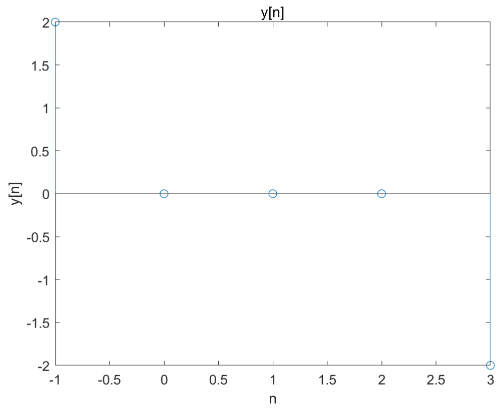
Lab 4

Problem 1

(a) Since $h[n] = 2\delta[n+1] - 2\delta[n-1]$ is nonzero on the range $[-1, 1]$ and $x[n]$ is nonzero on the range $[0, 2]$, we can see the time indexing of $y[n]$ is $[-1, 3]$.

The image of $y[n]$ is shown below



(b) Analytically

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k] = \delta[n-a-c] + \delta[n-a-d] + \delta[n-b-c] + \delta[n-b-d]$$

So $y[n]$ is nonzero on the interval $[a+c, b+d]$ When $a = 0, b = N-1, c = 0, d = M-1,\ n_y = [0 : M+N-2]$ which is coincident with the expected result.

(c) The analytical output is

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=2}^{n+2} (\frac{1}{2})^{k-2} = 2 - (\frac{1}{2})^n$$

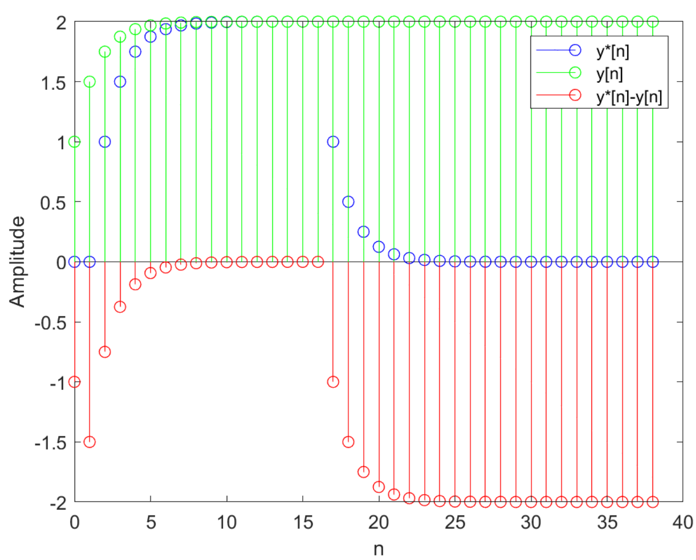
Since $x[n]$ is pegged in range of $0 \le n \le 24$ and $h[n]$ is pegged in range of $0 \le n \le 14$, the output for truncated signal

$$y'[n] = \sum_{k=\max\{2,n-14\}}^{\min\{24,n\}} (\frac{1}{2})^{k-2}$$

$y[n]$ and $y'[n]$ may be different for some n, so only a portion of the output is valid.

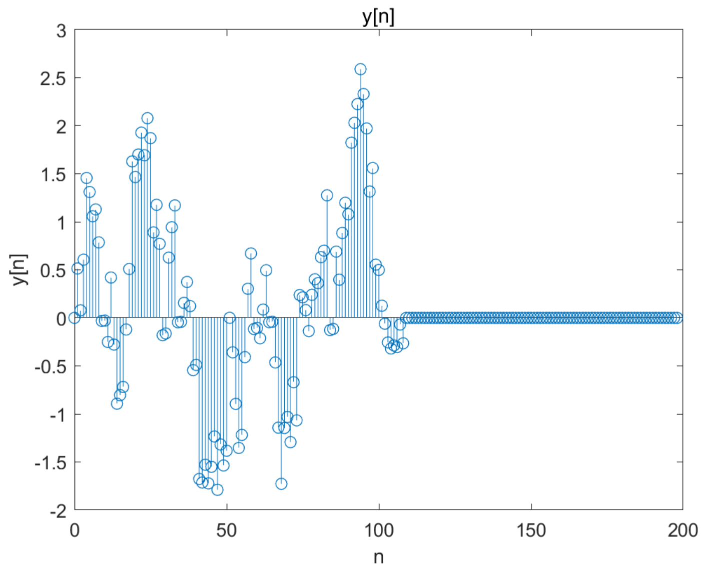
According to the given condition we can obtain $a = 0, b = 24, c = 0, d = 14$, so the time indices for $y[n]$ is $[0 : 38]$.

