Low-Noise Amplifiers (1/2)

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Noise Figure

- The Noise Figure (NF) of an amplifier is a measure of the degradation of the SNR

$$F = \frac{SNR_i}{SNR_o}$$

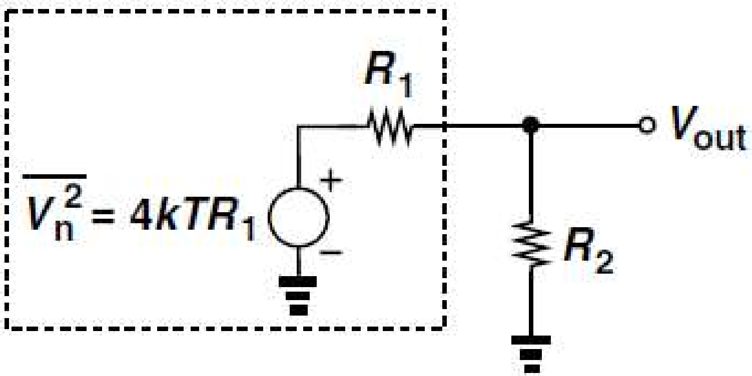
$$NF = 10 \cdot \log(F)\ (\mathrm{dB})$$

- The noise figure is measured (or calculated) by specifying a standard input noise level through the source resistance R_S and the temperature
- For RF communication systems, this is usually specified as R_S = 50 and T = 300 K.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Noise Figure

❑The Noise Figure (NF) of an amplifier is a measure of the degradation of the SNR



$$P_{R2} = \frac{\overline{V_{out}^2}}{R_2}$$

$$= \overline{V_n^2}\left(\frac{R_2}{R_1 + R_2}\right)^2 \frac{1}{R_2}$$

$$= 4kT\frac{R_1 R_2}{(R_1 + R_2)^2}.$$

❑If the input impedance of LNA is matched to the antenna, then

$$R_2 = R_1 \qquad P_{R2,max} = kT$$

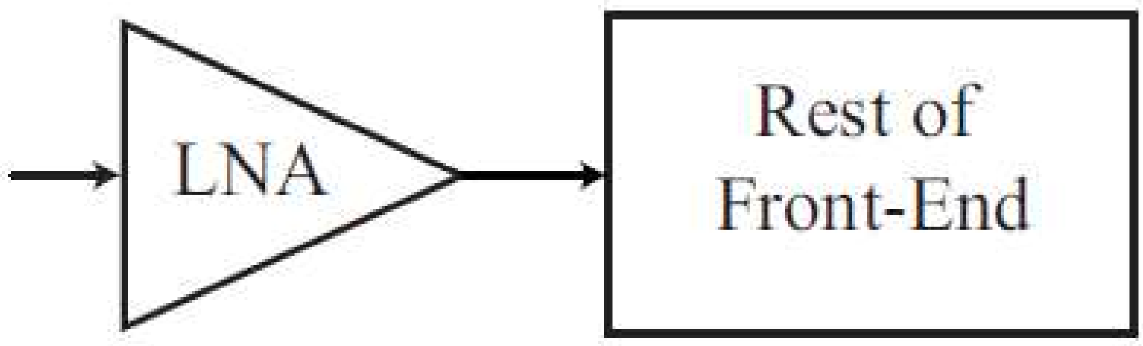
❑It can proved that kT=-174 dBm/Hz at T=300 K

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Noise Figure (NF)

❑The noise figure of the LNA directly adds to that of the receiver.

❑For a typical RX noise figure of 6 to 8 dB, it is expected that the antenna switch or duplexer contributes about 0.5 to 1.5 dB, the LNA about 2 to 3 dB, and the remainder of the chain about 2.5 to 3.5 dB.



[B. Razavi, RF Microelectronics]

Noise Figure of an Amplifier

❑Suppose an amplifier has a gain G and apply the definition of NF

$$SNR_i = \frac{P_{sig}}{N_s}$$

$$SNR_o = \frac{GP_{sig}}{GN_s + N_{amp,o}}$$

❑The term $N_{amp,o}$ is the total output noise due to the amplifier in absence of any input noise.

$$SNR_o = \frac{P_{sig}}{N_s + \frac{N_{amp,o}}{G}}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Noise Figure of an Amplifier

- Let $N_{amp,i}$ denote the total input referred noise of the amplifier

$$SNR_o = \frac{P_{sig}}{N_s + N_{amp,i}}$$

- The noise figure is therefore

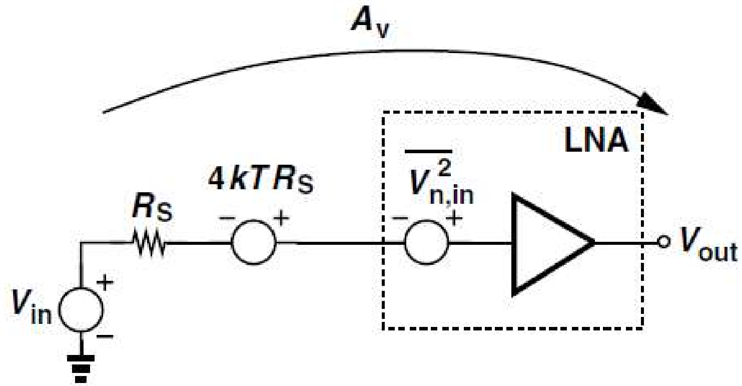
$$F = \frac{SNR_i}{SNR_o} = \frac{P_{\cancel{sig}}}{N_s} \times \frac{N_s + N_{amp,i}}{P_{\cancel{sig}}}$$

$$F = 1 + \frac{N_{amp,i}}{N_s} \geq 1$$

- All amplifiers have a noise figure ≥ 1. Any real system degrades the SNR since all circuit blocks add additional noise.

