Signals and Systems – Spring 2024

Problem Set 7

Issued: May 28th 2024 Due: June 10th , 2024

Reading Assignment:

Chap.7

Problem 1:

1. **Sampling CT sinusoids**

Consider 3 CT signals:

$$x_1(t) = \cos(3000t)\ ,$$
$$x_2(t) = \cos(4000t)\ , \text{and}$$
$$x_3(t) = \cos(5000t)\ .$$

Each of these is sampled as follows

$$x_1[n] = x_1(nT)\ ,$$
$$x_2[n] = x_2(nT)\ ,\ \text{and}$$
$$x_3[n] = x_3(nT)\ ,$$

where $T = 0.001$. Which of the resulting DT signals has the highest DT frequency? Which has the lowest DT frequency?

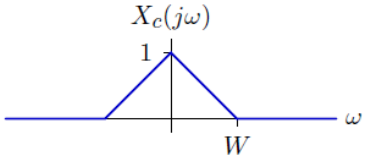
Problem 2:

2. **Sampling with alternating impulses**

A CT signal $x_c(t)$ is converted to a DT signal $x_d[n]$ as follows:

$$x_d[n] = \begin{cases} x_c(nT) & n \text{ even} \\ -x_c(nT) & n \text{ odd} \end{cases}$$

a. Assume that the Fourier transform of $x_c(t)$ is $X_c(j\omega)$ shown below.



Determine the DT Fourier transform $X_d(e^{j\Omega})$ of $x_d[n]$.

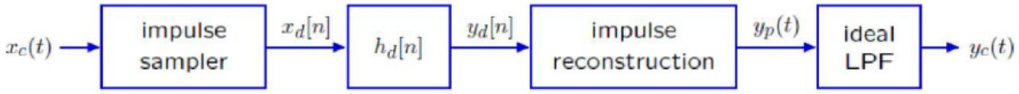
b. Assume that $x_c(t)$ is bandlimited to $-W \leq \omega \leq W$. Determine the maximum value of W for which the original signal $x_c(t)$ can be reconstructed from the samples $x_d[n]$.

c. Make a diagram of a system to reconstruct $x_c(t)$ from $x_d[n]$.

Problem 3:

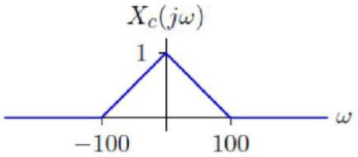
DT processing of CT signals

Sampling and reconstruction allow us to process CT signals using digital electronics as shown in the following figure.



The "impulse sampler" and "impulse reconstuction" use sampling interval $T = \pi/100$. The unit-sample function $h_d[n]$ represents the unit-sample response of an ideal DT low-pass filter with gain of 1 for frequencies in the range $-\frac{\pi}{2} < \Omega < \frac{\pi}{2}$. The "ideal LPF" passes frequencies in the range $-100 < \omega < 100$. It also has a gain of T throughout its pass band.

Assume that the Fourier transform of the input $x_c(t)$ is $X(j\omega)$ shown below.



Determine $Y_c(j\omega)$.

Problem 4: OWN Problem 7.6

Problem 5: OWN Problem 7.23