Lab3

Problem 1

(a) Use `roots` to find the poles and zeros of following three system functions, then plot them:

- $H(s) = \frac{s+5}{s^2+2s+3}$

  zeros: $s = -5$ **poles:** $s_1 = -1 + \sqrt{2}j, s_2 = -1 - \sqrt{2}j$

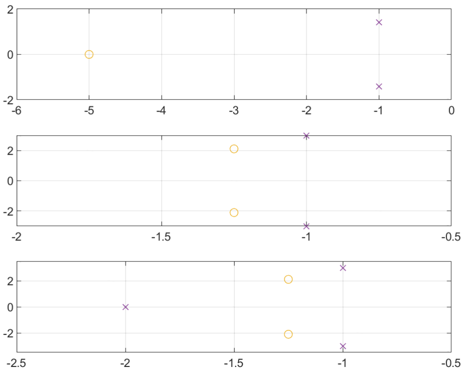
- $H(s) = \frac{2s^2+5s+12}{s^2+2s+10}$

  zeros: $s_1 = -1.25 + 2.1065j, s_2 = -1.25 - 2.1065j$ **poles:** $s_1 = -1 + 3j, s_2 = -1 - 3j$

- $H(s) = \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}$

  zeros: $s_1 = -1.25 + 2.1065j, s_2 = -1.25 - 2.1065j$ **poles:** $s_1 = -1 + 3j, s_2 = -1 - 3j, s_3 = -2$

The figure of poles and zeros of the system functions is shown below, circles represent zeros and crosses represent poles.



(b) For each of the rational expressions in **Part (a)**, determine the ROC corresponding to the stable system:

- $H(s) = \frac{s+5}{s^2+2s+3}$ $\quad ROC : Re(s) > -1$
- $H(s) = \frac{2s^2+5s+12}{s^2+2s+10}$ $\quad ROC : Re(s) > -1$
- $H(s) = \frac{2s^2+5s+12}{(s^2+2s+10)(s+2)}$ $\quad ROC : Re(s) > -1$

(c) For the new casual LTI system

$$\frac{dy(t)}{dt} - 3y(t) = \frac{d^2x(t)}{dt^2} + 2\frac{dx(t)}{dt} + 5x(t)$$

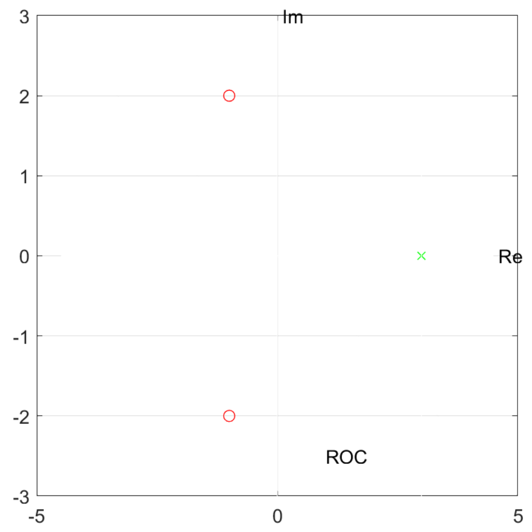
find the poles and zeros of the system and make an appropriately labeled pole-zero diagram:

From the equation above we can obtain

$$H(s) = \frac{s^2 + 2s + 5}{s - 3}$$

pole: $s = 3$ **zeros:** $s_1 = -1 + 2j, s_2 = -1 - 2j$

Use function `pzplot` to plot the figure:



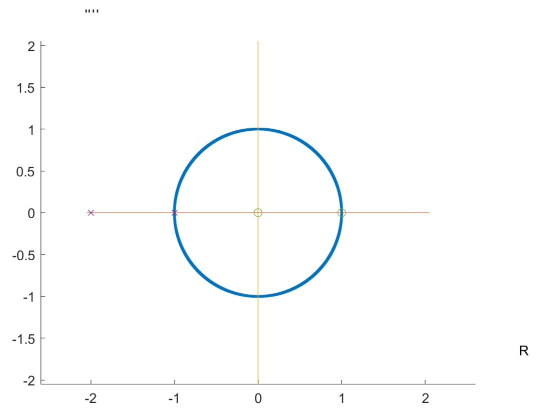
Red circles represent zeros and green cross represent the pole, which is coincide with analytical result.