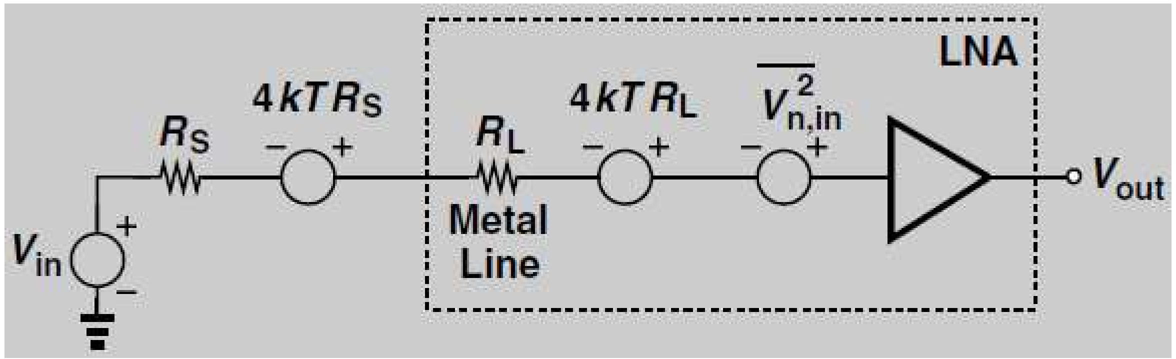
[Ali Niknejad, EE142&EE242 of UC Berkeley]

Noise Figure of LNA



$$\mathrm{NF} = 1 + \frac{\overline{V_{n,in}^2}}{4kTR_S}$$

- A noise figure of 2 dB with respect to a source impedance of 50 translates to $\sqrt{\overline{V_{n,in}^2}} = 0.696\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$, an extremely low value.
- The metal line resistance R_L contribute noise:



$$\mathrm{NF}_{\mathrm{tot}} = 1 + \frac{\overline{V_{n,in}^2} + 4kTR_L}{4kTR_S}$$

$$= \mathrm{NF}_{\mathrm{LNA}} + \frac{R_L}{R_S}$$

[B. Razavi, RF Microelectronics]

LNA Gain

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1} \cdots A_{P(m-1)}}$$

❑**The gain of the LNA must be large enough to minimize the noise contribution of subsequent stages.**

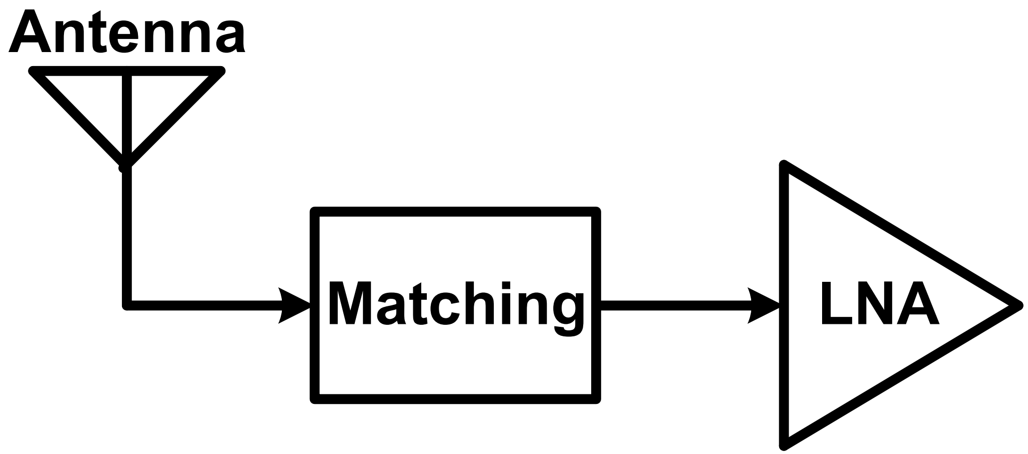
❑**For the linearity (IP3), a high gain leads to lower IP3:**

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{IP3,3}^2} + \cdots$$

❑**The choice of this gain leads to a compromise between the noise figure and the linearity of the receiver as a higher gain makes the nonlinearity of subsequent stages more pronounced.**

[B. Razavi, RF Microelectronics]

LNA Input



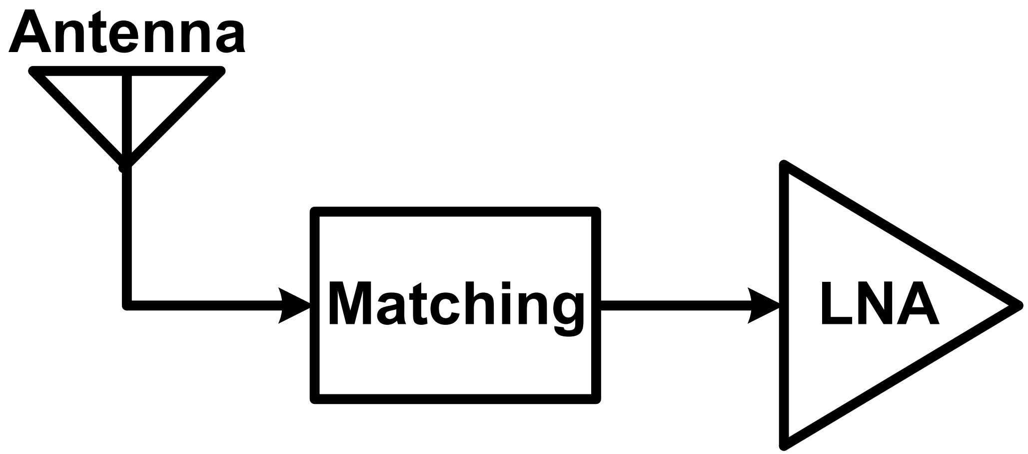
❑Considering the LNA as a voltage amplifier, we may expect that its input impedance must ideally be infinite.

❑From the noise point of view, we may precede the LNA with a transformation network to obtain minimum NF.

❑From the signal power point of view, we may realize conjugate matching between the antenna and the LNA.

[B. Razavi, RF Microelectronics]

LNA Input



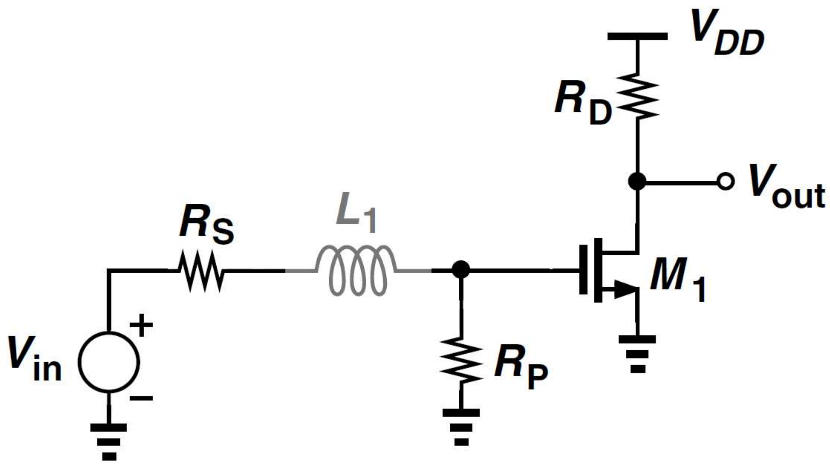
- If the load impedance of the antenna deviates from 50 Ohm significantly, then the signal transmission suffers from uncharacterized loss.
- The input "return loss" is defined as the reflected power divided by the incident power:

$$\Gamma = \left| \frac{Z_{in} - R_S}{Z_{in} + R_S} \right|^2$$

[B. Razavi, RF Microelectronics]

LNA Input

- The key point in the foregoing study is that the LNA must provide a 50 Ohm input resistance without the thermal noise of a physical 50 Ohm resistor. This becomes possible with the aid of active devices.



