Problem Set 8

Problem 1

Part a. According to BPF, $\Omega_c = \frac{1}{2}(\Omega_l + \Omega_h) = 0.25\pi$, so

$$f_c = \frac{w_c}{2\pi} = \frac{\Omega_c}{2\pi T} = \frac{0.25\pi}{2\pi \times 0.1us} = 1.25MHz$$

Part b.

(1) Aliasing occurs when $2\pi < 2w_r T$, that is , $w_{aliasing} > 10\pi MHz$

Since $1.5w_r = 9.4\pi < 10\pi$, so b1 is not correct.

(2) The max frequency station broadcast $w_{max} = 2\pi(f_c + 5k) = 2\pi \times 1.255MHz < w_r/2$

So decreasing the cutoff frequency w_r of LPF1 by a factor 2 has no effect on $y_r(t)$, (2) is correct.

(3)(4) $x_d[n]$ can not pass through BPF if sampling interval T changes, so (3)(4) are not correct.

Part c.

(1) The unwanted signals has been cut off by BPF, so increasing Ω_d will not add these signals, (1) is not correct.

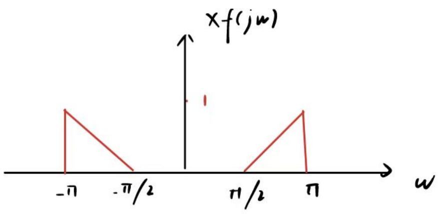
(2) LPF2 is low pass filter, which has on effect on the occurrence of aliasing, so (2) is not correct.

(3) Since all unwanted signals has been cut off by BPF, doubling Ω_d have no effect on $y_r(t)$, (3) is correct.

(4) The decreasing of Ω_c can lead to the cutoff of some valid frequency such as $f = f_c + 5kHz$, so (4) is not correct.

Problem 2

(a)



- $0 < w < \frac{\pi}{2}$ $\quad X(jw) = 0$
- $\frac{\pi}{2} < w < \pi$ $\quad X(jw) = \frac{2}{\pi}w - 1$
- $\pi < w < \frac{3\pi}{2}$ $\quad X(jw) = 0$
- $\frac{3\pi}{2} < w < 2\pi$ $\quad X(jw) = 0$

(b) $T = \frac{2}{7}$ and $K = 1$

Problem 3

(a) Since $x(t)$ $\sin w_c t$ $\cos w_c t$ are all real signals, we only need to demonstrate $x(t) * h(t)$ is a real signal, that is

$$(X(jw)H(jw))^* = X(-jw)H(-jw)$$

Since $X^*(jw) = X(-jw)$ and $H^*(jw) = H(-jw)$, we can conclude $y(t)$ is also real signal.

(b) $x(t)$ can be recovered by

$$x(t) = [y(t)sin(w_c t)] * \frac{2\sin w_c t}{\pi t}$$

Problem 4

(a) $w(t) = x(t)\cos^2(w_c t + \theta_c) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2w_c t + 2\theta_c)$

(b) we have to guarantee that

$$w_{co} > w_M \quad and \quad w_{co} < 2w_s - w_M$$

It is easy to see that the result doesn't depend on θ_c.

Problem 5

(a) $Z(jw) = \frac{1}{2}(X(j(w-w_c)) + X(j(w-w_c)))$

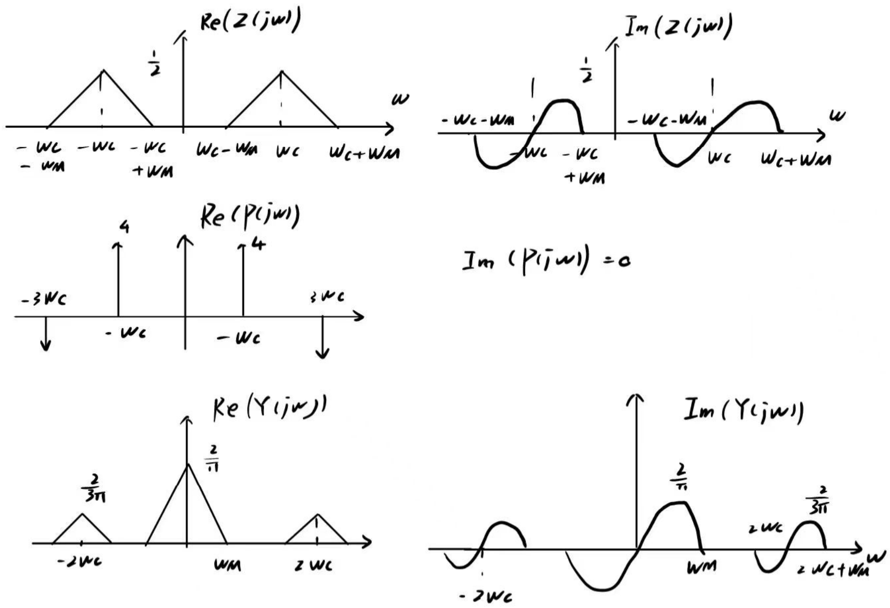
The Fourier coefficients of $p(t)$

$$a_k = \frac{2\sin(\pi k/2)}{\pi k} \qquad and \qquad a_0 = 0$$

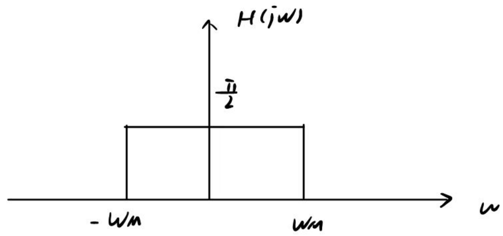
we can obtain

$$P(jw) = 2\pi \sum_{k=-\infty, k\neq 0}^{+\infty} \frac{2\sin(\pi k/2)}{\pi k}\delta(w - w_0)$$

$Y(jw) = \frac{1}{2\pi}\left(Z(jw) * P(jw)\right)$

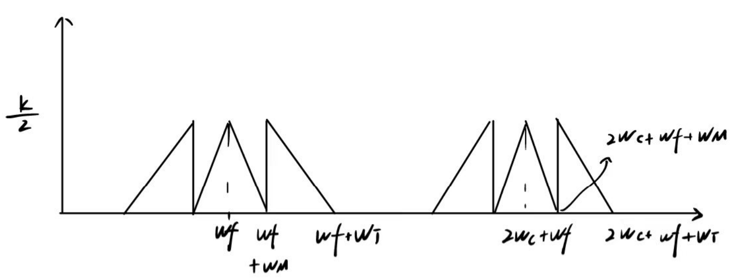


(b)



Problem 6

(a)



(b) According to (a), we have to guarantee

$$\begin{aligned} w_f + w_M &< 2w_c + w_f - w_T \\ - w_f + w_T &< w_f - w_M \end{aligned}$$

So

$$\begin{aligned} w_T &< 2w_c - w_M \\ w_T &< 2w_f - w_M \end{aligned}$$

(c) To avoid the occurrence of aliasing we have to guarantee

$$G = 2/K \qquad \alpha = w_f - w_M \qquad \beta = w_f + w_M$$