Problem Set 1

Problem 1 Solution

(a) Consider an input $x[n]$ of system S_1, the output $y_1[n] = 2x[n] + 4x[n-1]$, which will be the input of system S_2.

According to the input-output relationship of S_2

$$y_2[n] = y_1[n-2] + \frac{1}{2}y_1[n-3] = 2x[n-2] + 4x[n-3] + \frac{1}{2}(2x[n-3] + 4x[n-4]) = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

So the input-output relationship of S is

$$y[n] = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

(b) Consider an input $x[n]$ of system S_2, the output $y_2[n] = x_2[n-2] + \frac{1}{2}x_2[n-3]$, which will be the input of system S_1.

According to the input-output relationship of S_1, we can eventually deduce that

$$y[n] = 2(x[n-2] + \frac{1}{2}x[n-3]) + 4(x[n-3] + \frac{1}{2}x[n-4]) = 2x[n-2] + 5x[n-3] + 2x[n-4]$$

So the relationship doesn't change if the order is reversed.

Problem 2 Solution

(a) The output of a memoryless system at a given time depends only on the input at the same time, **so this system is not memoryless.**

(b) The only nonzero value of $\delta[n]$ is at the point at which n is zero, **so $y[n]$ is always zero** because n and $n-2$ can not be zero at the same time.

(c) No, beacuse we can't assure that for any different n_1 and n_2, the output $y_1 = x[n_1]x[n_1-2]$ and $y_2 = x[n_2]x[n_2-2]$ are definitely different.

Problem 3 Solution

(a)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$, $x_2(t) = x(t-t_0)$.

The output corresponding to x_2 is

$$y_2(t) = x_2(t-2) + x_2(2-t) = x(t-2-t_0) + x(2-t-t_0) \neq y_1(t-t_0)$$

So the system is **time-varying.**

(3) It's easy to see that the system is **linear.**

(4) If $t < 1$, then $2-t > t$, so the system is **noncausal.**

(5) $|y(t)| \leq |x(t-2)| + |x(2-t)| \leq 2|x(t)|_{max}$. If $x(t)$ is bounded, the output is also bounded, so the system is **stable.**

(b)

(1) The system is **memoryless** because the output at a given time depends only on the input at the same time.

(2) Let $y_1(t-t_0) = [cos(3t-3t_0)]x(t-t_0)$, $x_2(t) = x(t-t_0)$.

The output corresponding to x_2 is

$$y_2(t) = [cos(3t)]x_2(t) = [cos(3t)]x(t-t_0) \neq y_1(t-t_0)$$

So the system is **time-varying.**

(3) The output corresponding to $ax_1(t) + bx_2(t)$ is

$$y(t) = [cos(3t)](ax_1(t) + bx_2(t)) = a[cos(3t)]x_1(t) + b[cos(3t)]x_2(t) = ay_1(t) + by_2(t)$$

So the system is **linear.**

(4) It's easy to see that the system is **causal.**

(5) $|y(t)| \leq |cos(3t)||x(t)| \leq |x(t)|$. If $x(t)$ is bounded, the output is also bounded, so the system is **stable.**

(c)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1(t-t_0) = \int_{-\infty}^{2t-2t_0} x(\tau)d\tau$, $x_2(t) = x(t-t_0)$.

The output corresponding to x_2 is

$$y_2(t) = \int_{-\infty}^{2t} x_2(\tau)d\tau = \int_{-\infty}^{2t} x(\tau - t_0)d\tau = \int_{-\infty}^{2t-t_0} x(m)dm \neq y_1(t-t_0)$$

So the system is **time-varying**.