[B. Razavi, RF Microelectronics]

Stability

- If the user of a cell phone wraps his/her hand around the antenna, the antenna impedance changes. For this reason, the LNA must remain stable for all source impedances at all frequencies.
- A parameter often used to characterize the stability of circuits is the "Stern stability factor," defined as

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{21}||S_{12}|}$$

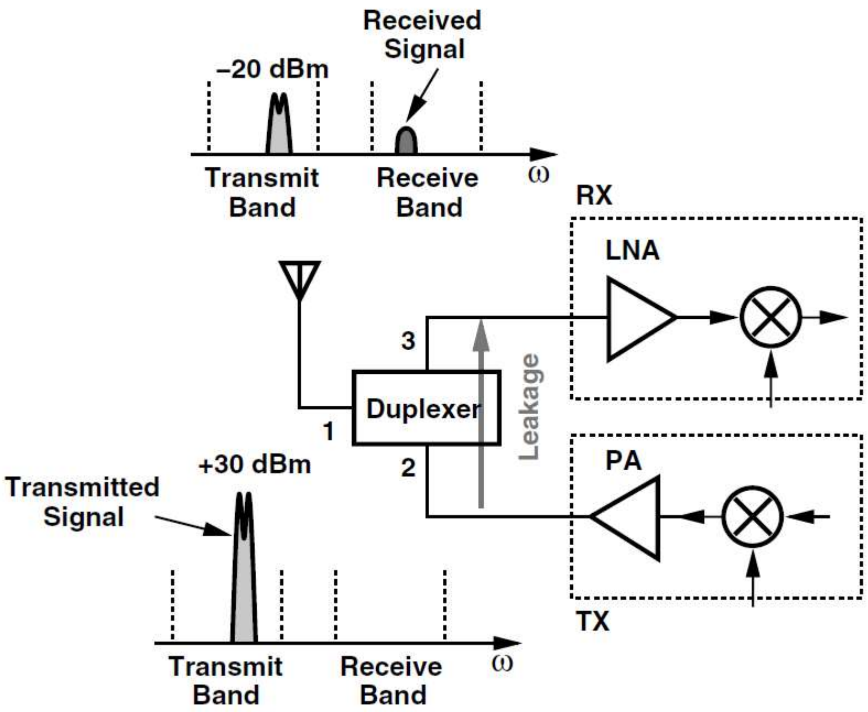
- where $\Delta = S_{11}S_{22} - S_{12}S_{21}$. If $K > 1$ and $\Delta < 1$, then the circuit is unconditionally stable, i.e., it does not oscillate with any combination of source and load impedances

[B. Razavi, RF Microelectronics]

Linearity

❑In most applications, the LNA does not limit the linearity of the receiver. We design LNAs with little concern for their linearity.

❑An exception to the above rule arises in "full-duplex" systems

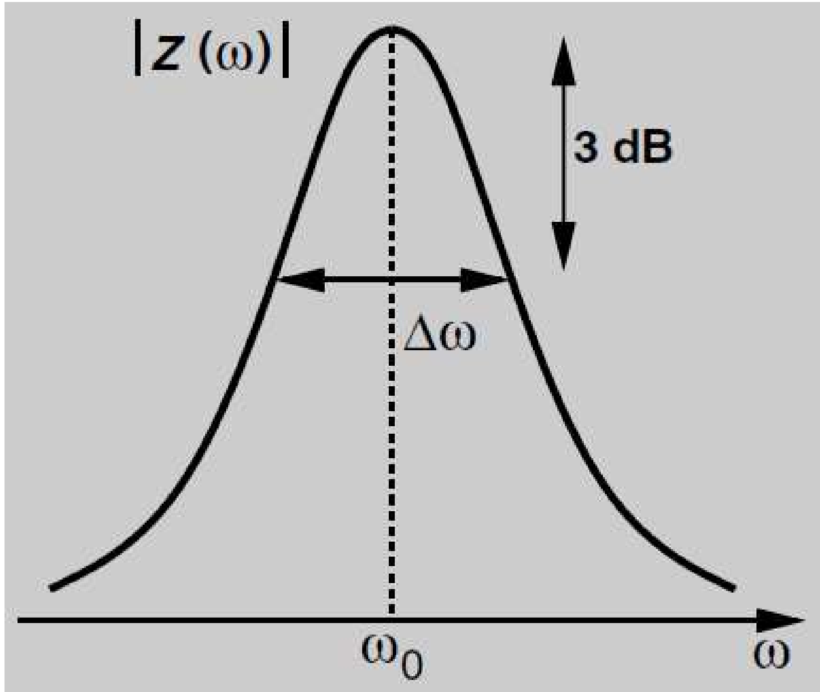


❑The linearity of the LNA also becomes critical in wideband receivers that may sense a large number of strong interferers.

[B. Razavi, RF Microelectronics]

Bandwidth

❑The LNA must provide a relatively flat response for the frequency range of interest, and the LNA 3-dB bandwidth must meet the requirements.



