电子电路基础

第六讲~正弦交流电路 part1

课程纲要

- 3.1 正弦交流电的基本概念
  - 3.1.1 正弦量的定义
  - 3.1.2 正弦量的重要特性参数：幅值、有效值、峰峰值、平均值、频率、周期和相位等
- 3.2 正弦交流电路的相量分析方法
  - 3.2.1 正弦量的相量表示
  - 3.2.2 电路元件基本物理量的相量描述
  - 3.2.3 基尔霍夫定律的相量形式
  - 3.2.4 叠加定理、戴维南定理和诺顿定理在相量域的推广
  - 3.2.5 网孔电流法和节点电压法在相量域的推广

Chapter 9 Sinusoids and Phasors 正弦交流电、相量

9.1 Motivation

9.2 Sinusoids’ features 正弦交流电的特性

9.3 Phasors 相量

9.4 Phasor relationships for circuit elements 电路元件基本物理量的相量描述

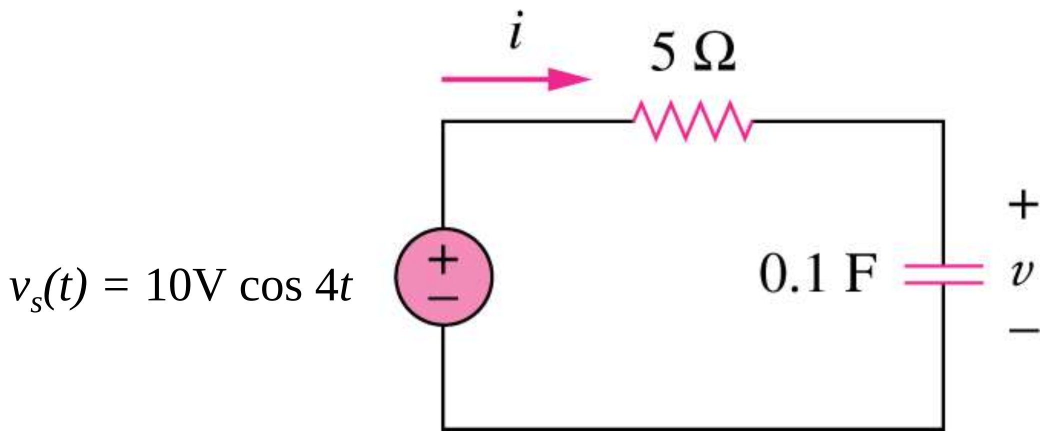
9.5 Impedance and admittance 阻抗和导纳

9.6 Kirchhoff’s laws in the frequency domain 基尔霍夫定理的相量形式

9.7 Impedance combinations 阻抗混合

9.1 Motivation (1)

How to determine *v(t)* and *i(t)*?



How can we apply what we have learned before to determine *i(t)* and *v(t)*?

根据电路的基本定律 VCR、KCL 和 KVL，编写含有储能元件的线性非时变电路的电路方程时，将获得一组常微(积)分方程。现以图 8－7 所示的 RLC 串联电路为例，电路的 KVL 方程为

$$u_R + u_L + u_C = u_S$$

其中

$$u_R = Ri \quad u_L = L\frac{\mathrm{d}i}{\mathrm{d}t} \quad u_C = \frac{1}{C}\int i\mathrm{d}t$$

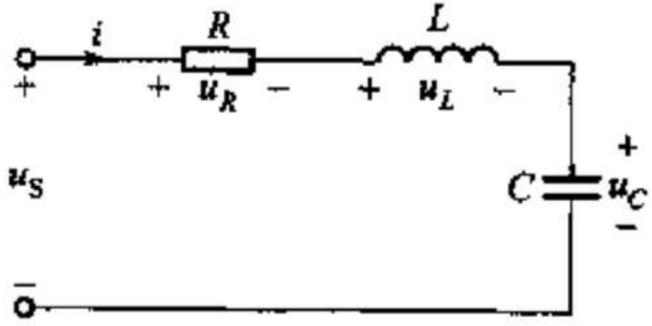


图 8－7　RLC 串联电路

将上述元件的 VCR 代入 KVL 方程有

$$Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{C}\int i\mathrm{d}t = u_S \qquad (8-3)$$

由数学理论可知，当 u_S(激励)为正弦量时，上述微分方程中的电流变量 i 的特解(响应的强制分量)也一定是与 u_S同一频率的正弦量，反之亦然。这一重要结论具有普遍意义，即线性非时变电路在正弦电源激励下，各支路电压、电流的特解都是与激励同频率的正弦量，当电路中存在有多个同频率的正弦激励时，该结论也成立。工程上将电路的这一特解状态称为正弦电流电路的稳定状态，简称正弦稳态。电路处于正弦稳态时，同频率的各正弦量之间，仅在有效值(或振幅)、初相上存在“差异和联系”，这种“差异和联系”正是正弦稳态分析求解中的关键问题。

稳态：“t → 无穷大”时的解；线性时不变：不产生新的频率

9.2 Sinusoids正弦信号

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid, $v(t) = V_m \sin(\omega t + \phi)$

where

V_m = the **amplitude** of the sinusoid，振幅

ω = the angular frequency in **radians/s**，角频率

ϕ = the phase，相位【或“初始相位”】

$\omega t + \phi$ = the argument, 幅角【或“相位”】

$$T = \frac{2\pi}{\omega}$$

T: 周期

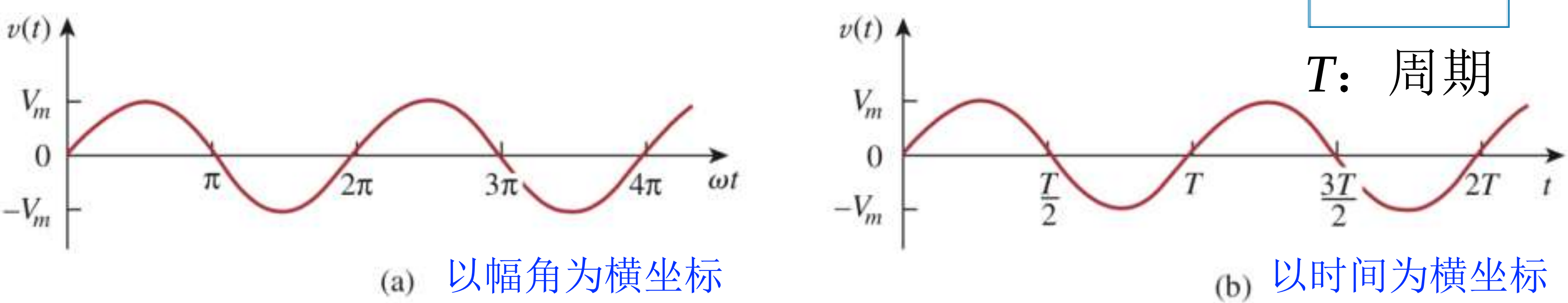
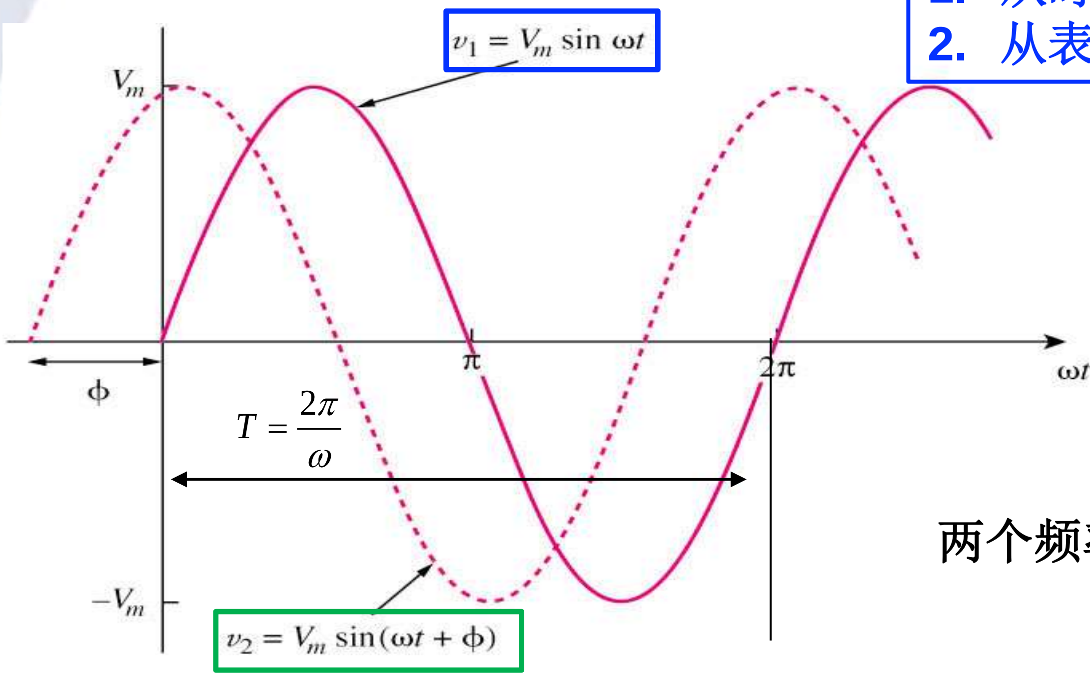


Figure 9.1
A sketch of $V_m \sin \omega t$: (a) as a function of ωt, (b) as a function of t.

9.2 Sinusoids (2)

A <u>periodic function</u> （周期函数）**is one that satisfies**

$v(t) = v(t + nT)$, **for all *t* and for all integers *n*.**



1. 从时域图中看，靠左的曲线相位领先
2. 从表达式看，幅角(ωt + ϕ) 比 (ωt) 领先 ϕ

如图所示，我们说

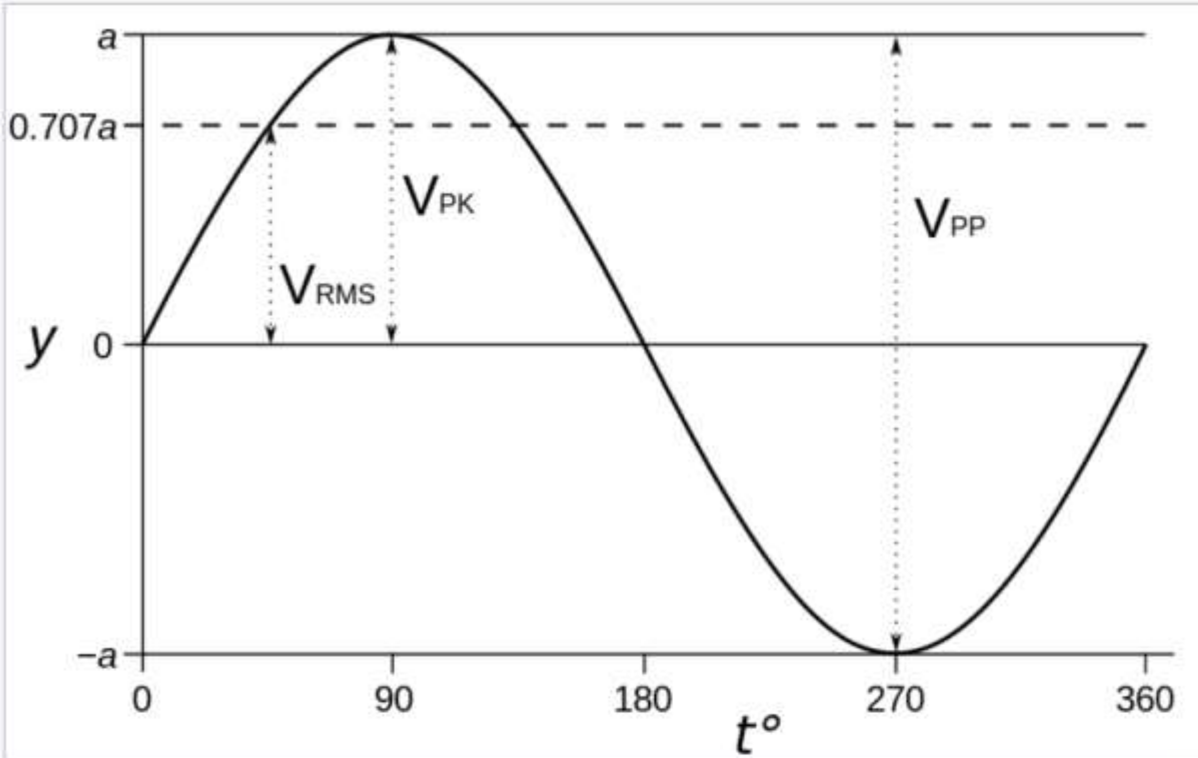
- v_2领先(lead) v_1 Φ相位; 或：
- v_1滞后(lag) v_2 Φ相位;

两个频率量：

$$f = \frac{1}{T}$$ 频率，Hz

$$\omega = 2\pi f$$ 角频率，rad/s

- Only two sinusoidal values with the <u>same frequency</u> can be compared by their amplitude and phase difference. 两个正弦信号**只有频率相同时**，比较它们的振幅、相位关系(领先，还是滞后)才有意义。
- If phase difference is zero, they are **in phase**; if phase difference is not zero, they are **out of phase**. 相位差为零~同相；相位差180° ~反相



Graph of a sine wave's voltage vs. time (in degrees), showing RMS, peak (PK), and peak-to-peak (PP) voltages.

