Problem Set 7

Problem 1

$x_1[n]$ has the highest DT frequency and $x_3[n]$ has the lowest DT frequency.

Problem 2

(a) Consider a discrete signal $x[n] = (-1)^n x_d[n] = e^{jn\pi} x_d[n] = x_c(nT)$

According to frequency shifting property we can obtain

$$X(e^{j\Omega}) = X_d(e^{j(\Omega-\pi)})$$

The relationship between Fourier transforms of $x[n]$ and $x_c(nT)$ is

$$X(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c(j(\Omega - 2\pi k)/T)$$

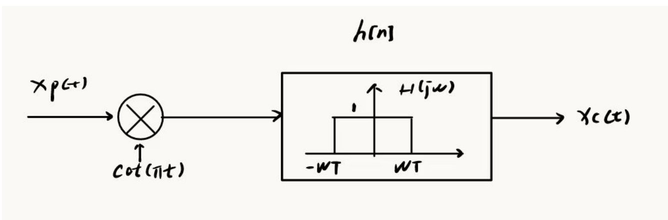
So we can obtain

$$X_d(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c(j(\Omega + \pi - 2\pi k)/T)$$

(b) To avoid the occurrence of aliasing, we have to ensure $2\pi \geq 2WT$, so

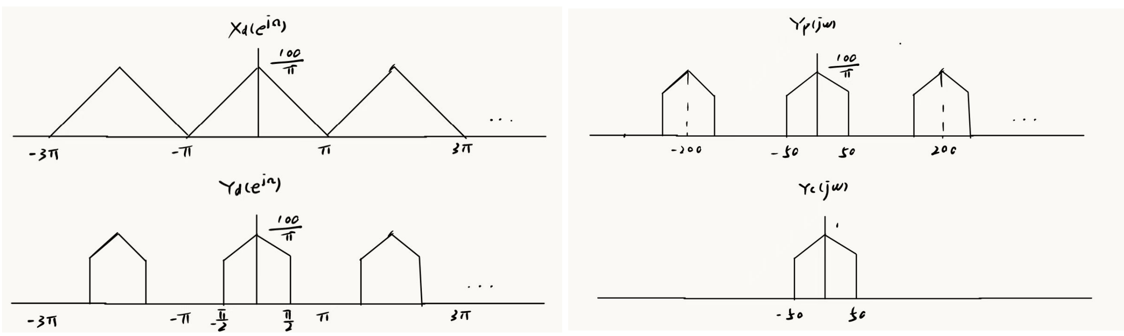
$$W_{max} = \frac{\pi}{T}$$

(c) The process of reconstruction is shown below



Problem 3

$$X_d(e^{j\Omega}) = \frac{1}{T} X_c(jw)\big|_{w=\frac{\Omega}{T}} = \frac{100}{\pi} X_c(j\frac{100\Omega}{\pi})$$



Problem 4

Since $w(t) = x_1(t)x_2(t)$, we can obtain

$$W(jw) = \frac{1}{2\pi} X_1(jw) * X_2(jw)$$

So $W(jw) = 0$ when $|w| \geq w_1 + w_2$

According to Sampling Theorem

$$T_{max} = \frac{2\pi}{w_{smin}} = \frac{2\pi}{2w} == \frac{\pi}{w_1 + w_2}$$

Problem 5

(a) $p(t)$ is a periodic signal, of which the Fourier coefficients

$$a_k = \frac{1}{2\Delta}\int_{-\frac{\Delta}{2}}^{\frac{3\Delta}{2}} p(t)e^{-jk\frac{\pi}{\Delta}t}dt = \frac{1}{2\Delta}(1 - e^{-jk\pi})$$

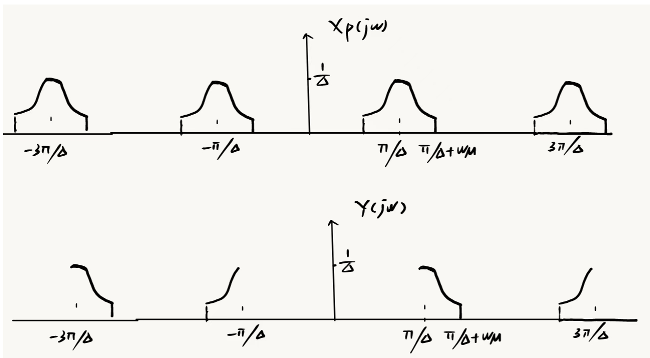
So the Fourier transform

$$P(jw) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0) = \sum_{k=-\infty}^{+\infty} \frac{\pi}{\Delta}(1 - e^{-jk\pi})\delta(w - k\frac{\pi}{\Delta})$$

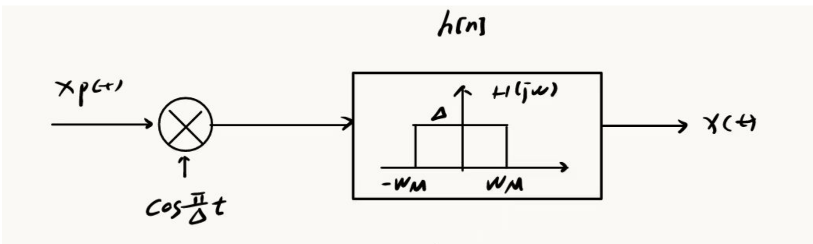
According to the property of convolution

$$X_p(jw) = \frac{1}{2\pi}P(jw) * X(jw) = \frac{1}{2\Delta}\sum_{k=-\infty}^{+\infty}(1 - e^{-jk\pi})X(j(w - k\frac{\pi}{\Delta}))$$

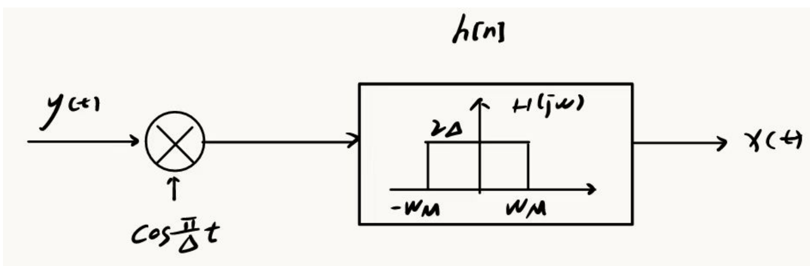
Since $\Delta < \frac{\pi}{2w_m}$ we can plot the figure of $X_p(jw)$ and $Y(jw)$



(b) The process of reconstruction is shown below



(c) The process of reconstruction is shown below



(d) We have to ensure $\frac{\pi}{\Delta} - w_M \geq 0$ so

$$\Delta_{max} = \frac{\pi}{w_M}$$