❑An 802.11a LNA must achieve a 3-dB bandwidth from 5 GHz to 6 GHz

❑The Q of the tank must remain Q=5.5GHz/1 GHz=5.5

[B. Razavi, RF Microelectronics]

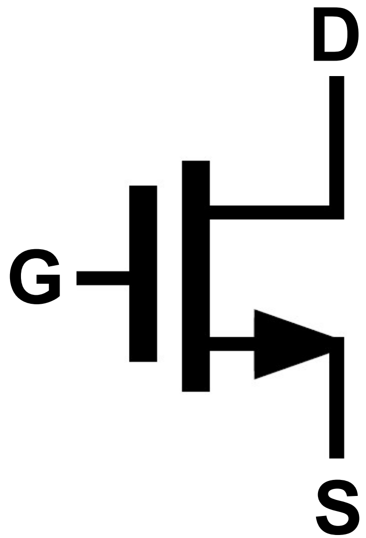
LNA Typologies

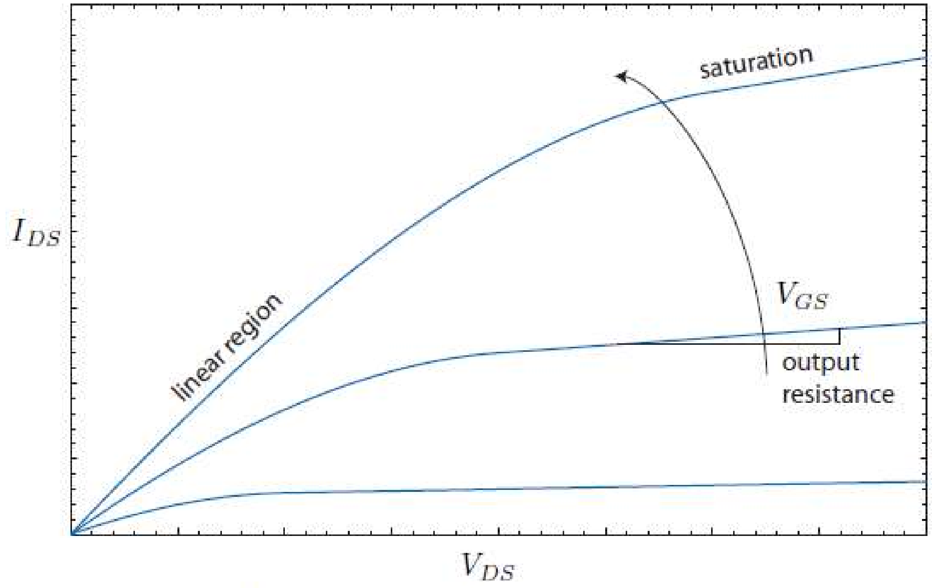
❑Proper input (conjugate) matching of LNAs requires certain circuit techniques that yield a real part of 50 in the input impedance without the noise of a 50Ohm resistor.

Common-Source Stage with	Common-Gate Stage with	Broadband Topologies
■ Inductive Load ■ Resistive Feedback ■ Cascode, Inductive Load, Inductive Degeneration	■ Inductive Load ■ Feedback ■ Feedforward ■ Cascode and Inductive Load	■ Noise-Cancelling LNAs ■ Reactance-Cancelling LNAs

[B. Razavi, RF Microelectronics]

MOSFET Basics





If $V_{DS} \le V_{GS} - V_T$

$$I_{DS} = \mu C_{ox} \frac{W}{L} \left((V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$

If $V_{DS} > V_{GS} - V_T$

$$I_{DS} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left((V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \right)$$

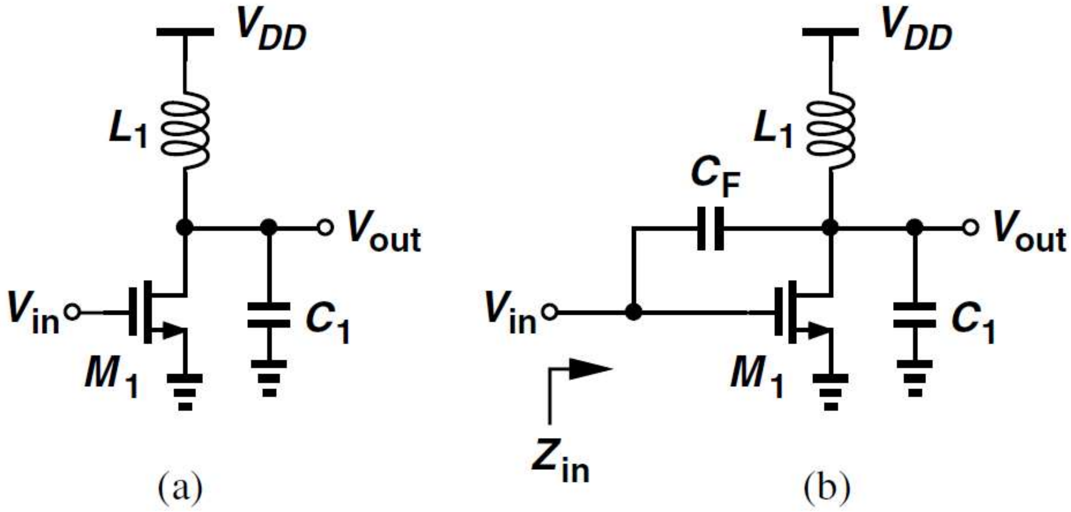
$$g_m = \frac{dI_{DS}}{dV_{GS}} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)(1 + \lambda V_{DS})$$

$$g_m = \frac{2I_{DS}}{V_{GS} - V_T} = \frac{2I_{DS}}{\sqrt{\frac{2I_{DS}}{\mu C_{ox} \frac{W}{L}}}}$$

$$g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_{DS}} \propto \sqrt{I_{DS}}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Inductive-Load CS LNA

