Problem Set 3

Problem 1 Solution

(a) $y[n]=x[n]*h[n]=\sum\limits_{k=-\infty}^{+\infty}x[k]h[n-k]=-h[n+3]+2h[n+1]+h[n]+h[n-1]$

- if $n+3\le -3$, i.e. $n\le -6$, $y[n]=0$
- if $n-1\ge 4$, i.e. $n\ge 5$, $y[n]=0$
- $y[-5]=-h[-2]+2h[-4]+h[-5]+h[-6]=-1$
- $y[-4]=-h[-1]+2h[-3]+h[-4]+h[-5]=1$
- $y[-3]=-h[0]+2h[-2]+h[-3]+h[-4]=2$
- $y[-2]=-h[1]+2h[-1]+h[-2]+h[-3]=-1$
- $y[-1]=-h[2]+2h[0]+h[-1]+h[-2]=-1$
- $y[0]=-h[3]+2h[1]+h[0]+h[-1]=-2$
- $y[1]=-h[4]+2h[2]+h[1]+h[0]=2$
- $y[2]=-h[5]+2h[3]+h[2]+h[1]=3$
- $y[3]=-h[6]+2h[4]+h[3]+h[2]=2$
- $y[4]=-h[7]+2h[5]+h[4]+h[3]=1$

Overall, we can conclude that

$$y[n]=\begin{cases}-2, & n=0\\ -1, & n=-5,-2,-1\\ 0, & n\le -6\ or\ n\ge 5\\ 1, & n=-4,4\\ 2, & n=-3,1,3\\ 3, & n=2\end{cases}$$

(b) $x[n]=u[n+4]-n[n-1]=\begin{cases}1, & -4\le n\le 0\\ 0, & otherwise\end{cases}$, so

$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{+\infty}x[k]h[n-k]=\sum_{k=-4}^{0}h[n-k]=2^n\left(u[2-n]+2u[1-n]+4u[-n]+8u[-n-1]+16u[-n-2]\right)$$

- if $-n-2\ge 0$, i.e. $n\le -2$, $y[n]=31\cdot 2^n$
- if $2-n<0$, i.e. $n\ge 3$, $y[n]=0$
- if $n=-1$, $y[n]=15\cdot 2^n$
- if $n=0$, $y[n]=7\cdot 2^n$
- if $n=1$, $y[n]=3\cdot 2^n$
- if $n=2$, $y[n]=2^n$

Overall, we can conclude that

$$y[n]=\begin{cases}0, & n\ge 3\\ 8-2^n, & -2\le n\le 2\\ 31\cdot 2^n, & n<-2\end{cases}$$

Problem 2 Solution

(a) $y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}e^{-\tau}u(\tau+1)e^{2t-2\tau}u(\tau-t)d\tau$

- if $t>-1$, $y(t)=\int_{t}^{+\infty}e^{-\tau}e^{2t-2\tau}d\tau=\int_{t}^{+\infty}e^{2t-3\tau}d\tau=\frac{1}{3}e^{-t}$
- if $t\le -1$, $y(t)=\int_{-1}^{+\infty}e^{-\tau}e^{2t-2\tau}d\tau=\int_{-1}^{+\infty}e^{2t-3\tau}d\tau=\dfrac{e^{2t+3}}{3}$

Overall, we can conclude that

$$y(t)=\begin{cases}\frac{1}{3}e^{-t}, & t>-1\\ \frac{e^{2t+3}}{3}, & t\le -1\end{cases}$$

(b) $y(t)=x(t)*h(t)=\int_{-1}^{1}x(\tau)h(t-\tau)d\tau$

- if $t+1\le 0$, i.e. $t\le -1$, $y(t)=0$
- if $t-1>4$, i.e. $t>5$, $y(t)=0$

- if $0 < t+1 \leq 1$, i.e. $-1 < t \leq 0$, $y(t) = \int_{-1}^{t} x(\tau)h(t-\tau)d\tau = \int_{-1}^{t} x(\tau)d\tau = (t+1)^2/2$
- if $0 < t \leq 1$, $y(t) = \int_{-1}^{t} x(\tau)d\tau = -\frac{1}{2}t^2 + t + \frac{1}{2}$
- if $1 < t < 2$, $y(t) = \int_{t-1}^{1} x(\tau)d\tau = \frac{t(t-1)}{2}$
- if $2 \leq t < 3$, $y(t) = \int_{t-1}^{1} x(\tau)d\tau + x(t-3) = t-2$
- if $3 \leq t \leq 4$, $y(t) = x(t-3) + x(t-4) = 1$
- if $4 < t \leq 5$, $y(t) = x(t-4) = 5-t$