The image of $y[n]$ (green) and $y'[n]$ (blue) is shown below

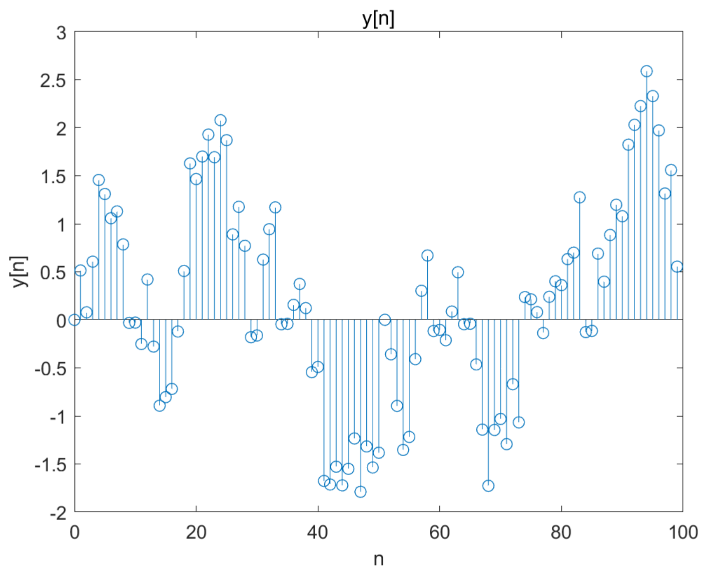


The red curve represent the difference between $y[n]$ and $y'[n]$, from which we can see the value of $y'[n]$ is nearly correct when for $8 \le n \le 16$

(d) The image of $y[n]$ is as follows



(e) Since $L = 50$, we can know that $k = L = 50$, and the image of $y[n]$ by using the overlap-add method is as follows



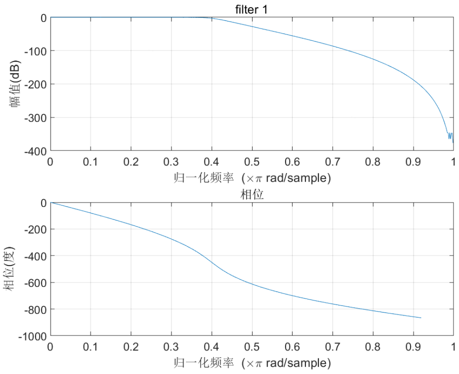
We can see it is the same as the result we obtain in **part(d)**

