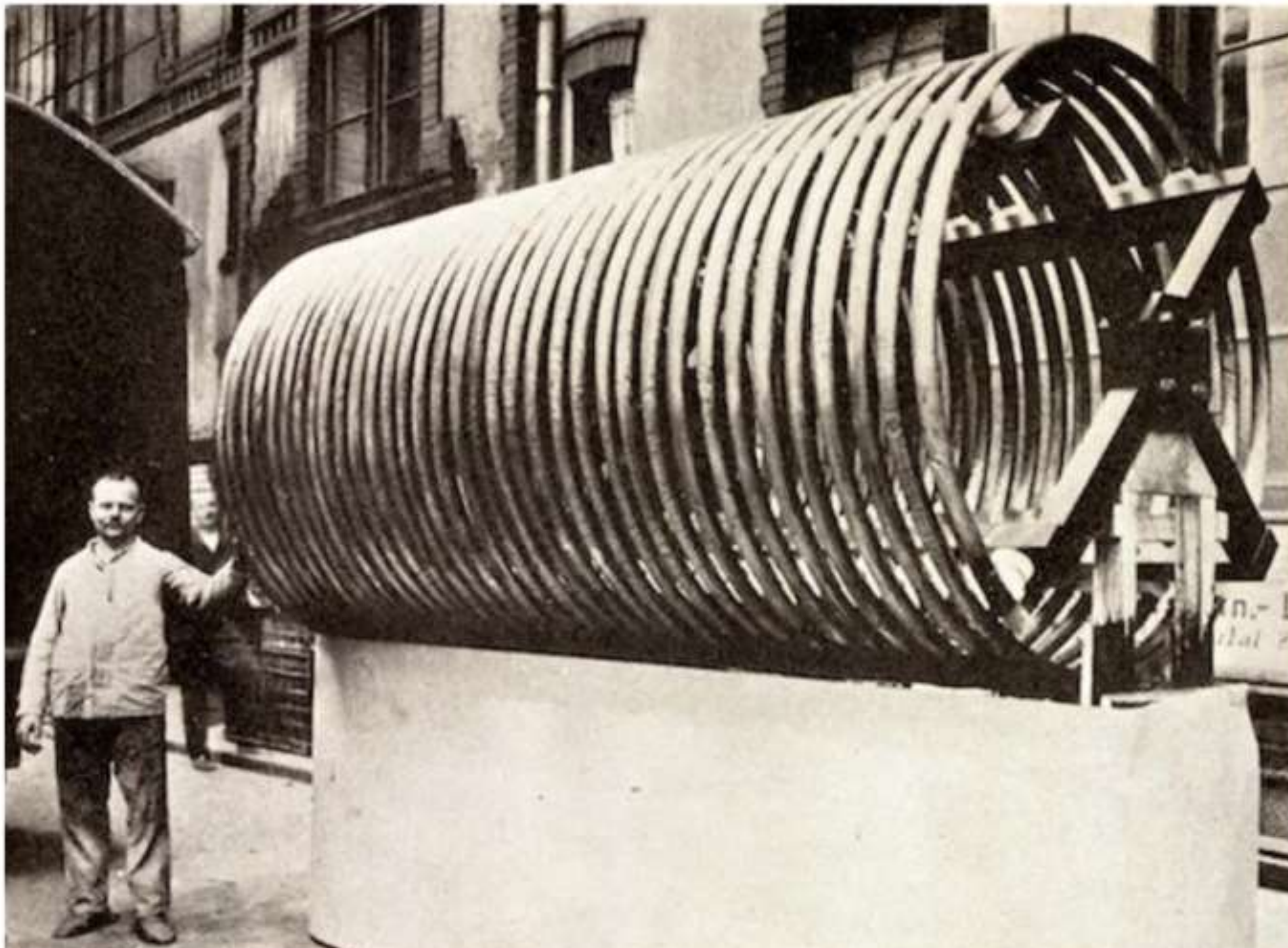
Figure 6.23
Circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core.

$$L = \frac{N^2 \mu A}{\ell}$$

磁导率

copper coil (an inductor) was part of a wireless telegraph station built in New Jersey, USA in 1912. It could send a message 4000 miles (6400 km), all the way across the Atlantic Ocean to Germany. Wow. Needless to say, most inductors are *much* smaller.





