第一章：集合

2. (1) $\{a \mid a = a_i,\ 1 \le i \le 5 且 i \in N\}$

(2) $\{a \mid a = 2k,\ k \in N\}$

(3) $\{a \mid a = 2k,\ k \in Z_{51}\}$

6. (1) 解：$\because A = \{a, \{b\}\}$

$\therefore 2^A = \{\phi, \{a\}, \{\{b\}\}, \{a, \{b\}\}\}$

(2) 解：$2^A = \{\phi, \{\phi\}, \{a\}, \{\{a\}\}, \{\phi, a\}, \{\phi, \{a\}\}, \{a, \{a\}\}, \{\phi, a, \{a\}\}\}$

14. (2) 解：$\because A \cap C \subseteq A,\ A \subseteq B$

$\therefore A \cap C \subseteq B$

同理，$A \cap C \subseteq D$ $\quad \therefore A \cap C \subseteq B \cap D$

$\therefore$ 该论断是正确的

(4) 解：若 $B \cap D = \phi$，则显然不成立

$\therefore$ 该论断是错误的

16. (1) 解：论断错误

如 $A = \{2\}$, $B = \{1\}$，则 $A \cup B = \{1, 2\}$

$2^{A \cup B} = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$，$2^A = \{\phi, \{2\}\}$，$2^B = \{\phi, \{1\}\}$

$2^A \cup 2^B \neq 2^{A \cup B}$

(2) 论断正确：若 $m \in 2^{A \cap B}$，则 $m \subseteq A \cap B$，即 $m \subseteq A, m \subseteq B$

即 $m \in 2^A$ 且 $m \in 2^B$，故 $2^{A \cap B} \subseteq 2^A \cap 2^B$．若 $m \in 2^A \cap 2^B$，则 $m \subseteq A$ 且 $m \subseteq B$

$\therefore m \subseteq A \cap B$，即 $m \in 2^{A \cap B}$．综上：$2^{A \cap B} = 2^A \cap 2^B$

20. 解：假设 $A_1, A_2, \ldots, A_r$ 两两互斥，则此时子集数量最多

除去 ϕ，则 A_i 的子集数量为 $2^{\# A_i} - 1$

故子集总数 $N_{max} = \left(\sum_{i=1}^{r} 2^{\# A_i} - 1\right) + 1 = \sum_{i=1}^{r} 2^{\# A_i} - (r-1)$

22(2) 证明：$M_1 = A\cup B$，$M_2 = B\cup C$，$M_3 = C\cup A$

$M_1\cap M_2\cap M_3 = M_1\cap(M_2\cap M_3) = M_1\cap(M_2\cap(C\cup A))$

$= M_1\cap\big((M_2\cap C)\cup(M_2\cap A)\big) = M_1\cap(C\cup(B\cup C\cap A))$

$= (M_1\cap C)\cup(M_1\cap(B\cup C\cap A))$

$M_1\cap C = C\cap(A\cup B) = (C\cap A)\cup(B\cap C)$

$M_1\cap(B\cup C\cap A) = (A\cup B)\cap[A\cap(B\cup C)]$

$= (A\cup B)\cap\big((A\cap B)\cup(A\cap C)\big) = [(A\cup B)\cap(A\cap B)]\cup[(A\cup B)\cap(A\cap C)]$

$= (A\cap B)\cup(A\cap C)$

$\therefore M_1\cap M_2\cap M_3 = (C\cap A)\cup(B\cap C)\cup(A\cap B)\cup(A\cap C) =$ 右边，证毕

24. 证明：设 $a\in\bigcup_{i=1}^{\infty}A_i$，则 $a\le 1-\frac{1}{m}$，$m=1,2,\dots$

而 $1-\frac{1}{m}<1$．故 $a<1$，即 $a\in A_0$ $\therefore \bigcup_{i=1}^{\infty}A_i\subseteq A_0$

设 $b\in A_0$．则 $b<1$，取 $i=[\frac{1}{1-b}]+1$

则 $i>\frac{1}{1-b}$，$-\frac{1}{i}>b-1$ $\therefore 1-\frac{1}{i}>b$

即 对 $\forall b\in A_0$，$\exists i=[\frac{1}{1-b}]+1$，s.t. $b\le 1-\frac{1}{i}$

$\therefore b\in\bigcup_{i=1}^{\infty}A_i$，即 $A_0\subseteq\bigcup_{i=1}^{\infty}A_i$ $\therefore A_0=\bigcup_{i=1}^{\infty}A_i$

25. 证明：设 $A_{i_1}\cap B$，$A_{i_2}\cap B$，…，$A_{i_k}\cap B$ 为 $\{A_{ix}\cap B \mid x=1,2,\dots r\}$ 中的所有非空元素

① 对 $\forall i_m\ne i_n$，$(A_{i_m}\cap B)\cap(A_{i_n}\cap B) = A_{i_m}\cap A_{i_n}\cap B = \phi\cap B=\phi$

② $A\cap B = B\cap(A_{i1}\cup A_{i2}\cup\dots\cup A_{ir}) = (B\cap A_{i1})\cup\dots\cup(B\cap A_{ik})\cup\dots\cup(B\cap A_{ir})$

$=\left(\bigcup_{m=1}^{k}(B\cap A_{im})\right)\cup\phi = \bigcup_{m=1}^{k}(B\cap A_{im})$ 综上：结论成立