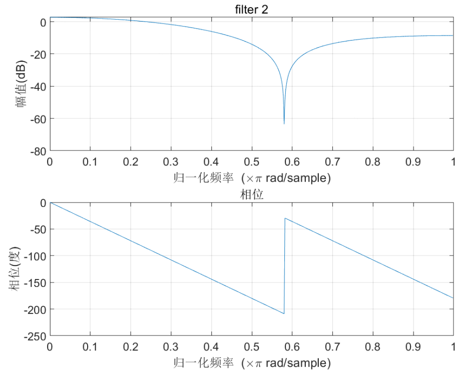
Problem 2

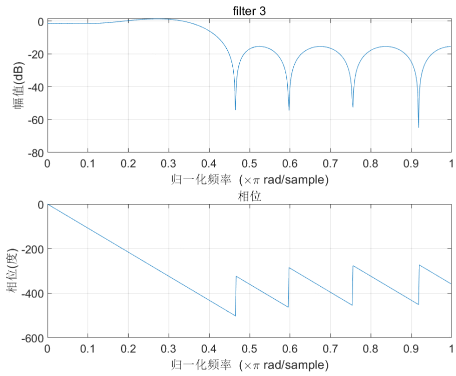
(b) The determination of three filters

```
wc = 0.4; % cutoff frequency
n1 = 10;
n2 = 4;
n3 = 12; % orders

[b1,a1] = butter(n1,wc); % coefficients
a2 = 1;b2 = remez(n2,[0 wc-0.04 wc+0.04 1],[1 1 0 0]);
a3 = 1;b3 = remez(n3,[0 wc-0.04 wc+0.04 1],[1 1 0 0]);
```

The magnitude and phase of the three filters are shown below



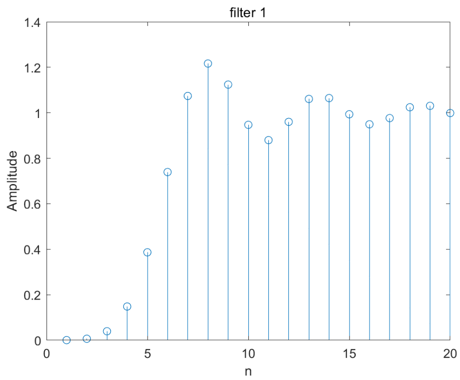


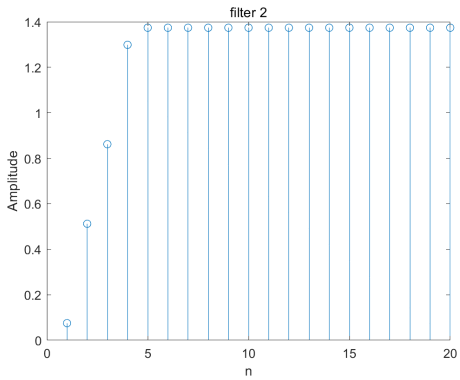


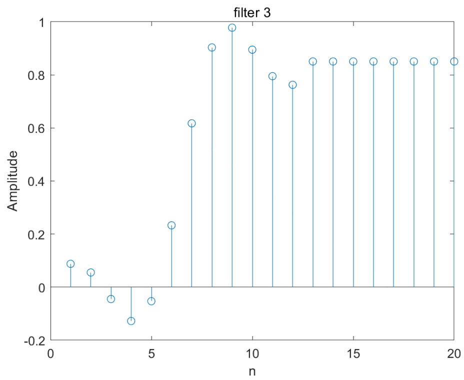
Obviously we can see that w_c is the approximate cutoff frequency of each filter from the magnitude plot.

Filter 2 and filter 3 have linear phase.

(c) The step response of the filter is shown as follows





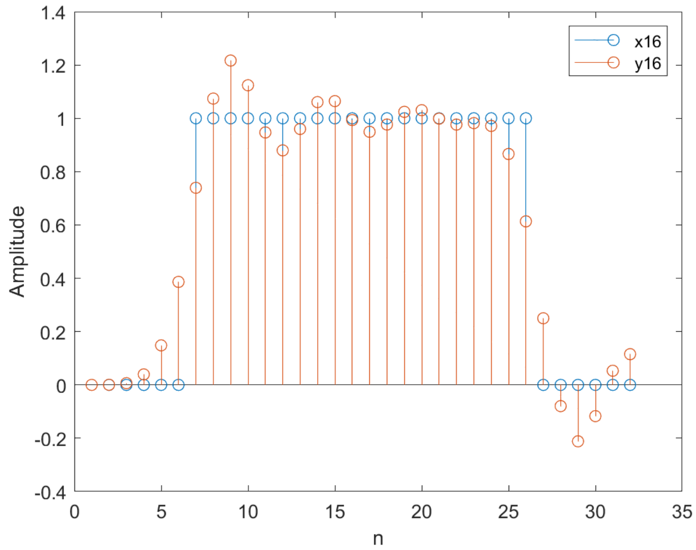


Filter 1 has the largest overshoot.