Overall, we can conclude that

$$y(t) = \begin{cases} 0, & t \leq -1 \; or \; t > 5 \\ \frac{(t+1)^2}{2}, & -1 < t \leq 0 \\ -\frac{1}{2}t^2 + t + \frac{1}{2}, & 0 < t \leq 1 \\ \frac{t(t-1)}{2}, & 1 < t < 2 \\ t-2, & 2 \leq t < 3 \\ 1 & 3 \leq t \leq 4 \\ 5-t, & 4 < t \leq 5 \end{cases}$$

Problem 3 Solution

(a) A discrete-time LTI system is stable if

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$

and is causal if

$$h[n] = 0 \quad for \; n < 0$$

since

$$\sum_{k=-\infty}^{+\infty} |h[k]| = \sum_{k=-\infty}^{3} 2^k = 16$$

So it is easy to see that the system is **noncausal and stable**

(b) A continuous-time LTI system is stable if

$$\int_{-\infty}^{+\infty} |h(\tau)| d\tau < \infty$$

and is causal if

$$h(t) = 0 \quad for \; n < 0$$

since

$$\int_{-\infty}^{+\infty} |h(\tau)| d\tau \geq \int_{-\infty}^{0} |h(\tau)| d\tau = \int_{-\infty}^{0} 1 d\tau = \infty$$

so it is easy to see that the system is **noncausal and unstable**.

(c) $h[n] < 0$ when $n < 0$

$$\sum_{k=-\infty}^{+\infty} |h[k]| = \sum_{k=0}^{+\infty} 1 - (0.99)^k = \infty$$

So the system is **causal and unstable**.

(d) $h(t) < 0$ when $t < 0$

$$\int_{-\infty}^{+\infty} |h(\tau)| d\tau = \int_{1}^{99} e^{15\tau} d\tau < \infty$$

So the system is **causal and stable**.

Problem 4 Solution

Let $h_3[n] = h_1[n] * h_2[n]$, so $x[n] * h_3[n] = y[n]$.

Since $h_2[n] = \delta[n] - \delta[n-1]$, we can obtain

$$h_3[n] = (\delta[n] - \delta[n-1]) * h_2[n] = \delta[n] * h_1[n] - \delta[n-1] * h_1[n] = h_1[n] - h_1[n-1]$$

Notice that $x[n]$ is always zero except $n = 0, 1$, so

$$y[n] = x[n] * h_3[n] = x[0](h_1[n] - h_1[n-1]) + x[1](h_1[n-1] - h_1[n-2]) = h_1[n] - h_1[n-2]$$

Finally we can calculate that

$$h_1[n] = \begin{cases} 2, & n = -2, 0 \\ 1, & n = -1 \\ 0, & otherwise \end{cases}$$

Problem 5 Solution

(a) $y(t) = Ax(t - t_0) = Acos(w_0 t + \phi_0 - w_0 t_0)$, $|Y(jw)| = |X(jw)||H(jw)| = A|X(jw)|$, so $A = H(jw_0)$

(b) It is easy to see that $-w_0 t_0 = \sphericalangle H(jw_0)$, so $t_0 = -\frac{\sphericalangle H(jw_0)}{w_0}$

Problem 6 Solution

(a) $A = |H(jw)| = |\frac{1-jw}{1+jw}| = 1$

(b)

$$H(jw) = \frac{1 - jw}{1 + jw} = \frac{(1 - jw)^2}{(1 + w)^2} = \frac{1 - w^2 - 2jw}{(1 + w)^2} \qquad \sphericalangle H(jw) = -arctan\frac{2w}{1 - w^2}$$

So $\tau(w) = -d(\sphericalangle H(jw))/dw = 2/(1 + w^2)$, and **2** is true.

Problem 7 Solution

From Bode magnitude plots of $H_1(jw)$, we can deduce that H_1 has a zero $w_0 = 1$ and two poles $w_1 = 8,\ w_2 = 40$, respectively.