(3) The output corresponding to $ax_1(t) + bx_2(t)$ is

$$y(t) = \int_{-\infty}^{2t} (ax_1(\tau) + bx_2(\tau))d\tau = a\int_{-\infty}^{2t} x_1(\tau)d\tau + b\int_{-\infty}^{2t} x_2(\tau)d\tau = ay_1(t) + by_2(t)$$

So the system is **linear.**

(4) If $t > 0$, then $2t > t$, so the system is **noncausal.**

(5) Let $x(t) = 1$, then $y(t)$ will be infinite. So the system is **not stable.**

(d)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1(t-t_0) = \begin{cases} 0, & t-t_0 < 0 \\ x(t-t_0) + x(t-t_0-2), & t-t_0 \geq 0 \end{cases}$ $\qquad x_2(t) = x(t-t_0)$

The output corresponding to x_2 is

$$y_2(t) = \begin{cases} 0, & t < 0 \\ x_2(t) + x_2(t-2), & t \geq 0 \end{cases} = \begin{cases} 0, & t < 0 \\ x(t-t_0) + x(t-t_0-2), & t \geq 0 \end{cases} \neq y_1(t-t_0)$$

So the system is **time-varying**.

(3) The output corresponding to $ax_1(t) + bx_2(t)$ is

$$y(t) = \begin{cases} 0, & t < 0 \\ ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2), & t \geq 0 \end{cases} = a\begin{cases} 0, & t < 0 \\ x_1(t) + x_1(t-2), & t \geq 0 \end{cases} + b\begin{cases} 0, & t < 0 \\ x_2(t) + x_2(t-2), & t \geq 0 \end{cases} = ay_1(t) + by_2(t)$$

So the system is **linear.**

(4) It's easy to see that the system is **causal.**

(5) When $t \geq 0$, $|y(t)| = |x(t) + x(t-2)| \leq |x(t)| + |x(t-2)| \leq 2|x(t)|_{max}$.

If $x(t)$ is bounded, the output is also bounded, so the system is **stable.**

(e)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1(t-t_0) = \begin{cases} 0, & x(t-t_0) < 0 \\ x(t-t_0) + x(t-t_0-2), & x(t-t_0) \geq 0 \end{cases} \qquad x_2(t) = x(t-t_0)$

The output corresponding to x_2 is

$$y_2(t) = \begin{cases} 0, & x_2(t) < 0 \\ x_2(t) + x_2(t-2), & x_2(t) \geq 0 \end{cases} = \begin{cases} 0, & x(t-t_0) < 0 \\ x(t-t_0) + x(t-t_0-2), & x(t-t_0) \geq 0 \end{cases} = y_1(t-t_0)$$

So the system is **time-invariant**.

(3) The output corresponding to $ax_1(t) + bx_2(t)$ is

$$y(t) = \begin{cases} 0, & ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2) < 0 \\ ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2), & ax_1(t) + bx_2(t) + ax_1(t-2) + bx_2(t-2) \geq 0 \end{cases}$$

The output corresponding to $x_1(t)$ is

$$y_1(t) = \begin{cases} 0, & x_1(t) < 0 \\ x_1(t) + x_1(t-2), & x_1(t) \geq 0 \end{cases}$$

The output corresponding to $x_2(t)$ is

$$y_1(t) = \begin{cases} 0, & x_2(t) < 0 \\ x_2(t) + x_2(t-2), & x_2(t) \geq 0 \end{cases}$$

$ay_1(t) + by_2(t) \neq y(t)$, so So the system is **nonlinear.**

(4) It's easy to see that the system is **causal.**

(5) Similar to **Problem 3 (d)** , the system **is stable.**

(f)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1(t-t_0) = x(t/3 - t_0/3)$, $x_2(t) = x(t-t_0)$

The output corresponding to x_2 is

$$y_2(t) = x_2(t/3) = x(t/3 - t_0) \neq y_1(t-t_0)$$

So the system is **time-varying**.

(3) The output corresponding to $ax_1(t) + bx_2(t)$ is

$$y(t) = ax_1(t/3) + bx_2(t/3) = ay_1(t) + by_2(t)$$

So the system is **linear.**

(4) If $t < 0$, then $t/3 > t$, so the system is **noncausal.**

(5) $|y(t)| = |x(t/3)| \leq |x(t)|_{max}$. If $x(t)$ is bounded, the output is also bounded, so the system is **stable.**

(g)

(1) $y(t) = \lim\limits_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$, which conflict the definition of memoryless system, so the system is **with memory**.