(d) we can plot y_{16} by the following method

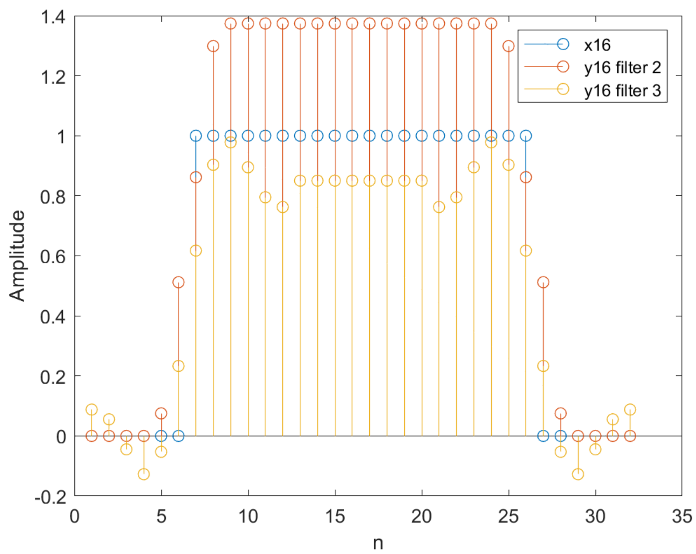
```
x16 = x(:,16); % the column input
y16 = filter(b1,a1,[x16;zeros(n1/2,1)]); % filter response

stem(x16);
stem(y16(n1/2+1:end));
```

The image we get is shown below



It is easy to see that the discontinuities in $x16$ line up with the smoothed discontinuities in $y16$

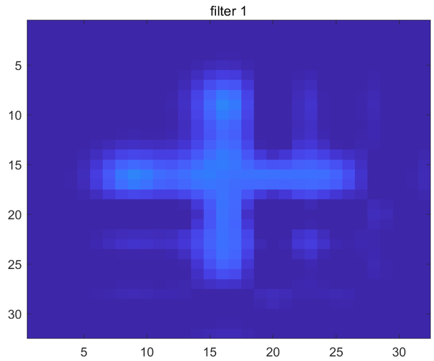
(e) Similar to (d) we can obtain the images of responses of filter two and three

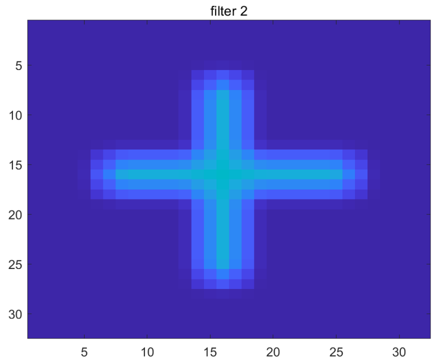


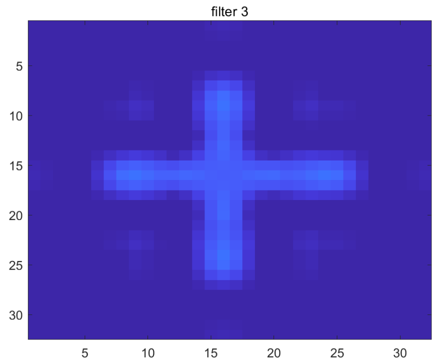
(f) The M-file is designed as follows

```
function y = filt2d(b,a,d,x)
 %d = n/2;n is the order of the 1D filter
 %a b is the coefficients of the 1Dfilter
 %size fo x y z is N*N

 [m,n] = size(x);
 y = zeros(m, n);
 z = zeros(m, n);

 for i = 1:n    %each column
     xi = x(:,i);
     z1 = filter(b,a,[xi;zeros(d,1)]);
     z(:,i) = z1(d+1:end);
 end

 for j = 1:m    %each row
     zj = z(j,:);
     y1 = filter(b,a,[zj zeros(1,d)]);
     y(j,:) = y1(d+1:end);
 end

end
```

(g) The filtered images for the three filters are shown below respectively







(h) Filter 1 leads to more distortion due to the nonlinearity of the filter phase.

