1. Prove that the Q of the circuit shown in Fig. 7. 1 is given by Eq. (7.1).

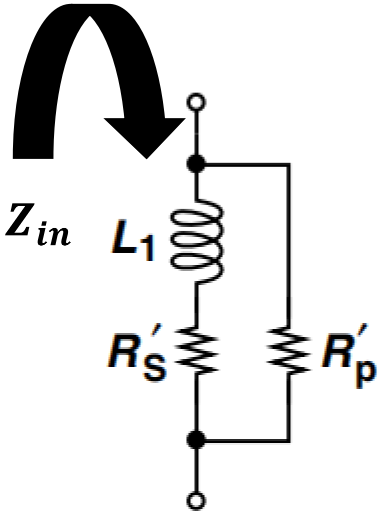


Fig. 7.1

$$Q = \frac{L_1 \omega R_p'}{L_1^2 \omega^2 + R_S'(R_S' + R_p')}.$$

Eq. (7.1)

$$Z_{in}(s) = (sL_1 + R_s') \parallel R_p' = \frac{sL_1 R_p' + R_s' R_p'}{sL_1 + R_s' + R_p'}$$

$$Z_{in}(j\omega) = \frac{R_s' R_p' + j\omega L_1 R_p'}{R_s' + R_p' + j\omega L_1} = \frac{(R_s' R_p' + j\omega L_1 R_p')(R_s' + R_p' - j\omega L_1)}{\left(R_s' + R_p'\right)^2 - \omega^2 L_1^2}$$

$$Q \triangleq \frac{Im(Z_{in})}{Re(Z_{in})} = \frac{\omega L_1 R_p'\left(R_s' + R_p'\right) - \omega L_1\left(R_s' R_p'\right)}{(R_s')^2 R_p' + \left(R_p'\right)^2 R_s' + \omega^2 L_1^2 R_p'} = \frac{L_1 \omega R_p'}{L_1^2 \omega^2 + R_s'(R_s' + R_p')}$$

2. **Repeat Example 7.19 for four turns.**

Example 7.19

Estimate the equivalent lumped interwinding capacitance of the three-turn spiral shown in Fig. 7.39(a).

