15. (1) 解: $P(N(1)=1, N(4)\geq 1) = P(N(1)=1)\,P(N(4)-N(1)\geq 1 \mid N(1))$

$N(1) \sim \pi(10)$, $N(4)-N(1) \sim \pi(30)$

$\therefore P(N(1)=1, N(4)\geq 1) = e^{-10}\cdot 10 \cdot (1-e^{-30}) = 10e^{-10} - 10e^{-40}$

(2) 解: $P(3 < W_3 \leq 4 \mid W_1=1, W_2=2) = P(N(3)-N(2)=0, N(4)-N(3)\geq 1)$

$= e^{-10}(1-e^{-10})$

(3) 解: 理赔钱数超过5500概率 $p = \frac{10000-5500}{9000} = \frac{1}{2}$

$\lambda' = \frac{1}{2}\times 10 = 5$

$P(W_1 \leq t) = P(N(t)\geq 1) = 1-e^{-5t}$

17.

	不足1kg	达到1kg	总
鲫鱼	$N_{11}(t)$	$N_{12}(t)$	$N_{13}(t)$
鳊鱼	$N_{21}(t)$	$N_{22}(t)$	$N_{23}(t)$
总	$N_{31}(t)$	$N_{32}(t)$	$N_{33}(t)$

(1) 解: $N_{33}(1) \sim \pi(3)$

$P(N(1)=2) = \frac{e^{-3}\cdot 3^2}{2!} = \frac{9}{2}e^{-3}$

(2) 解: $N_{31}(1) \sim \pi(\frac{3}{2})$, $N_{32}(1) \sim \pi(\frac{3}{2})$

$\therefore P(N_{31}(1)=2, N_{32}(1)=2) = \left(\frac{e^{-\frac{3}{2}}(\frac{3}{2})^2}{2!}\right)^2 = \frac{81}{64}e^{-3}$

(3) 解: $P(N_{32}(1)=1, N_{32}(2)-N_{32}(1)=1, N_{31}(2)=0)$

$= e^{-3}\cdot e^{-3} = e^{-6}$

(4) 解: $P(N_{12}(2)=2 \mid N_{33}(2)=2) = \frac{P(N_{12}(2)=2)P(N_{22}(2)=0)P(N_{31}(2)=0)}{P(N_{33}(2)=2)}$

$= \frac{\frac{e^{-2}2^2}{2!}\, e^{-1}\cdot e^{-3}}{\frac{e^{-6}6^2}{2!}} = \frac{1}{9}$

18. (1) 解：强度 $m(2) = \int_0^2 \lambda(t)\,dt = \int_0^2 t\,dt = 2$

$\therefore P(N(2)=3) = \frac{e^{-2} 2^3}{3!} = \frac{4}{3}e^{-2}$

(2) 解：$m(1) = \int_0^1 t\,dt = \frac{1}{2}$，

$N(2)-N(1)$ 服从强度为 $\int_1^2 t\,dt = \frac{3}{2}$ 的泊松分布

$\therefore P = \frac{e^{-\frac{1}{2}}(\frac{1}{2})^2}{2!} \cdot \frac{e^{-\frac{3}{2}}(\frac{3}{2})^2}{2!} = \frac{9}{64}e^{-2}$

(3) 解：$N(2)$ 服从强度为 $\int_0^2 t\,dt = 2$ 的泊松分布

$\therefore P(N(2)=4) = \frac{e^{-2}\ 2^4}{4!} = \frac{2}{3}e^{-2}$

$\therefore$ 原式 $= \frac{\frac{9}{64}e^{-2}}{\frac{2}{3}e^{-2}} = \frac{27}{128}$

21. (1) 解：原式 $= P(B(3.6)-B(2.39) \le 1.1) = \Phi(\frac{1.1-0}{1.1}) = \Phi(1)$

(2) 解：$Cov(B(8)-B(4), B(6)) = C_B(8,6) - C_B(4,6)$

$= \min\{8,6\} - \min\{4,6\} = 6-4 = 2$

(3) 解：$D(2B(1)+B(2)) = 4D(B(1)) + D(B(2)) + C_B(1,2) = 4\times1+2+4\times1 = 10$

22. 证明：$\because B(t)$ 为标准布朗运动 $\therefore \{(t+1)B(\frac{1}{t+1}); t \ge -1\}$ 是布朗运动

$\therefore \{X(t); t\ge 0\}$ 是布朗运动 $D(X(t)) = D[(t+1)B(\frac{1}{t+1}) - B(1)]$

$= (t+1)^2 \cdot \frac{1}{t+1} - 1 = t$ $\quad\therefore \sigma = 1$ $\therefore \{X(t); t\ge 0\}$ 为标准布朗运动

26. 解：$u_W(t) = E(e^{B(t)}) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}} e^x dx$

$= e^{\frac{t}{2}} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{1}{2}(\frac{x}{\sqrt{t}}-1)^2} dx = e^{\frac{t}{2}}$

$D_W(t) = E(e^{2B(t)}) - E^2(e^{B(t)}) = e^{2t} - e^t$

27. (1) 解：令 $\widetilde{B}(t) = tB(\frac{1}{t})$，则原式等价于

$P(\widetilde{B}(10) \ge 15 \mid \widetilde{B}(6) = 12, \widetilde{B}(4) = 9.6) = P(\widetilde{B}(10) - \widetilde{B}(6) \ge 3) = 1-\Phi(1.5)$

(2) 解: $\widetilde{B}(10) = \widetilde{B}(10) - \widetilde{B}(6) + 12 \sim N(12, 4)$

$\therefore \quad B(\frac{1}{10}) = \frac{1}{10}\widetilde{B}(10) \sim N(1.2, 0.04)$

29. (1) 解: $P(|B(t)| \le x) = P(-x \le B(t) \le x) = 2\Phi(\frac{x}{\sqrt{t}}) - 1$

(2) 解: $P(\max\limits_{0\le s\le t} B(s) - B(t) \le x) = P(\min\limits_{0\le s\le t} B(t) - B(s) \ge -x)$

$= P(\min\limits_{0\le s\le t} B(t) - B(s) \le x) = 2\Phi(\frac{x}{\sqrt{t}}) - 1$