电子电路基础

第五讲~过渡过程的经典解法
~part2

课程纲要

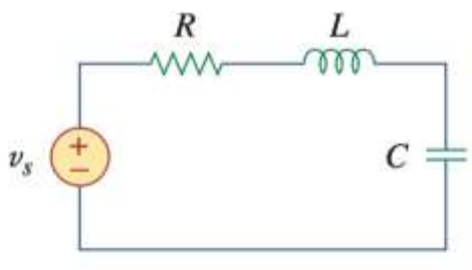
- 4.2 二阶电路的响应
- 4.2.1 二阶电路的零输入和零状态响应
- 4.2.2 二阶电路的阶跃响应和冲激响应

Second-Order Circuits
Chapter 8

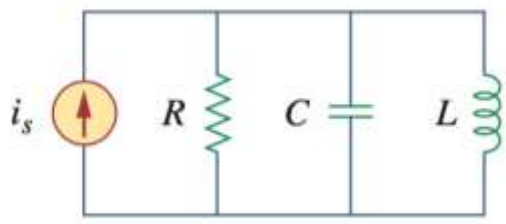
二阶电路的概念

- 一阶电路：电路中只有一个储能元件，解一阶微分方程；
- 二阶电路：电路中有两个储能元件，需解二阶微分方程；

化简之后仍有两个储能元件



(a)



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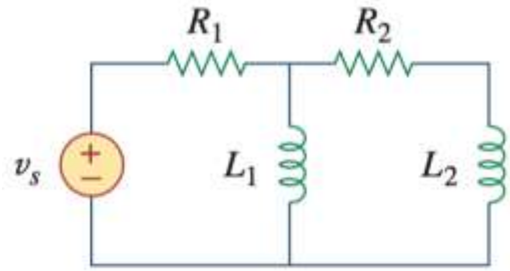
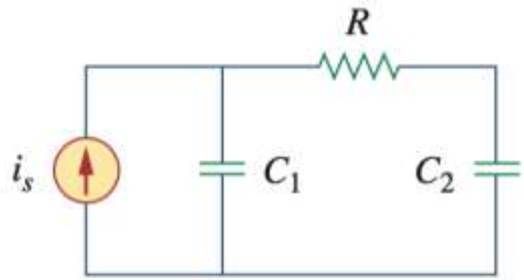


(c)

A **second-order circuit** is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



(d)

- 解决思路：
  - 零输入响应：无外部独立源，有初始储能
  - 阶跃响应（全响应）：外部独立源突然加入（初始储能可有可无）

Finding Initial & Final Values

- 解二阶微分方程，需要知道的初始条件为
  - 变量的初始值；
  - 变量导数的初始值；
- 二阶电路的初始条件及稳态条件为：

  $v(0)$, $i(0)$, $dv(0)/dt$, $di(0)/dt$, $i(\infty)$, and $v(\infty)$
- 需注意的几点：
  - 电压和电流的极性；
  - 默认时，电压指电容两端的电压；电流指流经电感的电流；
  - 电容两端的电压不能突变；
  - 流经电感的电流不能突变；
  - Focus on 上述不能突变的量（关键参数），寻找初始条件

The switch in Fig. 8.2 has been closed for a long time. It is open at $t = 0$. Find: (a) $i(0^+)$, $v(0^+)$, (b) $di(0^+)/dt$, $dv(0^+)/dt$, (c) $i(\infty)$, $v(\infty)$.

Example 8.1

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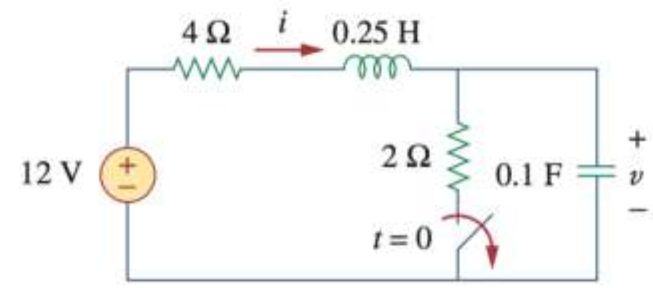


Figure 8.2
For Example 8.1.

抓住不突变量后，导数量要根据变量之间的关系推出。小心题目中的量应处以或乘以C、L

电容电压不能突变，所以0+=0-

①电压电流初始值，看0-时刻

电感电流不能突变，所以同理

②电容两端电压的导数初始值，看0+时刻的 i_C ,利用i_L、v_C不变求解

③流经电感电流的导数初始值，看0+时刻的 v_L ,利用i_L、v_C不变求解

④稳态值

In the circuit of Fig. 8.5, calculate: (a) $i_L(0^+)$, $v_C(0^+)$, $v_R(0^+)$, (b) $di_L(0^+)/dt$, $dv_C(0^+)/dt$, $dv_R(0^+)/dt$, (c) $i_L(\infty)$, $v_C(\infty)$, $v_R(\infty)$.

1. 看0-时刻，求i_L，v_C初始值

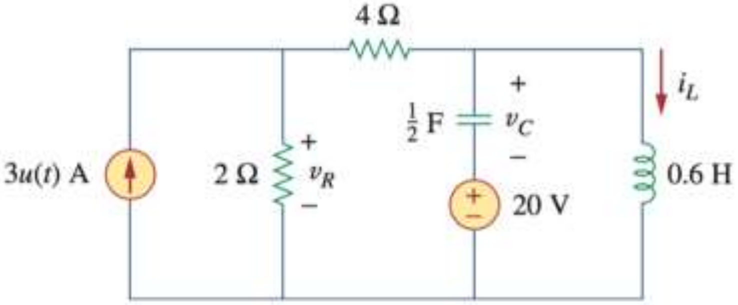
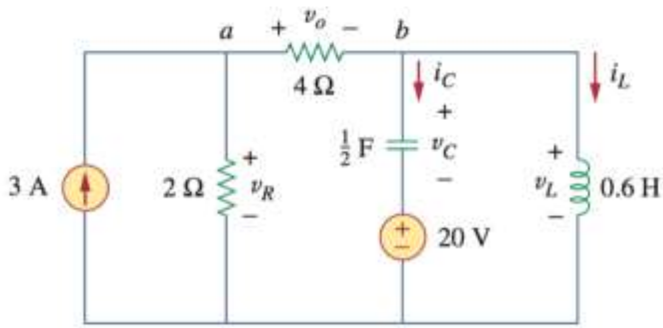


Figure 8.5
For Example 8.2.

求电阻初始状态值只是要求而已，因为电感电容求出后其他都会知道

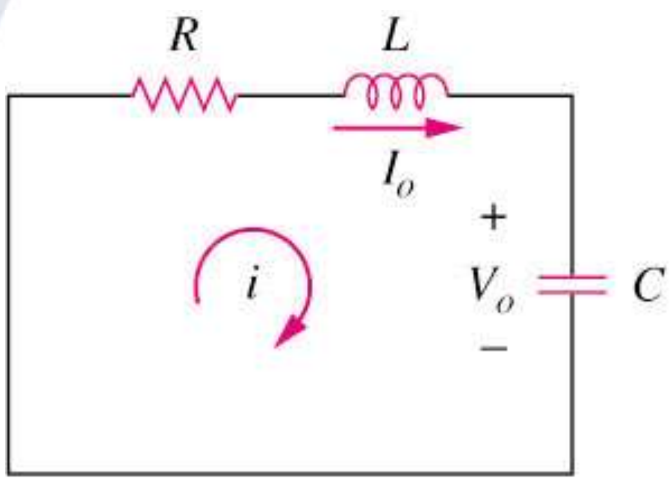


(c) As $t \to \infty$, the circuit reaches steady state. We have the equivalent circuit in Fig. 8.6(a) except that the 3-A current source is now operative. By current division principle,

$$i_L(\infty) = \frac{2}{2+4} 3\text{ A} = 1\text{ A}$$

$$v_R(\infty) = \frac{4}{2+4} 3\text{ A} \times 2 = 4\text{ V}, \qquad v_C(\infty) = -20\text{ V} \tag{8.2.12}$$

8.2 Source-Free Series RLC Circuits



- The solution of the **source-free** series RLC circuit is called as the natural response of the circuit. 自然响应（零输入响应）

- The circuit is excited by the energy initially stored in the capacitor and inductor. 电路的响应由 *LC* 的初始储能决定

The energy is represented by the initial capacitor voltage V_0 and initial inductor current I_0. Thus, at $t = 0$,

$$v(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0 \qquad \textbf{(8.2a)}$$

$$i(0) = I_0 \qquad \textbf{(8.2b)}$$

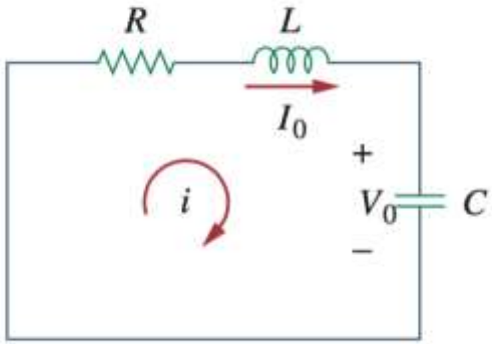


Figure 8.8
A source-free series *RLC* circuit.

Applying KVL around the loop in Fig. 8.8,

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

To eliminate the integral, we differentiate with respect to t and rearrange terms. We get

二阶微分方程
$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0 \tag{8.4}$$

我们的目标：求解上述微分方程，得到 i，从而得到其他量

求解二阶微分方程，我们需要两个初始条件：i、i 的导数

$$i(0) = I_0$$ ① i 的初始条件【根据电感电流不能突变原理，从0-时刻找】

② i 导数的初始条件【基于 $\boxed{v = L\frac{di}{dt}}$ 从0+时刻的电感两端电压找】

$$v(0) = \frac{1}{C}\int_{-\infty}^{0} i\,dt = V_0$$

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

$$\Rightarrow \frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$

0+时刻：电容电压已知、电阻电流已知 → 电感电压可计算

$i(t)$ 的表达式中可以有两个待定系数，根据两个初始条件（i、i 的导数），可以求出这两个待定系数，从而得到 $i(t)$ 的表达式

常微分方程【回顾】

- 齐次线性微分方程

$$\frac{d^n y}{dx^n} + A_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + A_n y = 0$$

1）求特征方程

$$z^n + A_1 z^{n-1} + \cdots + A_n = 0.$$

获得n个根 $z_1, \ldots, z_n$

若没有重根，n个独立解 $e^{z_i x}$

若z是 m_z 重根，m_z 个独立解

$$y = x^k e^{zx} \qquad k \in \{0, 1, \ldots, m_z - 1\}$$

依旧可获得总共 n个独立解

2）写出通解

n个独立解的线性组合就是方程的通解

- 非齐次线性微分方程

$$y'' + py' + qy = f(x)$$

1）求出对应齐次方程的通解

2）求出非齐次方程的一个特解（电路分析中一般选取 t = ∞ 稳态时刻的解为特解）

3）两者相加，获得非齐次方程的通解

8.2 Source-Free Series RLC Circuits

There are **three possible solutions** for the following 2nd order differential equation:

串联RLC的微分方程： $$\frac{d^2 i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$$

$$\rightarrow \quad \frac{d^2 i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0$$

where $\alpha=\frac{R}{2L}$ and $\omega_0=\sqrt{\frac{1}{LC}}$

General 2nd order Form

特征方程

$$s^2+2\alpha s+\omega_0^2=0$$

特征根

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0^2}, \qquad s_2=-\alpha-\sqrt{\alpha^2-\omega_0^2}$$

The types of solutions for $i(t)$ depend on the relative values of α and ω.

