调制性质讲义

① 调制性质

若 $x_1(t) \xrightarrow{F} X_1(jw)$

$x_2(t) \xrightarrow{F} X_2(jw)$

则 $x_1(t)x_2(t) \xrightarrow{F} \frac{1}{2\pi} X_1(jw) * X_2(jw)$

证明：设 $X(jw) = \frac{1}{2\pi} X_1(jw) * X_2(jw)$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(j\tau)\, X_2(j(w-\tau))\, d\tau$$

则 $x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(jw) e^{jwt}\, dw$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1(j\tau) X_2(j(w-\tau))\, d\tau\right] e^{jwt}\, dw$$

$$= \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(j\tau) e^{j\tau t}\, d\tau\right] \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X_2(j(w-\tau))\, e^{j(w-\tau)t}\right.$$

设 $\begin{cases} \tau' = \tau \\ w' = w - \tau \end{cases}$，则用二元积分换元得

$$= \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(j\tau') e^{j\tau' t}\, d\tau'\right] \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X_2(jw') e^{jw't}\right.$$

$$\cdot \left|\begin{matrix} \frac{\partial \tau}{\partial \tau'} & \frac{\partial w}{\partial \tau'} \\ \frac{\partial \tau}{\partial w'} & \frac{\partial w}{\partial w'} \end{matrix}\right|$$

$$= x_1(t)x_2(t)$$

② 应用：P198 双边带正弦载波幅度调制与解调

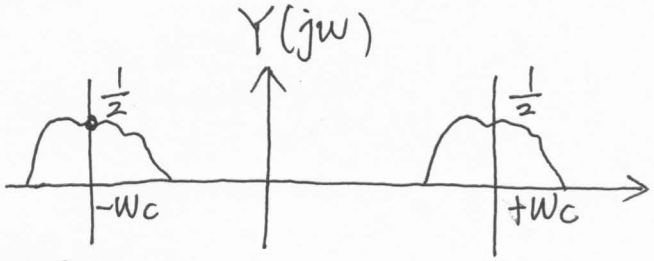
调制：

$$x(t) \xrightarrow{\text{调制}} y(t) = x(t)\cos(\underline{w_c t})$$

载波 (Carrier)

若

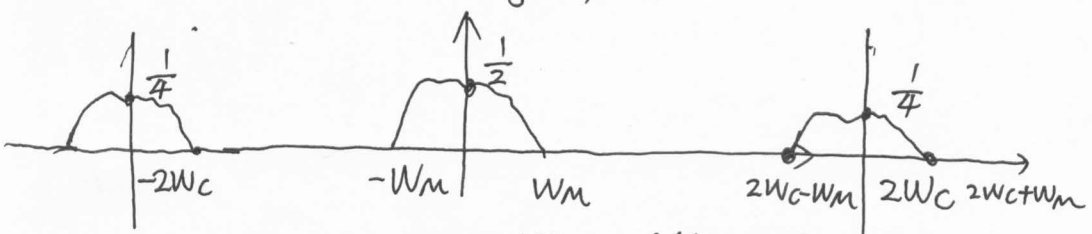
则有：



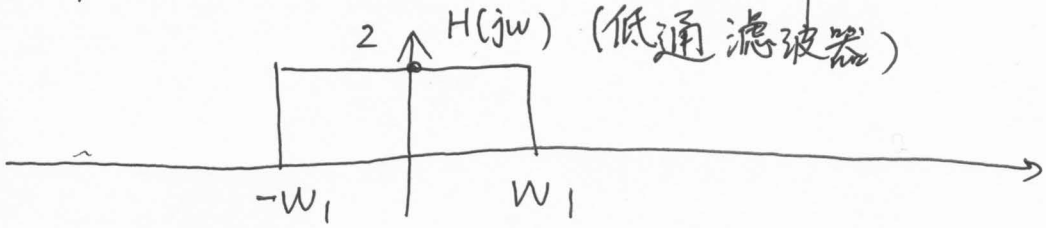
解调：

$$w(t) = y(t)\cos(w_c t)$$

则有：$W(jw)$



$(w_c > w_m)$



$(w_m < w_1 < 2w_c - w$

则有：$X(jw) = W(jw)H(jw)$

即：$x(t) = w(t) * h(t)$

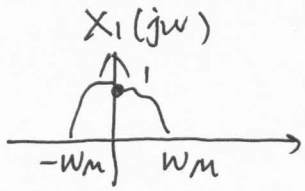
$$= \left[x(t)\cos^2(w_c t)\right] * \left[\frac{2\sin(w_1 t)}{\pi t}\right]$$