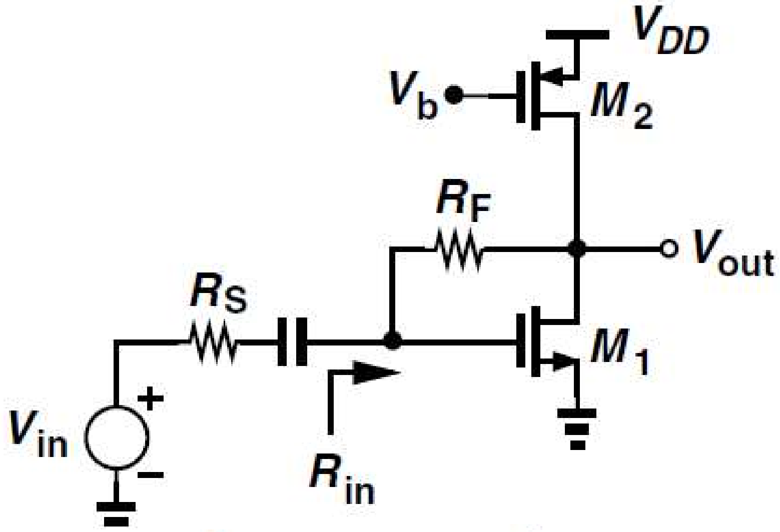


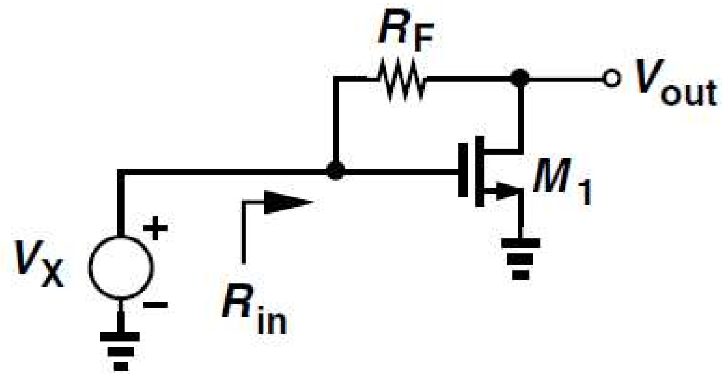
❑Such a topology operates with very low supply voltages because the inductor sustains a smaller DC voltage drop than a resistor does.

❑L_1 resonates with the total capacitance at the output node, providing high gain and a filter against interferences.

[B. Razavi, RF Microelectronics]

Resistive-Feedback CS LNA





$$R_{in} = \frac{1}{g_{m1}} \quad g_{m1} = \frac{1}{R_S} \qquad V_X - g_{m1}V_XR_F = V_{out}$$

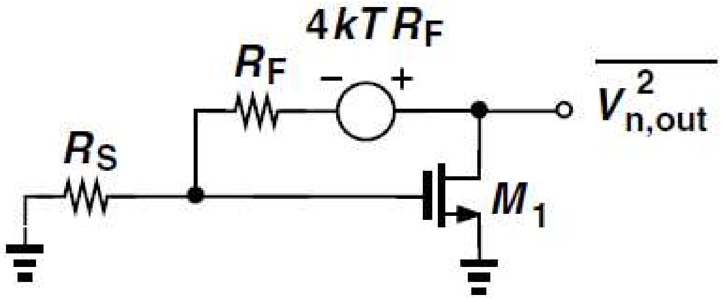
$$\frac{V_{out}}{V_X} = 1 - g_{m1}R_F = 1 - \frac{R_F}{R_S}$$

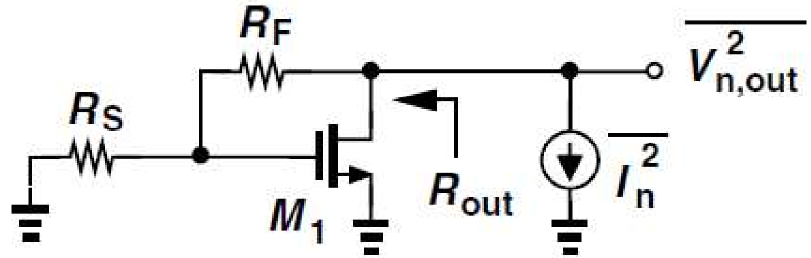
- The voltage gain from V_{in} to V_{out} can be expressed by

$$A_v = \frac{1}{2}\left(1 - \frac{R_F}{R_S}\right)$$

[B. Razavi, RF Microelectronics]

Noise Figure





- The noise of R_F appears at the output in its entirety:

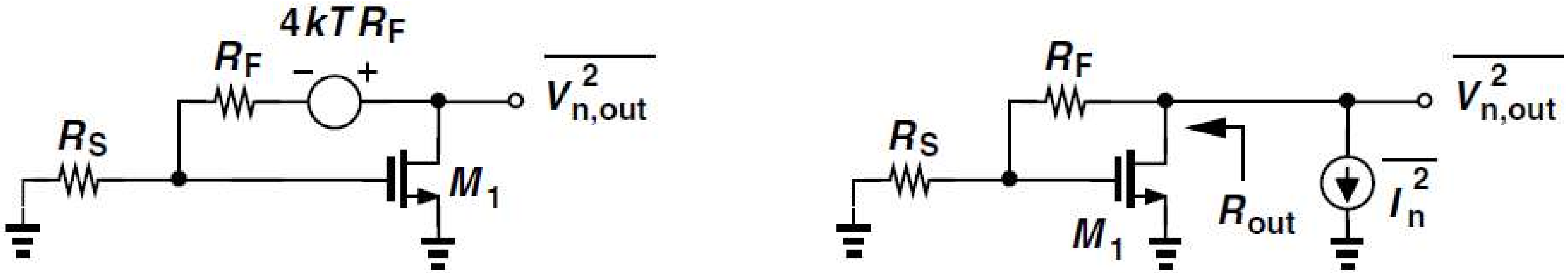
$$\overline{V^2_{n,out}}|_{RF} = 4kTR_F$$

- The noise currents of M_1 and M_2 flow through the output impedance of the circuit, R_{out}, which can be expressed by

$$R_{out} = \left[\frac{1}{g_{m1}}\left(1 + \frac{R_F}{R_S}\right)\right] ||(R_F + R_S) \ = \frac{1}{2}(R_F + R_S)$$

[B. Razavi, RF Microelectronics]

Noise Figure



❑The noise of MOSFETs appears at the output:

$$\overline{V_{n,out}^2}|_{M1,M2} = 4kT\gamma(g_{m1} + g_{m2})\frac{(R_F + R_S)^2}{4}$$