- **幅值**，V_{PK} 或 V_P
- **有效值**， 计算平均功率时采用有效值
- **峰峰值**， V_{PP}
- **平均值**，0

$$V_{\mathrm{RMS}} = \frac{V_{\mathrm{p}}}{\sqrt{2}}$$

$$P_{\mathrm{Avg}} = \frac{V_{\mathrm{RMS}}^2}{R}$$

$$P_{\mathrm{Avg}} = V_{\mathrm{RMS}} I_{\mathrm{RMS}}$$

有效值：一个时变周期电压在一个周期内消耗在电阻上的功率，可以用一个恒定的电压来等效，该恒定电压就是该时变电压的有效值。当时变电压为正弦函数时，有效值为其峰值的1/sqrt(2)

9.2 Sinusoids (3)

Example

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Solution:

Amplitude = 5, phase = –60°, angular frequency = 4π rad/s, Period = 0.5 s, frequency = 2 Hz.

三角函数关系式

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(\omega t \pm 180^\circ) = -\sin \omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos \omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos \omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp\sin \omega t$$

Step 1：看起点；Step 2：看变化的角度

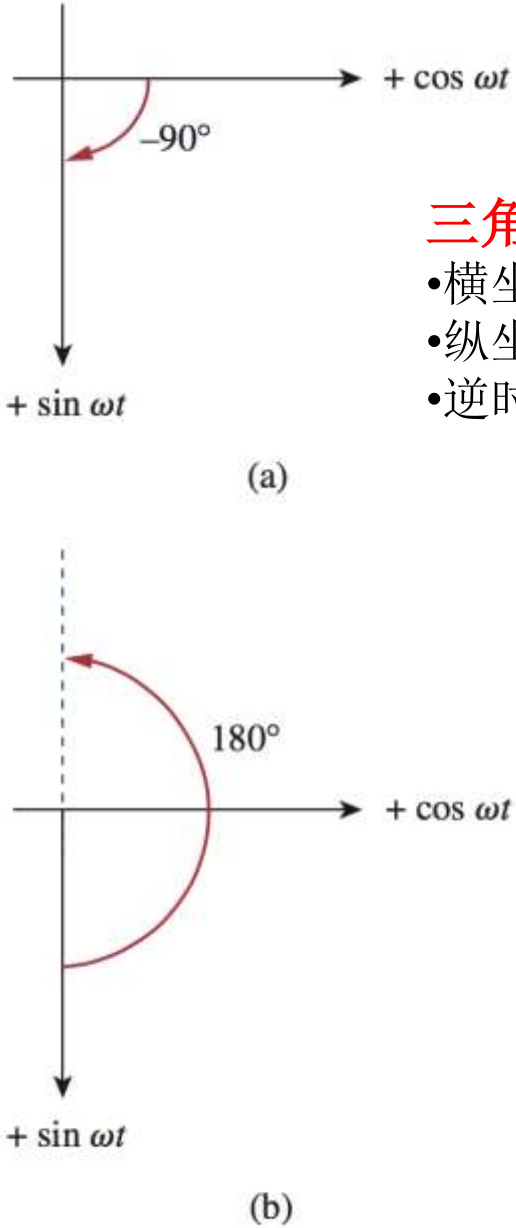


Figure 9.3
A graphical means of relating cosine and sine: (a) $\cos(\omega t - 90°) = \sin\omega t$, (b) $\sin(\omega t + 180°) = -\sin\omega t$.

三角函数关系的图示记忆法

- 横坐标为cos（与复平面一致）
- 纵坐标向下为sin（与复平面相反）
- 逆时针相位增加（与复平面一致）

$$\sin(\omega t \pm 180°) = -\sin\omega t$$

$$\cos(\omega t \pm 180°) = -\cos\omega t$$

$$\sin(\omega t \pm 90°) = \pm\cos\omega t$$

$$\cos(\omega t \pm 90°) = \mp\sin\omega t$$

$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1}\frac{B}{A}$$

$$3\cos\omega t - 4\sin\omega t = 5\cos(\omega t + 53.1°)$$

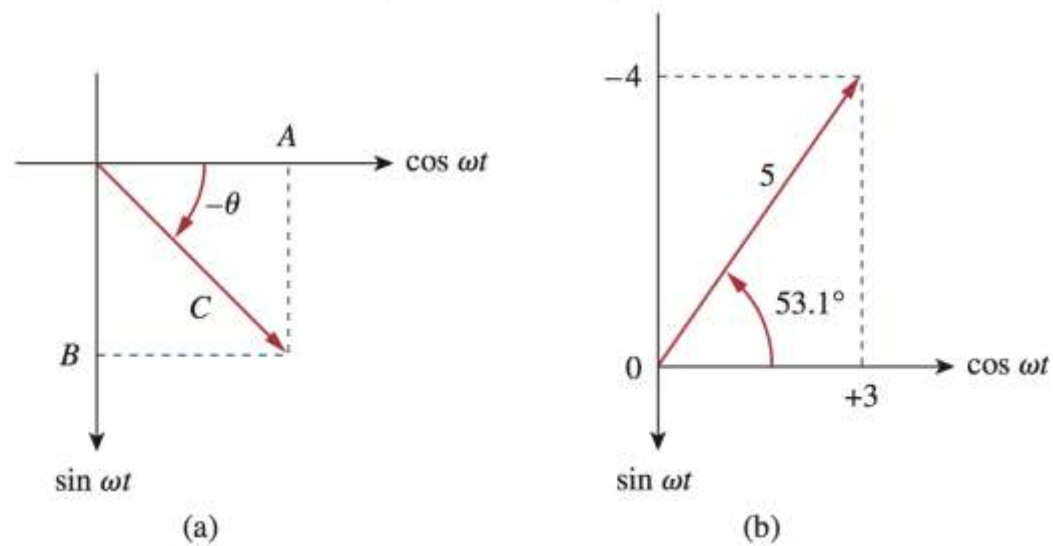


Figure 9.4
(a) Adding $A\cos\omega t$ and $B\sin\omega t$, (b) adding $3\cos\omega t$ and $-4\sin\omega t$.

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Example 9.2

METHOD 1

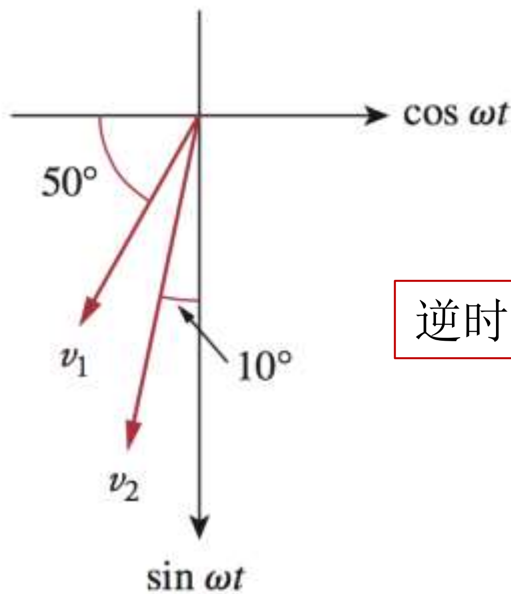
比较两个三角函数，要先化简成相同形式的cos或sin

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ) = 10\cos(\omega t - 130^\circ)$$

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ) = 12\cos(\omega t - 100^\circ)$$

$\Rightarrow$ v_2 leads v_1 by 30°

METHOD 2



逆时针领先

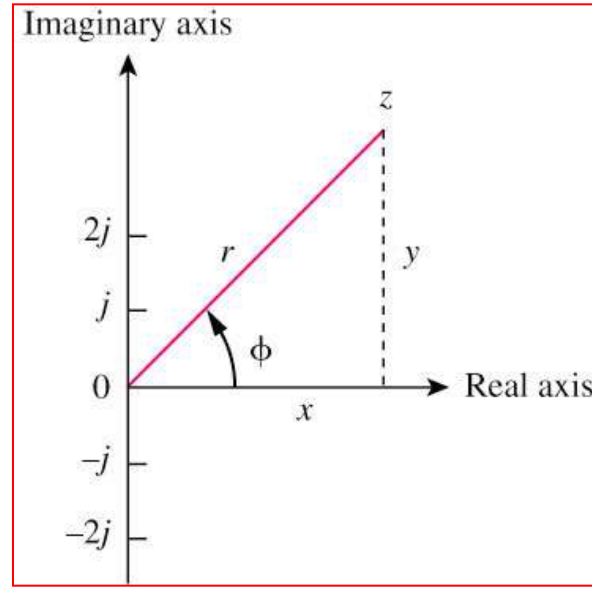
Figure 9.5
For Example 9.2.

9.3 Phasor 相量

- A **phasor** is a **complex number** that represents the amplitude and phase of a sinusoid. 相量是由正弦信号的**振幅**和**相位**构成的一个**复数**

$$v(t) = \underline{V_m} \sin(\omega t + \underline{\phi})$$

r



- 复数的三种表述形式:

a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$ 适合于复数的+、−

b. Polar $z = r\angle\phi$ 适合于复数的×、÷

c. Exponential $z = re^{j\phi}$

$$\boxed{z = x + jy = \underline{r\angle\phi} = r(\cos\phi + j\sin\phi)}$$

where $r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\dfrac{y}{x}$

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

复数的运算

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \tag{9.18a}$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \tag{9.18b}$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2 \tag{9.18c}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2 \tag{9.18d}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \angle -\phi \tag{9.18e}$$

Square Root:

$$\sqrt{z} = \sqrt{r} \angle \phi/2 \tag{9.18f}$$

Complex Conjugate:

$$z^* = x - jy = r\angle -\phi = re^{-j\phi} \tag{9.18g}$$

Example 9.3

Evaluate these complex numbers:

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

复数的运算：加减用直角坐标形式；乘除及其他用极坐标形式

Solution:

(a) Using polar to rectangular transformation,

$$40\angle 50^\circ = 40(\cos 50^\circ + j \sin 50^\circ) = 25.71 + j30.64$$

$$20\angle -30^\circ = 20[\cos(-30^\circ) + j \sin(-30^\circ)] = 17.32 - j10$$

Adding them up gives

$$40\angle 50^\circ + 20\angle -30^\circ = 43.03 + j20.64 = 47.72\angle 25.63^\circ$$

Taking the square root of this,

$$(40\angle 50^\circ + 20\angle -30^\circ)^{1/2} = 6.91\angle 12.81^\circ$$

(b) Using polar-rectangular transformation, addition, multiplication, and division,

$$\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 + j5)}$$

$$= \frac{11.66 - j9}{-14 + j22} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ}$$

$$= 0.565\angle -160.13^\circ$$

欧拉等式（Euler's Identity）：

$$e^{\pm j\phi} = \cos\phi \pm j\sin\phi$$

$$\cos\phi = \mathrm{Re}(e^{j\phi}) \tag{9.20a}$$

$$\sin\phi = \mathrm{Im}(e^{j\phi}) \tag{9.20b}$$

where Re and Im stand for the *real part of* and the *imaginary part of*. Given a sinusoid $v(t) = V_m\cos(\omega t + \phi)$, we use Eq. (9.20a) to express $v(t)$ as

$$v(t) = V_m\cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)}) \tag{9.21}$$

or

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t}) \tag{9.22}$$

Thus,

注意：
1.相量**V**是个复数，由正弦信号$v(t)$的振幅V_m和相位Φ构成
2.因为相量**V**是复数，故用粗体表示

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t}) \tag{9.23}$$

where

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle\phi \tag{9.24}$$

9.3 Phasor

- **相量的意义？**
  - 时域←→相量域（频域）的相互转换

时域 → 相量域，好处是少了时间量，代价是需要复数运算

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

  - 电路分析中，电压、电流常常是正弦信号，或正弦信号的叠加，我们将其统一用余弦信号 $V_m \cos(\omega t + \phi)$ 表示，其中振幅和相位是电路分析中主要关心的物理量
  - 若用时域的形式分析电路中电压、电流信号，数学上会比较复杂（处理三角函数）；而转换为相量域后，会比较简单（处理复数）