(2) Let $y_1(t-t_0) = \frac{dx(t-t_0)}{dt}$, $x_2(t) = x(t-t_0)$

The output corresponding to x_2 is

$$y_2(t) = \frac{dx_2(t)}{dt} = \frac{dx(t-t_0)}{dt} = y_1(t)$$

So the system is **time-invariant**.

(3) The output corresponding to $ax_1(t)+bx_2(t)$ is

$$y(t)=\frac{d(ax_1(t)+bx_2(t))}{dt}=a\frac{dx_1(t)}{dt}+b\frac{dx_2(t)}{dt}=ay_1(t)+by_2(t)$$

So the system is **linear.**

(4) $y(t)=\lim\limits_{\Delta t\to 0^+}\frac{x(t+\Delta t)-x(t)}{\Delta t}$, which demonstrate the output anticipate future values of the input, so the system is **noncausal.**

(5) If $x(t)=u(t)$, then $y(t)=\delta(t)$, which is infinite, so the system is **not stable.**

Problem 4 Solution

(a)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1[n-n_0]=x[-n+n_0]$, $x_2[n]=x[n-n_0]$.

The output corresponding to x_2 is

$$y_2[n]=x_2[-n]=x[-n-n_0]\neq y_1[n-n_0]$$

So the system is **time-varying**.

(3) The output corresponding to $ax_1[n]+bx_2[n]$ is

$$y[n]=ax_1[-n]+bx_2[-n]=ay_1[n]+by_2[n]$$

So the system is **linear.**

(4) If $n<0$, then $n<-n$, so the system is **noncausal.**

(5) If the input is bounded, then the output is also bounded, so the system is **stable**.

(c)

(1) It's easy to see that the system is **memoryless.**

(2) Let $y_1[n-n_0]=(n-n_0)x[n-n_0]$, $x_2[n]=x_1[n-n_0]$

The output corresponding to x_2 is

$$y_2[n]=nx_2[n]=nx_1[n-n_0]\neq y_1[n-n_0]$$

So the system is **time-varying**.

(3) The output corresponding to $ax_1[n]+bx_2[n]$ is

$$y[n]=n(ax_1[n]+bx_2[n])=anx_1[n]+bnx_2[n]=ay_1[n]+by_2[n]$$

So the system is **linear.**

(4) It's easy to see that the system is **causal**.

(5) Let $x[n]=1$, $|y[n]|=n$, so the system is **not stable.**

(d)

(1) $y[n]=\frac{1}{2}x[n-1]+\frac{1}{2}x[-n-1]$, so the system is **with memory.**

(2) Let $y_1[n-n_0]=\frac{1}{2}x[n-n_0-1]+\frac{1}{2}x[-n+n_0-1]$, $x_2[n]=x[n-n_0]$

The output corresponding to x_2 is

$$y_2[n]=\frac{1}{2}x_2[n-1]+\frac{1}{2}x_2[-n-1]=\frac{1}{2}x_2[n-n_0-1]+\frac{1}{2}x_2[-n-1-n_0]\neq y_1[n-n_0]$$

So the system is **time-varying**.

(3) Similar to **Problem 4 Solution (a)** , the system is **linear**.

(4) If $n < -1$, then $n < -n-1$, so the system is **noncausal.**

(5) $|y[n]| = \frac{1}{2}|x[n-1] + x[-n-1]| \leq \frac{1}{2}(|x[n-1]| + |x[-n-2]|) \leq 2|x[n]|_{max}$

If $x(t)$ is bounded, the output is also bounded, so the system is **stable.**

(g)

(1) It's easy to see that the system is **with memory.**

(2) Let $y_1[n-n_0] = x[4n-4n_0+1]$, $x_2[n] = x_1[n-n_0]$

The output corresponding to x_2 is

$$y_2[n] = x_2[4n+1] = x_1[4n+1-n_0] \neq y_1[n-n_0]$$

So the system is **time-varying**.

(3) Similar to **Problem 4 Solution (a)** , the system is **linear**.

