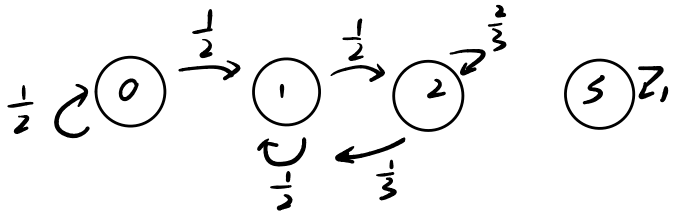
10.



(1)解: $P(X_1=1, X_3=2) = \left(\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{2}\right)\times\left(\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{2}\right) = \frac{7}{36}$

$P(X_2=1) = \frac{1}{3}\left(\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\right)+\frac{1}{3}\left(\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{3}\right) = \frac{11}{36}$

$P(X_{10}=0) = \frac{1}{3}\times\frac{1}{2^{10}}$

(2)解: $f_{00}^{(1)} = \frac{1}{2}$，$f_{00}^{(n)} = 0, n>1$ $\therefore f_{00}<1$ ∴状态0暂留

$f_{11}^{(1)} = \frac{1}{2}$，$f_{11}^{(n)} = \frac{1}{2}\times\frac{1}{3}\times\left(\frac{2}{3}\right)^{n-2} = \frac{1}{6}\cdot\left(\frac{2}{3}\right)^{n-2}$

$\therefore f_{11} = \frac{1}{2}+\frac{1}{6}\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n = \frac{1}{2}+\frac{1}{6}\times\frac{1}{1-\frac{2}{3}} = 1$

又 状态1和2互达 ∴$\{1,2\}$常返

$u_1 = \sum_{n=1}^{\infty} n f_{11}^{(n)} = \frac{5}{2}$ 同理可求得 $u_2 = \frac{5}{3}$

易知状态3常返，平均回转时为1

14. (1)解: $I = \{0,1,2,\dots,N\}$

$p_{ii} = \frac{i}{N}p + \frac{N-i}{N}(1-p)$

$p_{i,i+1} = \frac{N-i}{N}p$

$p_{i,i-1} = \frac{i}{N}(1-p)$

(2)解: $N=3$，则一步转移矩阵

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

记 $\vec{x} = \vec{x}P$，得 $\vec{x} = \left(\frac{1}{8},\frac{3}{8},\frac{3}{8},\frac{1}{8}\right)$ $u_0 = \frac{1}{x_0} = 8$

16. 解: 平稳分布时

$$(x_1,x_2,x_3,x_4)\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (x_1,x_2,x_3,x_4)$$

$$\therefore \begin{cases} \frac{1}{2}x_1 = x_1 \\ \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 = x_2 \\ \frac{1}{2}x_2 + \frac{2}{3}x_3 = x_3 \\ x_1 + x_2 + x_3 + x_4 = 1 \end{cases} \text{解得} \begin{cases} x_1 = 0 \\ x_2 = \frac{4}{15} \\ x_3 = \frac{2}{5} \\ x_4 = \frac{1}{3} \end{cases}$$

$\therefore \lim_{n\to\infty} P(X_n=0)=0 \quad \lim_{n\to\infty} P(X_n=2)=\frac{2}{5}$

$\lim_{n\to\infty} P(X_n=1)=\frac{4}{15} \quad \lim_{n\to\infty} P(X_n=3)=\frac{1}{3}$

17. 解：(1) ~~互达~~等价类：$\{0,1,2,3\}$ $\{4,5\}$ $\{6,7\}$

其中 $\{0,1,2,3\}$、$\{6,7\}$ 为闭的

(2) 解：$p_{00}^{(1)}=0$，$p_{00}^{(2)}>0$，$p_{00}^{(3)}=0$，$p_{00}^{(4)}>0$ …… $p_{00}^{(2m)}>0$

$\therefore d(0)=d(1)=d(2)=d(3)=2$

而 $p_{44}=\frac{1}{3}>0$，$p_{77}=\frac{1}{2}>0$ $\therefore d(4)=d(5)=d(6)=d(7)=1$

对状态 0，$f_{00}^{(2)}=\frac{1}{2}$，当 $n>2$，记 $n=2(m+k)=2p$

状态在 1、2 间切换了 m 次，在 2、3 间切换了 k 次

则 $f_{00}^{(n)}=\frac{1}{2}\sum_{m=1}^{p-1}(\frac{1}{4})^m\times(\frac{1}{2})^{p-1-m}=(\frac{1}{2})^p\sum_{m=1}^{p-1}\frac{1}{2^m}=\frac{1}{2^p}\frac{\frac{1}{2}-\frac{1}{2^{p-1}}}{1-\frac{1}{2}}=\frac{1}{2^p}-\frac{2}{4^p}$

$\therefore f_{00}=1$，$\{0,1,2,3\}$ 常返，平稳分布 $(\frac{1}{6},\frac{1}{3},\frac{1}{3},\frac{1}{6})$ $\therefore \mu_0=\mu_3=6$，$\mu_1=\mu_2=3$

对状态 7：$f_{77}^{(1)}=\frac{1}{2}$，$f_{77}^{(2)}=\frac{1}{2}$，$f_{77}^{(n)}=0$，$n\geq 3$ $\therefore \{6,7\}$ 常返

平稳分布时 $(x_6,x_7)=(\frac{1}{3},\frac{2}{3})$ $\therefore \mu_6=3$，$\mu_7=\frac{3}{2}$

对状态 4：$f_{44}=f_{44}^{(1)}+f_{44}^{(2)}=\frac{5}{6}\neq 1$ $\therefore \{4,5\}$ 暂留

(3) 解：$\because \{4,5\}$ 暂留 $\therefore \lim_{n\to\infty} p_{45}^{(n)}=0$

而 $\lim_{n\to\infty} p_{67}^{(n)}=\pi_7=\frac{2}{3}$

(4) 解：$\because \{4,5\}$ 暂留 $\therefore$ 对 $i=4,5$，$\lim_{n\to\infty} P(X_n=i)=0$

设 p_4 为状态 4 经有限步到 6 的概率，p_5 为状态 5 经有限步到 5 的概率

则 $p_4=\frac{1}{3}p_4+\frac{1}{3}p_5$，$p_5=\frac{1}{2}+\frac{1}{2}p_4$ $\therefore p_4=\frac{1}{3}$，$p_5=\frac{2}{3}$

$\therefore \lim_{n\to\infty} P\{X_n=6\}=\frac{1}{18}$ $\therefore \lim_{n\to\infty} P\{X_n=7\}=\frac{1}{18}\times 2=\frac{1}{9}$

19. 解：由题意，1为吸收态，设h_i为从$X_0=i$出发在有限时间内到达1的概率，则

$$\begin{cases} h_1=1 \\ h_2=0 \\ h_3=\frac{1}{4}h_3+\frac{1}{4}h_2+\frac{1}{4}h_1+\frac{1}{4}h_4 \\ h_4=\frac{1}{8}h_4+\frac{1}{8}h_1+\frac{3}{8}h_2+\frac{3}{8}h_3 \end{cases}$$

$$\therefore \begin{cases} h_1=1 \\ h_2=0 \\ h_3=\frac{4}{9} \\ h_4=\frac{1}{3} \end{cases}$$

$\therefore P(T_1<\infty \mid X_0=3)=h_3=\frac{4}{9}$