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

  - 相量分析法，先去除时域变量 t ，简化运算，最后可再用Re{}函数把 t 加回来。
  - 将与时间有关的同频的正弦（余弦）函数的电路方程 → 与时间无关的复代数形式的方程

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation		Phasor domain representation
$V_m \cos(\omega t + \phi)$		$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	先转换成cos形式	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$		$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	先转换成cos形式	$I_m \angle \theta - 90^\circ$

9.3 Phasor

Example

Transform the following sinusoids to phasors:

$i = 6\cos(50t - 40^o)$ A

$v = -4\sin(30t + 50^o)$ V

时域信号→相量信号：
1.统一表示成 cos 形式
2.再用振幅和相位构成相量（复数）

Solution:

a. $\mathrm{I} = 6\angle -40^\circ$ A

b. Since $-\sin(x) = \cos(x + 90^o)$;

$v(t) = 4\cos(30t + 50^\circ + 90^o) = 4\cos(30t + 140^o)$ V

Transform to phasor => $\mathrm{V} = 4\angle 140^\circ$ V

9.3 Phasor

Find the sinusoids represented by these phasors:

(a) $\mathbf{I} = -3 + j4$ A
(b) $\mathbf{V} = j8e^{-j20^\circ}$ V

相量信号→时域信号：
•先将相量表示成极坐标形式，再写成 cos 形式；
•或 $v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$

Solution:

(a) $\mathbf{I} = -3 + j4 = 5\angle 126.87^\circ$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

(b) Since $j = 1\angle 90^\circ$,

$$\mathbf{V} = j8\angle -20^\circ = (1\angle 90^\circ)(8\angle -20^\circ)$$
$$= 8\angle 90^\circ - 20^\circ = 8\angle 70^\circ \text{ V}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

Example 9.6

Given $i_1(t) = 4\cos(\omega t + 30°)$ A and $i_2(t) = 5\sin(\omega t - 20°)$ A, find their sum.

相量分析法，先去除时域变量 t ，简化运算，最后再用Re{}函数把 t 加回来。

9.3 Phasor

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t}) = V_m \cos(\omega t + \phi)$$

$$\frac{dv}{dt} = -\omega V_m \sin(\omega t + \phi) = \omega V_m \cos(\omega t + \phi + 90^\circ)$$

$$= \mathrm{Re}(\omega V_m e^{j\omega t} e^{j\phi} e^{j90^\circ}) = \mathrm{Re}(j\omega \mathbf{V} e^{j\omega t})$$

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega\mathbf{V} \qquad (9.27)$$

(Time domain) (Phasor domain)

$$\int v\,dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega} \qquad (9.28)$$

(Time domain) (Phasor domain)

$\boxed{\text{采用 phasor, 可以使微分、积分的计算变得简单}}$

Example 9.7

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

回到最初的 motivation →

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

Solution:

We transform each term in the equation from time domain to phasor domain. Keeping Eqs. (9.27) and (9.28) in mind, we obtain the phasor form of the given equation as

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\underline{/75^\circ}$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\underline{/75^\circ}$$

$$\mathbf{I} = \frac{50\underline{/75^\circ}}{4 - j10} = \frac{50\underline{/75^\circ}}{10.77\underline{/-68.2^\circ}} = 4.642\underline{/143.2^\circ}\text{ A}$$

Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2^\circ)\text{ A}$$

Keep in mind that this is only the steady-state solution, and it does not require knowing the initial values.

注意：用相量法，求出的是稳态解。求稳态解不需要初始值。

9.3 Phasor

The differences between $v(t)$ and $\mathbf{V}$:

- $v(t)$ is instantaneous or time-domain representation $\mathbf{V}$ is the frequency or phasor-domain representation.
- $v(t)$ is time dependent, $\mathbf{V}$ is not.
- $v(t)$ is always real with no complex term, $\mathbf{V}$ is generally complex.

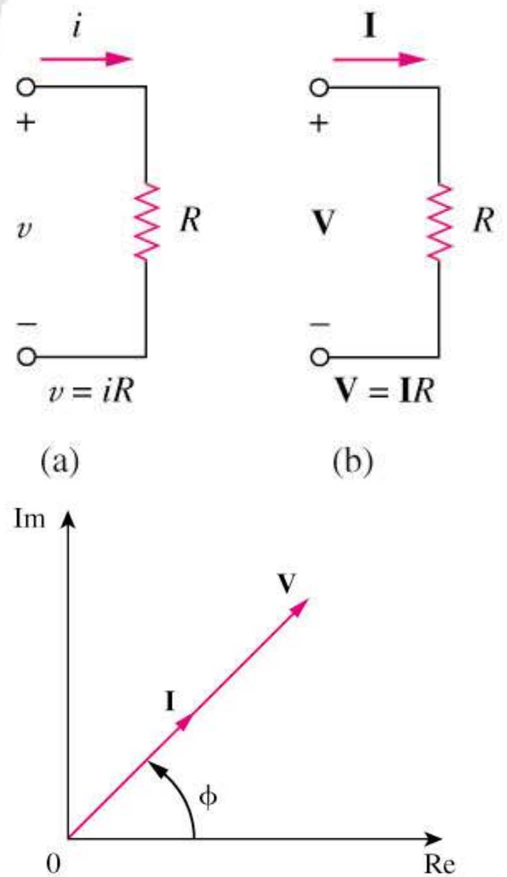
Note: Phasor analysis **applies only when frequency is constant**;

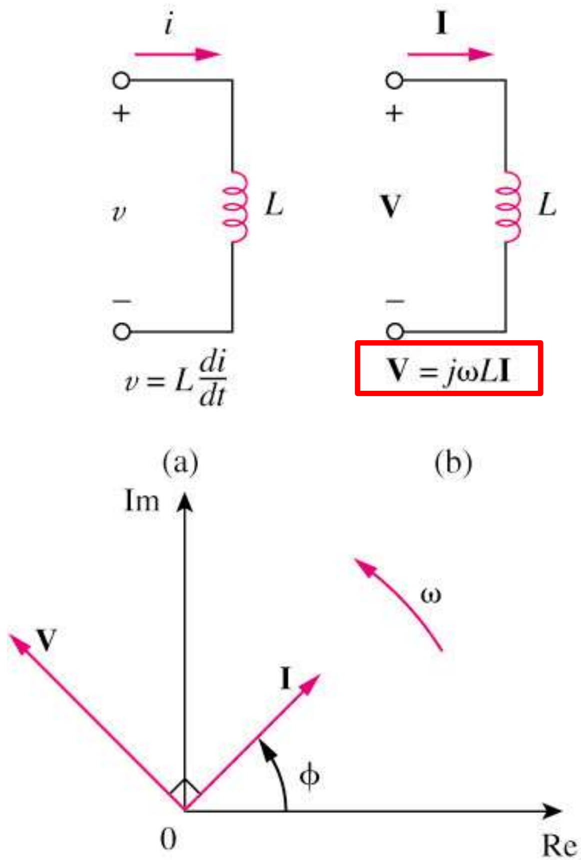
it is applied to two or more sinusoidal signals only if they have the same frequency.

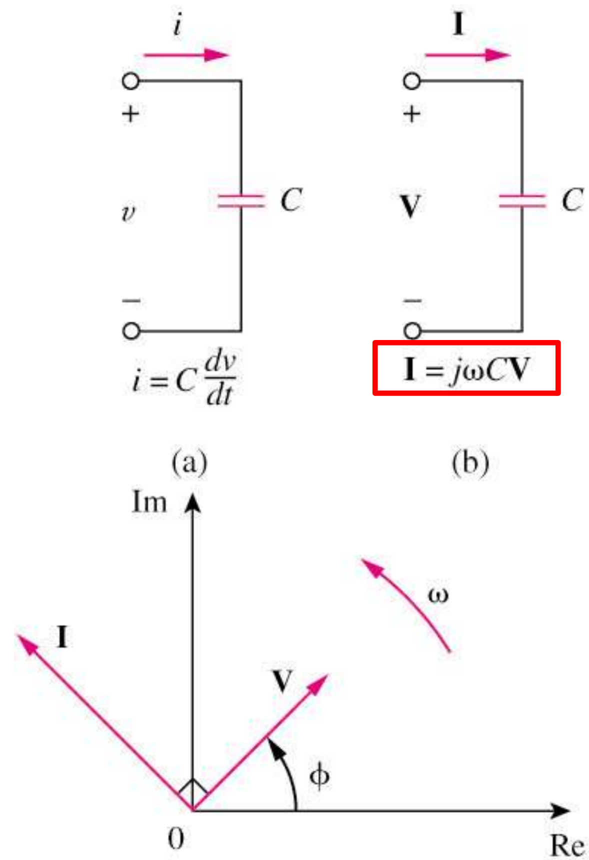
因为所有的相量运算，都隐含着$e^{j\omega t}$，最后变换回时域时，要把$e^{j\omega t}$加回去，如果ω不相同，则没法回到时域。怎么办？单独分析，在时域上相加

9.4 电路元件的相量表达

Resistor: Inductor: Capacitor:







电感：**I** lags **V** 电容：**I** leads **V**

因为一般是施加电压，求电流，所以在描述相位关系时，常用“电流领先/滞后电压”

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Example 9.8

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45^\circ}$ V. Hence,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45^\circ}}{j60 \times 0.1} = \frac{12\underline{/45^\circ}}{6\underline{/90^\circ}} = 2\underline{/-45^\circ}\ \text{A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45^\circ)\ \text{A}$$

9.5 Impedance（阻抗） and Admittance （导纳）

- The impedance $\mathbf{Z}$ of a circuit is the ratio of the phasor voltage $\mathbf{V}$ to the phasor current $\mathbf{I}$, measured in ohms Ω.

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta}$$

where R = Re{$\mathbf{Z}$} is the resistance(电阻) and X = Im{$\mathbf{Z}$} is the reactance(电抗). $X > 0$ for L and $X < 0$ for C.

- The admittance $\mathbf{Y}$ is the reciprocal of impedance, measured in siemens (S).
  - G: conductance, 电导；
  - B: susceptance, 电纳；

$$\boxed{\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}}$$

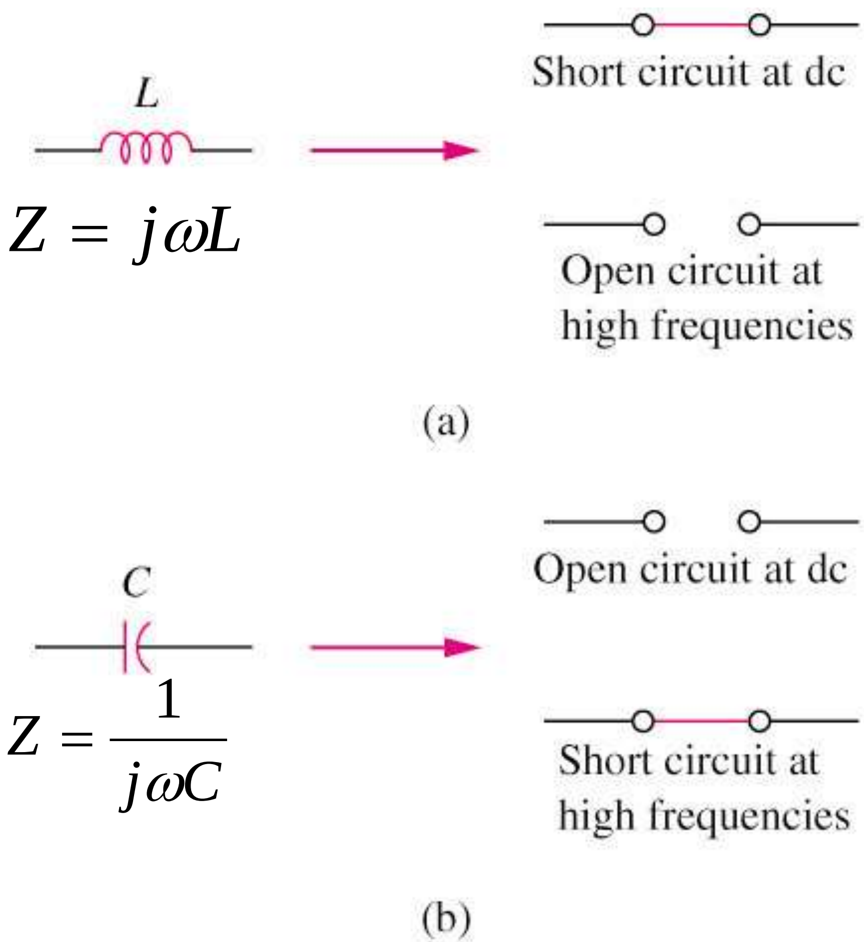
$$\boxed{\mathbf{Y} = G + jB}$$

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

9.5 Impedance and Admittance



$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

Find $v(t)$ and $i(t)$ in the circuit shown in Fig. 9.16.