1）$\alpha=\omega_0$，重根；$i(t)=(A_2+A_1 t)e^{-\alpha t}$ 2）$\alpha>\omega_0$，两负实数根 $i(t)=A_1e^{s_1 t}+A_2e^{s_2 t}$

3）$\alpha<\omega_0$，一对共轭复数根 $i(t)=A_1e^{-(\alpha-j\omega_d)t}+A_2e^{-(\alpha+j\omega_d)t}=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$

8.2 Source-Free Series RLC Circuits

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0$$

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0^2},\qquad s_2=-\alpha-\sqrt{\alpha^2-\omega_0^2}$$

1. If $\alpha > \omega_o$, overdamped case 过阻尼

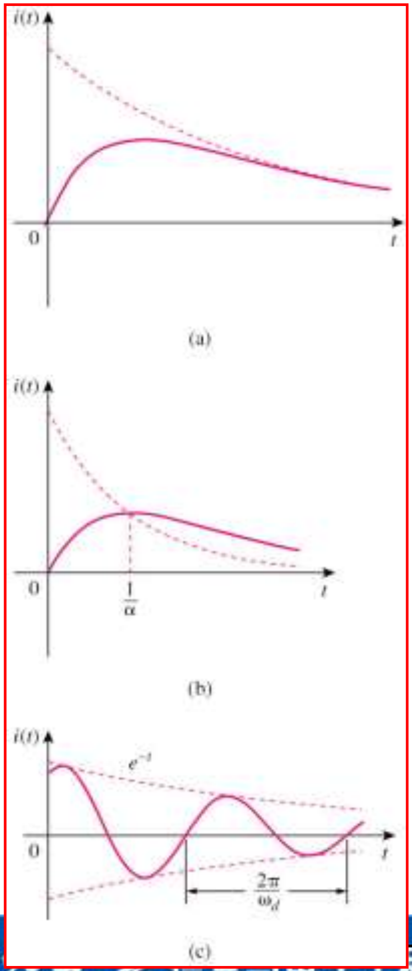
$$i(t)=A_1e^{s_1t}+A_2e^{s_2t}$$ **where** $$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0{}^2}$$

2. If $\alpha = \omega_o$, critically damped case临界阻尼

$$i(t)=(A_2+A_1t)e^{-\alpha t}$$ **where** $$s_{1,2}=-\alpha$$

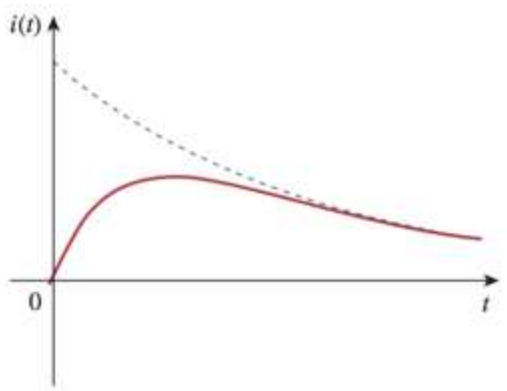
3. If $\alpha < \omega_o$, underdamped case 欠阻尼

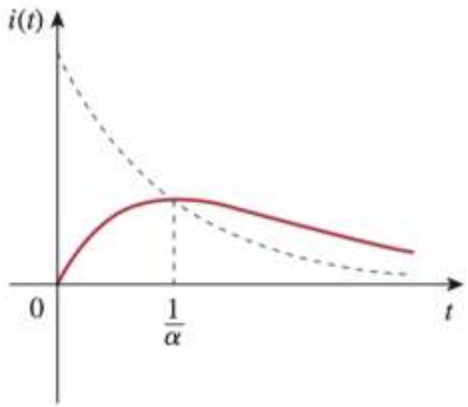
$$i(t)=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$$ **where** $$\omega_d=\sqrt{\omega_0^2-\alpha^2}$$

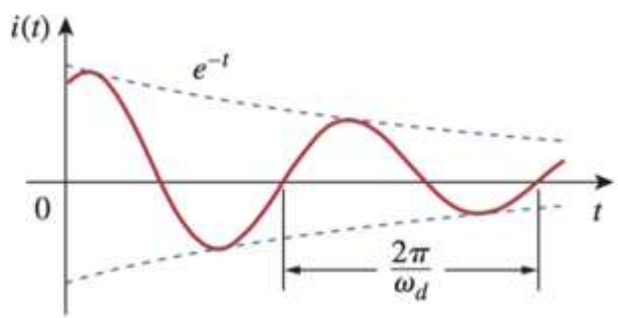


$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

- 阻尼衰减，初始储存于 LC 中的能量逐渐衰减的过程；
  - 阻尼衰减的原因：电路中存在表征损耗的电阻 R；
  - 若 $R = 0$，则是无损耗电路（lossless）
  - α 决定阻尼衰减的速率；$\alpha = 0$，$R = 0$，则 LC 振荡，振荡频率 $1/\sqrt{LC}$
  - 调节 R 的值，可调节 α 值 → 调节电路是过阻尼、临界阻尼、或欠阻尼
  - **R 从零开始增加，谐振频率从ω_0开始逐渐下降** $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
- 振荡时，能量在 L 和 C 中来回传输；
  - 从振荡曲线很难直接判断出是过阻尼，还是临界阻尼；
  - 临界阻尼是过阻尼和欠阻尼的临界状态，衰减最快；





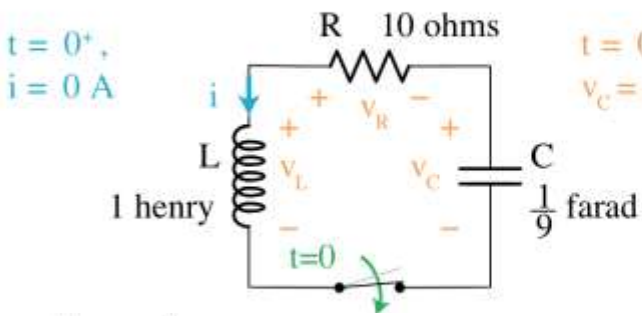


overdamped $\alpha > \omega_o$

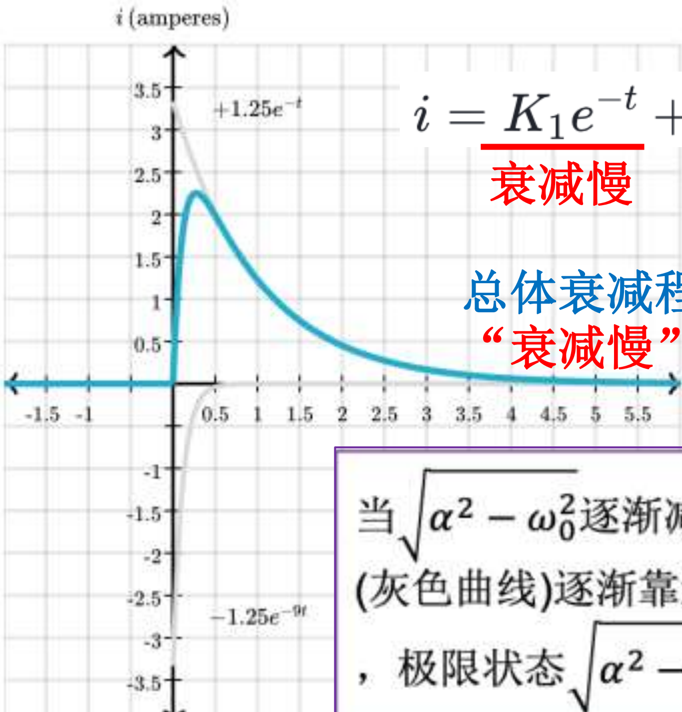
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$s_1 = -\text{real number}_1$ and $s_2 = -\text{real number}_2$

$i = K_1 e^{-\text{real}_1 t} + K_2 e^{-\text{real}_2 t}$ 两项指数衰减叠加



$\alpha = 5$
$\omega_0 = 3$



$$i = \underbrace{K_1 e^{-t}}_{衰减慢} + \underbrace{K_2 e^{-9t}}_{衰减快}$$

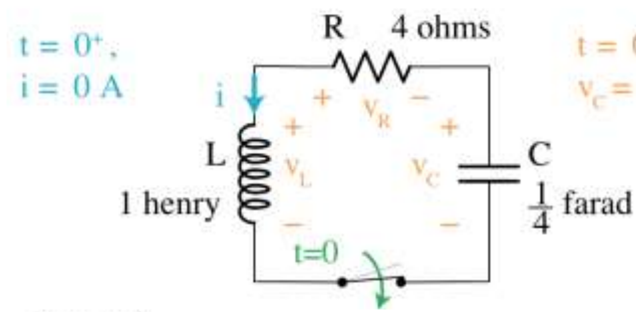
总体衰减程度取决于“衰减慢”的一项

当$\sqrt{\alpha^2-\omega_0^2}$逐渐减小时，两个衰减项(灰色曲线)逐渐靠近，总体衰减变快，极限状态$\sqrt{\alpha^2-\omega_0^2}=0$时衰减最快

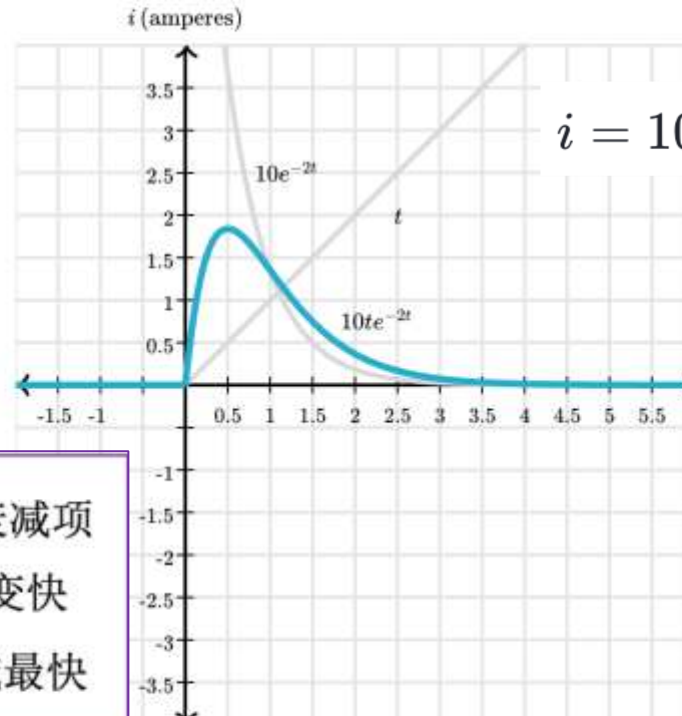
critically damped $\alpha = \omega_o$

$$s_{1,2} = -\alpha \pm \cancel{\sqrt{\alpha^2 - \omega_o^2}}^{\,0}$$

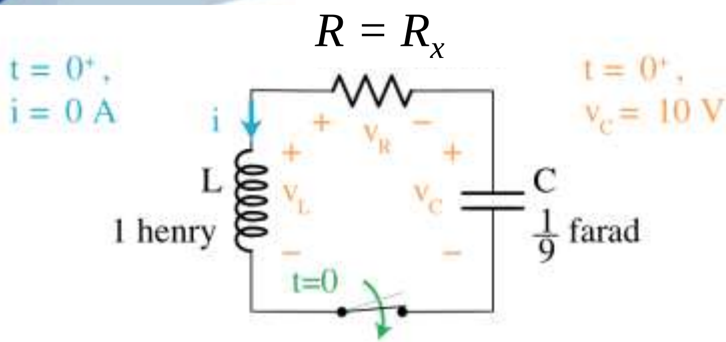
$s_{1,2} = -\alpha$ $\qquad i(t) = (A_2 + A_1 t)e^{-\alpha t}$



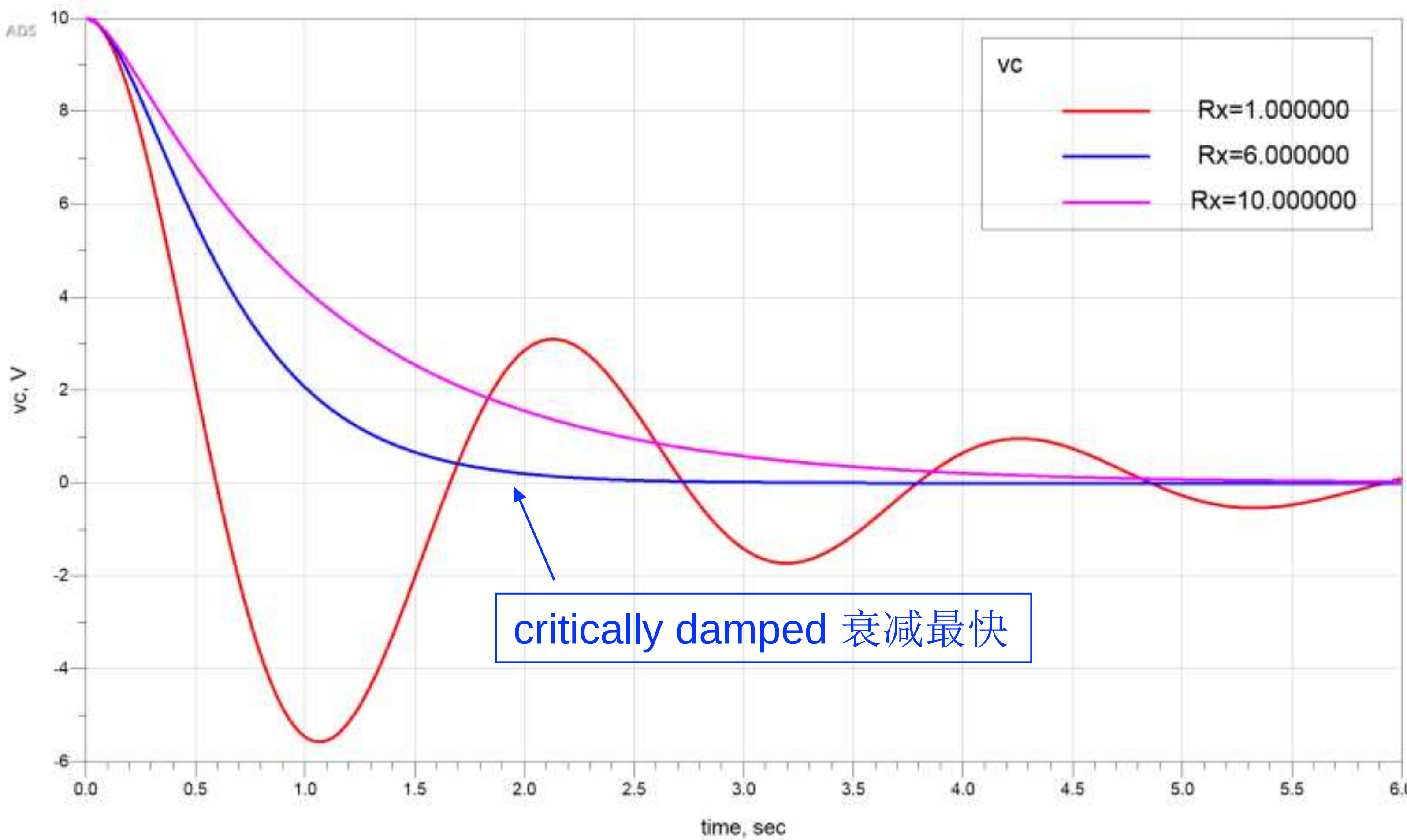
$\alpha = 2$
$\omega_0 = 2$



$$i = 10\, t\, e^{-2t}$$



- $\omega_0 = 3$，$R = 10$ （$\alpha = 5$），overdamped
- $\omega_0 = 3$，$R = 6$ （$\alpha = 3$），critically damped
- $\omega_0 = 3$，$R = 1$ （$\alpha = 0.5$），underdamped



Example 8.3

In Fig. 8.8, $R = 40\ \Omega$, $L = 4$ H, and $C = 1/4$ F. Calculate the characteristic roots of the circuit. Is the natural response overdamped, underdamped, or critically damped?