Assume $H_1(jw) = A\frac{jw+1}{(jw+8)(jw+40)}$, we can get

$$20log\, H_1(0) = 20log\,\frac{A}{320} = 6 \longrightarrow A = 640$$

So $H_1(jw) = 640\frac{jw+1}{(jw+8)(jw+40)}$

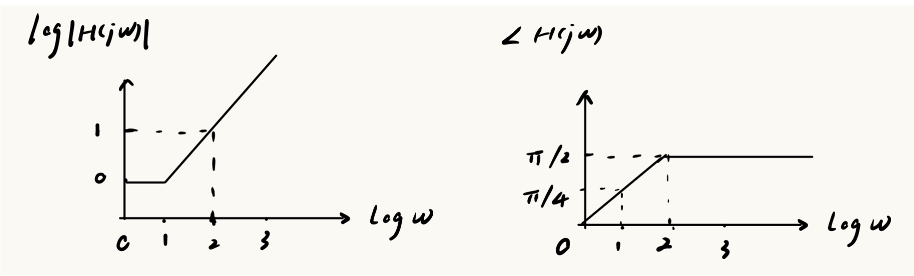
Similarly, we can obtain the expression of $H(jw)$

$$H(jw) = \frac{6.4}{(jw + 8)^2}$$

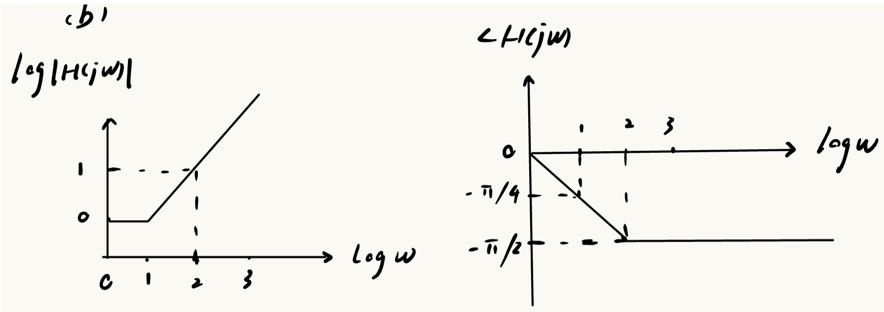
So $H_2(jw) = \frac{H(jw)}{H_1(jw)} = 0.01\frac{jw+40}{(jw+1)(jw+8)}$

Problem 8 Solution

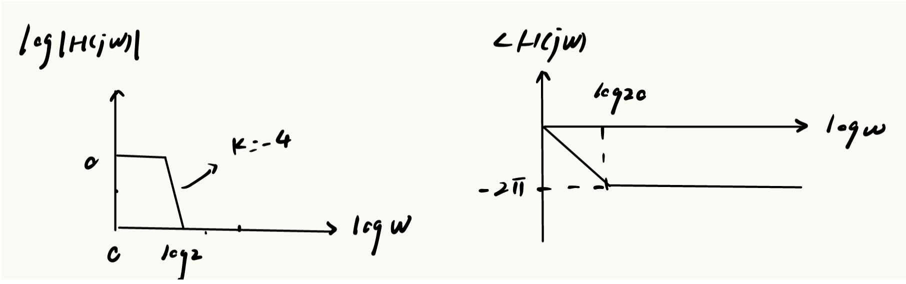
(1)



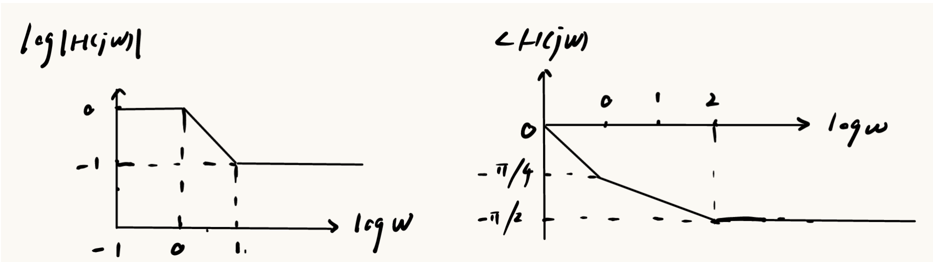
(2)