Example 9.9

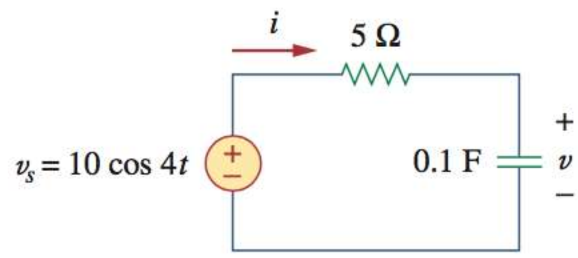


Figure 9.16
For Example 9.9.

Solution:

From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} = 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A} \quad \textbf{(9.9.1)}$$

The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1} = \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V} \quad \textbf{(9.9.2)}$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

电路的相量分析：
① 时域信号→相量信号；
② 电路元件用阻抗或导纳表达，计算复电流、复电压；
③ 相量信号→时域信号

只要是RC组合，电流永远领先电压，领先多少看电阻值

容性负载都是电流leads电压

9.5 频域（相量域）的基尔霍夫定律

- 将电路中的元件（R、L、C、电源）用相量表述后，我们就可以对电路在相量域中进行分析了

- KCL、KVL定律，以及其他的电路定理，在相量域中也同样适用

9.6 Kirchhoff's Laws in the Frequency Domain (1)

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain. 以后，我们会更多地称之为"频域"

- Moreover, the variables to be handled are phasors, which are complex numbers. 频域中的变量是相量（复数）

- All the mathematical operations involved are now in complex domain.所有的数学运算都在复数域中进行

9.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.

- For example:
  a. voltage division 分压
  b. current division 分流
  c. circuit reduction 串并联等电路简化
  d. impedance equivalence 等效阻抗
  e. Y-Δ transformation Y-Δ转换

频域分析中，这些原则都仍适用！

用阻抗统一**RLC**后，串并联原则略有不同

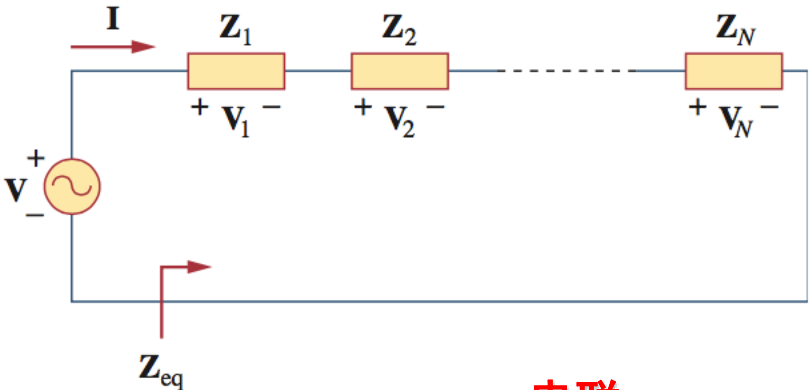


Figure 9.18
N impedances in series.

串联

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

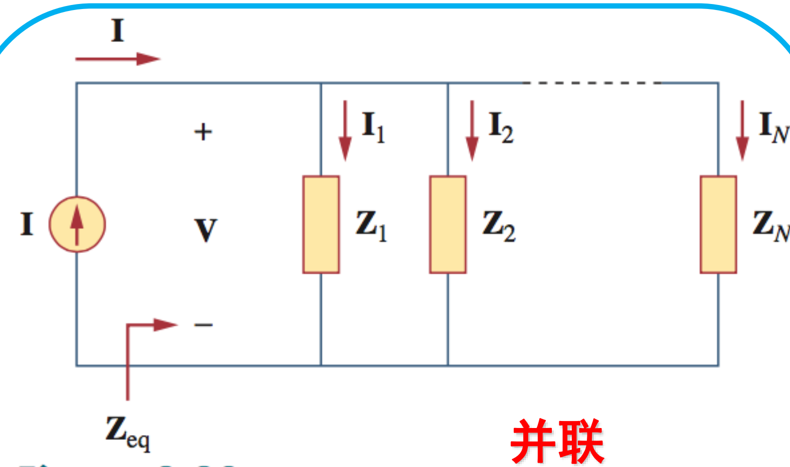


Figure 9.20
N impedances in parallel.

并联

$$\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N$$

在频域中可以把RLC都当作阻抗来分析

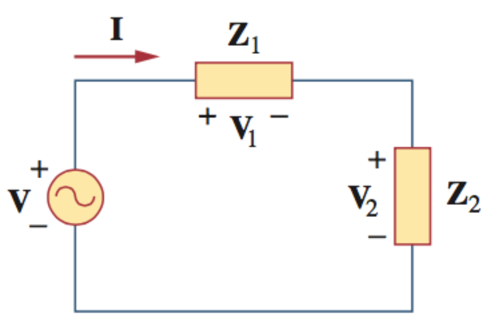


Figure 9.19
Voltage division.

分压

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$

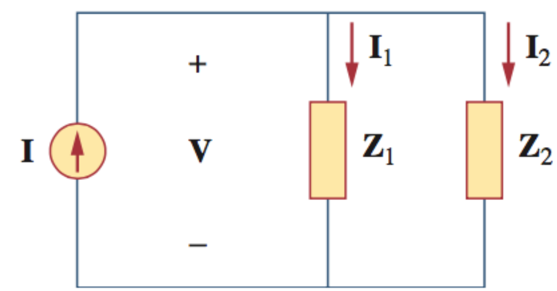


Figure 9.21
Current division.

分流

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$$

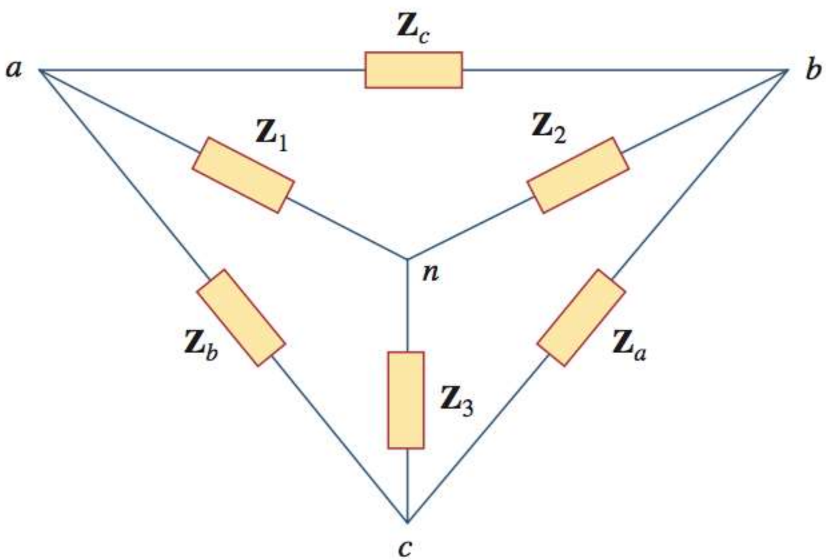


Figure 9.22
Superimposed Y and Δ networks.

Y-Δ Conversion:

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$
$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$
$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y Conversion:

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$
$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$
$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

若阻值相等，则称为平衡，平衡时delta的阻值是wye的3倍

A delta or wye circuit is said to be **balanced** if it has equal impedances in all three branches.

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

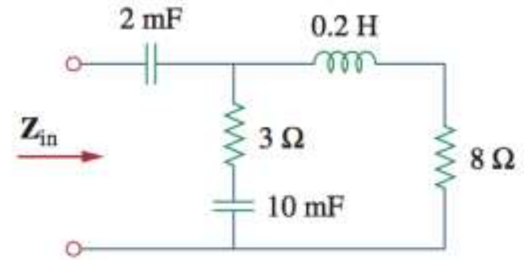


Figure 9.23
For Example 9.10.

频域的阻抗计算

Example 9.11

Determine $v_o(t)$ in the circuit of Fig. 9.25.

频域分压

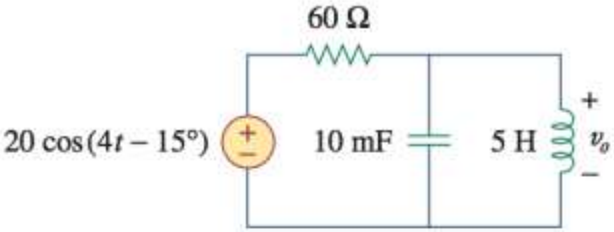


Figure 9.25
For Example 9.11.

Find current $\mathbf{I}$ in the circuit of Fig. 9.28.

Example 9.12

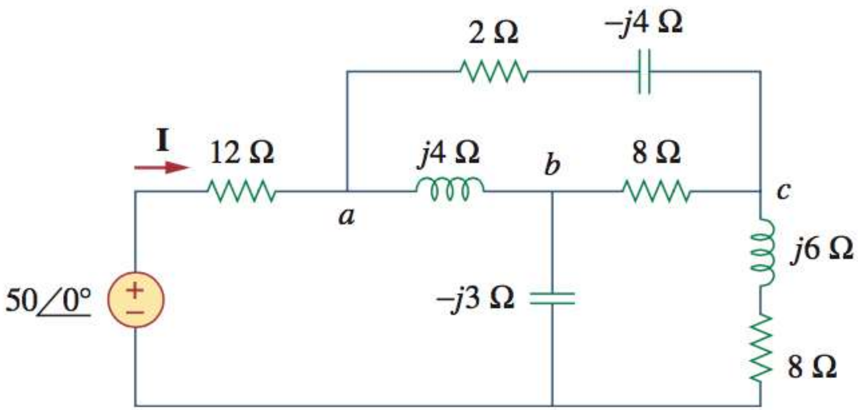


Figure 9.28
For Example 9.12.

频域Y-Δ转换

Solution:
The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.29. We obtain the Y impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned}
\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\
&= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\
&= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}\\
&= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega
\end{aligned}$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$

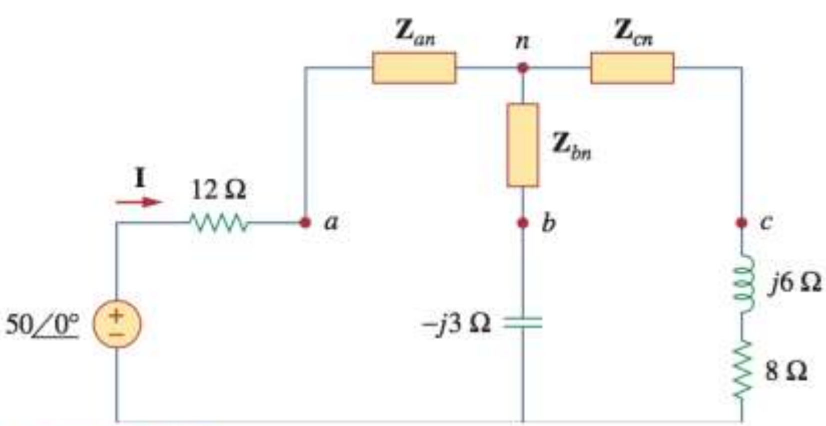


Figure 9.29
The circuit in Fig. 9.28 after delta-to-wye transformation.