Problem 2

(a) Use **dpzplot** to plot the poles and zeros for

$$H(z) = \frac{z^2 - z}{z^2 + 3z + 2}$$

The figure is shown below



We can see that the poles are $s_1 = -1, s_2 = -2$ and the zeros are $s_1 = 0, s_2 = 1$

(b) Use **dpzplot** to plot the poles and zeros for a filter which satisfies the difference equation

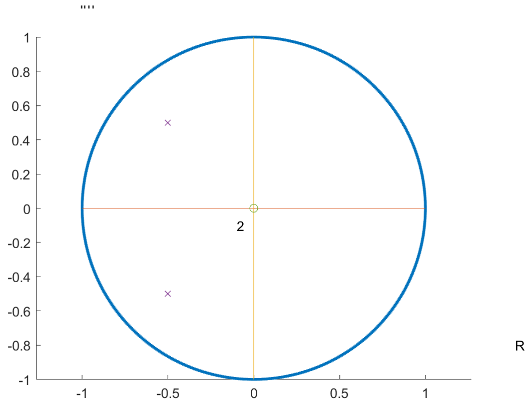
$$y[n] + y[n-1] + 0.5y[n-2] = x[n]$$

From the equation above we can obtain

$$H(z) = \frac{1}{1 + z^{-1} + 0.5z^{-2}}$$

```
a = [1 1 0.5];
b = 1;
dpzplot(b,a);
```

The figure is shown below

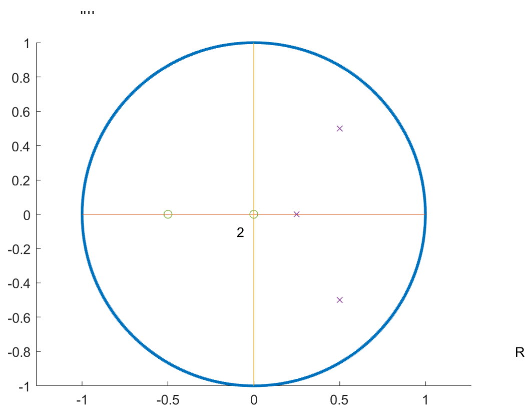


The system has zero at $s = 0$ and poles at $s_1 = -0.5 - 0.5j$, $s_2 = -0.5 + 0.5j$

(c) Use **dpzplot** to plot the poles and zeros for a filter which satisfies the difference equation

$$y[n] - 1.25y[n-1] + 0.75y[n-2] - 0.125y[n-3] = x[n] + 0.5x[n-1]$$

```
a = [1 -1.25 0.75 -0.125];
b = [1,0.5];
dpzplot(b,a)
```



Problem 3

(a) If $Hc(s)$ is the system function of Eq.(1) and $Hac(s)$ is the system function of Eq.(2) , how are the poles of these tow system functions related? How is $Hc(s)$ ralated to $Hac(s)$?

$$\sum_{k=0}^{K} a_k \frac{d^k y(t)}{dt^k} = \sum_{m=0}^{M} b_m \frac{d^k x(t)}{dt^m} \tag{1}$$

$$\sum_{k=0}^{K} (-1)^k a_k \frac{d^k y(-t)}{dt^k} = \sum_{m=0}^{M} (-1)^m b_m \frac{d^k x(-t)}{dt^m} \tag{2}$$