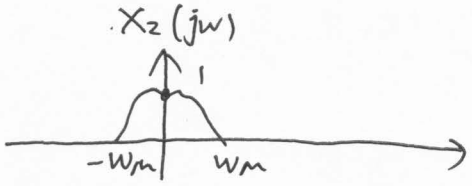
③ 另一个角度看调制解调（时域观点）

$$w(t) = x(t)\cos^2(w_c t)$$

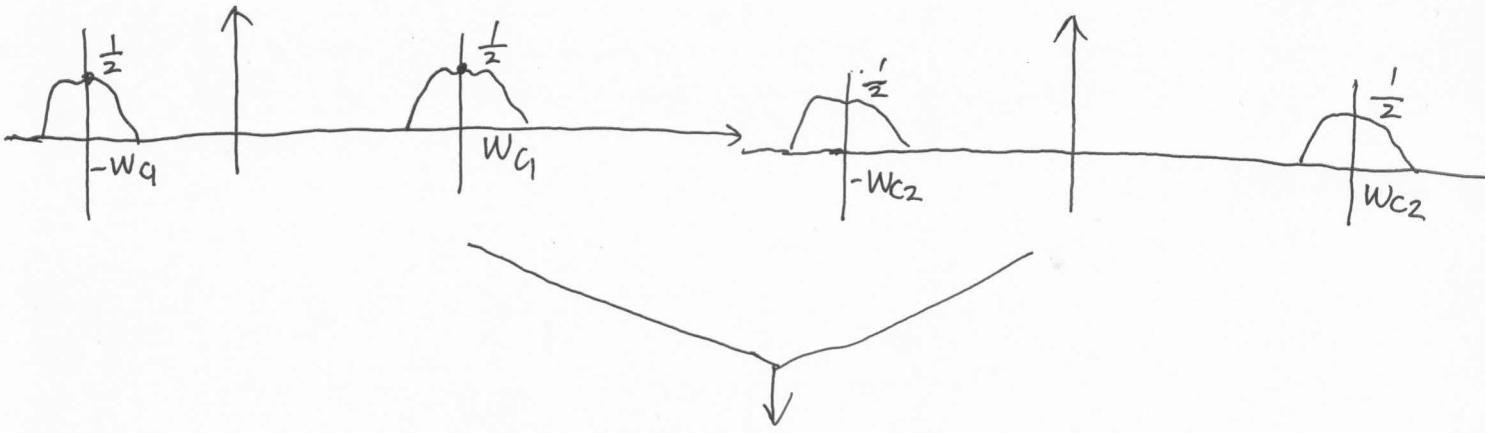
$$= \underbrace{\frac{1}{2}x(t)}_{\text{保留项}} + \underbrace{\frac{1}{2}x(t)\cos(2w_c t)}_{\text{低通滤波滤掉了}}$$

④ 多个信号的调制与解调



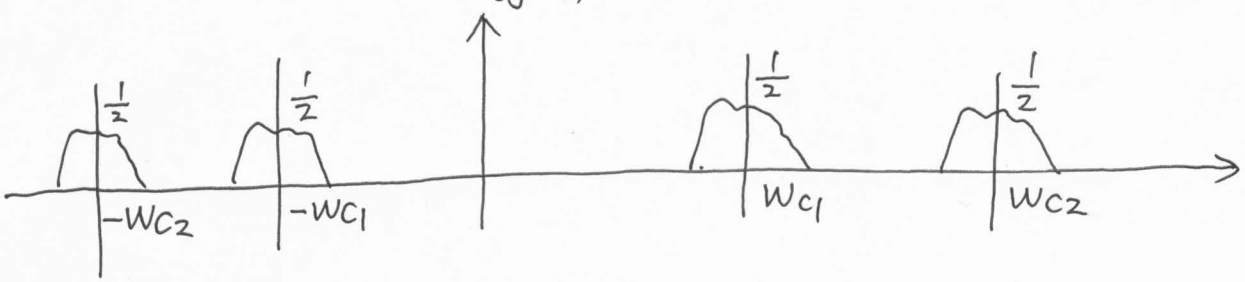


$Y_1(jw)$ $(y_1(t) = x_1(t)\cos(w_{c1}t))$ $Y_2(jw)$ $(y_2(t) = x_2(t)\cos(w_{c2}t))$



$$x(t) = x_1(t)\cos(w_{c1}t) + x_2(t)\cos w_{c2}(t)$$

$X(jw)$



ⓐ 恢复 $X_1(t)$

$$w_1(t) = x(t)\cos(w_{c1}t)$$

$W_1(jw)$

低通滤波后获得 $x_1(t)$

ⓑ 恢复 $x_2(t)$

$$w_2(t) = x(t)\cos(w_{c2}t)$$


W2(jw)
1/T
1/T
1/T
1/2
1/T
1/T
1/T
-2Wc2
-(Wc2+Wc1)
-(Wc2-Wc1)
Wc2-Wc1
Wc2+Wc1
2Wc2

低通滤波后获得 $x_2(t)$