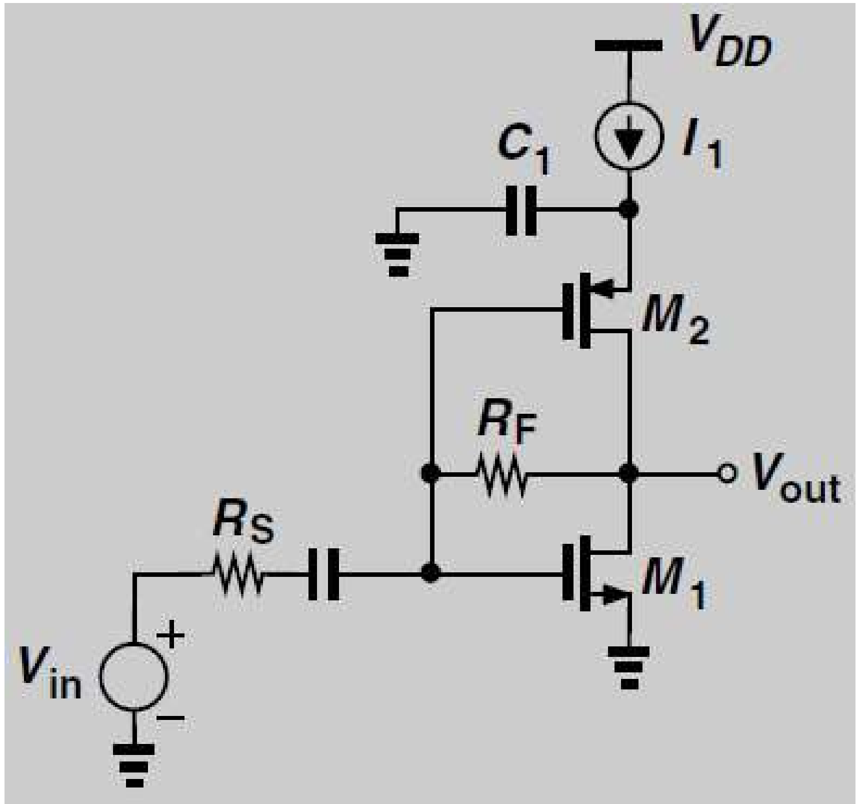
❑The noise figure:

$$\mathrm{NF} = 1 + \frac{4R_F}{R_S\left(1 - \frac{R_F}{R_S}\right)^2} + \frac{\gamma(g_{m1} + g_{m2})(R_F + R_S)^2}{\left(1 - \frac{R_F}{R_S}\right)^2 R_S} \approx 1 + \frac{4R_S}{R_F} + \gamma(g_{m1} + g_{m2})R_S$$

❑For γ ≈ 1, the NF exceeds 3 dB

[B. Razavi, RF Microelectronics]

Noise Figure



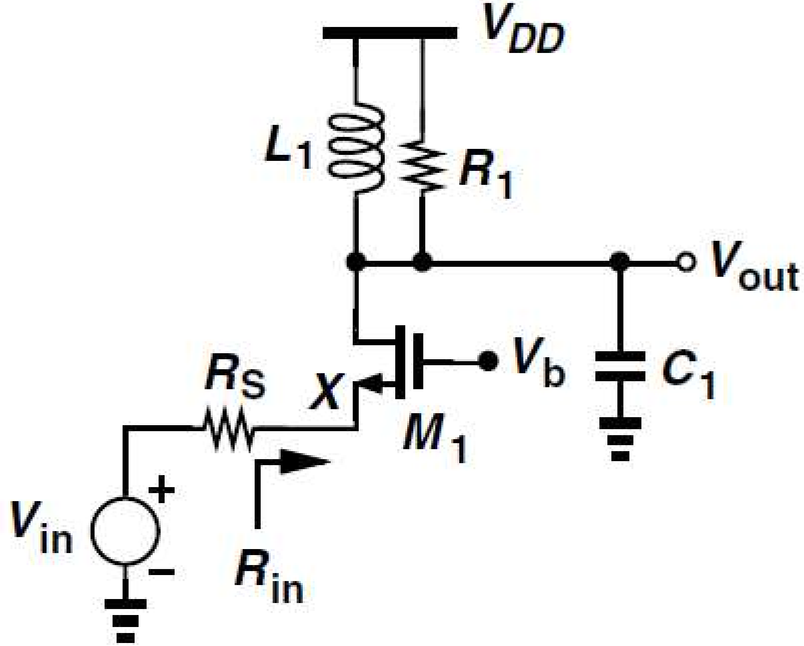
❑For small-signal operation, M_1 and M_2 appear in parallel, behaving as a single transistor with a transconductance of $g_{m1}+g_{m2}$. Thus, for input matching, $g_{m1}+g_{m2}=1/R_S$. Then, we have $\gamma(g_{m1}+g_{m2})R_S= \gamma$, and the noise figure:

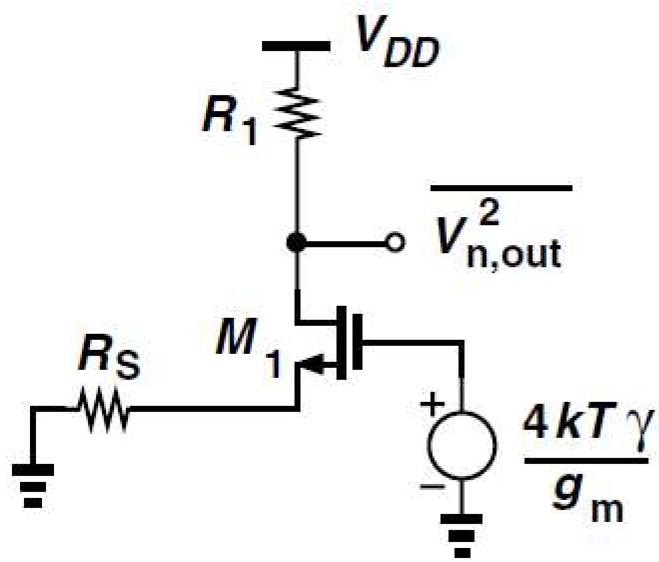
$$\mathrm{NF} \approx 1 + \frac{4R_S}{R_F} + \gamma$$

❑The circuit has a lower noise figure and higher gain, but requires a higher supply voltage

[B. Razavi, RF Microelectronics]

Common-Gate (CG) LNA





❑The input resistance is R_{in}=1/g_m. Thus, the dimensions and bias current of M_1 are chosen so as to yield g_m=1/R_S=1/50Ohm.

$$\frac{V_{out}}{V_X} = g_m R_1 = \frac{R_1}{R_S} \qquad V_{out}/V_{in} = R_1/(2R_S)$$

[B. Razavi, RF Microelectronics]

Noise Figure

❑The thermal noise of M_1 transferred to the output:

$$\overline{V_{n,out}^2}|_{M1} = \frac{4kT\gamma}{g_m}\left(\frac{R_1}{R_S + \frac{1}{g_m}}\right)^2 = kT\gamma\frac{R_1^2}{R_S}$$