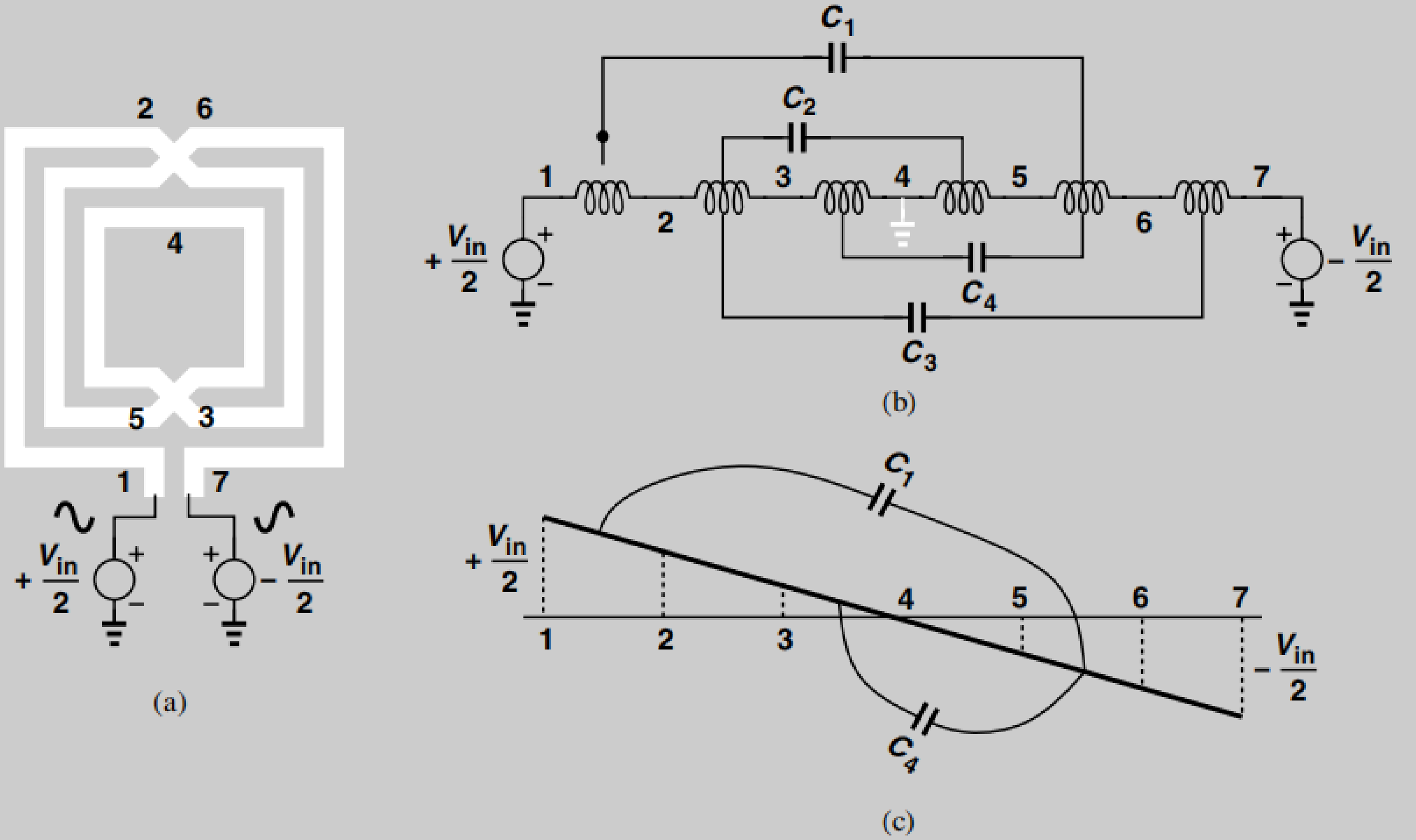
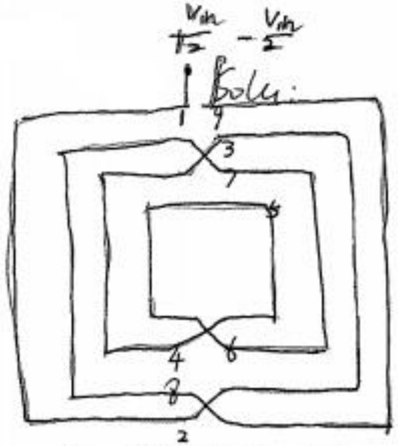
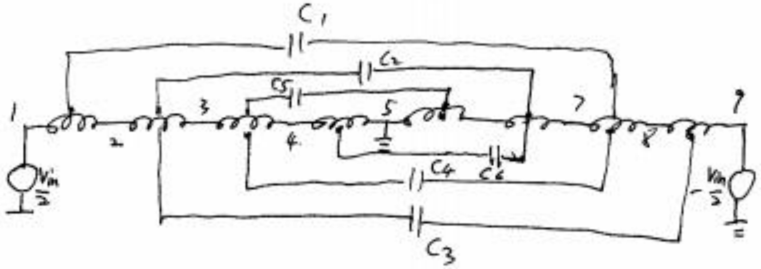
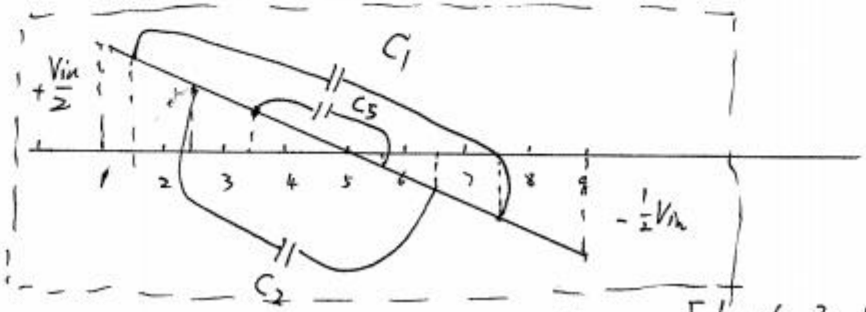


Figure 7.39 *(a) Three-turn symmetric inductor, (b) equivalent circuit, (c) voltage profile along the ladder.*

Solu:







$C_1 \& C_3$ sustains $\frac{6}{8}V_{in}$

$C_2 \& C_4$ sustain $\frac{4}{8}V_{in}$

$C_5 \& C_6$ sustain $\frac{2}{8}V_{in}$ $\Rightarrow$

$$E_{tot} = 2\cdot\left[\frac{1}{2}C\left(\frac{6}{8}V_{in}\right)^2 + \frac{1}{2}C\cdot\left(\frac{4}{8}V_{in}\right)^2 + \frac{1}{2}C\left(\frac{2}{8}V_{in}\right)^2\right]$$

$$= \frac{7}{8}\cdot C\cdot V_{in}^2$$

$\because$ $C_1 + C_2 + \cdots + C_6 = C_{tot} \Rightarrow C = \frac{C_{tot}}{6}$

$\therefore$ $E_{tot} = \frac{7}{8}\cdot\frac{C_{tot}}{6}\cdot V_{in}^2$

$$\Rightarrow \boxed{C_{eq} = \frac{7}{24}C_{tot}.}$$