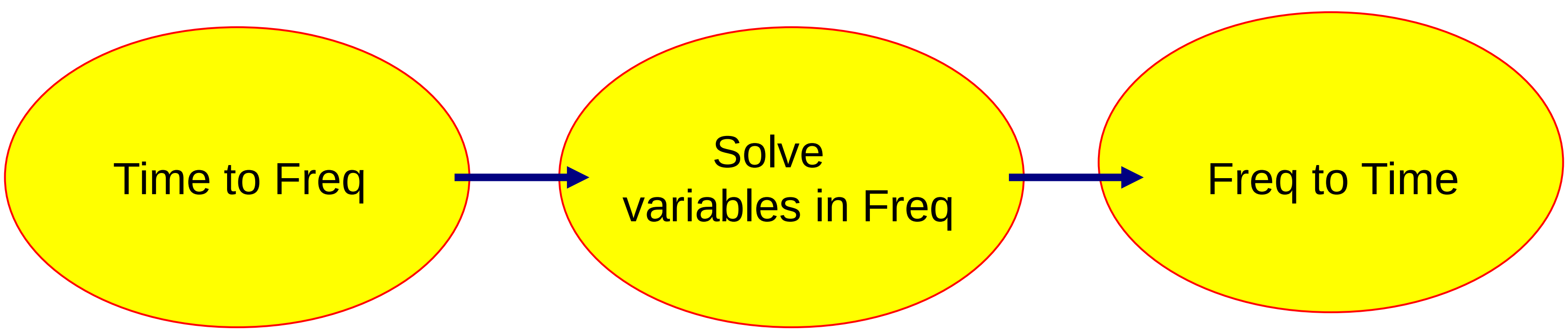
Sinusoidal Steady-State Analysis
Chapter 10 正弦稳态分析

10.1 Basic Approach
10.2 Nodal Analysis 节点电压法@频域
10.3 Mesh Analysis 网孔电流法@频域
10.4 Superposition Theorem 叠加定理@频域
10.5 Source Transformation 电源转换@频域
10.6 Thevenin and Norton Equivalent Circuits 戴维南定理、诺顿定理@频域

10.1 Basic Approach (1)

Steps to Analyze AC Circuits:

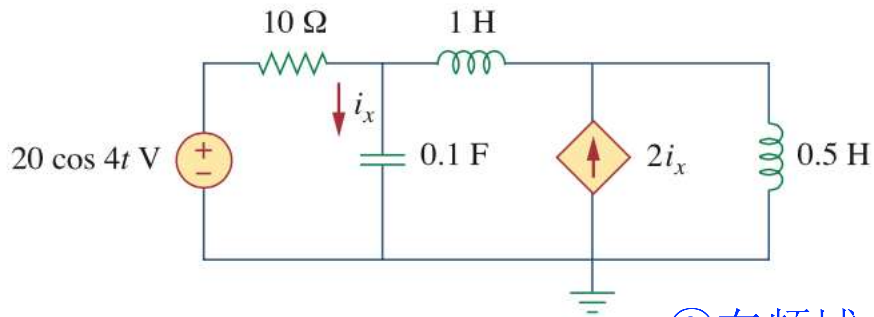
1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.



Example 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

频域节点电压法



Solution:

We first convert the circuit to the frequency domain:

$$20\cos 4t \quad\Rightarrow\quad 20\underline{/0^\circ}, \qquad \boxed{\omega = 4 \text{ rad/s}}$$

$$1\text{ H} \quad\Rightarrow\quad j\omega L = j4$$

$$0.5\text{ H} \quad\Rightarrow\quad j\omega L = j2$$

$$0.1\text{ F} \quad\Rightarrow\quad \frac{1}{j\omega C} = -j2.5$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.

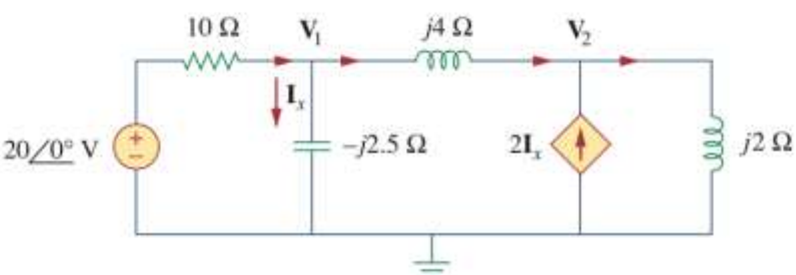


Figure 10.2
Frequency domain equivalent of the circuit in Fig. 10.1.

①将元件写成相量形式

②在频域中分析

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \qquad \textbf{(10.1.1)}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \qquad \textbf{(10.1.2)}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

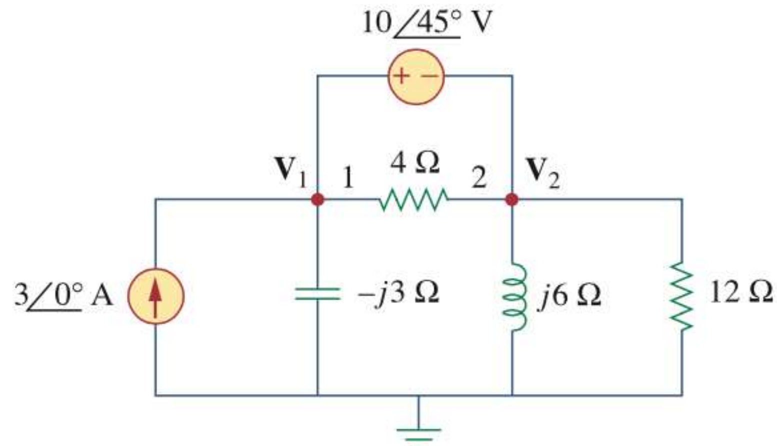
③写回时域表达式

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ)\text{ A}$$

Example 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.



频域节点电压法~supernode

Solution:

Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \tag{10.2.1}$$

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} \tag{10.2.2}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135^\circ} = (1 + j2)\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 31.41\underline{/-87.18^\circ} \text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} = 25.78\underline{/-70.48^\circ} \text{ V}$$

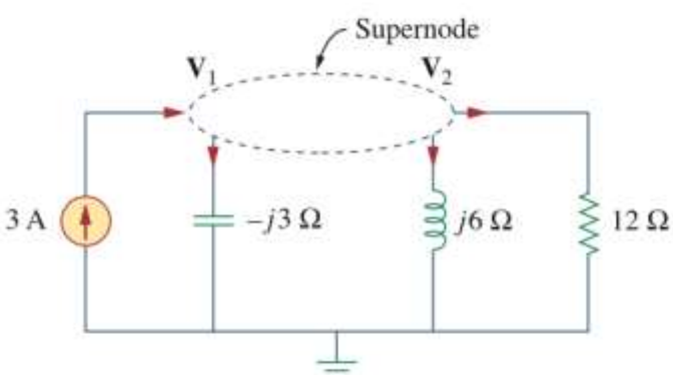
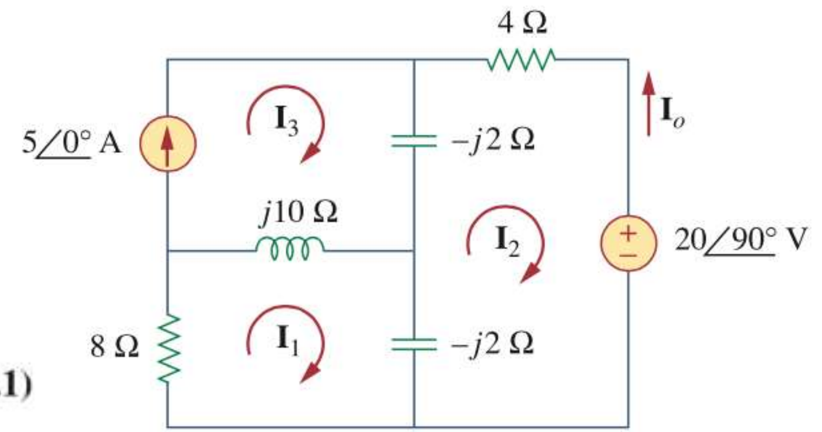


Figure 10.5
A supernode in the circuit of Fig. 10.4.

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Example 10.3

频域网孔电流法



Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78^\circ} \text{ A}$$

Example 10.4

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

频域网孔电流法~supermesh

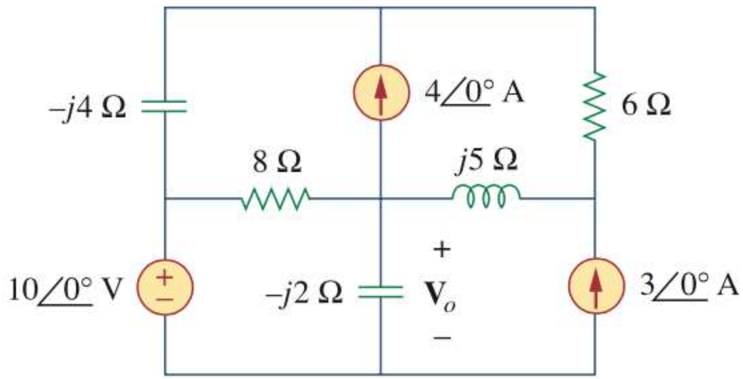


Figure 10.9

Solution:

As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \tag{10.4.1}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \tag{10.4.2}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \tag{10.4.3}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \tag{10.4.4}$$

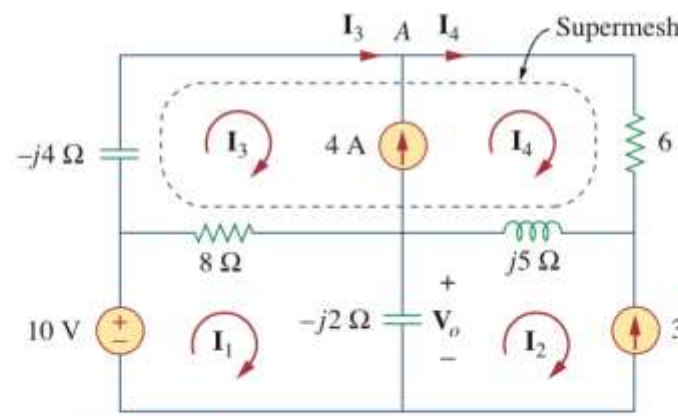


Figure 10.10
Analysis of the circuit in Fig. 10.9.

$$\begin{bmatrix} 8 - j2 & j2 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ -8 & -j5 & 8 - j4 & 6 + j5 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 0 \\ 4 \end{bmatrix}$$

10.4 Superposition Theorem (1)

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.
- 若电路中有不同的频率，只能时域叠加，不能相量(频域)叠加，**Why**？

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

Example 10.6

Find v_o of the circuit of Fig. 10.13 using the superposition theorem.

频域叠加定理：
注意，电路中若有不同的频率，
只能时域叠加，不能相量叠加

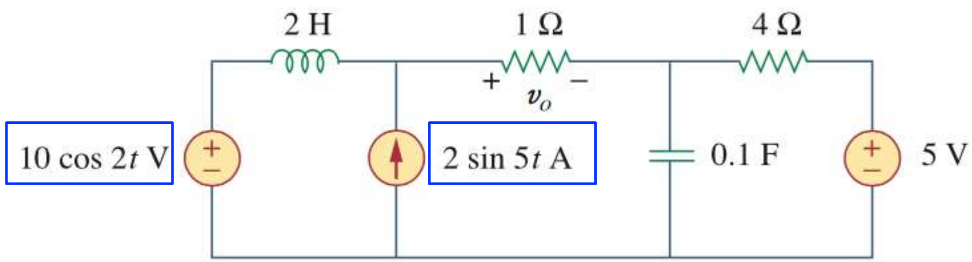


Figure 10.13
For Example 10.6.