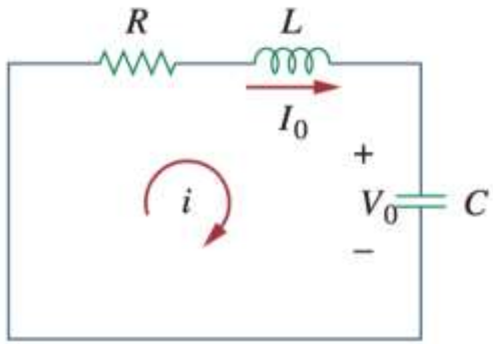


Figure 8.8
A source-free series *RLC* circuit

Solution:

We first calculate

$$\boxed{\alpha = \frac{R}{2L}} = \frac{40}{2(4)} = 5, \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{4 \times \frac{1}{4}}} = 1$$

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

or

$$s_1 = -0.101, \qquad s_2 = -9.899$$

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

Example 8.4

Find $i(t)$ in the circuit of Fig. 8.10. Assume that the circuit has reached steady state at $t = 0^-$.

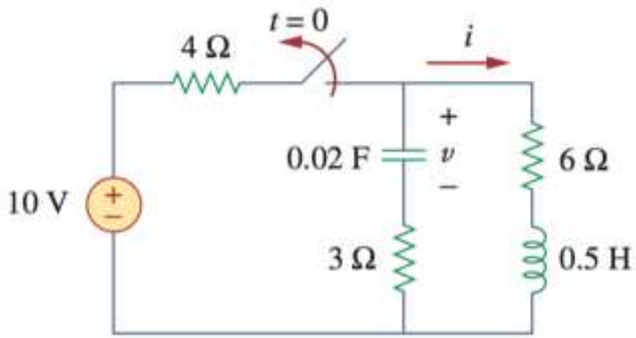
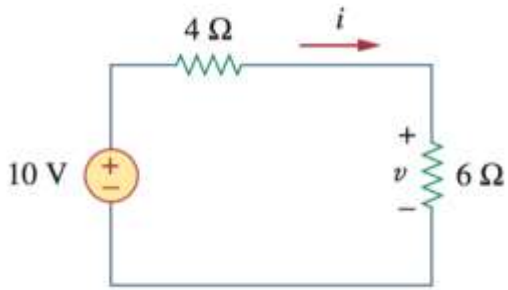


Figure 8.10
For Example 8.4.

Solution: 0+时刻，串联RLC → 先求 *i* 及其导数的初始值

For $t < 0$, the switch is closed. The capacitor acts like an open circuit while the inductor acts like a shunted circuit. The equivalent circuit is shown in Fig. 8.11(a). Thus, at $t = 0$,

$$i(0) = \frac{10}{4+6} = 1\text{ A}, \qquad v(0) = 6i(0) = 6\text{ V}$$



(a)

For $t > 0$, the switch is opened and the voltage source is disconnected. The equivalent circuit is shown in Fig. 8.11(b), which is a source-free series *RLC* circuit. Notice that the 3-Ω and 6-Ω resistors, which are in series in Fig. 8.10 when the switch is opened, have been combined to give $R = 9\ \Omega$ in Fig. 8.11(b). The roots are calculated as follows:

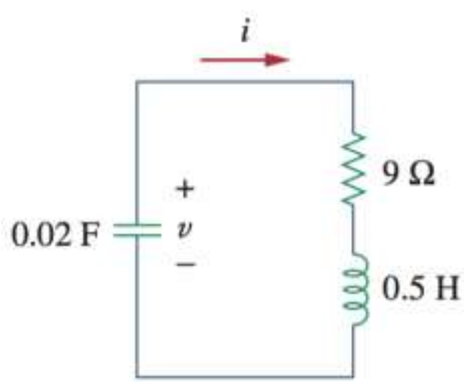
$$\boxed{\alpha = \frac{R}{2L}} = \frac{9}{2(\frac{1}{2})} = 9, \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{\frac{1}{2}\times\frac{1}{50}}} = 10$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -9 \pm \sqrt{81 - 100}$$

or

①求特征根，写出通解

$$s_{1,2} = -9 \pm j4.359$$



(b)

Hence, the response is underdamped ($\alpha < \omega$); that is,

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t) \qquad \textbf{(8.4.1)}$$

2个待定系数，需2个初始条件求解

Example 8.4

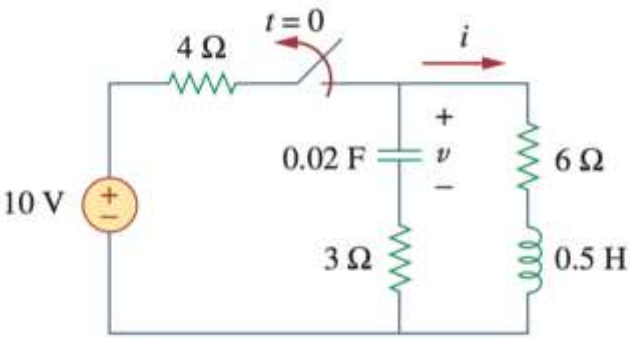
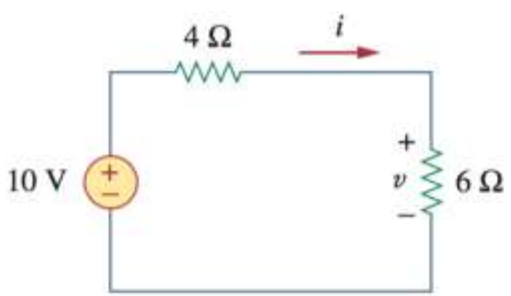
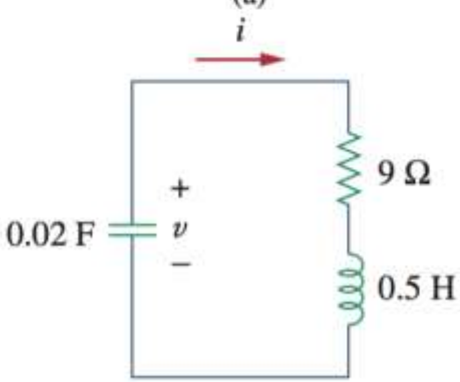


Figure 8.10
For Example 8.4.



(a)



(b)

Find $i(t)$ in the circuit of Fig. 8.10. Assume that the circuit has reached steady state at $t = 0^-$.

$$i(t) = e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$

We now obtain A_1 and A_2 using the initial conditions. At $t = 0$,

$$i(0) = 1 = A_1 \qquad \textbf{(8.4.2)}$$

From Eq. (8.5),

电感电流导数的初始条件由 v_L 决定

$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}[Ri(0) + v(0)] = -2[9(1) - 6] = -6 \text{ A/s} \qquad \textbf{(8.4.3)}$$

Note that $v(0) = V_0 = -6$ V is used, because the polarity of v in Fig. 8.11(b) is opposite that in Fig. 8.8. Taking the derivative of $i(t)$ in Eq. (8.4.1),

②根据初始条件求出待定系数

$$\frac{di}{dt} = -9e^{-9t}(A_1 \cos 4.359t + A_2 \sin 4.359t)$$
$$+ e^{-9t}(4.359)(-A_1 \sin 4.359t + A_2 \cos 4.359t)$$

Imposing the condition in Eq. (8.4.3) at $t = 0$ gives

$$-6 = -9(A_1 + 0) + 4.359(-0 + A_2)$$

But $A_1 = 1$ from Eq. (8.4.2). Then

$$-6 = -9 + 4.359A_2 \qquad \Rightarrow \qquad A_2 = 0.6882$$

Substituting the values of A_1 and A_2 in Eq. (8.4.1) yields the complete solution as

$$i(t) = e^{-9t}(\cos 4.359t + 0.6882 \sin 4.359t) \text{ A}$$

Source-free parallel RLC circuits

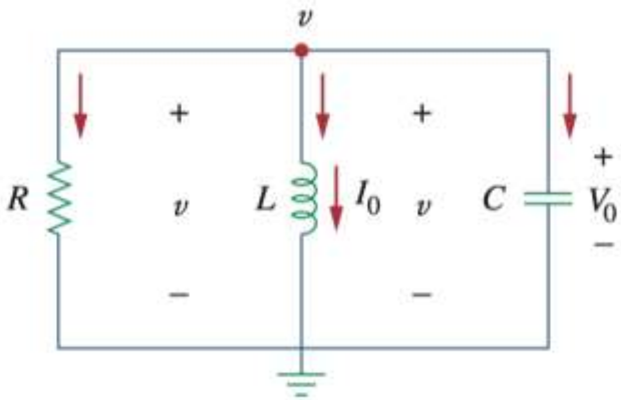


Figure 8.13
A source-free parallel *RLC* circuit.

Consider the parallel *RLC* circuit shown in Fig. 8.13. Assume initial inductor current I_0 and initial capacitor voltage V_0,

$$i(0) = I_0 = \frac{1}{L}\int_{\infty}^{0} v(t)\,dt \qquad (8.27a)$$

$$v(0) = V_0 \qquad (8.27b)$$

applying KCL at the top node gives

以 *v* 作为关键变量

$$\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau + C\frac{dv}{dt} = 0 \qquad (8.28)$$

Taking the derivative with respect to *t* and dividing by *C* results in

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0 \qquad (8.29)$$

注意：串并联RLC，ω_0相同，α不同
串联，R越大，损耗越大，R在分子中
并联：R越大，损耗越小，R在分母中

特征方程： $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$

特征根：

$$\boxed{s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$

$$\boxed{\alpha = \frac{1}{2RC}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}}$$

8.3 Source-Free Parallel RLC Circuits

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case 过阻尼

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ **where** $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

与串联RLC相比，只是 α 的表达式不同

2. If $\alpha = \omega_o$, critical damped case 临界阻尼

$$v(t) = (A_2 + A_1 t)\,e^{-\alpha t}$$ **where** $$s_{1,2} = -\alpha$$

因此，解题思路也类似
1.求特征值，写出通解；
2.根据初始条件求出待定系数

3. If $\alpha < \omega_o$, under-damped case 欠阻尼

$$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$ **where** $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

串联 *RLC* Vs. 并联 *RLC*

- 串联 *RLC*
  - 不同
    - $$\alpha = \frac{R}{2L}$$
    - 关键变量：电感电流
  - 相同
    - 解题思路（先求特征值、写出通解；再根据初始条件求待定系数）
    - 过阻尼、临界阻尼、欠阻尼的判断
- 并联 *RLC*
  - 不同
    - $$\alpha = \frac{1}{2RC}$$
    - 关键变量：电容电压
  - 相同
    - 解题思路（先求特征值、写出通解；再根据初始条件求待定系数）
    - 过阻尼、临界阻尼、欠阻尼的判断

Example 8.5

In the parallel circuit of Fig. 8.13, find $v(t)$ for $t > 0$, assuming $v(0) = 5$ V, $i(0) = 0$, $L = 1$ H, and $C = 10$ mF. Consider these cases: $R = 1.923\ \Omega$, $R = 5\ \Omega$, and $R = 6.25\ \Omega$.

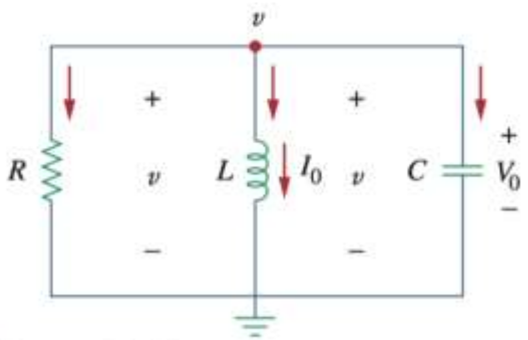


Figure 8.13
A source-free parallel *RLC* circuit.

Solution:

CASE 1 If $R = 1.923\ \Omega$,

$$\boxed{\alpha = \frac{1}{2RC}} = \frac{1}{2 \times 1.923 \times 10 \times 10^{-3}} = 26$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{1 \times 10 \times 10^{-3}}} = 10$$

Since $\alpha > \omega_0$ in this case, the response is overdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2, -50$$

①求特征根，写出通解

and the corresponding response is

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t} \qquad \textbf{(8.5.1)}$$

We now apply the initial conditions to get A_1 and A_2.

$$\boxed{v(0)} = 5 = A_1 + A_2$$

$$\boxed{\frac{dv(0)}{dt}} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{1.923 \times 10 \times 10^{-3}} = -260$$

But differentiating Eq. (8.5.1),

$$\frac{dv}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

②根据初始条件，求待定系数

At $t = 0$,

$$-260 = -2A_1 - 50A_2 \qquad \textbf{(8.5.3)}$$

From Eqs. (8.5.2) and (8.5.3), we obtain $A_1 = -0.2083$ and $A_2 = 5.208$. Substituting A_1 and A_2 in Eq. (8.5.1) yields

$$v(t) = -0.2083e^{-2t} + 5.208e^{-50t} \qquad \textbf{(8.5.4)}$$

Example 8.5

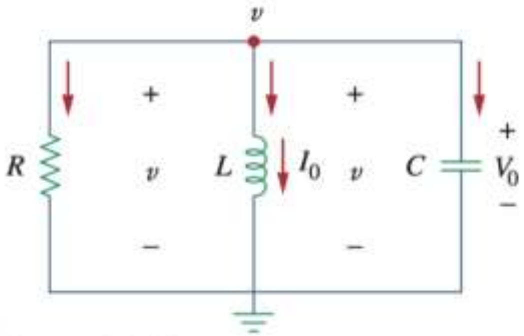


Figure 8.13
A source-free parallel *RLC* circuit.