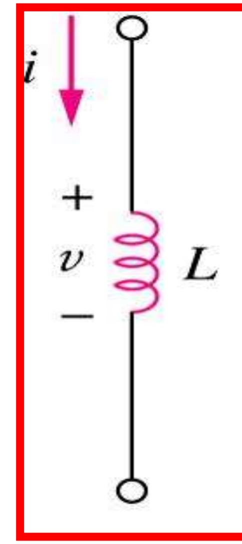
6.3 Inductors (2)

- 电感也是基本的电路元件，我们需要熟悉流过它的电流和它两端电压之间的关系，就如熟悉电阻一样。电感的“电压-电流”关系：

$$\boxed{v = L\frac{di}{dt}} \Longleftrightarrow \boxed{i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

6.3 Inductors (3)



- The power stored by an inductor:

$$p = vi = \left(L\frac{di}{dt}\right)i$$

$$w = L\int_{-\infty}^{t}\frac{di}{d\tau}\,i\,d\tau = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

$$\Rightarrow \boxed{w = \frac{1}{2}Li^2}$$

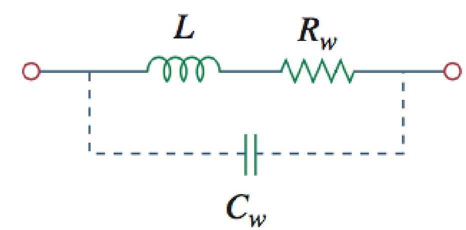
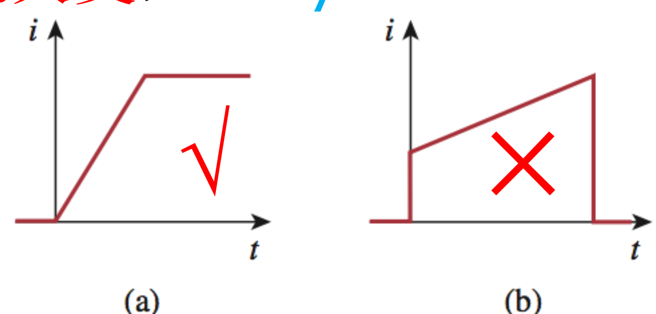


Figure 6.26
Circuit model for a practical inductor.

- 电感的两个基本属性：

①直流短路(di/dt = 0)；②电流不能突变，Why？

$$v = L\frac{d\,i}{d\,t}$$，i突变，则v无穷大



Example 6.8

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

$$\boxed{v = L\frac{di}{dt}}$$

$$\boxed{w = \frac{1}{2}Li^2}$$

Example 6.9

Find the current through a 5-H inductor if the voltage across it is

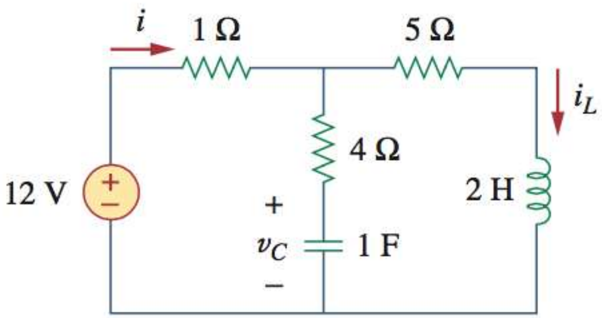
$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

$$\boxed{i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)}$$

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

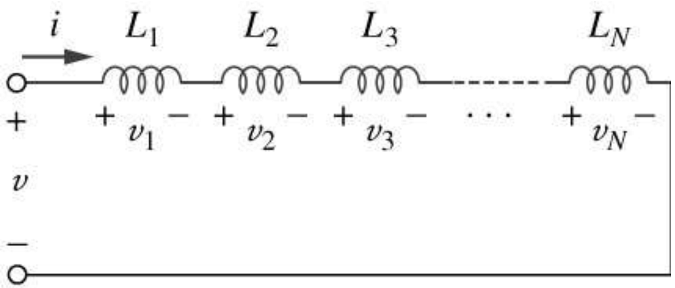


(a)

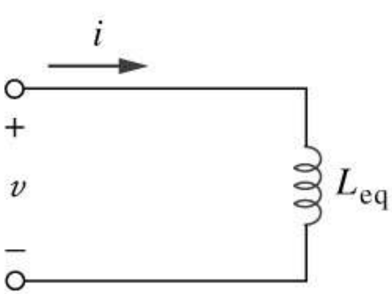
电容直流开路；电感直流短路

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)



(b)

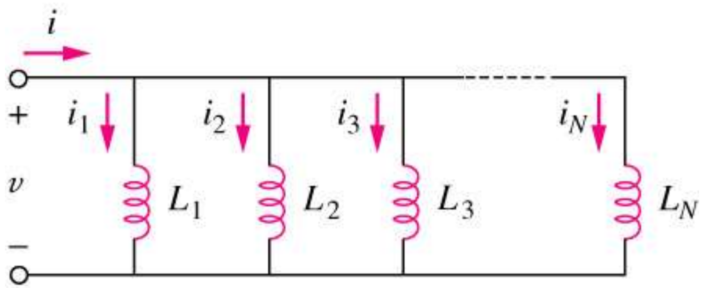
$$v = L\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