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V}$$

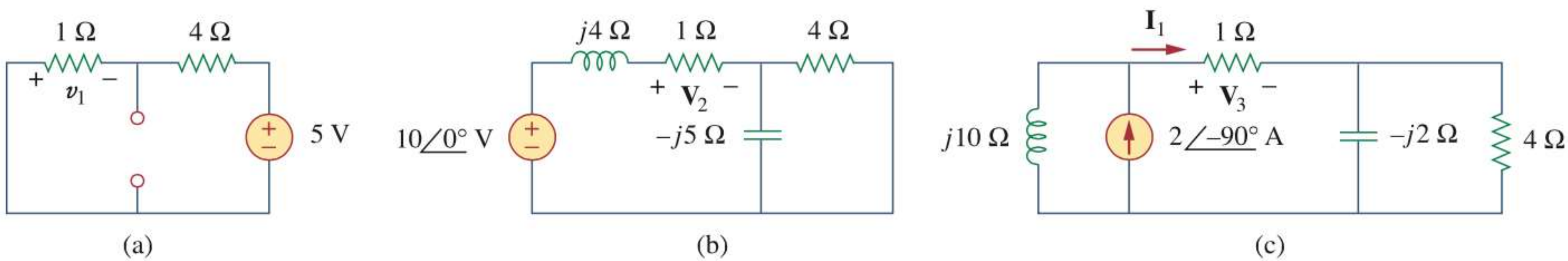


Figure 10.14
Solution of Example 10.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

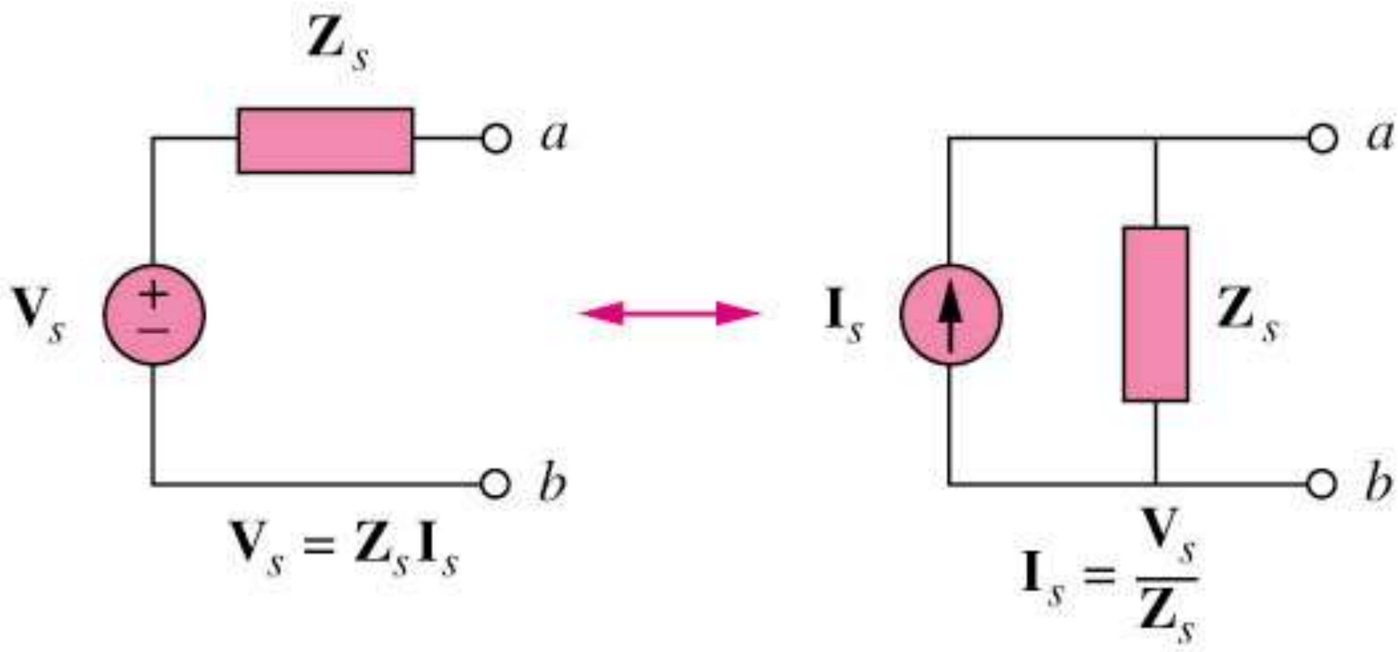
$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ} \text{ V}$$

$$v_2 = 2.498 \cos(2t - 30.79^\circ)$$

$$v_3 = 2.33 \cos(5t - 80^\circ) = 2.33 \sin(5t + 10^\circ) \text{ V}$$

$$v_o(t) = -1 + 2.498 \cos(2t - 30.79^\circ) + 2.33 \sin(5t + 10^\circ) \text{ V}$$

10.5 Source Transformation (1)



Example 10.7

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.

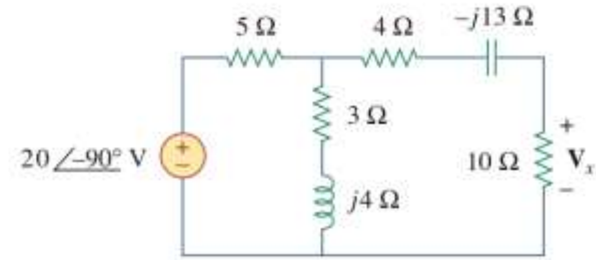


Figure 10.17
For Example 10.7.

Solution:

We transform the voltage source to a current source and obtain the circuit in Fig. 10.18(a), where

$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$

The parallel combination of 5-Ω resistance and $(3 + j4)$ impedance gives

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

频域电源转换定理

Converting the current source to a voltage source yields the circuit in Fig. 10.18(b), where

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

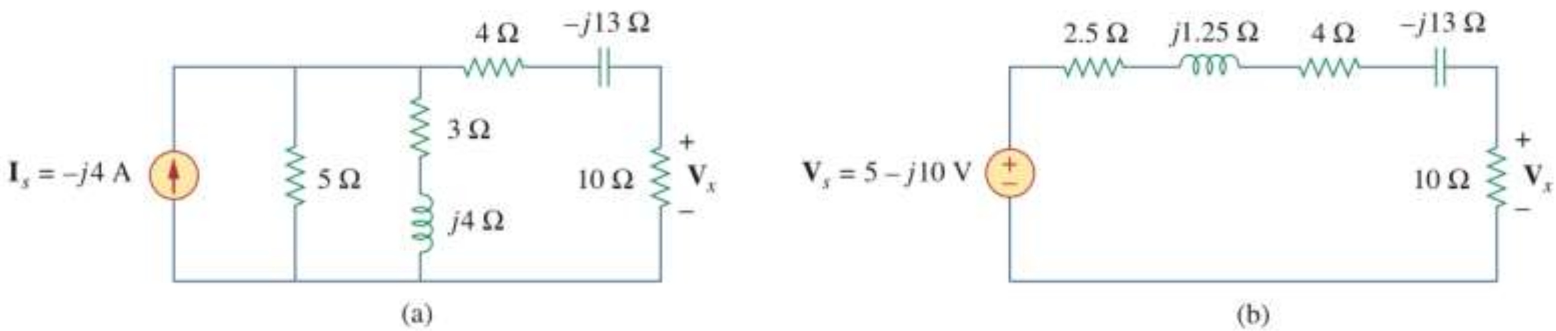
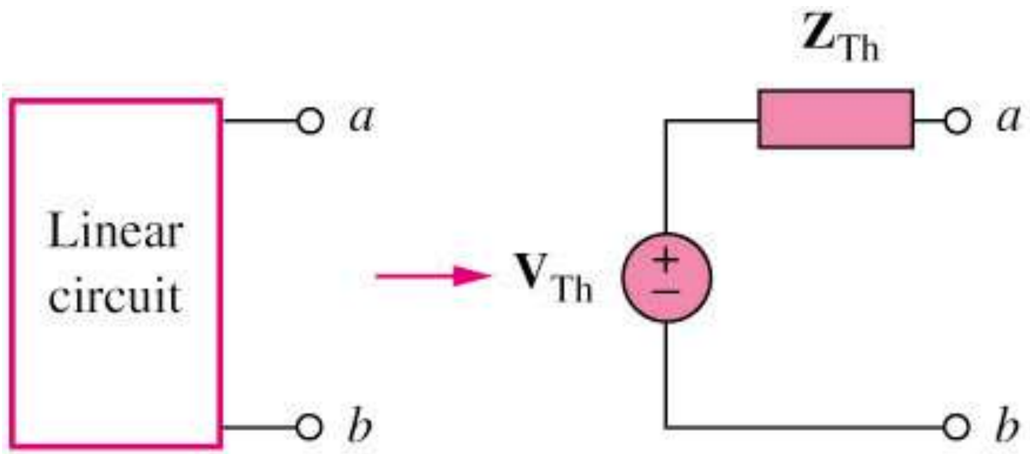


Figure 10.18
Solution of the circuit in Fig. 10.17.

By voltage division,

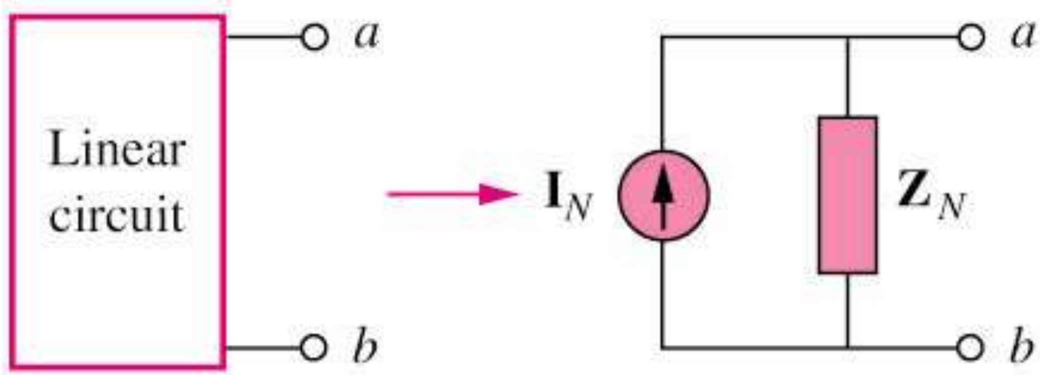
$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle -28^\circ \text{ V}$$

10.6 Thevenin and Norton Equivalent Circuits (1)



Thevenin transform

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_N$$



Norton transform

Example 10.8

Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.22.

频域戴维南定理

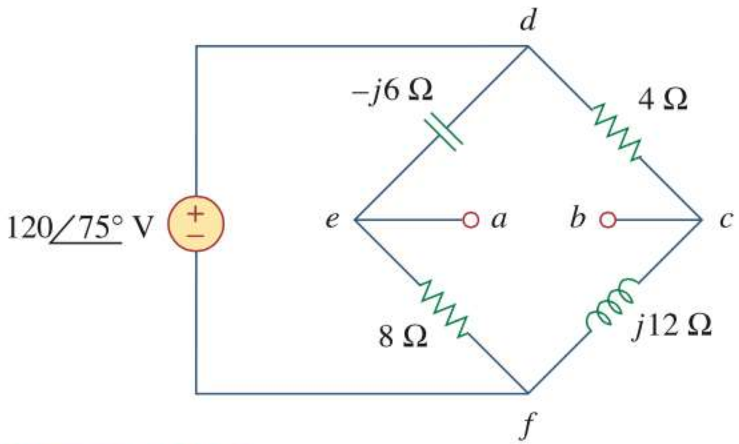


Figure 10.22
For Example 10.8.