(4) If $n > 0$, then $n < 4n+1$, so the system is **noncausal.**

(5) Similar to **Problem 4 Solution (a)** , the system is **stable**.

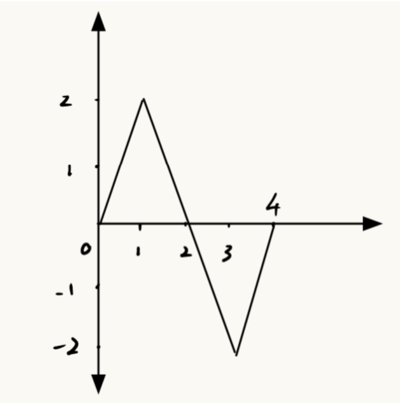
Problem 5 Solution

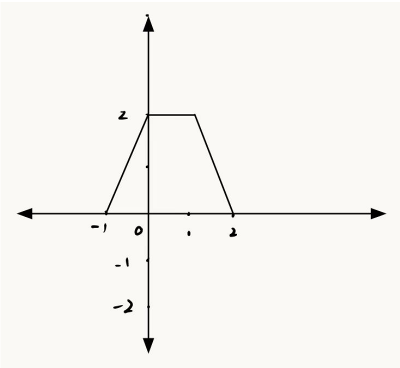
(a) For $0 \leq t < 2$, $x_2(t) = x_1(t)$, so $y_2(t) = y_1(t)$ when $0 \leq t < 2$.

For $2 \leq t \leq 4$, $x_2(t) = -x_1(t-2)$, according to linear and time-invariant properties, $y_2(t) = -y_1(t-2)$.

(b) $x_3(t) = x_1(t) + x_1(t+1)$, so

$$y_3(t) = y_1(t) + y_1(t+1)$$





Problem 6 Solution

(a) First, linearly stretch $b(t)$ to $b(\frac{t}{T})$. Then $x_c(t)$ can be represented as

$$x_c(t) = x_d[0]b(\frac{t}{T}) + x_d[1]b(\frac{t}{T}-1) + \ldots + x_d[10]b(\frac{t}{T}-10) = \sum_{i=0}^{10} x_d[i]b(\frac{t}{T}-i)$$

(b) First, linearly stretch $a(t)$ to $a(\frac{t}{T})$. Then $y_c(t)$ can be represented as

$$y_c(t) = \left\{a(\frac{t}{T})(y_d[1]-y_d[0]) + y_d[0]\right\} + \ldots + \left\{a(\frac{t}{T}-9)(y_d[10]-y_d[9]) + y_d[9]\right\} = \sum_{i=0}^{9} y_d[i][1-a(\frac{t}{T}-i)] + y_d[i+1]a(\frac{t}{T}-i)$$

(c) Similar to **(a)** and **(b)** , $\frac{dy_c(t)}{dt}$ can be represented as

$$\frac{dy_c(t)}{dt} = \frac{y_d[1]-y_d[0]}{T}b(\frac{t}{T}) + \frac{y_d[2]-y_d[1]}{T}b(\frac{t}{T}-1)+\ldots+\frac{y_d[10]-y_d[9]}{T}b(\frac{t}{T}-9) = \frac{1}{T}\sum_{i=0}^{9}b(\frac{t}{T}-i)(y_d[i+1]-y_d[i])$$

Problem 7 Solution

According to the Block Diagram

$$\alpha(X-\mathcal{R}\frac{Y}{\beta})\frac{\mathcal{R}}{1-\frac{3}{2}\mathcal{R}} = \frac{Y}{\beta}$$

What we can deduce from the equation above

$$\alpha\beta x[n-1] = \alpha y[n-2] + y[n] - \frac{3}{2}y[n-1]$$

Let $n = 1, 3$ then substituting the value of $x[n]$ and $y[n]$ into the equation

$$\begin{cases} \alpha\beta = 0+1-0 \\ 0 = \alpha + \frac{7}{4} - \frac{9}{4} \end{cases}$$

So $\alpha = \frac{1}{2},\ \beta = 2$