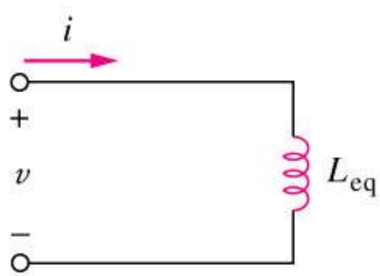
电感串联：
电流相等，电压相加 → L相加

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



(a)



(b)

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,d\,t + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

若N=2： $L_{eq} = \dfrac{L_1 L_2}{L_1 + L_2}$

电感并联：
电压相等，电流相加 → 1/L 相加

电感的串并联与电阻器的电阻值计算一样！

Example 6.11

Find the equivalent inductance of the circuit shown in Fig. 6.31.

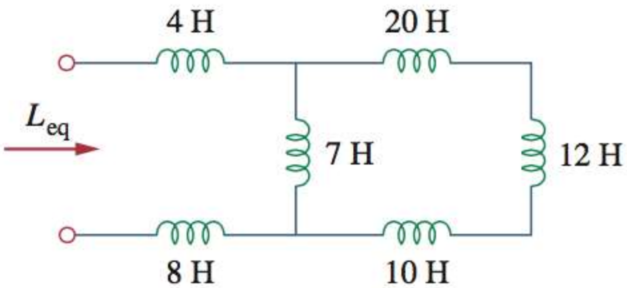


Figure 6.31
For Example 6.11.

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.

Example 6.12

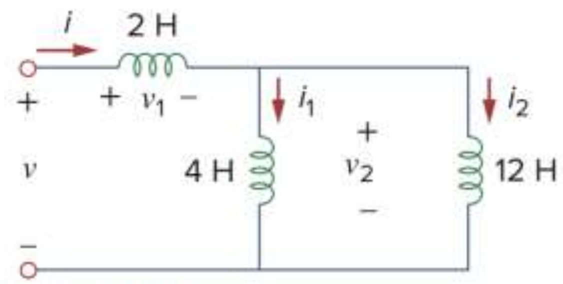


Figure 6.33
For Example 6.12.

Solution:

(a) From $i(t) = 4(2 - e^{-10t})$ mA, $i(0) = 4(2 - 1) = 4$ mA. Since $i = i_1 + i_2$,

$$\boxed{i_1(0) = i(0) - i_2(0)} = 4 - (-1) = 5 \text{ mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \| 12 = 2 + 3 = 5 \text{ H}$$

Thus,

$$\boxed{v(t) = L_{eq}\frac{di}{dt}} = 5(4)(-1)(-10)e^{-10t} \text{ mV} = 200e^{-10t} \text{ mV}$$

电感：根据电压求电流，需要先求出各电感的初始电流值

and

$$\boxed{v_1(t) = 2\frac{di}{dt}} = 2(-4)(-10)e^{-10t} \text{ mV} = 80e^{-10t} \text{ mV}$$

Since $v = v_1 + v_2$,

$$\boxed{v_2(t) = v(t) - v_1(t)} = 120e^{-10t} \text{ mV}$$

(c) The current i_1 is obtained as

$$\boxed{i_1(t) = \frac{1}{4}\int_0^t v_2\,dt + i_1(0)} = \frac{120}{4}\int_0^t e^{-10t}\,dt + 5 \text{ mA}$$

$$= -3e^{-10t}\Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$

Similarly,

$i_2(t)$ 也可以用 $i - i_1(t)$ 获得

$$\boxed{i_2(t) = \frac{1}{12}\int_0^t v_2\,dt + i_2(0)} = \frac{120}{12}\int_0^t e^{-10t}\,dt - 1 \text{ mA}$$

$$= -e^{-10t}\Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$\boxed{v = L\frac{di}{dt}}$
i-v:	$i = v/R$	$\boxed{i = C\frac{dv}{dt}}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$\boxed{w = \frac{1}{2}Cv^2}$	$\boxed{w = \frac{1}{2}Li^2}$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