(b)
log|H(jw)|
log w
∠H(jw)
log w
-π/4
-π/2

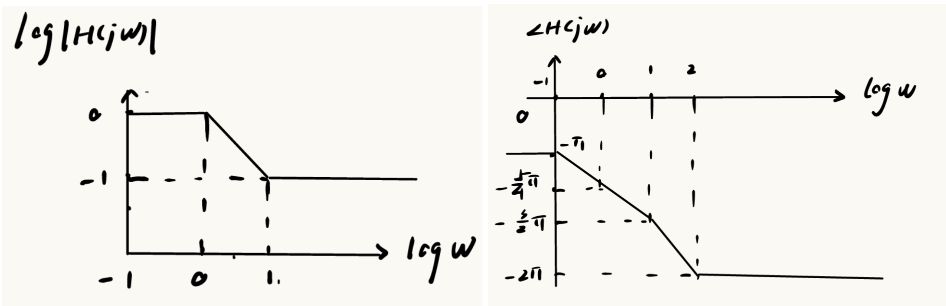
(3)


log|H(jw)|
k=-4
log w
log2
∠H(jw)
log20
log w
-2π

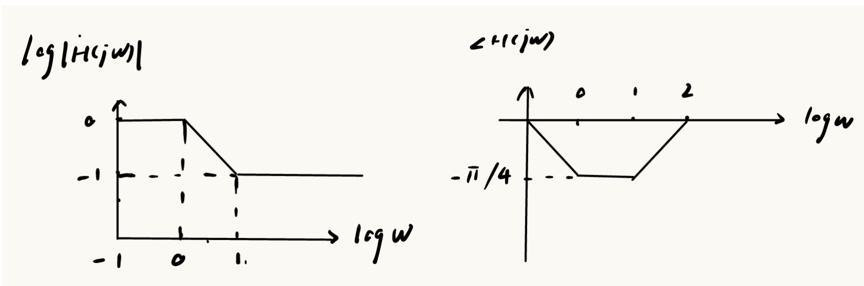
(4)


log|H(jw)|
log w
∠H(jw)
log w
-π/4
-π/2

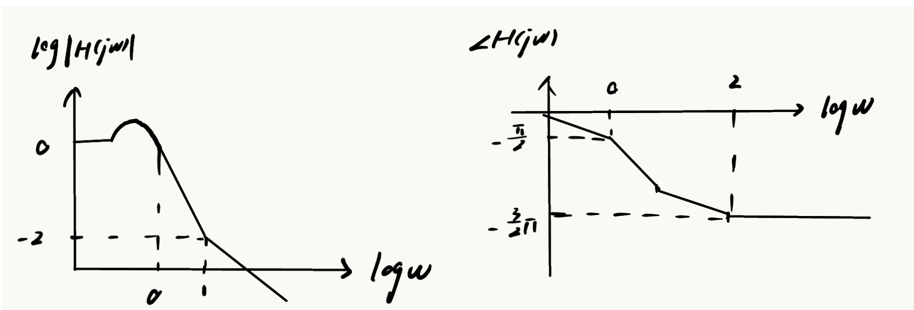
(5)


log|H(jw)|
log w
∠H(jw)
log w
-π
-2π

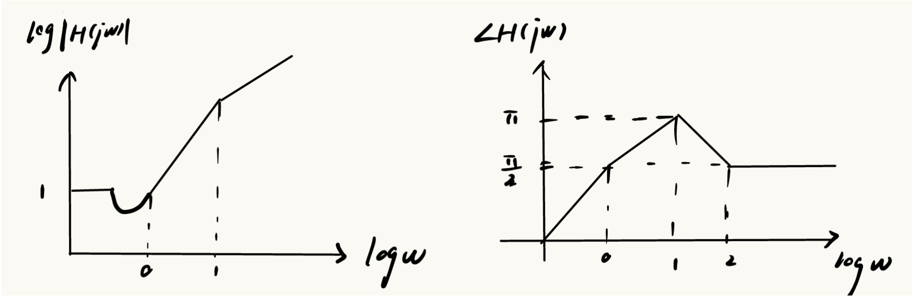
(6)


log|H(jw)|
log w
∠H(jw)
log w
-π/4

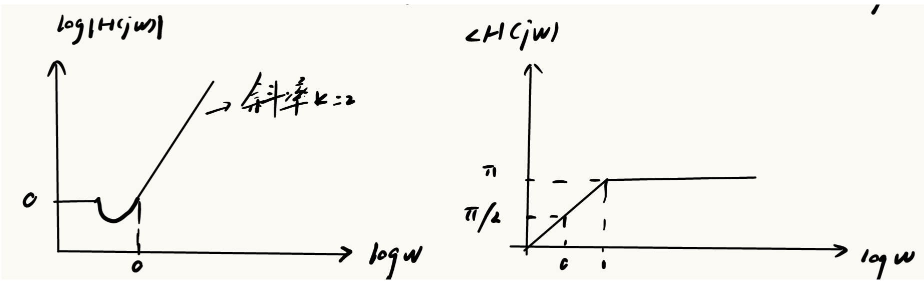
(7)