❑The output noise due to R_1 is simply equal to $4kTR_1$

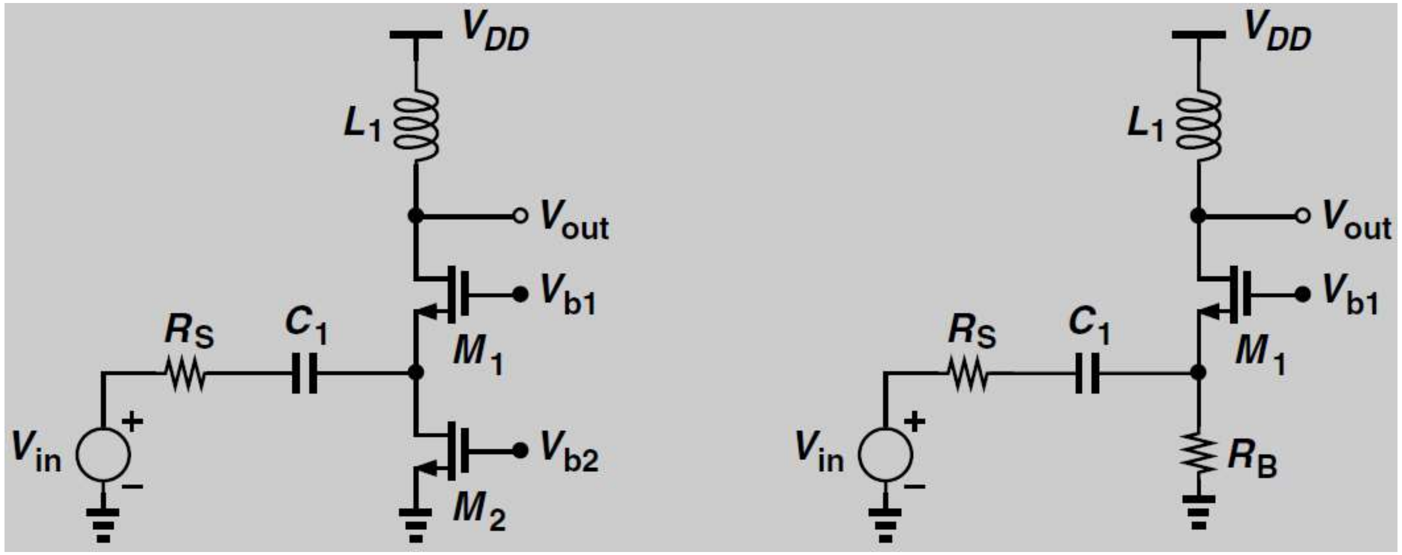
❑The overall noise figure:

$$\begin{aligned} NF &= 1 + \frac{\gamma}{g_m R_S} + \frac{R_S}{R_1}\left(1 + \frac{1}{g_m R_S}\right)^2 \\ &= 1 + \gamma + 4\frac{R_S}{R_1} \end{aligned}$$

❑The NF still reaches 3 dB with a large R_1

[B. Razavi, RF Microelectronics]

DC Basing of CG Stage



❑V_{DS2} is equal to the voltage drop across R_B(=V_{RB}). Operating in saturation, M_2 requires that $V_{DS2} \geq V_{GS2} - V_{TH2}$.

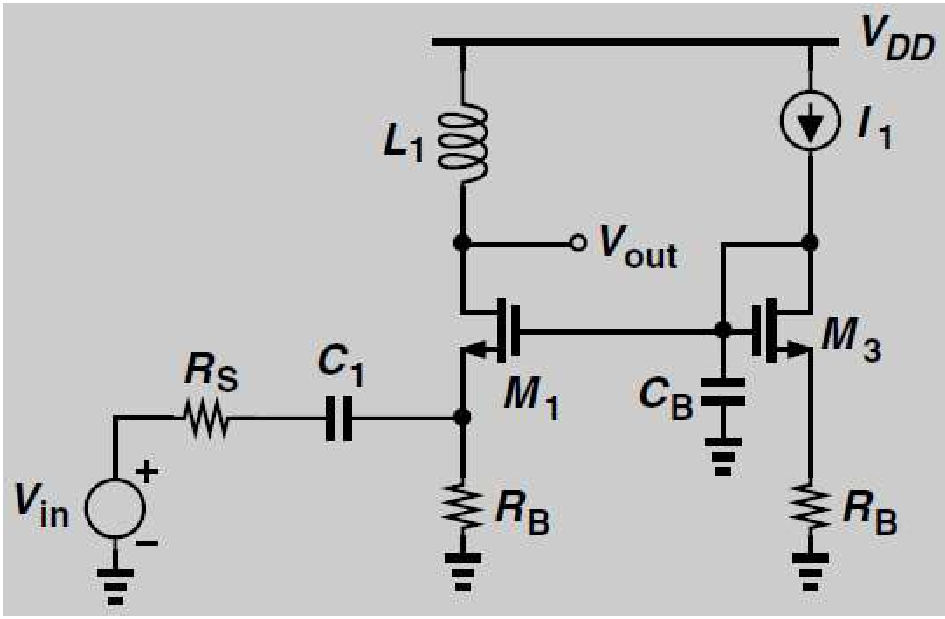
$$\overline{I^2_{n,M2}} = 4kT\gamma g_{m2} = 4kT\gamma \frac{I_D}{V_{GS2} - V_{TH2}}$$

$$\overline{I^2_{n,RB}} = \frac{4kT}{R_B} = 4kT\frac{I_D}{V_{RB}}$$

[B. Razavi, RF Microelectronics]