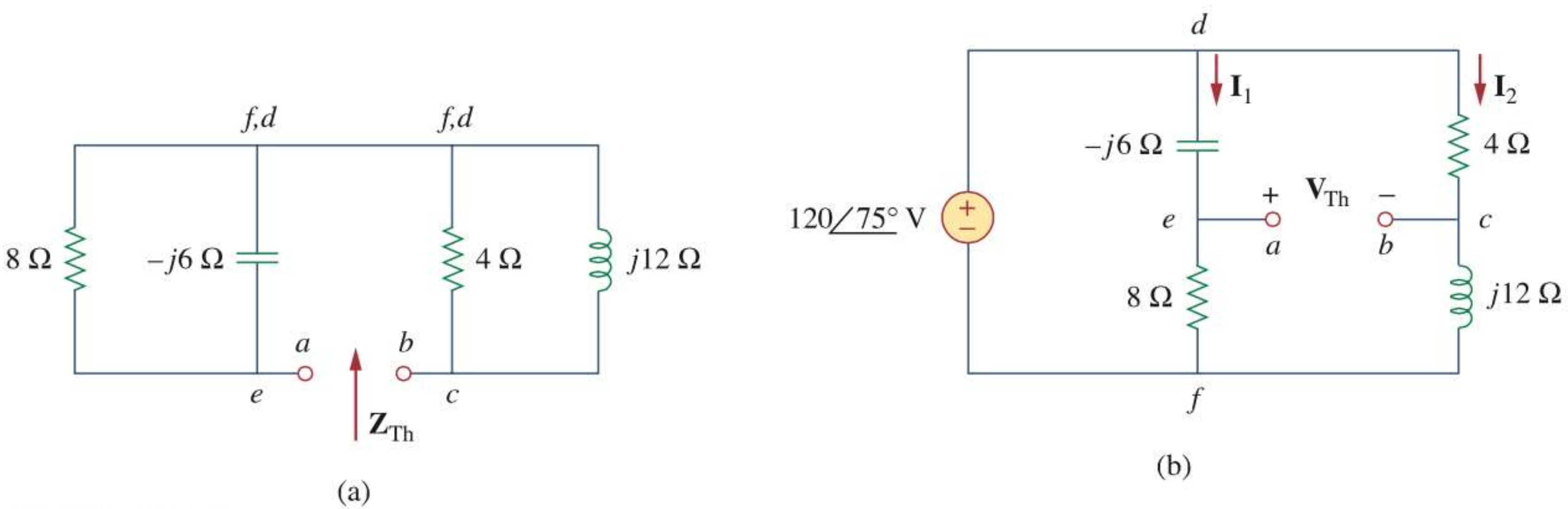


Figure 10.23
Solution of the circuit in Fig. 10.22: (a) finding $\mathbf{Z}_{\text{Th}}$, (b) finding $\mathbf{V}_{\text{Th}}$.

Example 10.9

Find the Thevenin equivalent of the circuit in Fig. 10.25 as seen from terminals *a*-*b*.

频域戴维南定理~含受控源

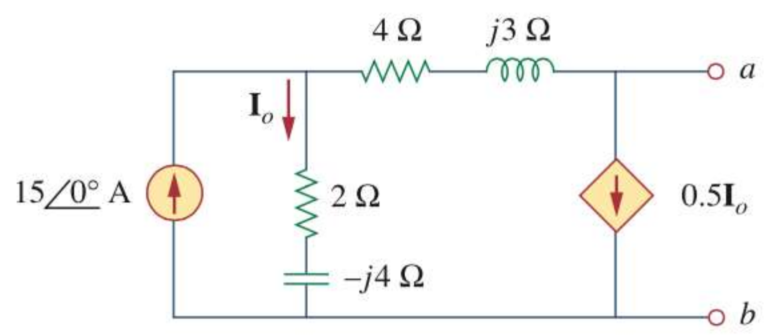


Figure 10.25
For Example 10.9.

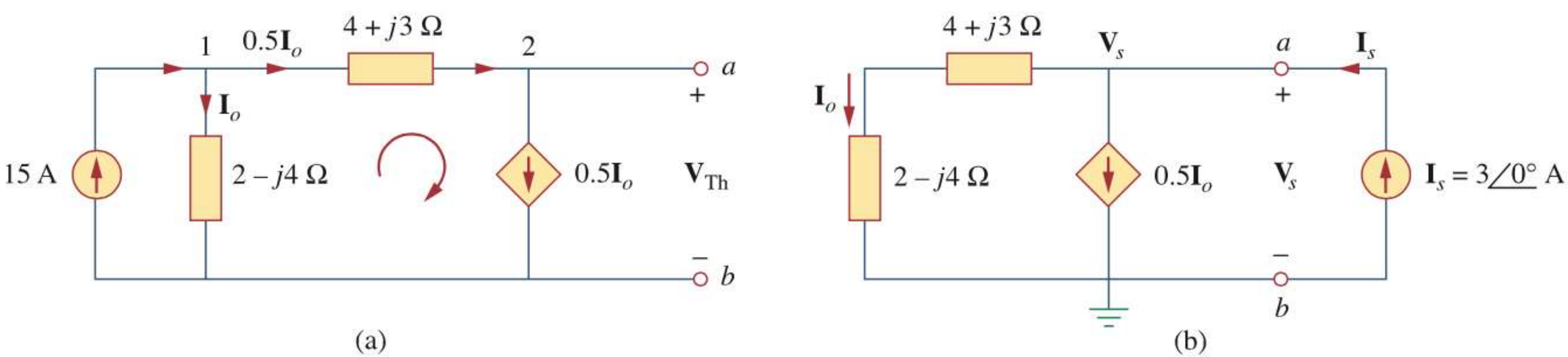


Figure 10.26
Solution of the problem in Fig. 10.25: (a) finding $\mathbf{V}_{Th}$, (b) finding $\mathbf{Z}_{Th}$.

Obtain current $\mathbf{I}_o$ in Fig. 10.28 using Norton's theorem.

Example 10.10

频域诺顿定理

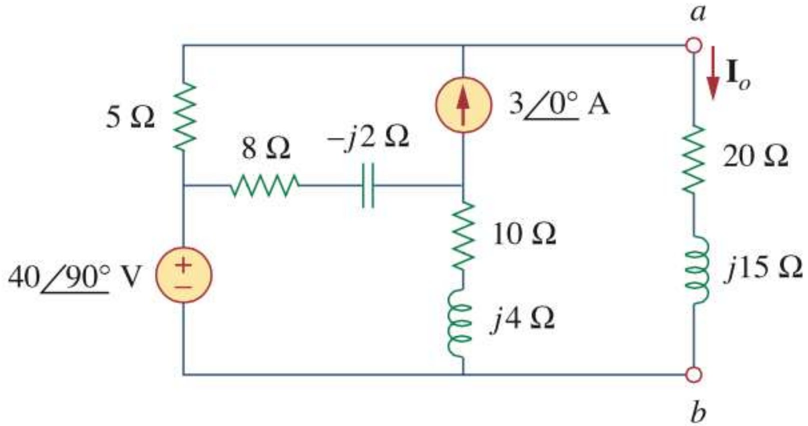


Figure 10.28
For Example 10.10.

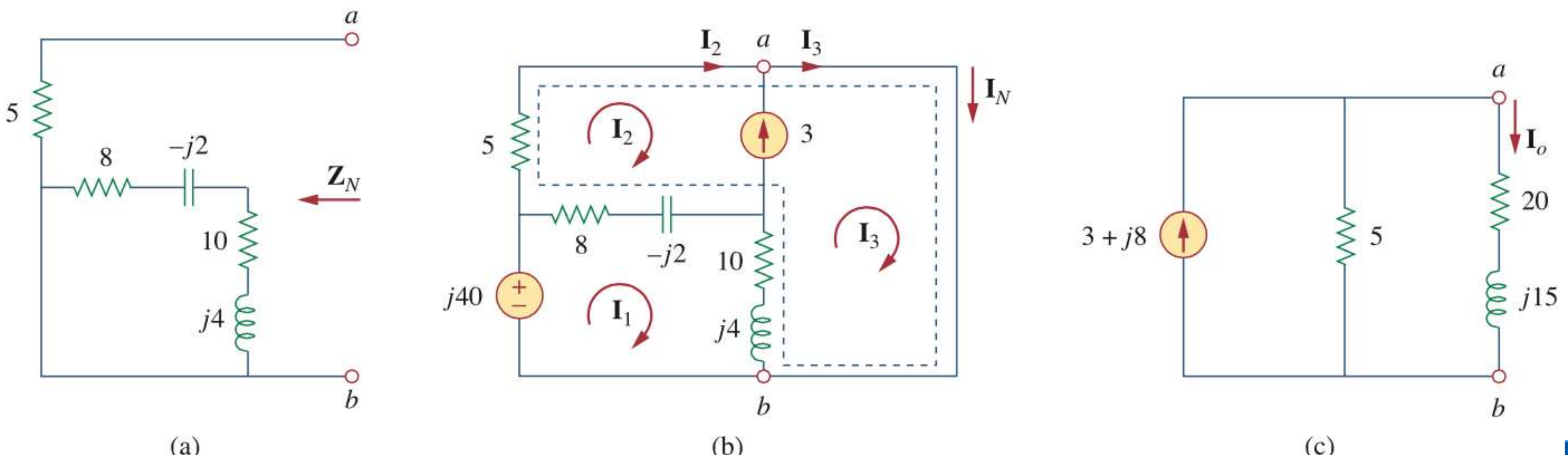


Figure 10.29
Solution of the circuit in Fig. 10.28: (a) finding $\mathbf{Z}_N$, (b) finding $\mathbf{V}_N$, (c) calculating $\mathbf{I}_o$.

应用——移向电路

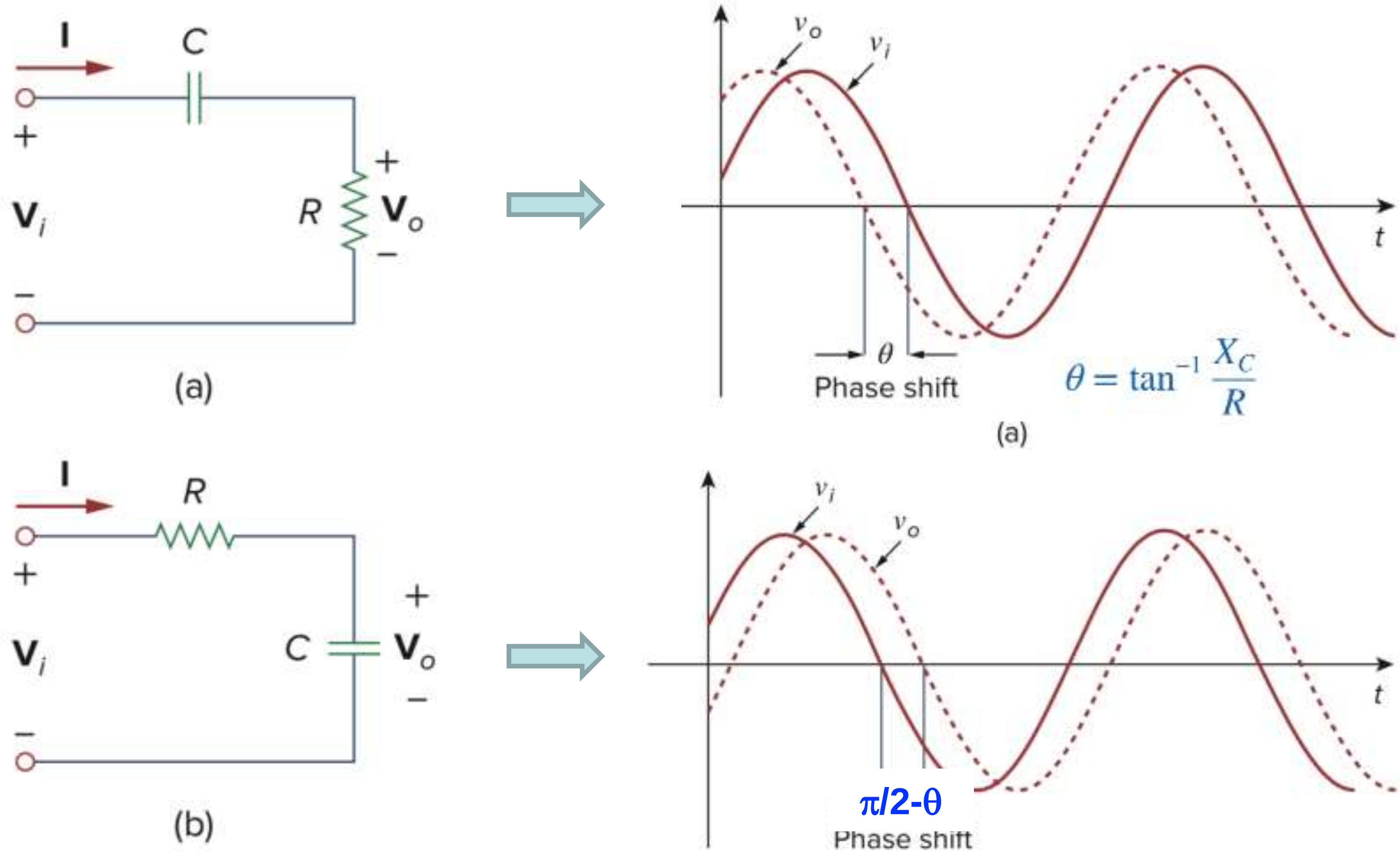
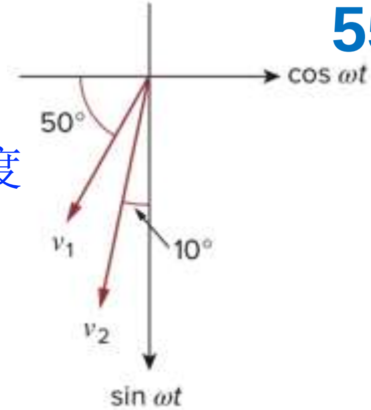


Figure 9.31 先分析 I 与 V_i 的关系，再分析 V_o 与 I 的关系，得出 V_o 与 V_i 的关系

Series *RC* shift circuits: (a) leading output, (b) lagging output.

