Signals and Systems – Spring 2024

Problem Set 2

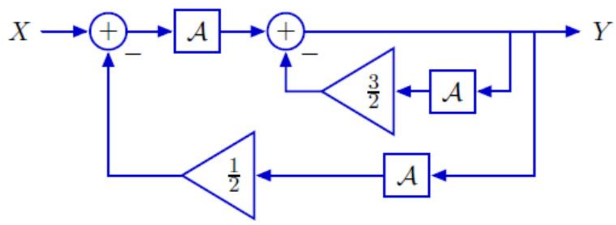
Issued: Mar. 14, 2024 Due: Mar. 21, 2024

Reading Assignments:

Supplementary notes, Chapter 1-5
Signals and Systems (OWN) Chapter 9 & 10

Problem 1 Characterizing Block Diagrams

Consider the system defined by the following block diagram:



a. Determine the system functional $H = \frac{Y}{X}$.

b. Determine the poles of the system.

c. Determine the impulse response of the system.

Problem 2 Finding A System

a. Determine the difference equation and block diagram representations for a system whose output is 10, 1, 1, 1, 1, … when the input is 1, 1, 1, 1, 1, … .

b. Determine the difference equation and block diagram representations for a system whose output is 1, 1, 1, 1, 1, . . . when the input is 10, 1, 1, 1, 1, ...

c. Compare the difference equations in parts a and b. Compare the block diagrams in parts a and b.

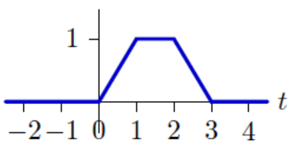
Problem 3

Determine the Laplace transforms (including the regions of convergence) of each of the following signals:

a. $x_1(t) = e^{-2(t-3)}u(t-3)$

b. $x_2(t) = |t|e^{-|t|}$

c. $x_3(t)$



Problem 4

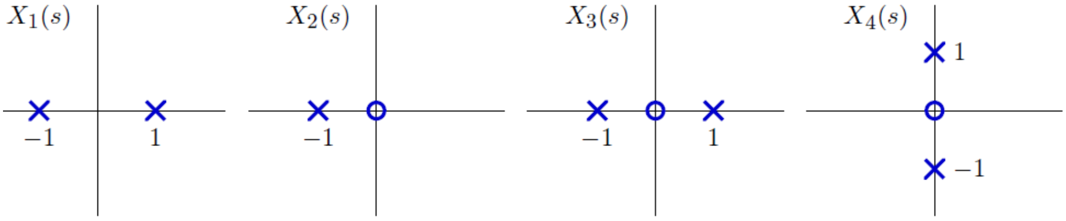
Determine and sketch all possible signals with Laplace transforms of the following forms. For each signal, indicate the associated region of convergence.

a. $X_1(s) = \dfrac{s+2}{(s+1)^2}$

b. $X_2(s) = \left(\dfrac{1-e^{-s}}{s}\right)^2$

Problem 5

Determine which of the following pole-zero diagrams could represent Laplace transforms of even functions of time. Determine expressions for the time functions of those that can represent even functions of time. For those that cannot, explain why they cannot.



How can you determine if a signal is even or not by looking at its Laplace transform and region of convergence?

Problem 6

a. Use the initial and final value theorems (where applicable) to find $x(0)$ and $x(\infty)$ for the signals with the following Laplace transforms:

1. $\dfrac{1-e^{-sT}}{s}$ 2. $\dfrac{(s+1)^2-1}{[(s+1)^2+1]^2}$

b. Find the inverse Laplace transforms for each of the previous parts and show that the time waveforms and initial and final values agree.

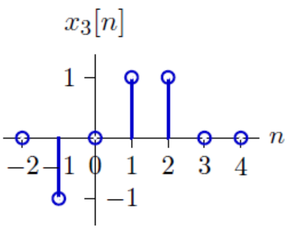
Problem 7

Determine the Z transform (including the region of convergence) for each of the following signals:

a. $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$

b. $x_2[n] = (1+n)\left(\frac{1}{3}\right)^n u[n]$

c.



Problem 8

Determine and sketch all possible signals with Z transforms of the following forms. For each signal, indicate the associated region of convergence.

a. $X_1(z) = \dfrac{1}{z(z-1)^2}$

b. $X_2(z) = \left(\dfrac{1-z^2}{z}\right)^2$

Problem 9

Let $X(z)$ represent the Z transform of $x[n]$, and let $r_0 < |z| < r_1$ represent its region of convergence (ROC).

Let $x[n]$ be represented as the sum of even and odd parts

$$x[n] = x_e[n] + x_o[n]$$

where $x_e[n] = x_e[-n]$ and $x_o[n] = -x_o[-n]$.

a. Under what conditions does the Z transform of $x_e[n]$ exist?

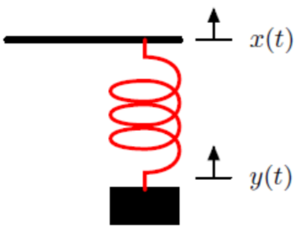
b. Assuming the conditions given in part a, find an expression for the Z transform of $x_e[n]$, including its region of convergence.

Problem 10

OWN, Problem 9.51

Problem 11 Masses and Springs, Forwards and Backwards

The following figure illustrates a mass and spring system. The input $x(t)$ represents the position of the top of the spring. The output $y(t)$ represents the position of the mass.



The mass is $M = 1\,\text{kg}$ and the spring constant is $K = 1\,\text{N/m}$. Assume that the spring obeys Hooke's law and that the reference positions are defined so that if the input $x(t)$ is equal to zero, then the resting position of $y(t)$ is also zero.

a. Determine a differential equation that relates the input $x(t)$ and output $y(t)$.

b. Calculate the step response of the system.

c. The differential equation in part a contains a second derivative (if you did part a correctly). We wish to develop a forward-Euler approximation for this derivatve. One method is to write the second-order differential equation in part a as a part of first order differential equations. Then apply the forward-Eular approximation to the first order derivatives:

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} \approx \frac{y[n+1]-y[n]}{T}.$$

Use this approach to find a difference equation to approximate the behavior of the mass and spring system. Calculate the step response of the system and compare your results to those in part b.

d. An alternative to the forward-Euler approximation is the backward-Euler approximation:

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} \approx \frac{y[n]-y[n-1]}{T}.$$

Repeat the exercise in the previous part, but using the backward-Euler approximation instead of the forward-Euler approximation.

e. The forward-Euler method approximates the second derivative at $t = nT$ as

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n+2]-2y[n+1]+y[n]}{T^2}.$$

The backward-Euler method approximates the second derivative at $t = nT$ as

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n]-2y[n-1]+y[n-2]}{T^2}.$$

Consider a compromise based on a centered approximation:

$$\left.\frac{d^2y(t)}{dt^2}\right|_{t=nT} = \frac{y[n+1]-2y[n]+y[n-1]}{T^2}.$$

Use this approximation to determine the step response of the system. Compare your results to those in the two previous parts of this problem.