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CASE 2 When $R = 5\ \Omega$,

$$\boxed{\alpha = \frac{1}{2RC}} = \frac{1}{2 \times 5 \times 10 \times 10^{-3}} = 10$$

while $\omega_0 = 10$ remains the same. Since $\alpha = \omega_0 = 10$, the response is critically damped. Hence, $s_1 = s_2 = -10$, and

①求特征根，写出通解

$$v(t) = (A_1 + A_2 t)e^{-10t} \tag{8.5.5}$$

To get A_1 and A_2, we apply the initial conditions

$$v(0) = 5 = A_1 \tag{8.5.6}$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{5 \times 10 \times 10^{-3}} = -100$$

But differentiating Eq. (8.5.5),

$$\frac{dv}{dt} = (-10A_1 - 10A_2 t + A_2)e^{-10t}$$

②根据初始条件，求待定系数

At $t = 0$,

$$-100 = -10A_1 + A_2 \tag{8.5.7}$$

From Eqs. (8.5.6) and (8.5.7), $A_1 = 5$ and $A_2 = -50$. Thus,

$$v(t) = (5 - 50t)e^{-10t}\ \text{V} \tag{8.5.8}$$

Example 8.5

In the parallel circuit of Fig. 8.13, find $v(t)$ for $t > 0$, assuming $v(0) = 5$ V, $i(0) = 0$, $L = 1$ H, and $C = 10$ mF. Consider these cases: $R = 1.923\ \Omega$, $R = 5\ \Omega$, and $R = 6.25\ \Omega$.

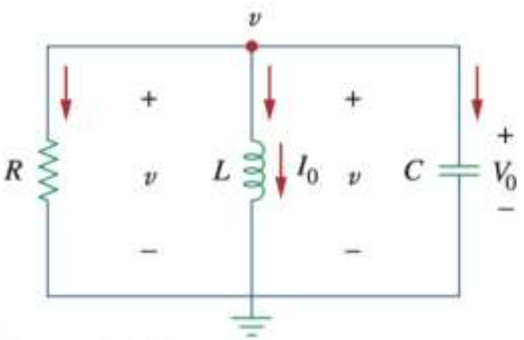


Figure 8.13
A source-free parallel *RLC* circuit.

CASE 3 When $R = 6.25\ \Omega$,

$$\boxed{\alpha = \frac{1}{2RC}} = \frac{1}{2 \times 6.25 \times 10 \times 10^{-3}} = 8$$

while $\omega_0 = 10$ remains the same. As $\alpha < \omega_0$ in this case, the response is underdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8 \pm j6$$

①求特征根，写出通解

Hence,

$$v(t) = (A_1 \cos 6t + A_2 \sin 6t)e^{-8t} \quad \textbf{(8.5.9)}$$

We now obtain A_1 and A_2, as

$$v(0) = 5 = A_1 \quad \textbf{(8.5.10)}$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{6.25 \times 10 \times 10^{-3}} = -80$$

But differentiating Eq. (8.5.9),

$$\frac{dv}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$$

②根据初始条件，求待定系数

At $t = 0$,

$$-80 = -8A_1 + 6A_2 \quad \textbf{(8.5.11)}$$

From Eqs. (8.5.10) and (8.5.11), $A_1 = 5$ and $A_2 = -6.667$. Thus,

$$v(t) = (5 \cos 6t - 6.667 \sin 6t)e^{-8t} \quad \textbf{(8.5.12)}$$

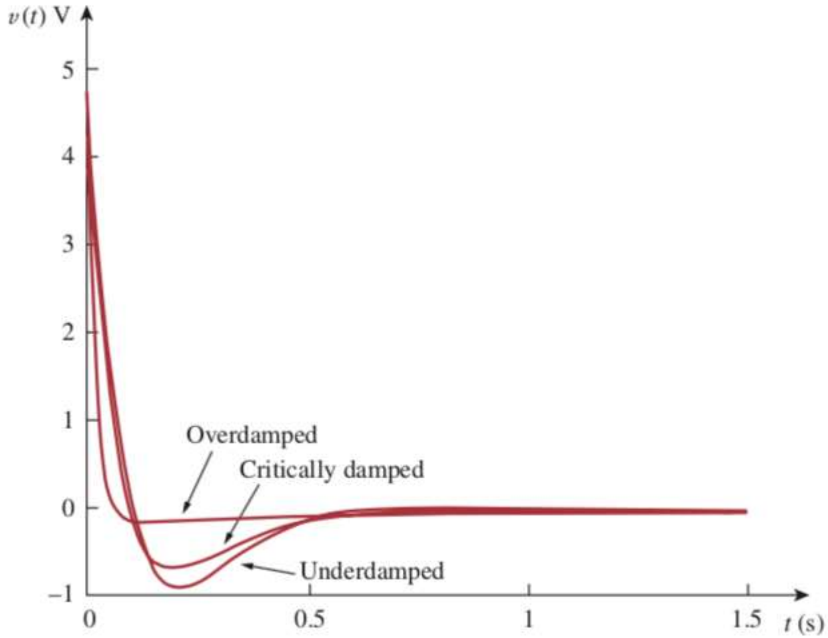
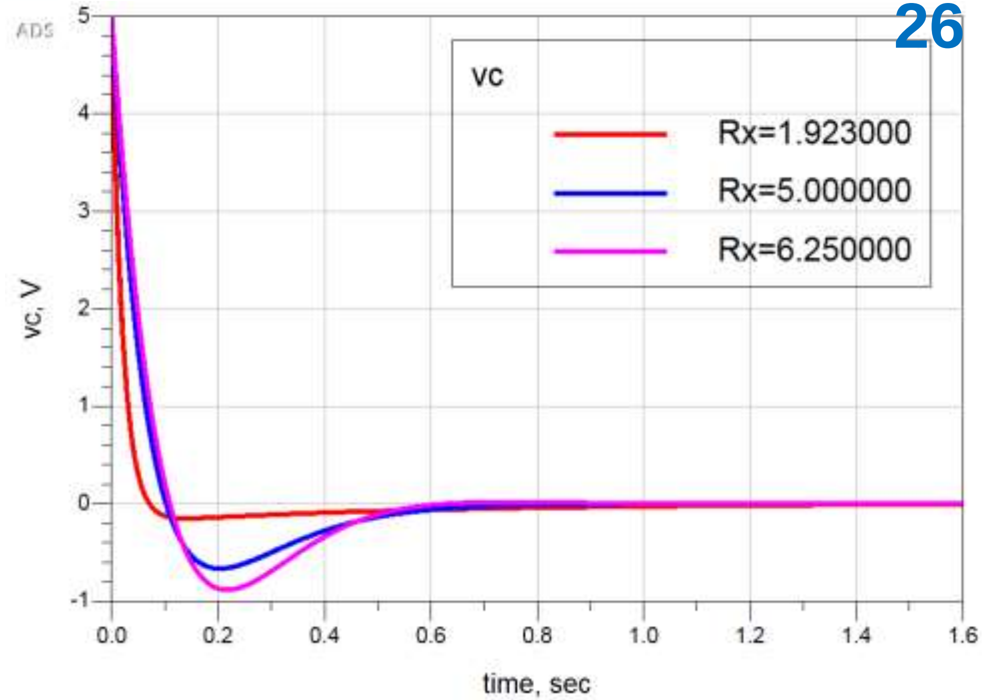
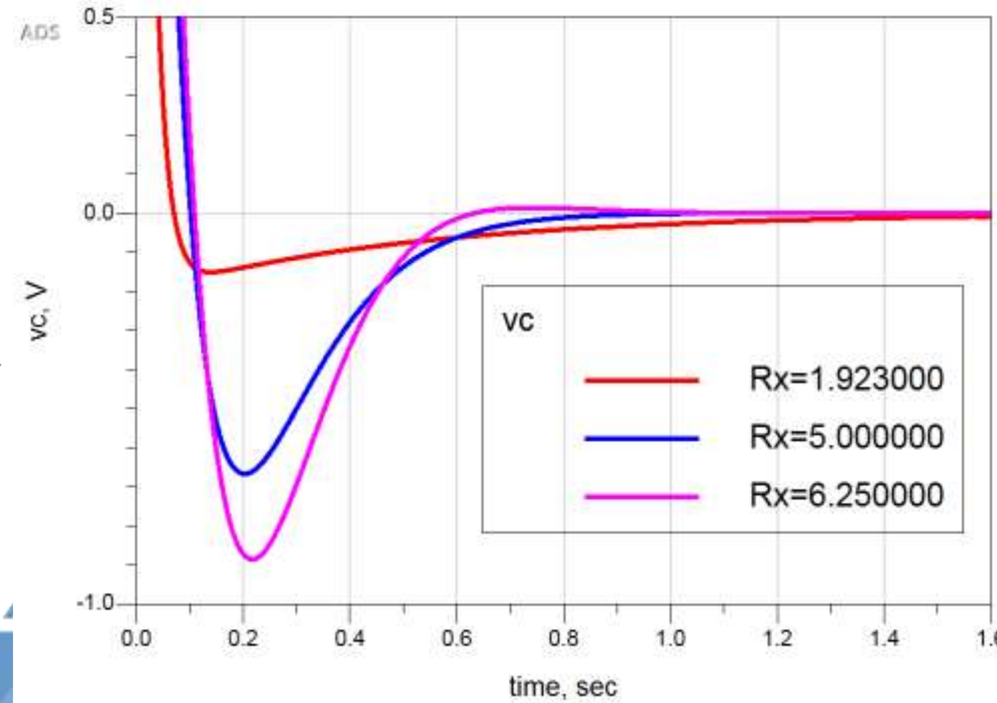


Figure 8.14
For Example 8.5: responses for three degrees of damping.

书上插图，貌似overdamped衰减最快

实际局部放大后，依然是 critically damped衰减最快





Example 8.6

Find $v(t)$ for $t > 0$ in the RLC circuit of Fig. 8.15.

0+时刻，并联RLC → 先求 v 及其导数的初始值

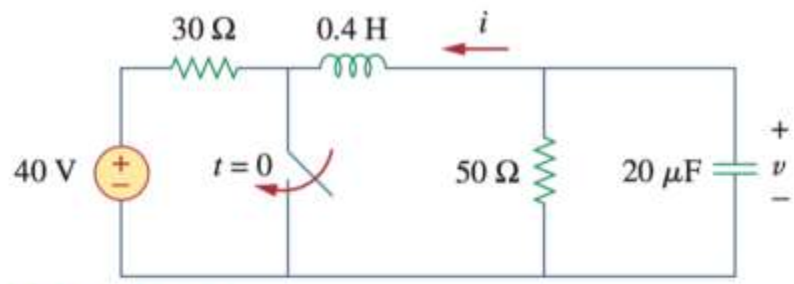


Figure 8.15
For Example 8.6.

Solution:
When $t < 0$, the switch is open; the inductor acts like a short circuit while the capacitor behaves like an open circuit. The initial voltage across the capacitor is the same as the voltage across the 50-Ω resistor; that is,

$$\boxed{v(0)} = \frac{50}{30+50}(40) = \frac{5}{8} \times 40 = 25 \text{ V} \qquad \textbf{(8.6.1)}$$

The initial current through the inductor is

$$i(0) = -\frac{40}{30+50} = -0.5 \text{ A}$$

求初始条件

The direction of i is as indicated in Fig. 8.15 to conform with the direction of I_0 in Fig. 8.13, which is in agreement with the convention that current flows into the positive terminal of an inductor (see Fig. 6.23). We need to express this in terms of dv/dt, since we are looking for v.

$$\boxed{\frac{dv(0)}{dt}} = -\frac{v(0) + Ri(0)}{RC} = -\frac{25 - 50 \times 0.5}{50 \times 20 \times 10^{-6}} = 0 \qquad \textbf{(8.6.2)}$$

Example 8.6

Find $v(t)$ for $t > 0$ in the *RLC* circuit of Fig. 8.15.