Problem3

(a) $x[n]$ is purely real because the DTFS coefficients of $x[n]$ is conjugation symmetric, which imply $x[n] = x^*[n]$

(b) Since the DTFS coefficients is periodic with period $N = 5$ we can obtain

$$a_0 = 1, a_1 = 2e^{-j\frac{\pi}{3}}, a_2 = e^{j\frac{\pi}{4}}, a_3 = e^{-j\frac{\pi}{4}}, a_4 = 2e^{j\frac{\pi}{3}}$$

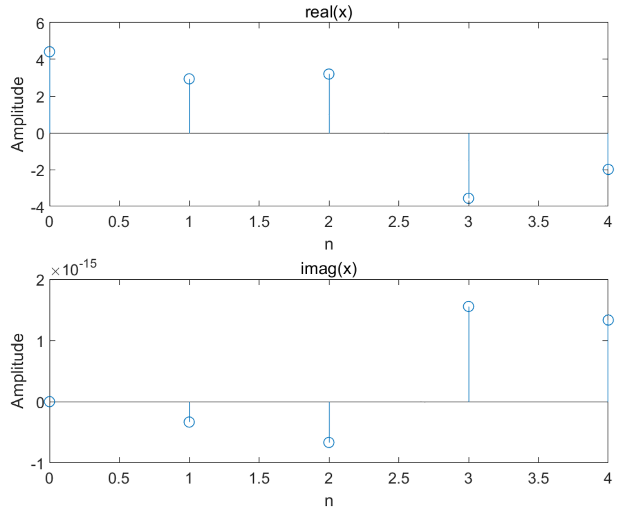
The vector

```
a=[1 2*e^(-1j*pi/3) e^(1j*pi/4) e^(-1j*pi/4) 2*e^(1j*pi/3)]
```

(c) Use a for loop to compute $x[n]$

```
a = [1 2*exp(-1j*pi/3) exp(1j*pi/4) exp(-1j*pi/4) 2*exp(1j*pi/3)];
x = zeros(1,5);
nx = 0:4;
for n = 1:5
   for k = 1:5
      x(n) = x(n)+a(k)*exp(1j*(k-1)*2*pi/5*(n-1)); % compute x[n]
   end
end
xr = real(x); % real and imaginary part
xi = imag(x);
```

The image is shown below

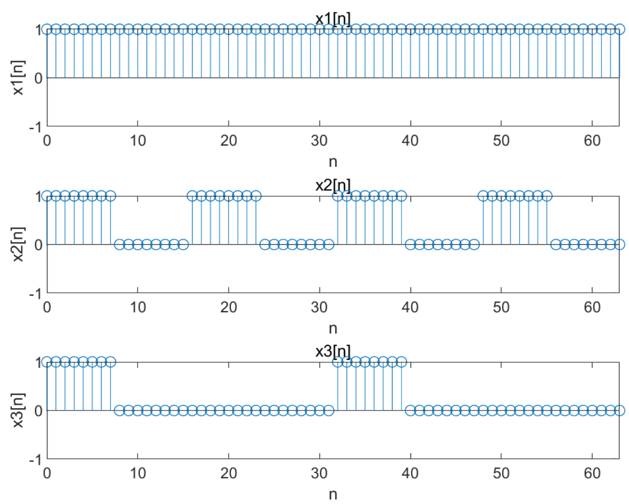


Because of the roundoff errors there exits a very small nonzero imaginary part in $x[n]$, which is negligible. So the conclusion in **(a)** is correct.