From the property of Laplace transform, $H_c(s) = H_{ac}(-s)$

so if s_0 is the pole of the $Hc(s)$, then $-s_0$ is the pole of $H_{ac}(s)$

(b) $\frac{dy(t)}{dt} + 2y(t) = x(t)$

Determine $H(s)$, and all of the possible ROC of $H(s)$. For each ROC, determine the impulse response of the corresponding LTI system.

$$H(s) = \frac{1}{s+2}$$

1. $ROC: Re(s) > -2$, the impulse response is $e^{-2t}u(t)$
2. $ROC: Re(s) < -2$, the impulse response is $-e^{-2t}u(-t)$

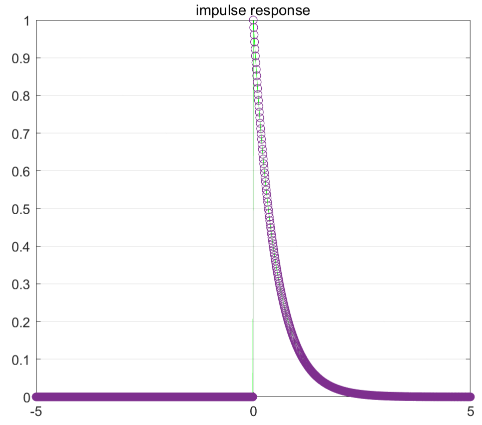
(c) For each ROC, determine auxiliary condition for the differential equation.

1. $ROC: Re(s) > -2$, the auxiliary conditions are initial rest conditions
2. $ROC: Re(s) < -2$, the auxiliary conditions are final rest conditions

(d) For the casual system, use **impulse** to verify the analytic expression.

```
a = [1,2];
b = 1;

t = -5:0.01:5;
ts = 0:0.01:5;

h_ana = exp(-2*t).*(t>=0);
h = impulse(b,a,ts);
h = [zeros(500, 1); h]; % to assure the length of h and t are the same
```

Run the `.m` file we can obtain the figure



We can see that the two curves are precisely coincide with each other.

(e) Repeat **(d)** for the anticausal system.

```
h_ana = -exp(-2*t).*(t<=0);
```

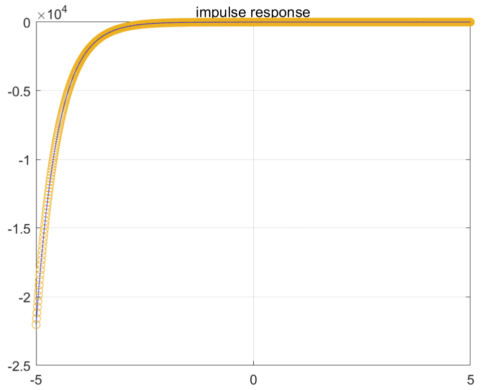
According to (a), we have to update the value of vector a

```
a = [-1,2];
b = 1;
```

We also need to flip the output h to obtain the correct result

```
plot(t,flip(h),'r');
```

Finally we can obtain the figure



We can see that the two curves are precisely coincide with each other.

(f) Analytically calculate the output of the anticausal LTI system when the input is $x(t) = e^{5t/2}u(-t)$

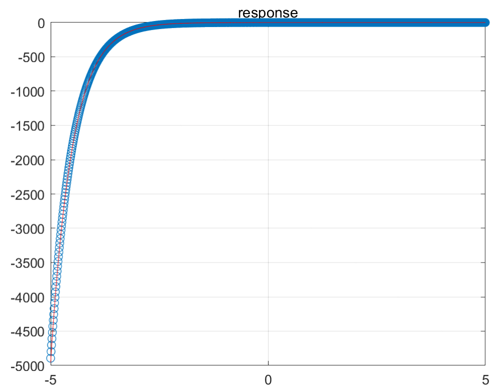
$$X(s) = -\frac{1}{s-2.5} \qquad Y(s) = H(s)X(s) = -\frac{1}{(s+2)(s-2.5)}$$