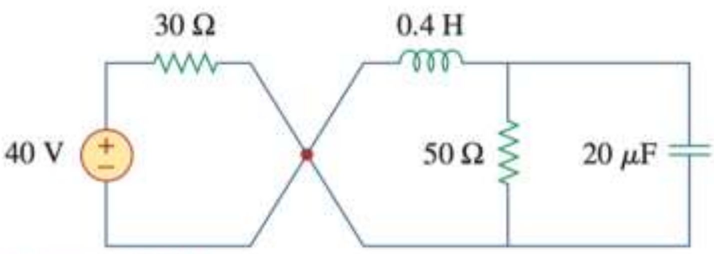


Figure 8.16

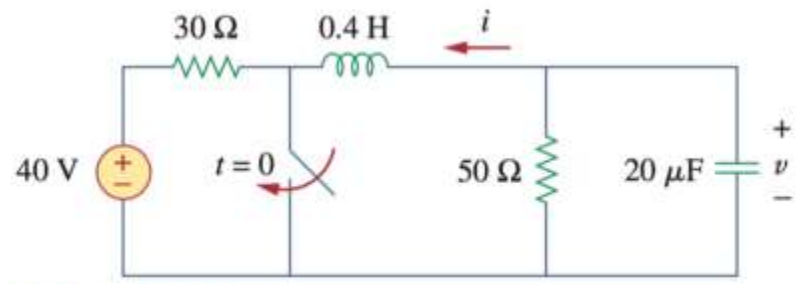


Figure 8.15
For Example 8.6.

When $t > 0$, the switch is closed. The voltage source along with the 30-Ω resistor is separated from the rest of the circuit. The parallel *RLC* circuit acts independently of the voltage source, as illustrated in Fig. 8.16. Next, we determine that the roots of the characteristic equation are

$$\boxed{\alpha = \frac{1}{2RC}} = \frac{1}{2 \times 50 \times 20 \times 10^{-6}} = 500$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{0.4 \times 20 \times 10^{-6}}} = 354$$

①求特征根，写出通解

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$= -500 \pm \sqrt{250{,}000 - 124{,}997.6} = -500 \pm 354$$

or

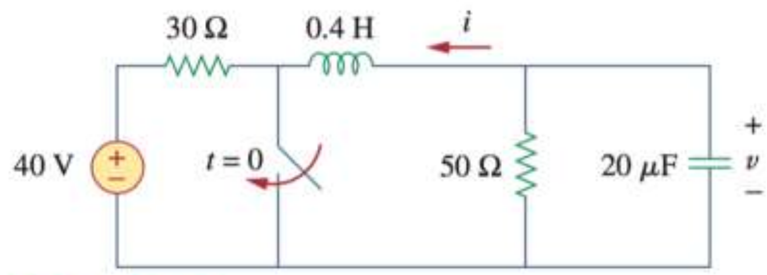
$$s_1 = -854, \qquad s_2 = -146$$

Since $\alpha > \omega_0$, we have the overdamped response

$$v(t) = A_1 e^{-854t} + A_2 e^{-146t} \tag{8.6.3}$$

Example 8.6

Find $v(t)$ for $t > 0$ in the RLC circuit of Fig. 8.15.



At $t = 0$, we impose the condition in Eq. (8.6.1),

$$v(0) = 25 = A_1 + A_2 \quad \Rightarrow \quad A_2 = 25 - A_1 \qquad \textbf{(8.6.4)}$$

Taking the derivative of $v(t)$ in Eq. (8.6.3),

$$\frac{dv}{dt} = -854A_1e^{-854t} - 146A_2e^{-146t}$$

Imposing the condition in Eq. (8.6.2),

$$\frac{dv(0)}{dt} = 0 = -854A_1 - 146A_2$$

②根据初始条件，求待定系数

or

$$0 = 854A_1 + 146A_2 \qquad \textbf{(8.6.5)}$$

Solving Eqs. (8.6.4) and (8.6.5) gives

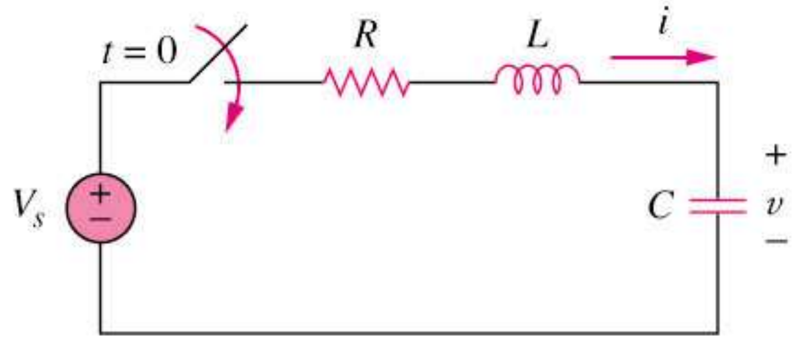
$$A_1 = -5.156, \qquad A_2 = 30.16$$

Thus, the complete solution in Eq. (8.6.3) becomes

$$v(t) = -5.156e^{-854t} + 30.16e^{-146t}\ \text{V}$$

8.4 Step-Response Series *RLC* Circuits

- The step response is obtained by the sudden application of a dc source.



$$L\frac{di}{dt} + Ri + v = V_s$$

$$i = C\frac{dv}{dt}$$

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{v_s}{LC}$$

* 因为有电压源，所以关键变量采用电压比较合适

Q: 以电感电流为关键变量可以吗？

A：OK，区别仅在于等号右侧的数值不同

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters). → 同样的特征方程、同样的特征根
- Different circuit variable in the equation.

8.4 Step-Response Series *RLC* Circuits

The solution of the equation should have two components:
the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

齐次微分方程的通解　　　　非齐次微分方程的特解（t=∞）

$$v(t) = \boxed{v_t(t)} + \boxed{v_{ss}(t)}$$

- The transient response v_t is the same as that for source-free case

$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (over-damped)

$v_t(t) = (A_1 + A_2 t)e^{-\alpha t}$ (critically damped)

$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$ (under-damped)

- The steady-state response is the final value of $v(t)$.
  - $v_{ss}(t) = v(\infty) = V_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - $v(0)$ and $\frac{dv(0)}{dt}$

$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(Overdamped)} \tag{8.44a}$$

$$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t} \quad \text{(Critically damped)} \tag{8.44b}$$

$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \quad \text{(Underdamped)} \tag{8.44c}$$

Alternatively, the complete response for any variable $x(t)$ can be found directly, because it has the general form

$$x(t) = x_{ss}(t) + x_t(t) \tag{8.45}$$

where the $x_{ss} = x(\infty)$ is the final value and $x_t(t)$ is the transient response.

Example 8.7

For the circuit in Fig. 8.19, find $v(t)$ and $i(t)$ for $t > 0$. Consider these cases: $R = 5\ \Omega$, $R = 4\ \Omega$, and $R = 1\ \Omega$.

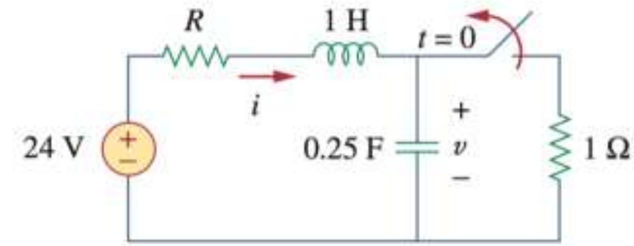


Figure 8.19
For Example 8.7.

Solution:

0+时刻，带激励源的串联RLC

→ 先求 *v*及其导数的初始值

CASE 1 When $R = 5\ \Omega$. For $t < 0$, the switch is closed for a long time. The capacitor behaves like an open circuit while the inductor acts like a short circuit. The initial current through the inductor is

$$i(0) = \frac{24}{5+1} = 4\text{ A}$$

and the initial voltage across the capacitor is the same as the voltage across the 1-Ω resistor; that is,

求初始条件

$$\boxed{v(0)} = 1i(0) = 4\text{ V}$$

For $t > 0$, the switch is opened, so that we have the 1-Ω resistor disconnected. What remains is the series *RLC* circuit with the voltage source. The characteristic roots are determined as follows:

$$\boxed{\alpha = \frac{R}{2L}} = \frac{5}{2\times 1} = 2.5, \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{1\times 0.25}} = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1, -4$$

Since $\alpha > \omega_0$, we have the overdamped natural response. The total response is therefore

$$v(t) = v_{ss} + (A_1 e^{-t} + A_2 e^{-4t})$$

①求特征根，写出通解

. 与零输入响应比，通解多了V_s

. 也可理解为：零输入响应时，变量最终都会衰减到零，即$v_{ss} = 0$

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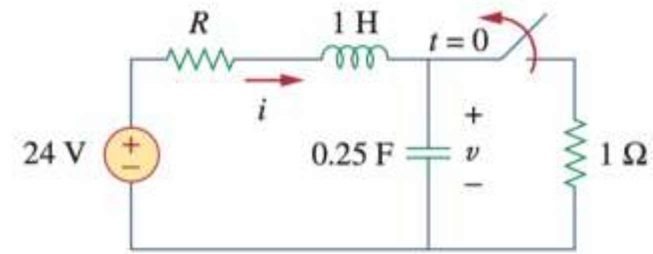


Figure 8.19
For Example 8.7.

$$v(t) = 24 + (A_1 e^{-t} + A_2 e^{-4t}) \tag{8.7.1}$$

We now need to find A_1 and A_2 using the initial conditions.

$$v(0) = 4 = 24 + A_1 + A_2$$

or

$$-20 = A_1 + A_2 \tag{8.7.2}$$

The current through the inductor cannot change abruptly and is the same current through the capacitor at $t = 0^+$ because the inductor and capacitor are now in series. Hence,

$$i(0) = C\frac{dv(0)}{dt} = 4 \quad \Rightarrow \quad \boxed{\frac{dv(0)}{dt}} = \frac{4}{C} = \frac{4}{0.25} = 16$$

②根据初始条件，
求待定系数

Before we use this condition, we need to take the derivative of v in Eq. (8.7.1).

$$\frac{dv}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t} \tag{8.7.3}$$

At $t = 0$,

$$\frac{dv(0)}{dt} = 16 = -A_1 - 4A_2 \tag{8.7.4}$$

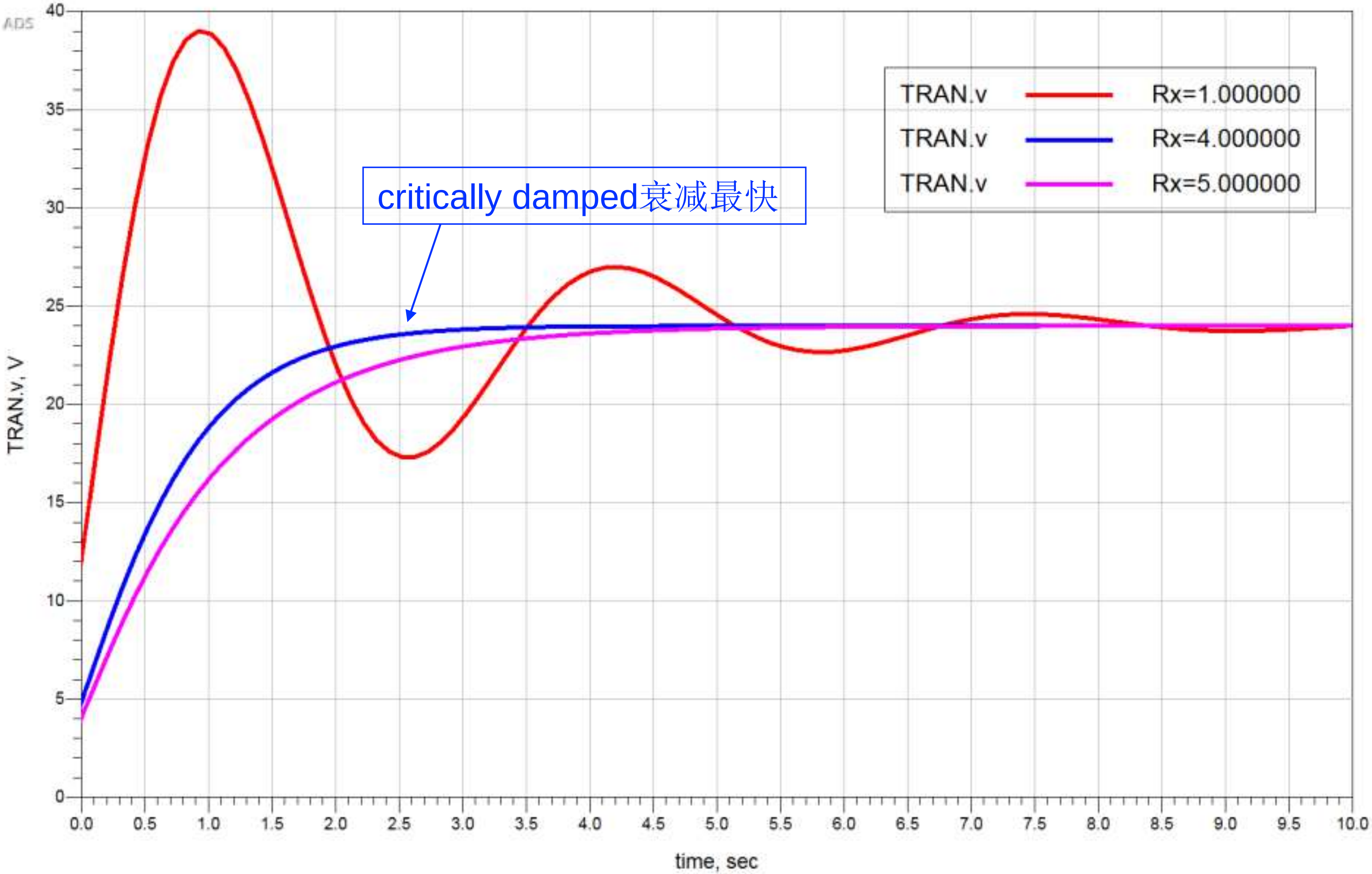
From Eqs. (8.7.2) and (8.7.4), $A_1 = -64/3$ and $A_2 = 4/3$. Substituting A_1 and A_2 in Eq. (8.7.1), we get

$$v(t) = 24 + \frac{4}{3}(-16e^{-t} + e^{-4t})\ \text{V} \tag{8.7.5}$$

$$i(t) = C\frac{dv}{dt}$$

⇩

$$i(t) = \frac{4}{3}(4e^{-t} - e^{-4t})\ \text{A}$$


ADS
critically damped衰减最快
TRAN.v Rx=1.000000
TRAN.v Rx=4.000000
TRAN.v Rx=5.000000
TRAN.v, V
time, sec
0
5
10
15
20
25
30
35
40
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
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8.5
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10.0

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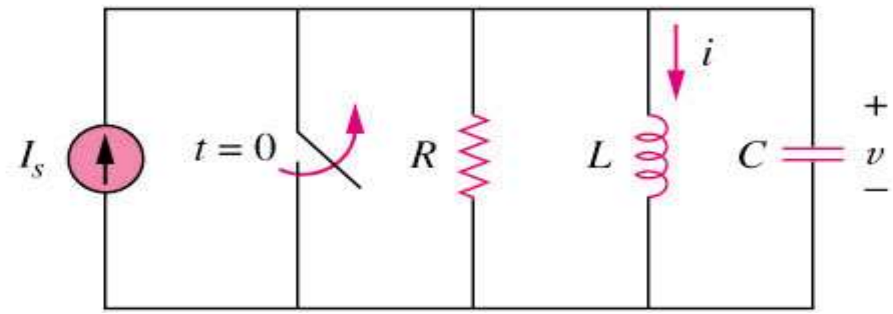

$$\frac{v}{R} + i + C\frac{dv}{dt} = I_s$$

$$v = L\frac{di}{dt}$$

* 因为有电流源，所以关键变量采用电流比较合适

$$\Rightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters). →同样的特征方程、同样的特征根
- Different circuit variable in the equation.