log|H(jw)|
log w
∠H(jw)
log w

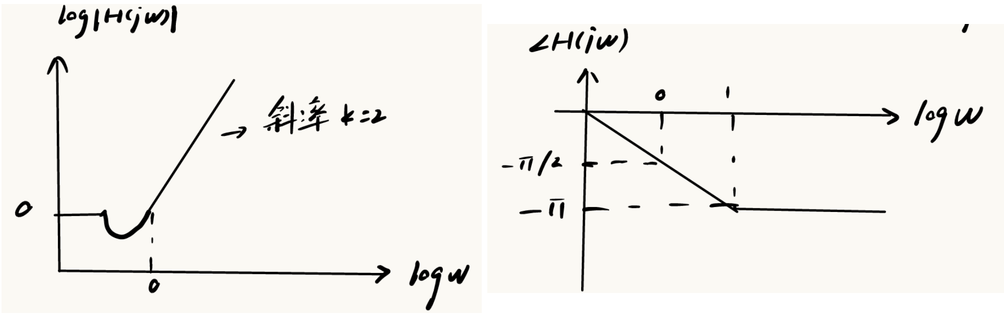
(8)



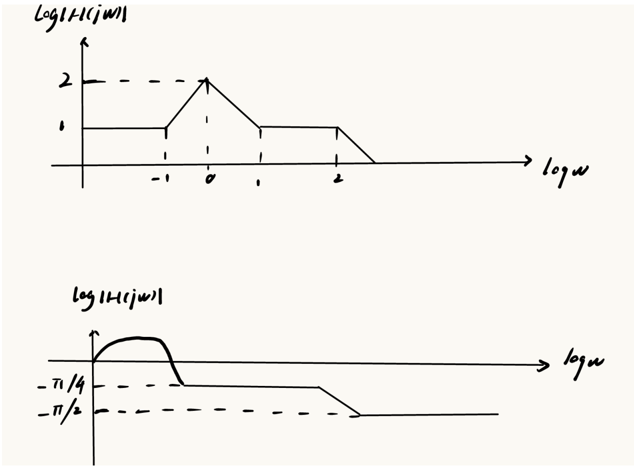
(9)



(10)



(11)



Problem 9 Solution

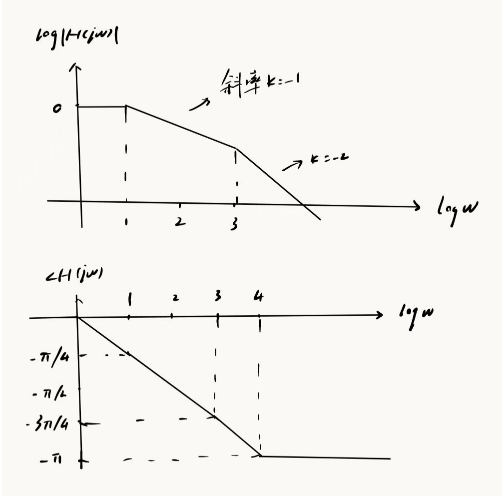
From the Bode plot we can know that

$$H(jw) = \frac{100}{(1+jw)(100+jw)}$$

so the Fourier transform of $10h(10t)$ is

$$H'(jw) = H(jw/10) = \frac{10000}{(jw+10)(jw+1000)}$$

The following is the Bode plot for $10h(10t)$



Problem 10 Solution

Group1: Pole-zero diagram 1/Impulse response 3/Bode Magnitude 5/Bode Angle 4

Group2: Pole-zero diagram 2/Impulse response 1/Bode Magnitude 2/Bode Angle 3

Group3: Pole-zero diagram 3/Impulse response 4/Bode Magnitude 3/Bode Angle 6

Group4: Pole-zero diagram 4/Impulse response 2/Bode Magnitude 6/Bode Angle 2

Group5: Pole-zero diagram 5/Impulse response 6/Bode Magnitude 1/Bode Angle 1

Group6: Pole-zero diagram 6/Impulse response 5/Bode Magnitude 4/Bode Angle 5