So $y(t) = \frac{2}{9}(e^{2.5t} - e^{-2t})u(-t)$

(g) Use **lsim** to verify the output of the anticausal LTI system derived in **(f)** at the time samples t.

```
a = [-1,2];
b = 1;

t = -5:0.01:5;
t_reversed = 5:0.01:-5;

x = exp(5*t/2).*(t<=0);
x_reversed = exp(5*(-t)/2).*(t>=0);

y_ana = 2/9*(-exp(-2*t)+exp(5/2*t)).*(t<=0);
y = lsim(b,a,x_reversed,t);

plot(t,y_ana,'o');
hold on;
plot(t,flip(y),'r');
title('impulse response');
grid on;
```

Run the `.m` file we can obtain the figure



We can see that the two curves are precisely coincide with each other.

(h)

$$\frac{d^3y(t)}{dt^3}+\frac{d^2y(t)}{dt^2}+24\,\frac{dy(t)}{dt}-26\,y(t)=\frac{d^2x(t)}{dt^2}+7\,\frac{dx(t)}{dt}+21\,x(t)\,. \qquad (9.14)$$

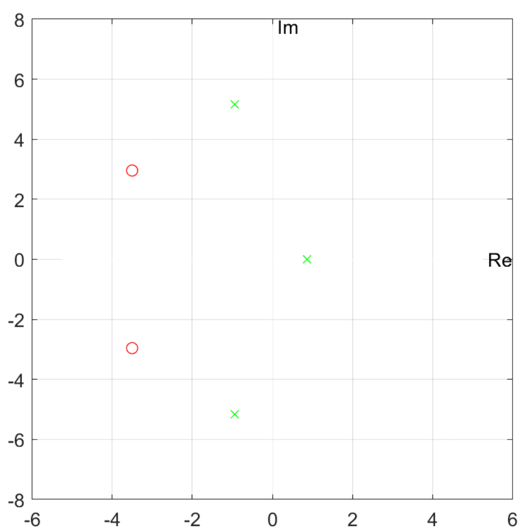
Determine the system function $H(s)$ associated with Eq.(9.14) and plot the poles and zeros. Determine all of the possible ROC. For which ROC is the system stable?

From the equation above we can obtain

$$H(s)=\frac{s^2+7s+21}{s^3+s^2+24s-26}$$

1. $ROC: Re(s)>1$
2. $ROC: Re(s)<-1$
3. $ROC: -1<Re(s)<1$, for which the system is stable.

The figure is shown below



(i) Use `residue` to determine the partial fraction expansion of $H(s)$. For each ROC, analytically determine the associated impulse response.

```
b = [1 7 21];
a = [1 1 24 -26];
[r,p,k] = residue(b,a)
```

the values of $[r, p, k]$ are demonstrated below

```
r =

   0.0000 - 0.5000i
   0.0000 + 0.5000i
   1.0000 + 0.0000i


p =

  -1.0000 + 5.0000i
  -1.0000 - 5.0000i
   1.0000 + 0.0000i


k =

     []
```

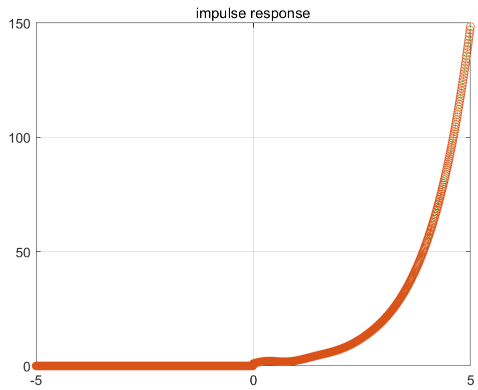
$$H(s) = \frac{1}{s-1} + \frac{-0.5j}{s-(-1+5j)} + \frac{0.5j}{s-(-1-5j)}$$

1. $ROC: Re(s) > 1, h(t) = [e^t - e^{-t} sin(5t)]u(t)$
2. $ROC: Re(s) < -1, h(t) = -[e^t - e^{-t} sin(5t)]u(-t)$
3. $ROC: -1 < Re(s) < 1, h(t) = -e^t u(-t) + e^{-t} sin(5t)u(t)$

(j) For casual system ,use **impulse** to verify the analytic expression. For each ROC, determine auxiliary condition.