8.5 Step-Response Parallel RLC Circuits

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齐次微分方程的通解　　非齐次微分方程的特解（t=∞）

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$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

$$i_t(t) = (A_1 + A_2 t)e^{-\alpha t} \quad \text{(critical damped)}$$

$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of $i(t)$.
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - $i(0)$ and $\frac{di(0)}{dt}$

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(Overdamped)}$$
$$i(t) = I_s + (A_1 + A_2 t)e^{-\alpha t} \qquad \text{(Critically damped)} \tag{8.49}$$
$$i(t) = I_s + (A_1 \cos\omega_d t + A_2 \sin\omega_d t)e^{-\alpha t} \qquad \text{(Underdamped)}$$

Alternatively, the complete response for any variable $x(t)$ may be found directly, using

$$x(t) = x_{ss}(t) + x_t(t) \tag{8.50}$$

where x_{ss} and x_t are its final value and transient response, respectively.

In the circuit of Fig. 8.23, find $i(t)$ and $i_R(t)$ for $t > 0$.

Example 8.8

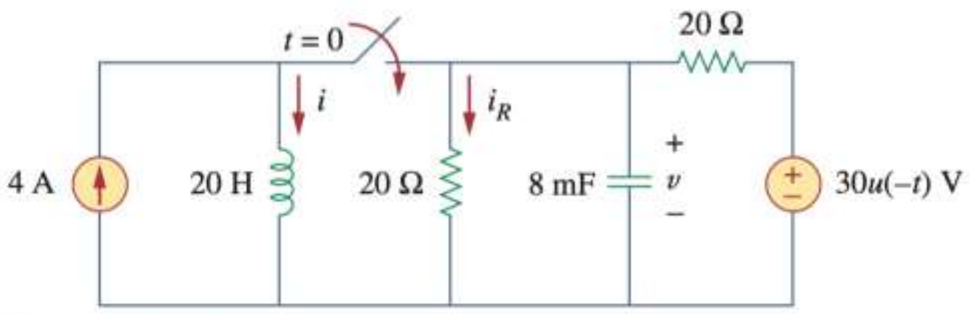


Figure 8.23
For Example 8.8.

Solution:
For $t < 0$, the switch is open, and the circuit is partitioned into two independent subcircuits. The 4-A current flows through the inductor, so that

$$\boxed{i(0)} = 4 \text{ A}$$

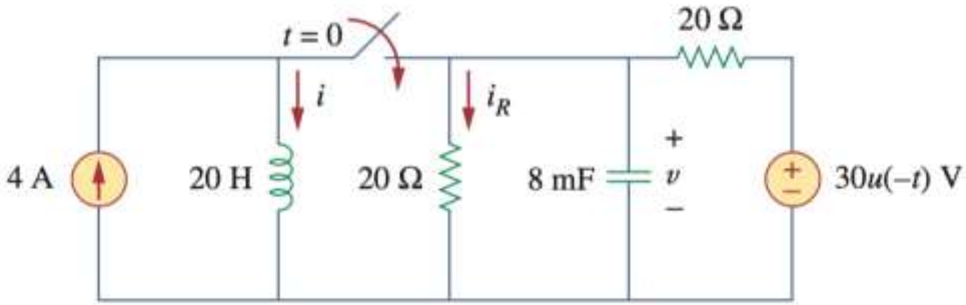
Since $30u(-t) = 30$ when $t < 0$ and 0 when $t > 0$, the voltage source is operative for $t < 0$. The capacitor acts like an open circuit and the voltage across it is the same as the voltage across the 20-Ω resistor connected in parallel with it. By voltage division, the initial capacitor voltage is

求初始条件

$$v(0) = \frac{20}{20 + 20}(30) = 15 \text{ V}$$

Example 8.8

In the circuit of Fig. 8.23, find $i(t)$ and $i_R(t)$ for $t > 0$.



For $t > 0$, the switch is closed, and we have a parallel *RLC* circuit with a current source. The voltage source is zero which means it acts like a short-circuit. The two 20-Ω resistors are now in parallel. They are combined to give $R = 20 \| 20 = 10\ \Omega$. The characteristic roots are determined as follows:

$$\boxed{\alpha = \frac{1}{2RC}} = \frac{1}{2 \times 10 \times 8 \times 10^{-3}} = 6.25$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} = \frac{1}{\sqrt{20 \times 8 \times 10^{-3}}} = 2.5$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm \sqrt{39.0625 - 6.25}$$
$$= -6.25 \pm 5.7282$$

or

$$s_1 = -11.978, \qquad s_2 = -0.5218$$

Since $\alpha > \omega_0$, we have the overdamped case. Hence,

$$i(t) = I_s + A_1 e^{-11.978t} + A_2 e^{-0.5218t} \tag{8.8.1}$$

①求特征根，写出通解

. 与零输入响应比，通解多了 I_s

. 也可理解为：零输入响应时，变量最终都会衰减到零，即$i_{ss} = 0$

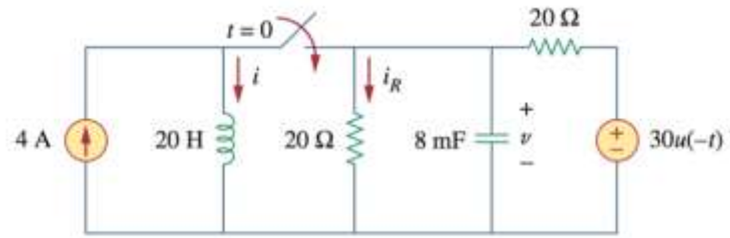
In the circuit of Fig. 8.23, find $i(t)$ and $i_R(t)$ for $t > 0$.

Example 8.8

where $I_s = 4$ is the final value of $i(t)$. We now use the initial conditions to determine A_1 and A_2. At $t = 0$,

$$i(0) = 4 = 4 + A_1 + A_2 \quad \Rightarrow \quad A_2 = -A_1 \qquad \textbf{(8.8.2)}$$

$$i(t) = I_s + A_1 e^{-11.978t} + A_2 e^{-0.5218t}$$



Taking the derivative of $i(t)$ in Eq. (8.8.1),

$$\frac{di}{dt} = -11.978A_1 e^{-11.978t} - 0.5218A_2 e^{-0.5218t}$$

so that at $t = 0$,

$$\frac{di(0)}{dt} = -11.978A_1 - 0.5218A_2 \qquad \textbf{(8.8.3)}$$

But

$$L\frac{di(0)}{dt} = v(0) = 15 \quad \Rightarrow \quad \boxed{\frac{di(0)}{dt}} = \frac{15}{L} = \frac{15}{20} = 0.75$$

②根据初始条件，求待定系数

Substituting this into Eq. (8.8.3) and incorporating Eq. (8.8.2), we get

$$0.75 = (11.978 - 0.5218)A_2 \quad \Rightarrow \quad A_2 = 0.0655$$

Thus, $A_1 = -0.0655$ and $A_2 = 0.0655$. Inserting A_1 and A_2 in Eq. (8.8.1) gives the complete solution as

$$i(t) = 4 + 0.0655(e^{-0.5218t} - e^{-11.978t}) \text{ A}$$

③ 电路中其他的量可依 $i(t)$ 求出

From $i(t)$, we obtain $v(t) = L\,di/dt$ and

$$i_R(t) = \frac{v(t)}{20} = \frac{L}{20}\frac{di}{dt} = 0.785e^{-11.978t} - 0.0342e^{-0.5218t} \text{ A}$$

二阶电路阶跃响应的通用解法

- 二阶电路除了串联 *RLC*、并联 *RLC* 之外，还有其他的电路类型。我们为所有二阶电路提供一种通用解法（当然，该解法也适用于串联 *RLC*、并联 *RLC*）：
  - 第一步：求变量（电容电压OR电感电流）初始值 $x(0)$ and $dx(0)/dt$
  - 第二步：求稳态响应 $\boxed{x_{ss}(t) = x(\infty)}$【非齐次方程在t=∞时的特解】
  - 第三步：0+时刻，**turn off 独立源**，求瞬态响应（零输入响应）$\boxed{x_t(t)}$。用KCL、KVL 列微分方程，求特征根，写出通解；【齐次方程通解】
  - 第四步：将稳态响应和瞬态响应相加，得到全响应

    $$x(t) = x_t(t) + x_{ss}(t)$$

  - 第五步：结合初始条件，求待定系数

Find the complete response v and then i for $t > 0$ in the circuit of Fig. 8.25.

0+时刻，电路非简单串并联，非齐次通解无法直接写出 → 采用二阶电路通用解法

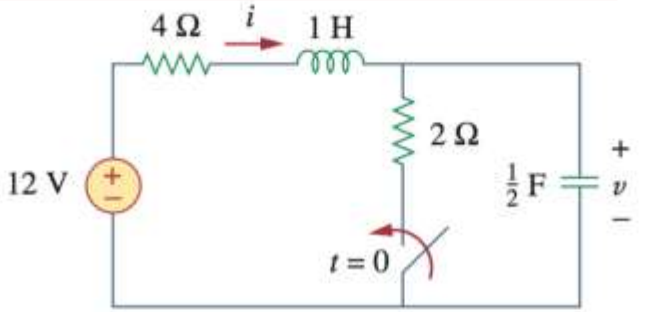


Figure 8.25
For Example 8.9.

Solution:

We first find the initial and final values. At $t = 0^-$, the circuit is at steady state. The switch is open; the equivalent circuit is shown in Fig. 8.26(a). It is evident from the figure that

$$v(0^-) = 12\text{ V}, \qquad i(0^-) = 0$$

At $t = 0^+$, the switch is closed; the equivalent circuit is in Fig. 8.26(b). By the continuity of capacitor voltage and inductor current, we know that

$$\boxed{v(0^+)} = v(0^-) = 12\text{ V}, \qquad i(0^+) = i(0^-) = 0 \tag{8.9.1}$$

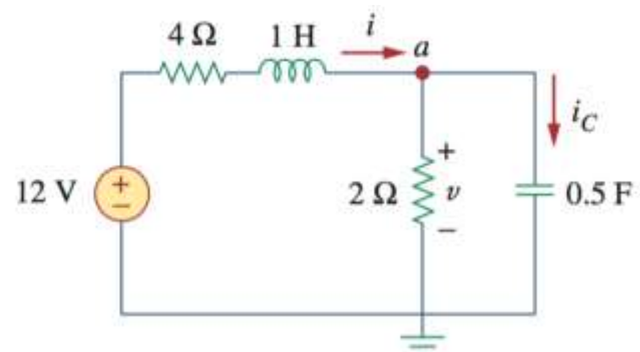
To get $dv(0^+)/dt$, we use $C\,dv/dt = i_C$ or $dv/dt = i_C/C$. Applying KCL at node a in Fig. 8.26(b),

$$i(0^+) = i_C(0^+) + \frac{v(0^+)}{2}$$

$$0 = i_C(0^+) + \frac{12}{2} \quad \Rightarrow \quad i_C(0^+) = -6\text{ A}$$

Hence,

$$\boxed{\frac{dv(0^+)}{dt}} = \frac{-6}{0.5} = -12\text{ V/s} \tag{8.9.2}$$



(b)

①求初始值和稳态值

The final values are obtained when the inductor is replaced by a short circuit and the capacitor by an open circuit in Fig. 8.26(b), giving

$$i(\infty) = \frac{12}{4+2} = 2\text{ A}, \qquad \boxed{v(\infty)} = 2i(\infty) = 4\text{ V} \tag{8.9.3}$$

Find the complete response v and then i for $t > 0$ in the circuit of Fig. 8.25.