† *Passive sign convention is assumed.*

小结

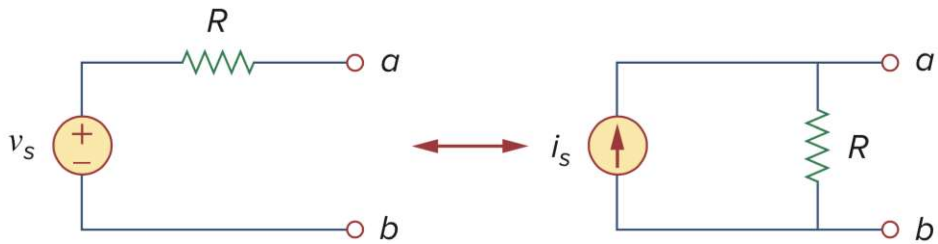
- **线性电路**：仅由线性元件构成的电路，比如仅由电源和电阻构成的电路。1）齐次性：输入放大k倍，则输出也放大k倍；2）可加性：输入相加，则输出也相加；3）线性电路的电压、电流具有线性性质，但功率不具有线性性质。

- **线性性质的应用1**: 如果从“因”推出“果”比较复杂，但从“果”反推出“因”比较简单，则可以先假设“果”为1，计算出“因”，再按比例缩放，得出实际的“果”。

- 线性性质的应用2: **叠加定理**。适合于电路中有多个独立源的情况，分别考虑每个独立源的作用，最后再线性相加得到共同作用的结果。Turn off 其他独立源 → set zero （电压源置零→短路；电流源置零→开路）

小结

- 电源变换



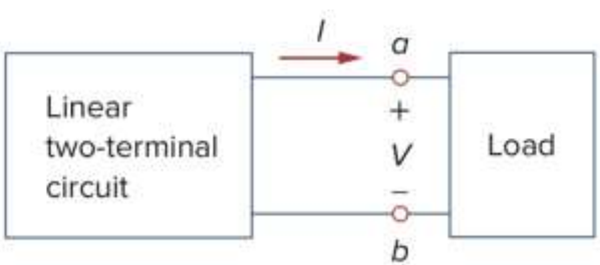
$$v_s = i_s R$$

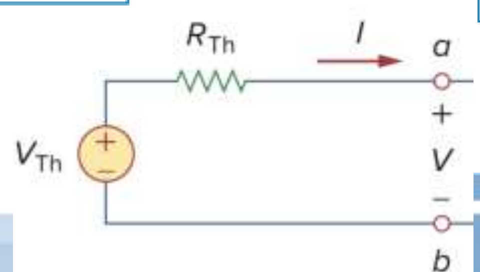
Figure 4.15
Transformation of independent sources.

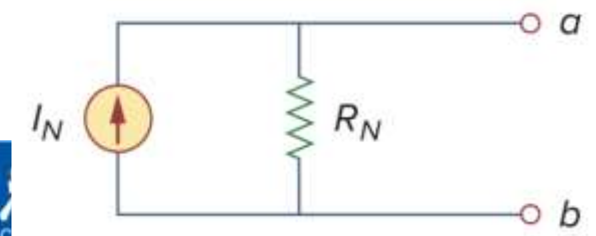
- 端口port：一对端子terminals
- 一端口网络的输入电阻/等效电阻：turn off独立源，若仅含电阻，则简化；若还含独立源，则施加激励求响应；
- 戴维南定理：一端口网络可用开路电压（V_{Th}）串联其等效电阻（R_{Th}）替代
- 诺顿定理：一端口网络可用短路电流（I_N）并联其等效电阻（R_N）替代

$$R_N = R_{\text{Th}}$$

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$







小结

- **最大功率传输：**【对于纯电阻电路】当负载等于电压源内阻时，功率传输最大；
- **实际电源：**电压源要求内阻小；电流源要求内阻大；
- **惠斯通电桥：**精确测量电阻、温度传感器的基本解决方案
- **电容：**电流从“+”端流入为充电；从“+”端流出为放电；直流相当于开路；电容两端电压不能突变。

$$\boxed{i = C\frac{dv}{dt}}$$

$$\boxed{w = \frac{1}{2}Cv^2}$$

小结

- **电感**：直流短路，流过电感的电流不能突变

$$\boxed{v = L\frac{di}{dt}}$$

$$\boxed{w = \frac{1}{2}Li^2}$$

- **电容并联**：C相加；**电容串联**：相当于电阻并联；

- **电感的串并联**相当于电阻的串并联

- 电容，根据电流求电压，需要先求电容两端电压的初始值；
- 电感，根据电压求电流，需要先求流过电感的电流初始值；

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\, d\tau + v(t_0)}$$

$$\boxed{i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)}$$

作业

Find i_o in the circuit of Fig. 4.19 using source transformation.