小结

v_2领先$v_1$30度

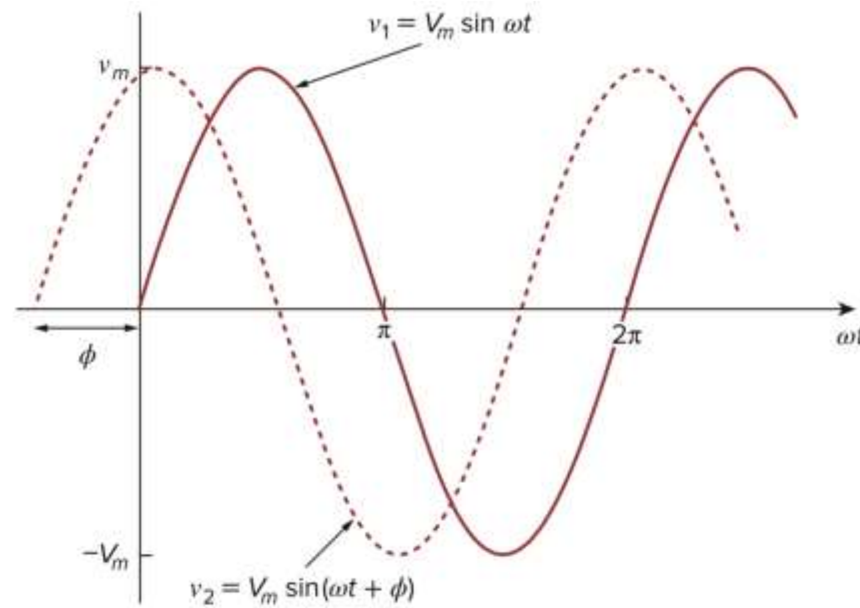


- 正弦信号的基本参数： $v(t) = V_m \sin(\omega t + \phi)$
  - 振幅（V_m）、角频率（ω）、幅角（ωt + ϕ）、相位（ϕ）、周期、频率、有效值（V_m/sqrt(2)）、峰峰值（2V_m）、平均值

$$\boxed{T = \frac{2\pi}{\omega}} \qquad \boxed{f = \frac{1}{T}}$$

- 相位关系。
  - 不同正弦信号的相位比较

1. **从时域图中看，靠左的曲线相位领先**
2. **从表达式看，幅角(ωt + ϕ) 比 (ωt) 领先 ϕ**
3. **从图示法或复平面看，逆时针领先**



  - 电路分析时，往往需要分析“电流_领先/滞后_电压____相位”。
    容性负载电流领先；感性负载电流滞后；

小结

- 相量
  - 从时域信号写出相量：先统一表示成cos形式，再拿出振幅和相位，构成一个复数
  - 从相量复原出时域信号：

$$v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})$$

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \underline{/\phi}$$

(Time-domain representation) (Phasor-domain representation)

  - 时域微分、积分在相量域中的对应关系

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega\mathbf{V}$$

(Time domain) (Phasor domain)

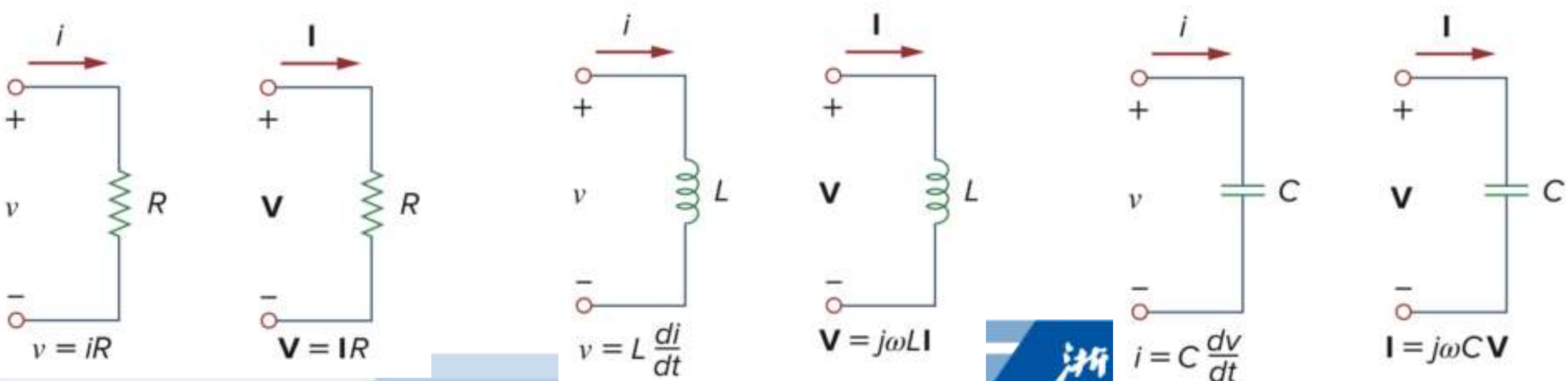
$$\int v\,dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega}$$

(Time domain) (Phasor domain)

小结

- 相量
  - 元件的“电压电流约束关系”在相量域中的表述；阻抗和导纳的定义

Element	Time domain	Frequency domain	Impedance	Admittance
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$



小结

- 电路的频域分析法

  ①将元件写成相量形式

  ②在频域中分析

  ③写回时域表达式

- 基尔霍夫定律，以及其他电路定理，在频域中同样适用
  - 要注意的是，若电路中有不同的频率，只能时域叠加，不能相量（频域）叠加

作业

Practice Problem 9.4

时域信号→相量信号：
统一表示成cos形式，再用幅度和相位构成相量（复数）

Express these sinusoids as phasors:

(a) $v = -14 \sin(5t - 22^\circ)$ V
(b) $i = -8 \cos(16t + 15^\circ)$ A

Answer: (a) $\mathbf{V} = 14\underline{/68^\circ}$ V, (b) $\mathbf{I} = 8\underline{/-165^\circ}$ A.

Practice Problem 9.5

Find the sinusoids corresponding to these phasors:

(a) $\mathbf{V} = -25\angle 40^\circ$ V
(b) $\mathbf{I} = j(12 - j5)$ A

Answer: (a) $v(t) = 25\cos(\omega t - 140^\circ)$ V or $25\cos(\omega t + 220^\circ)$ V,
(b) $i(t) = 13\cos(\omega t + 67.38^\circ)$ A.

相量信号→时域信号：

- 先将相量表示成极坐标形式，再写成cos形式；
- 或 $v(t) = \text{Re}(\mathbf{V}e^{j\omega t})$

Practice Problem 10.1

Using nodal analysis, find v_1 and v_2 in the circuit of Fig. 10.3.

电路的频域分析法：

① 时域信号→频域信号；

② 电路元件用阻抗表达，在频域中分析电路；

③ 频域信号→时域信号

频域节点电压法

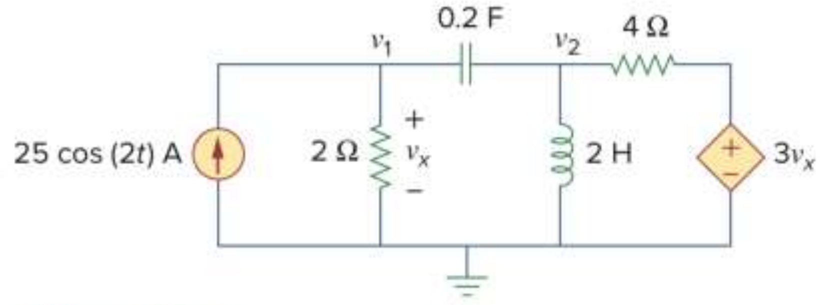


Figure 10.3
For Practice Prob. 10.1.

Answer: $v_1(t) = 28.31\cos(2t + 60.01^\circ)$ V,
$v_2(t) = 82.56\cos(2t + 57.12^\circ)$ V.

Practice Problem 10.4

Calculate current $\mathbf{I}_o$ in the circuit of Fig. 10.11.

Answer: $6.089\angle 5.94^\circ$ A.

频域网孔电流法~supermesh

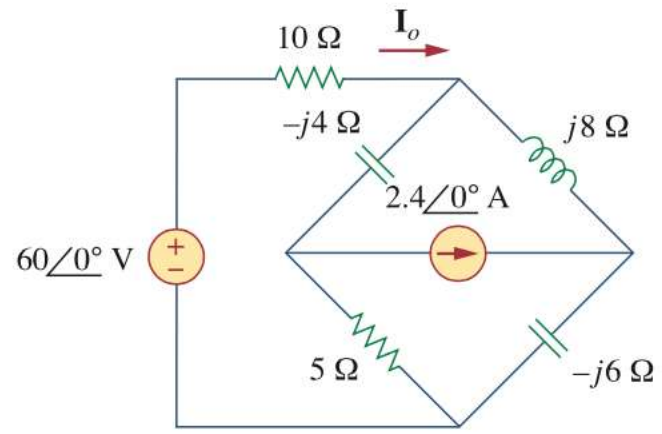


Figure 10.11
For Practice Prob. 10.4.

Practice Problem 10.6

Calculate v_o in the circuit of Fig. 10.15 using the superposition theorem.

频域叠加定理：

注意，电路中若有不同的频率，
只能时域叠加，不能相量叠加

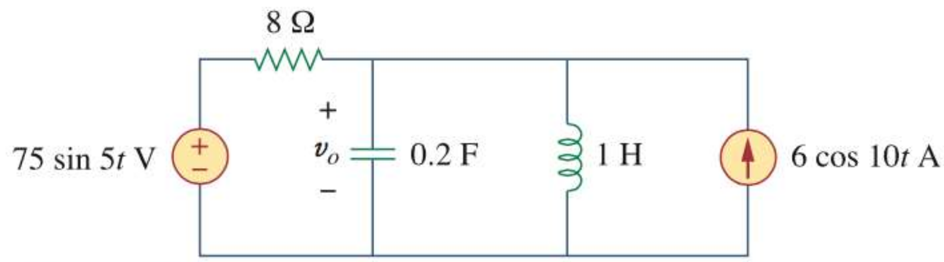


Figure 10.15
For Practice Prob. 10.6.

Answer: $11.577 \sin(5t - 81.12°) + 3.154 \cos(10t - 86.24°)$ V.

Practice Problem 10.9

Determine the Thevenin equivalent of the circuit in Fig. 10.27 as seen from the terminals *a*-*b*.

Answer: $\mathbf{Z}_{Th} = 4.473\underline{/-7.64^\circ}\ \Omega$, $\mathbf{V}_{Th} = 11.763\ \underline{/72.9^\circ}$ volts.

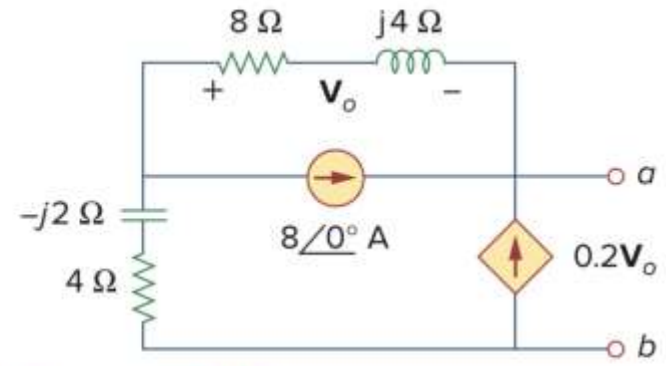


Figure 10.27
For Practice Prob. 10.9.

频域戴维南定理

Vth对不上！！！Zth有偏差
如果网孔电流法不能用，改用节点电流法！！！