Next, we obtain the form of the transient response for $t > 0$. By turning off the 12-V voltage source, we have the circuit in Fig. 8.27. Applying KCL at node a in Fig. 8.27 gives

$$i = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt} \tag{8.9.4}$$

Applying KVL to the left mesh results in

$$4i + 1\frac{di}{dt} + v = 0 \tag{8.9.5}$$

Since we are interested in v for the moment, we substitute i from Eq. (8.9.4) into Eq. (8.9.5). We obtain

$$2v + 2\frac{dv}{dt} + \frac{1}{2}\frac{dv}{dt} + \frac{1}{2}\frac{d^2v}{dt^2} + v = 0$$

or

$$\boxed{\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 0}$$

From this, we obtain the characteristic equation as

$$s^2 + 5s + 6 = 0$$

with roots $s = -2$ and $s = -3$. Thus, the natural response is

$$\boxed{v_n(t) = Ae^{-2t} + Be^{-3t}} \tag{8.9.6}$$

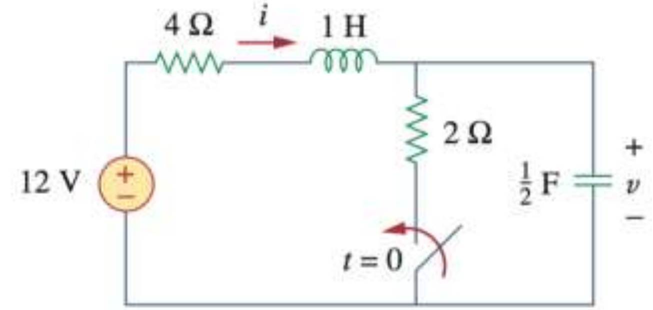


Figure 8.25
For Example 8.9.

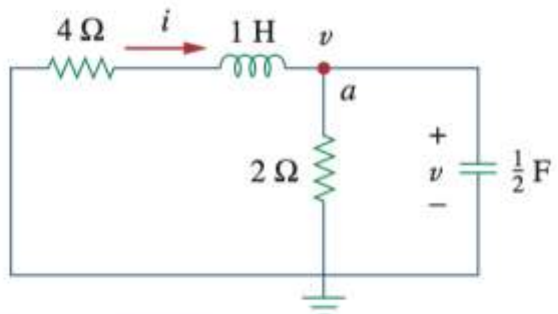


Figure 8.27
Obtaining the form of the transient response for Example 8.9.

②turn off独立源，求瞬态响应

Find the complete response v and then i for $t > 0$ in the circuit of Fig. 8.25.

where A and B are unknown constants to be determined later. The steady-state response is

$$v_{ss}(t) = v(\infty) = 4 \tag{8.9.7}$$

The complete response is ③ 将稳态响应和瞬态响应相加，得到全响应

$$\boxed{v(t) = v_t + v_{ss} = 4 + Ae^{-2t} + Be^{-3t}} \tag{8.9.8}$$

We now determine A and B using the initial values. From Eq. (8.9.1), $v(0) = 12$. Substituting this into Eq. (8.9.8) at $t = 0$ gives

$$12 = 4 + A + B \quad \Rightarrow \quad A + B = 8 \tag{8.9.9}$$

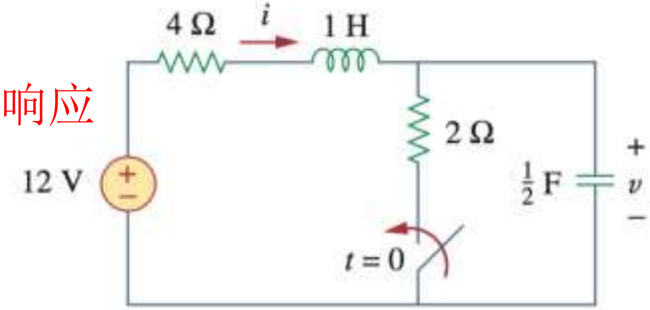


Figure 8.25
For Example 8.9.

Taking the derivative of v in Eq. (8.9.8),

$$\frac{dv}{dt} = -2Ae^{-2t} - 3Be^{-3t} \tag{8.9.10}$$

Substituting Eq. (8.9.2) into Eq. (8.9.10) at $t = 0$ gives

$$-12 = -2A - 3B \quad \Rightarrow \quad 2A + 3B = 12 \tag{8.9.11}$$

From Eqs. (8.9.9) and (8.9.11), we obtain

$$A = 12, \qquad B = -4$$

so that Eq. (8.9.8) becomes 结合初始条件，求待定系数

$$v(t) = 4 + 12e^{-2t} - 4e^{-3t}\ \text{V}, \qquad t > 0 \tag{8.9.12}$$

From v, we can obtain other quantities of interest by referring to Fig. 8.26(b). To obtain i, for example,

再计算其他量

$$\begin{aligned} i &= \frac{v}{2} + \frac{1}{2}\frac{dv}{dt} = 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t} \\ &= 2 - 6e^{-2t} + 4e^{-3t}\ \text{A}, \qquad t > 0 \end{aligned} \tag{8.9.13}$$

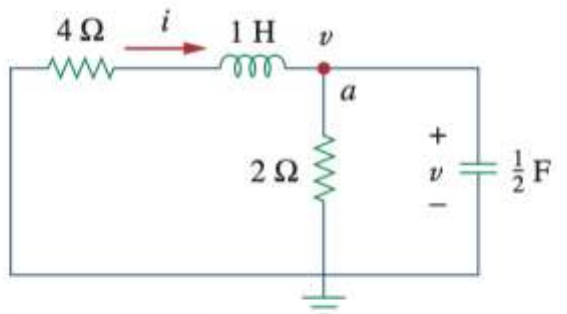


Figure 8.27
Obtaining the form of the transient response for Example 8.9.

Example 8.10

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 8.29.

Solution:

This is an example of a second-order circuit with two inductors. We first obtain the mesh currents i_1 and i_2, which happen to be the currents through the inductors. We need to obtain the initial and final values of these currents.

For $t < 0$, $7u(t) = 0$, so that $i_1(0^-) = 0 = i_2(0^-)$. For $t > 0$, $7u(t) = 7$, so that the equivalent circuit is as shown in Fig. 8.30(a). Due to the continuity of inductor current,

$$\boxed{i_1(0^+)} = i_1(0^-) = 0, \qquad i_2(0^+) = i_2(0^-) = 0 \qquad \textbf{(8.10.1)}$$

$$v_{L_2}(0^+) = v_o(0^+) = 1[(i_1(0^+) - i_2(0^+)] = 0 \qquad \textbf{(8.10.2)}$$

Applying KVL to the left loop in Fig. 8.30(a) at $t = 0^+$,

$$7 = 3i_1(0^+) + v_{L_1}(0^+) + v_o(0^+)$$

Since $L_1\, di_1/dt = v_{L_1}$,

$$\boxed{\frac{di_1(0^+)}{dt}} = \frac{v_{L_1}}{L_1} = \frac{7}{\frac{1}{2}} = 14 \text{ V/s} \qquad \textbf{(8.10.3)}$$

Similarly, since $L_2\, di_2/dt = v_{L_2}$,

$$\frac{di_2(0^+)}{dt} = \frac{v_{L_2}}{L_2} = 0 \qquad \textbf{(8.10.4)}$$

As $t \to \infty$, the circuit reaches steady state, and the inductors can be replaced by short circuits, as shown in Fig. 8.30(b). From this figure,

$$\Rightarrow \boxed{i_1(\infty)} = i_2(\infty) = \frac{7}{3} \text{ A}$$

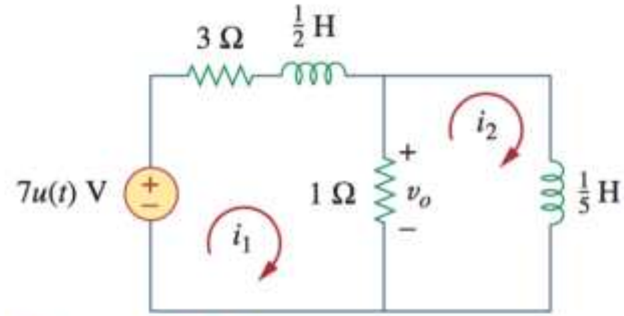
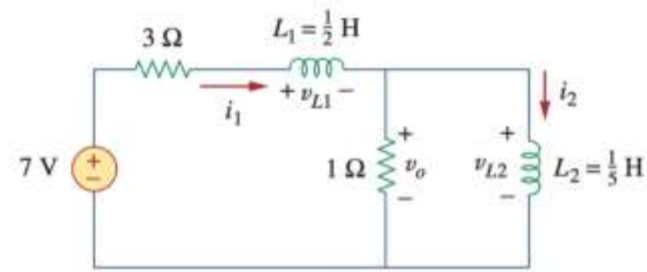


Figure 8.29
For Example 8.10.



(a)

①求初始值和稳态值

- i_1 和 i_2 作为关键变量；
- 先求一个（ i_1 ）

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 8.29.

Example 8.10

Next, we obtain the form of the transient responses by removing the voltage source, as shown in Fig. 8.31. Applying KVL to the two meshes yields

$$4i_1 - i_2 + \frac{1}{2}\frac{di_1}{dt} = 0 \quad \textbf{(8.10.6)}$$

and

$$i_2 + \frac{1}{5}\frac{di_2}{dt} - i_1 = 0 \quad \textbf{(8.10.7)}$$

From Eq. (8.10.6),

$$i_2 = 4i_1 + \frac{1}{2}\frac{di_1}{dt} \quad \textbf{(8.10.8)}$$

Substituting Eq. (8.10.8) into Eq. (8.10.7) gives

$$4i_1 + \frac{1}{2}\frac{di_1}{dt} + \frac{4}{5}\frac{di_1}{dt} + \frac{1}{10}\frac{d^2i_1}{dt^2} - i_1 = 0$$

$$\boxed{\frac{d^2i_1}{dt^2} + 13\frac{di_1}{dt} + 30i_1 = 0}$$

From this we obtain the characteristic equation as

$$s^2 + 13s + 30 = 0$$

which has roots $s = -3$ and $s = -10$. Hence, the form of the transient response is

$$\boxed{i_{1n} = Ae^{-3t} + Be^{-10t}} \quad \textbf{(8.10.9)}$$

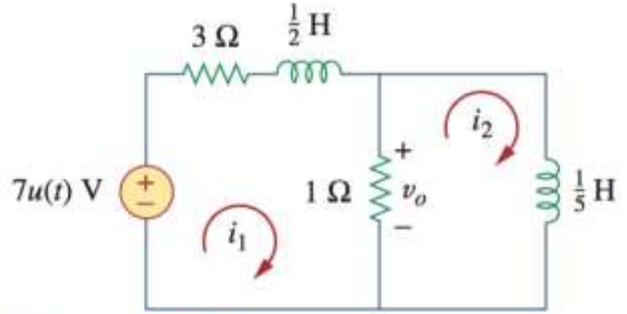


Figure 8.29
For Example 8.10.

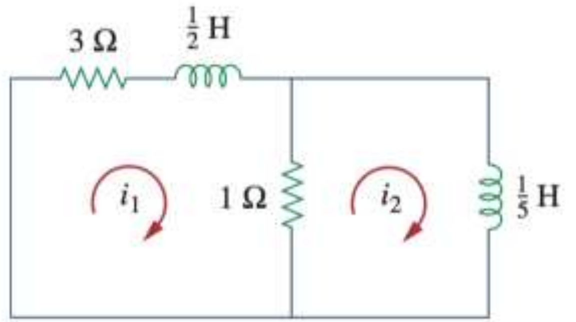


Figure 8.31
Obtaining the form of the transient response for Example 8.10.

②turn off独立源，求瞬态响应

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 8.29.

③将稳态响应和瞬态响应相加，得到全响应

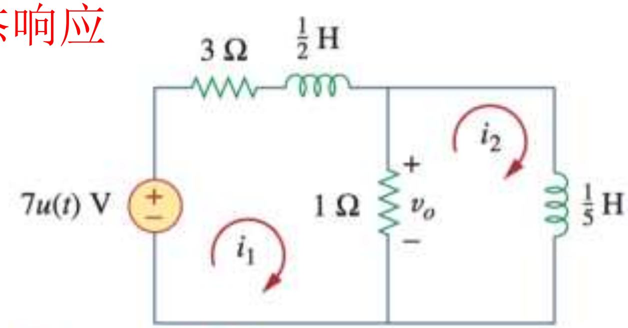


Figure 8.29
For Example 8.10.

where A and B are constants. The steady-state response is

$$i_{1ss} = i_1(\infty) = \frac{7}{3}\text{ A} \qquad \textbf{(8.10.10)}$$

From Eqs. (8.10.9) and (8.10.10), we obtain the complete response as

$$\boxed{i_1(t) = \frac{7}{3} + Ae^{-3t} + Be^{-10t}} \qquad \textbf{(8.10.11)}$$

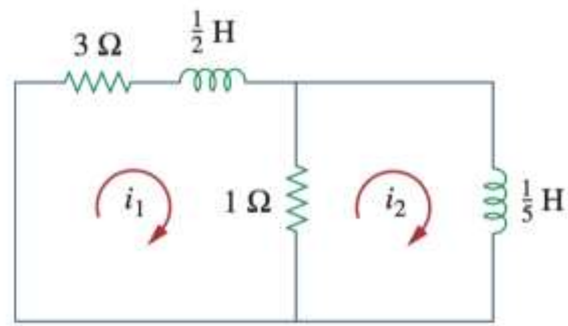


Figure 8.31
Obtaining the form of the transient response for Example 8.10.