Causal system correspond to case 1 in **(i)**, of which the auxiliary conditions are initial rest conditions.

```
a = [1 1 24 -26];
b = [1 7 21];
t = -5:0.01:5;
ts = 0:0.01:5;

h_ana = (exp(t)-exp(-t).*sin(5*t)).*(t>=0);
h = impulse(b,a,ts);
h = [zeros(500,1);h];
```

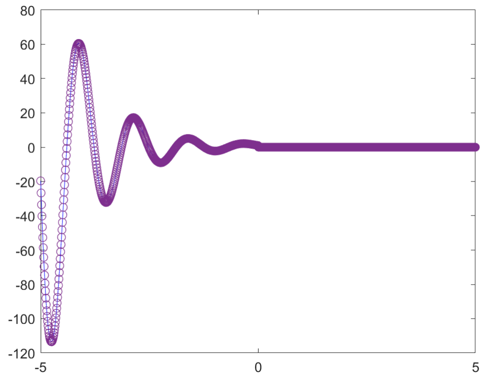


We can see that the two curves are precisely coincide with each other.

(k) Repeat **(j)** for the anticasual system.

Anticausal system correspond to case 2 in **(i)**, of which the auxiliary conditions are final rest conditions. Compared with **(j)**, we need to update vector **a**,**b** and flip **h**

```
a = [-1 1 -24 -26];
b = [1 -7 21];
t = -5:0.01:5;
ts = 0:0.01:5;

h_ana = (exp(t)-exp(-t).*sin(5*t)).*(t<=0);
h = impulse(b,a,ts);
h=[zeros(500,1);h];


plot(t,flip(h),'b');
hold on
plot(t,h_ana,'o');
```



(l) Decompose $H(s) = H_1(s) + H_2(s)$.

Since $H(s) = \frac{1}{s-1} + \frac{-0.5j}{s-(-1+5j)} + \frac{0.5j}{s-(-1-5j)}$, so $H_1(s) = \frac{-0.5j}{s-(-1+5j)} + \frac{0.5j}{s-(-1-5j)} = \frac{5}{s^2+2s+26}$, $H_2(s) = \frac{1}{s-1}$

- For $H_1(s)$, $ROC: Re(s) < 1$
- For $H_2(s)$, $ROC: Re(s) > -1$

(m) Determine $h_1(t)$ and $h_2(t)$.

1. $h_1(t) = e^{-t}sin(5t)u(t)$
2. $h_2(t) = -e^t u(-t)$

(n) Differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 26y(t) = 5x(t)$$

The auxiliary conditions are initial rest conditions.

(o) Differential equation

$$\frac{dy(t)}{dt} - y(t) = x(t)$$

The auxiliary conditions are final rest conditions.

(p)

```
a1 = [1 2 26];
b1 = 5;
a2 = [-1-1];
b2 = 1;

t = -10:0.01:10;
ts = 0:0.01:10;

x1 = [ones(201,1);zeros(800,1)];
y1 = lsim(b1,a1,x1,ts);
y1 = s[zeros(1000,1);y1];

x2 = [ones(301,1);zeros(700,1)];
y2 = lsim(b2,a2,x2,ts);
y2 = [zeros(1000,1);y2];
y2 = flip(y2)

y = y1 + y2;
```

```
subplot(3,1,1)
plot(t,y1)
title('y1')
subplot(3,1,2)
plot(t,y2)
title('y2')
subplot(3,1,3)
plot(t,y)
title('y')
```