1. $A_{z1} = C_1 \frac{e^{-jkr}}{r}$, $A_{z2} = C_2 \frac{e^{jkr}}{r}$

$$\therefore \frac{dA_{z1}}{dr} = C_1 e^{-jkr} \frac{(-jkr-1)}{r^2} = -jk A_{z1} - \frac{A_{z1}}{r} \quad ①$$

$$\frac{d^2 A_{z1}}{dr^2} = -jk \frac{dA_{z1}}{dr} - \frac{\frac{dA_{z1}}{dr} r - A_{z1}}{r^2} = -jk\left(-jkA_{z1} - \frac{A_{z1}}{r}\right) + \frac{jkA_{z1} + \frac{A_{z1}}{r}}{r}$$

$$+ \frac{A_{z1}}{r^2} = A_{z1}\left(-k^2 + \frac{2jk}{r} + \frac{2}{r^2}\right) \quad ②$$

$$\frac{dA_{z2}}{dr} = C_2 e^{jkr} \frac{jkr-1}{r^2} = jk A_{z2} - \frac{A_{z2}}{r} \quad ③$$

$$\frac{d^2 A_{z2}}{dr^2} = jk \frac{dA_{z2}}{dr} - \frac{\frac{dA_{z2}}{dr} r - A_{z2}}{r^2} = \left(jk - \frac{1}{r}\right)\left(jkA_{z2} - \frac{A_{z2}}{r}\right) + \frac{A_{z2}}{r^2}$$

$$= \left(-k^2 - \frac{2jk}{r} + \frac{2}{r^2}\right) A_{z2} \quad ④$$

将①②代入方程3-34，左边 $= A_{z1}\left(-k^2 + \frac{2jk}{r} + \frac{2}{r^2} - \frac{2jk}{r} - \frac{2}{r^2} + k^2\right) = 0$

同理,将③④代入方程3-34, 左边$=0$

$\therefore$ 3-35, 3-36是方程3-34的解

2. $\nabla \times \vec{E}_1 = -\vec{H}_1 - \vec{M}_1$, $\nabla \times \vec{H}_1 = \vec{E}_1 + J_1$

$\nabla \times \vec{E}_2 = -\vec{H}_2 - \vec{M}_2$, $\nabla \times \vec{H}_2 = \vec{E}_2 + \vec{J}_2$

$\vec{H}_2 \cdot \nabla \times \vec{E}_1 = -\vec{H}_2 \cdot \vec{H}_1 - \vec{H}_2 \cdot \vec{M}_1$

$\vec{E}_1 \cdot \nabla \times \vec{H}_2 = \vec{E}_1 \cdot \vec{E}_2 + \vec{E}_1 \cdot \vec{J}_2$

$\therefore \vec{E}_1 \cdot \nabla \times \vec{H}_2 - \vec{H}_2 \cdot \nabla \times \vec{E}_1 = \vec{E}_1 \cdot \vec{E}_2 + \vec{H}_2 \cdot \vec{H}_1 + \vec{E}_1 \cdot \vec{J}_2 + \vec{H}_2 \cdot \vec{M}_1$

$\therefore -\nabla \cdot (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) = \vec{E}_1 \cdot \vec{J}_1 + H_2 \cdot \vec{M}_1 - \vec{E}_2 \cdot \vec{J}_1 - \vec{H}_1 \cdot \vec{M}_2$ (3-60)

对3-60两边作体积分,由散度定理

(3-61)

$$-\oiint_S (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot d\vec{s} = \iiint_V (\vec{E}_1 \cdot \vec{J}_1 + H_2 \cdot \vec{M}_1 - \vec{E}_2 \cdot \vec{J}_1 - \vec{H}_1 \cdot \vec{M}_2)\, dV$$

4.1

a. $\sin\varphi = \sqrt{1-\cos^2\varphi} = \sqrt{1-(\sin\theta\cos\varphi)^2}$

$$E_\varphi = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cdot \sin\varphi = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \sqrt{1-(\sin\theta\cos\varphi)^2}$$

$$H_x = j \frac{kI_0 l e^{-jkr}}{4\pi r} \sin\varphi = \frac{E_\varphi}{\eta}$$

b. $U = U_0(1-\sin^2\theta\cos^2\varphi)$

$$P_{rad} = U_0 \int_0^{2\pi} \int_0^{\pi} (1-\sin^2\theta\cos^2\varphi)\sin\theta \, d\theta \, d\varphi$$

$$= U_0 \int_0^{2\pi} \int_0^{\pi} (\sin\theta - \cos^2\varphi\sin^3\theta) \, d\theta \, d\varphi = \frac{8}{3}\pi U_0$$

$\therefore D_0 = \frac{4\pi U_0}{P_{rad}} = \frac{4}{8/3} = 1.5$

4.5

(a) $\varphi = 0°$，则 $E_\varphi = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \sqrt{1-\sin^2\theta} = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos\theta$

$\therefore E_\varphi$ 只有 $\hat{\theta}$ 方向

4.29

(a) 对于半波偶极子天线，$P_{ad} = P_{in} = 1w$，$D_0 = 1.643$

$\therefore U_0 = \frac{P_{ad}}{4\pi} = \frac{1}{4\pi}$，$U_{max} = U_0 D_0 = 0.137\ w/rad$

(b) $W_{max} = \frac{U_{max}}{r^2} = \frac{0.137}{(5\times10^3)^2} = 5.48\times10^{-9}\ w/m^2$