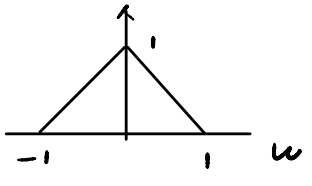
20. 解：$R_X(\tau) = e^{-|\tau|} + e^{-|\tau|}\cos\pi\tau$

$\therefore R_X(\tau) \xleftrightarrow{F} \frac{2}{\omega^2+1} + \frac{1}{(\omega-\pi)^2+1} + \frac{1}{(\omega+\pi)^2+1}$

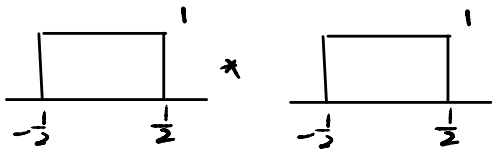
$\therefore S_X(\omega) = \frac{2}{\omega^2+1} + \frac{1}{(\omega-\pi)^2+1} + \frac{1}{(\omega+\pi)^2+1}$

22. 解：$\frac{1}{\pi} \xleftrightarrow{F} 2\delta(\omega)$

对于 $1-|\omega|$，其图像



$=$



记上图的为波为 f，则 $\frac{\sin\frac{1}{2}t}{\pi t} \xrightarrow{F} f$

$\therefore R_X(\tau) = \frac{1}{\pi} + 2\pi\left(\frac{\sin\frac{1}{2}t}{\pi t}\right)^2$

24. (1) 证明：$E[Y(t)] = E[X(t)]E[\cos(t+\theta)] = 0$

$R_Y(t, t+\tau) = E[X(t)X(t+\tau)\cos(t+\theta)\cos(t+\theta+\tau)]$

$= R_X(\tau)\cdot\frac{1}{2}\left[\cos(t+\frac{\pi}{4})\cos(t+\tau+\frac{\pi}{4}) + \cos(t-\frac{\pi}{4})\cos(t+\tau-\frac{\pi}{4})\right]$

$= \frac{1}{2}R_X(\tau)\cos\tau$

$\therefore \{Y(t); -\infty < t < +\infty\}$ 为平稳过程，$R_Y(t, t+\tau) = \frac{1}{2}R_X(\tau)\cos\tau$

(2) 证明：$S_Y(\omega) = \left(\frac{1}{2}S_X(\omega)\cdot\frac{1}{2\pi}\right) * \pi[\delta(\omega-1)+\delta(\omega+1)] = \frac{1}{4}[S_X(\omega+1)+S_X(\omega-1)]$

25. 证明：$R_Y(t, t+\tau) = E[(X(t+L)-X(t))(X(t+\tau+L)-X(t+\tau))]$

$= R_X(\tau) - R_X(\tau+L) + R_X(\tau) - \begin{cases} R_X(\tau-L), & \tau > L \\ R_X(L-\tau), & L > \tau \end{cases}$

而 $R_X(L-\tau)$ 是偶函数

$\therefore R_Y(\tau) = 2R_X(\tau) - R_X(\tau+L) - R_X(\tau-L)$

$\therefore S_Y(\omega) = R_Y(\tau) \xleftrightarrow{F} 2S_X(\omega) - S_X(\omega)[e^{-j\omega L} + e^{j\omega L}]$

$= 2S_X(\omega)(1-\cos\omega L)$