We finally obtain A and B from the initial values. From Eqs. (8.10.1) and (8.10.11),

$$0 = \frac{7}{3} + A + B \qquad \textbf{(8.10.12)}$$

Taking the derivative of Eq. (8.10.11), setting $t = 0$ in the derivative, and enforcing Eq. (8.10.3), we obtain

$$14 = -3A - 10B \qquad \textbf{(8.10.13)}$$

From Eqs. (8.10.12) and (8.10.13), $A = -4/3$ and $B = -1$. Thus,

④结合初始条件，求待定系数

$$i_1(t) = \frac{7}{3} - \frac{4}{3}e^{-3t} - e^{-10t} \qquad \textbf{(8.10.14)}$$

Example 8.10

Find $v_o(t)$ for $t > 0$ in the circuit of Fig. 8.29.

ISEE

We now obtain i_2 from i_1. Applying KVL to the left loop in Fig. 8.30(a) gives

$$7 = 4i_1 - i_2 + \frac{1}{2}\frac{di_1}{dt} \quad \Rightarrow \quad i_2 = -7 + 4i_1 + \frac{1}{2}\frac{di_1}{dt}$$

Substituting for i_1 in Eq. (8.10.14) gives

$$i_2(t) = -7 + \frac{28}{3} - \frac{16}{3}e^{-3t} - 4e^{-10t} + 2e^{-3t} + 5e^{-10t}$$
$$= \frac{7}{3} - \frac{10}{3}e^{-3t} + e^{-10t} \qquad \textbf{(8.10.15)}$$

From Fig. 8.29,

$$\boxed{v_o(t) = 1[i_1(t) - i_2(t)]} \qquad \textbf{(8.10.16)}$$

Substituting Eqs. (8.10.14) and (8.10.15) into Eq. (8.10.16) yields

$$v_o(t) = 2(e^{-3t} - e^{-10t}) \qquad \textbf{(8.10.17)}$$

Note that $v_o(0) = 0$, as expected from Eq. (8.10.2).

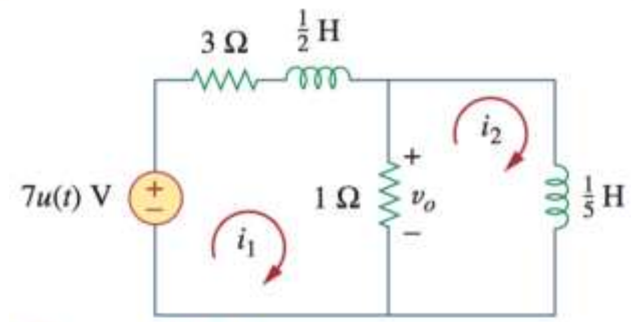
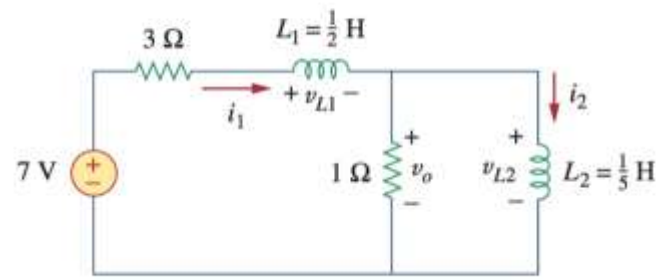


Figure 8.29
For Example 8.10.



(a)

根据 i_1 求得 i_2 及其他电路量

二阶电路的冲激响应

- 二阶电路对于单位冲激函数激励的零状态响应称为二阶电路的冲激响应。
  - 因冲激激励是阶跃激励的一阶导数 → 冲激响应也可以按阶跃激励的一阶导数求得

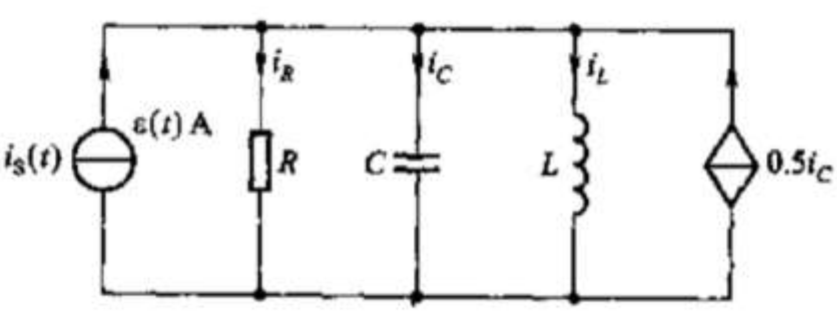


图 7-33　例 7-12 图

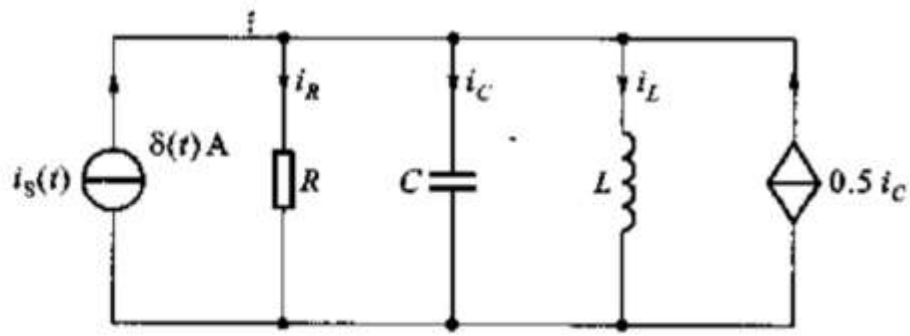


图 7-40　例 7-14 图

$$i_L(t)=s(t)=\left(1-\frac{4}{3}e^{-t}+\frac{1}{3}e^{-4t}\right)\varepsilon(t)\ \mathrm{A}$$

$$i_L(t)=h(t)=\frac{\mathrm{d}s(t)}{\mathrm{d}t}$$

$$=\left(\frac{4}{3}e^{-t}-\frac{4}{3}e^{-4t}\right)\varepsilon(t)\ \mathrm{A}$$

小结

- 初始值的计算
  - 利用“电容电压不能突变”、“电感电流不能突变”原则，从**0-时刻**发现 $v(0), i(0)$
  - $dv(0)/dt, di(0)/dt$ 根据电感电容的“电流-电压约束关系”，

$$\boxed{i = C\frac{dv}{dt}} \quad \boxed{v = L\frac{di}{dt}}$$

导数可以从**0+时刻**的“电容电流”、“电感电压”中获得

小结

- Source-free RLC 电路【二阶齐次微分方程】

	串联RLC	**并联RLC**
关键变量	电感电流	电容电压
微分方程	$\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$	$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$
特征方程	$s^2+2\alpha s+\omega_0^2=0$	
特征值	$s_1=-\alpha+\sqrt{\alpha^2-\omega_0^2},\qquad s_2=-\alpha-\sqrt{\alpha^2-\omega_0^2}$	
系数	$\alpha=\frac{R}{2L},\qquad \omega_0=\frac{1}{\sqrt{LC}}$	$\alpha=\frac{1}{2RC},\qquad \omega_0=\frac{1}{\sqrt{LC}}$
三种情况	1）$\alpha<\omega_0$，一对共轭复数根，欠阻尼，通解：$e^{-\alpha t}(A_1\cos\omega_d t+A_2\sin\omega_d t)$ 2）$\alpha=\omega_0$，重根，临界阻尼，通解：$(A_1+A_2t)e^{-\alpha t}$ 3）$\alpha>\omega_0$，两负实数根，过阻尼，通解：$A_1e^{s_1t}+A_2e^{s_2t}$ $\omega_d=\sqrt{\omega_0^2-\alpha^2}$	

小结

- RLC 电路阶跃响应（全响应）【二阶非齐次微分方程】

	串联RLC	并联RLC
激励源 & 关键变量	电压源激励 关键变量：电容电压	电流源激励 关键变量：电感电流
微分方程	$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{V_s}{LC}$	$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{i}{LC}=\frac{I_s}{LC}$
通解	齐次微分方程的通解 + 非齐次微分方程的特解（t=∞）	
三种情况	$v(t)=V_s+A_1e^{s_1t}+A_2e^{s_2t}$ (Overdamped) $v(t)=V_s+(A_1+A_2t)e^{-\alpha t}$ (Critically damped) $v(t)=V_s+(A_1\cos\omega_d t+A_2\sin\omega_d t)e^{-\alpha t}$ (Underdamped) $i(t)=I_s+A_1e^{s_1t}+A_2e^{s_2t}$ (Overdamped) $i(t)=I_s+(A_1+A_2t)e^{-\alpha t}$ (Critically damped) $i(t)=I_s+(A_1\cos\omega_d t+A_2\sin\omega_d t)e^{-\alpha t}$ (Underdamped)	

小结

- 二阶电路阶跃响应（全响应）的通用解法 $x(t) = x_t(t) + x_{ss}(t)$
  - 选择合适的变量（电容电压，或者电感电流）
  - 求该变量的初始值、其导数的初始值
  - 求该变量的稳态响应【非齐次方程的特解】
  - Turn off 独立源，求该变量的瞬态响应【齐次方程通解】
  - 将稳态响应和瞬态响应相加，得到该变量的全响应通解
  - 结合该变量、变量导数的初始条件，求待定系数
  - 再依据该变量的表达式，计算出电路中其他的量

作业

Practice Problem 8.4

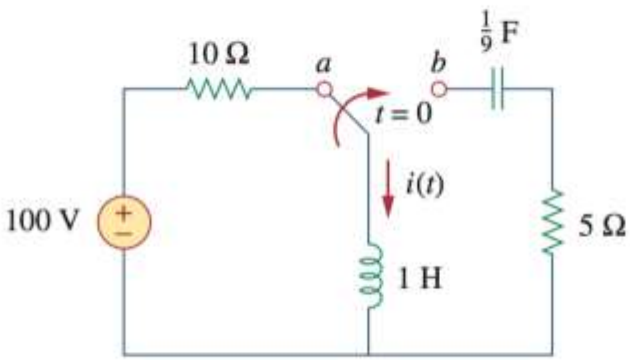


Figure 8.12
For Practice Prob. 8.4.

The circuit in Fig. 8.12 has reached steady state at $t = 0^-$. If the make-before-break switch moves to position b at $t = 0$, calculate $i(t)$ for $t > 0$.

Answer: $e^{-2.5t}(10\cos 1.6583t - 15.076\sin 1.6583t)$ A.

串联RLC二阶电路的零输入响应

Refer to the circuit in Fig. 8.17. Find $v(t)$ for $t > 0$.

Practice Problem 8.6

Answer: $50(e^{-10t} - e^{-2.5t})$ V.

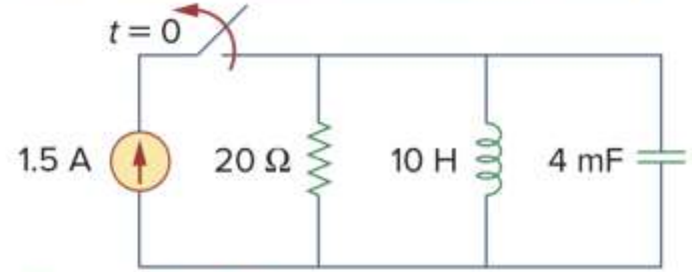


Figure 8.17

并联RLC二阶电路的零输入响应

Practice Problem 8.7

串联RLC的阶跃响应（全响应）

Having been in position a for a long time, the switch in Fig. 8.21 is moved to position b at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

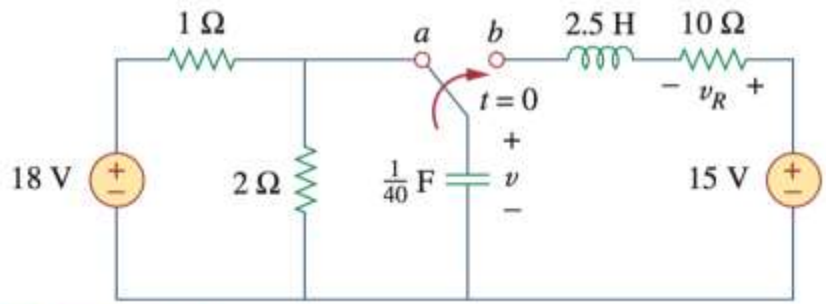


Figure 8.21
For Practice Prob. 8.7.

Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t}$ V, $3.464e^{-2t} \sin 3.464t$ V.

Practice Problem 8.9

Determine v and i for $t > 0$ in the circuit of Fig. 8.28. (See comments about current sources in Practice Prob. 7.5.)

Answer: $20(1 - e^{-5t})$ V, $5(1 - e^{-5t})$ A.

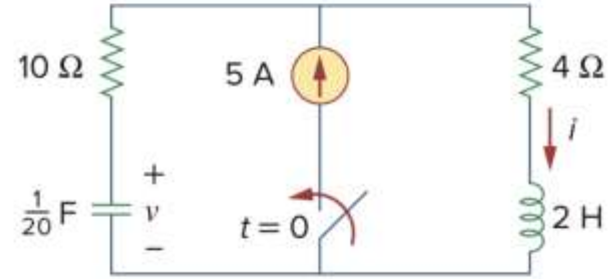


Figure 8.28
For Practice Prob. 8.9.

二阶电路的通用解法

列方程小心正负值！！！