(d) Repeat the vector to cover the range of samples

```
x1 = ones(1,8); % each periodic signal
x2 = [ones(1,8),zeros(1,8)];
x3 = [ones(1,8),zeros(1,24)];
n = 0:63;

x1_0 = [x1 x1 x1 x1 x1 x1 x1 x1];
x2_0 = [x2 x2 x2 x2];
x3_0 = [x3 x3]; % repeat the vectors
```

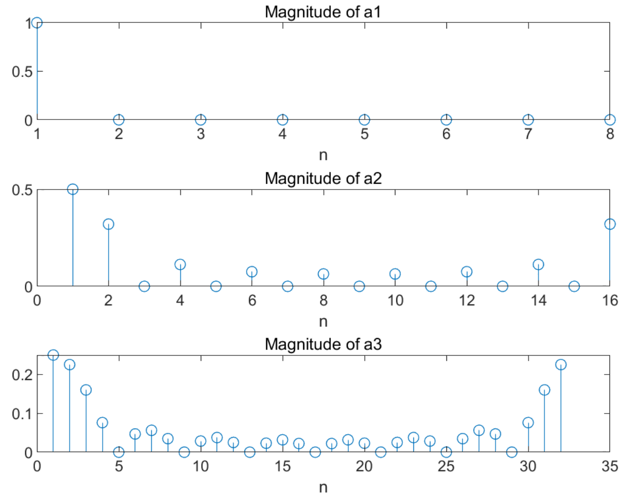
And the three signals is plotted as following



(e) The DC components of a signal is the average of its one period, so

$$a_1(1) = 1 \qquad a_2(1) = \frac{1}{2} \qquad a_3(1) = \frac{1}{4}$$

The plots of the magnitude of each of the DTFS coefficients



The figure above match the result we have predicted.

(f) Since $a_k = a_{k+32}$ we can determine $a3(18)\cdots\cdots a3(32)$ correspond to $a_{-15}\cdots\cdots a_{-1}$

(g) According to the condition

$$x_{3\_all}[n] = \sum_{k=-15}^{16} a_k e^{jk(2\pi/32)n}$$

The conjugation

$$x^*_{3\_all}[n] = \sum_{k=-15}^{16} a^*_k e^{-jk(2\pi/32)n}$$

Since $x[n]$ is purely real, the FS coefficients are conjugation symmetric. Substituting k with $-k$ in the equation above

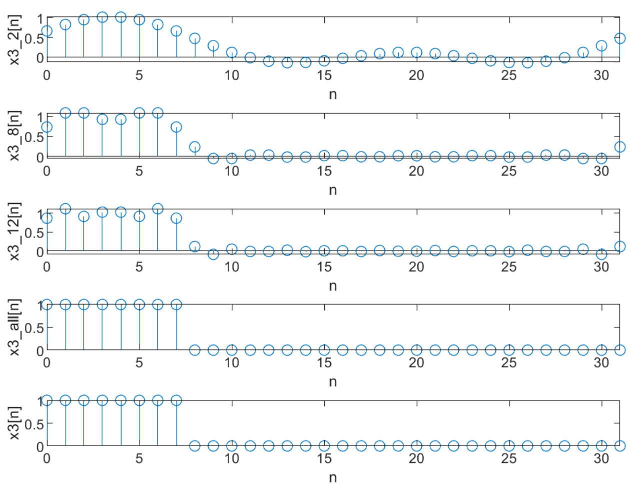
$$x^*_{3\_all}[n] = \sum_{k=-16}^{15} a^*_{-k} e^{jk(2\pi/32)n} = \sum_{k=-16}^{15} a_k e^{jk(2\pi/32)n}$$

Meanwhile $a_{16} = a_{-16}$

$$x^*_{3\_all}[n] = \sum_{k=-16}^{15} a^*_{-k} e^{jk(2\pi/32)n} = \sum_{k=-15}^{16} a_k e^{jk(2\pi/32)n} = x_{3\_all}[n]$$

So $x_{3\_all}[n]$ is a real signal.

(h) The images of x_{3_2} to x_{3_all} are shown below



As we can see, the signals converge to $x_3[n]$ as more of the DTFS coefficients are included in the sum.

There is no Gibb's phenomenon being displayed.