Practice Problem 4.6

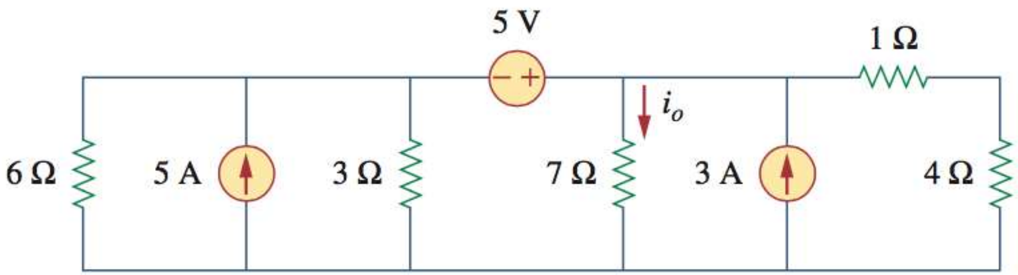


Figure 4.19
For Practice Prob. 4.6.

电源转换，简化电路再计算

Answer: 1.78 A.

Practice Problem 4.12

Find the Norton equivalent circuit of the circuit in Fig. 4.45 at terminals *a*-*b*.

Answer: $R_N = 1\ \Omega$, $I_N = 10$ A.

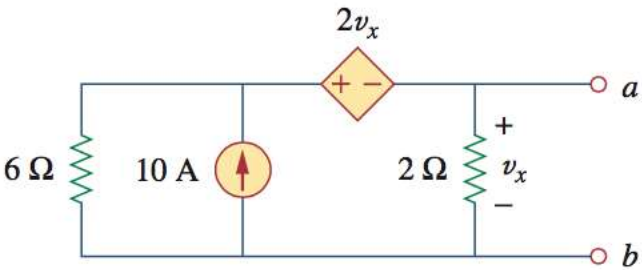


Figure 4.45
For Practice Prob. 4.12.

诺顿定理，含受控源：
①短路电流；②turn off独立源，外加测试电源，计算输入电阻

Practice Problem 4.13

Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 4.52. Calculate the maximum power.

Answer: 4.222 Ω, 2.901 W.

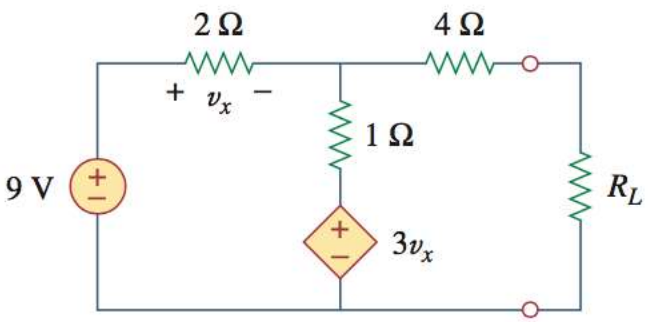


Figure 4.52
For Practice Prob. 4.13.

最大功率传输，先计算戴维南等效电路

Practice Problem 6.7

Find the voltage across each of the capacitors in Fig. 6.20.

Answer: $v_1 = 75$ V, $v_2 = 75$ V, $v_3 = 25$ V, $v_4 = 50$ V.

电容串并联；串联电容的q相等

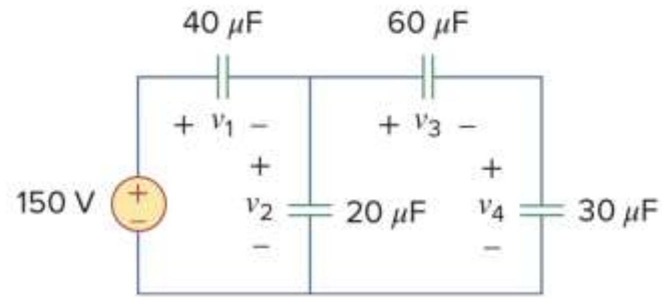


Figure 6.20
For Practice Prob. 6.7.

Practice Problem 6.10

Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of Fig. 6.28 under dc conditions.

Answer: 15 V, 7.5 A, 450 J, 168.75 J.

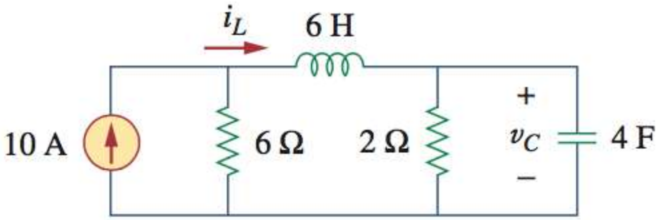


Figure 6.28
For Practice Prob. 6.10.

电容直流开路；电感直流短路