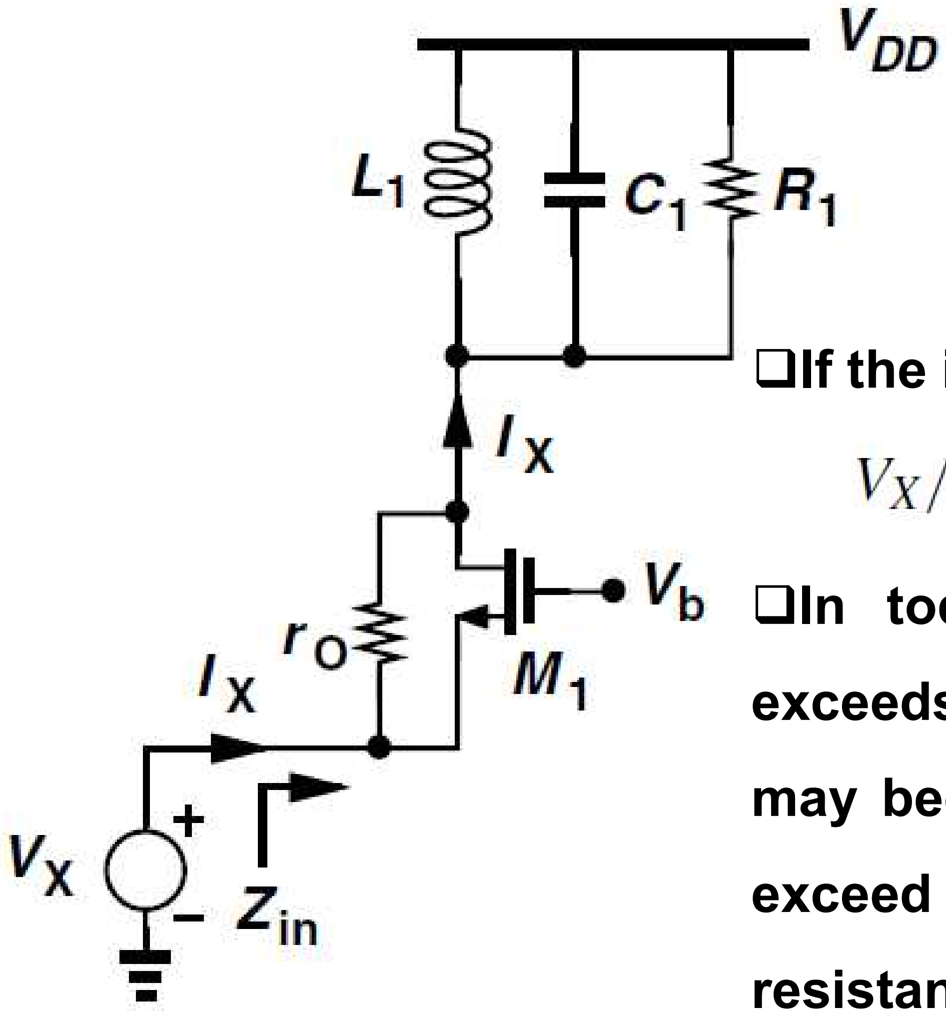
DC Basing of CG Stage

- V_{GS2} - $V_{TH2} \leq V_{RB}$, the noise contribution of M_2 is larger than R_B.
- M_2 may introduce significant capacitance at the input node
- The use of a resistor is therefore preferable, so long as R_B is much greater than R_S so that it does not attenuate the input signal.



[B. Razavi, RF Microelectronics]

Input Impedance of CG Stage



$$V_X = r_O(I_X - g_m V_X) + I_X R_1$$

$$\frac{V_X}{I_X} = \frac{R_1 + r_O}{1 + g_m r_O}$$

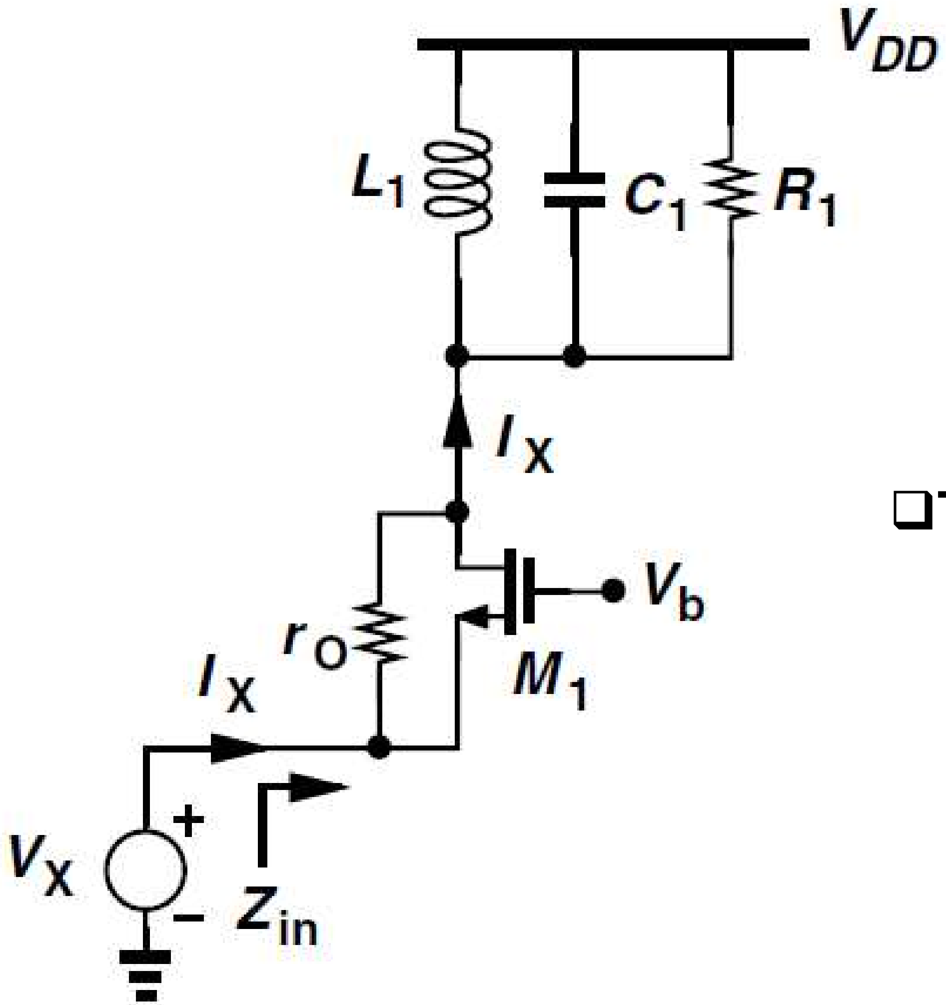
❑**If the intrinsic gain $g_m r_O$ >>1, then**

$$V_X/I_X \approx 1/g_m + R_1/(g_m r_O)$$

❑**In today's technology, $g_m r_O$ hardly exceeds 10. Thus, the term $R_1/(g_m r_O)$ may become comparable with or even exceed the term $1/g_m$, yielding an input resistance substantially higher than 50.**

[B. Razavi, RF Microelectronics]

Gain of CG Stage



- we must equate the actual input resistance to R_S to guarantee input matching:

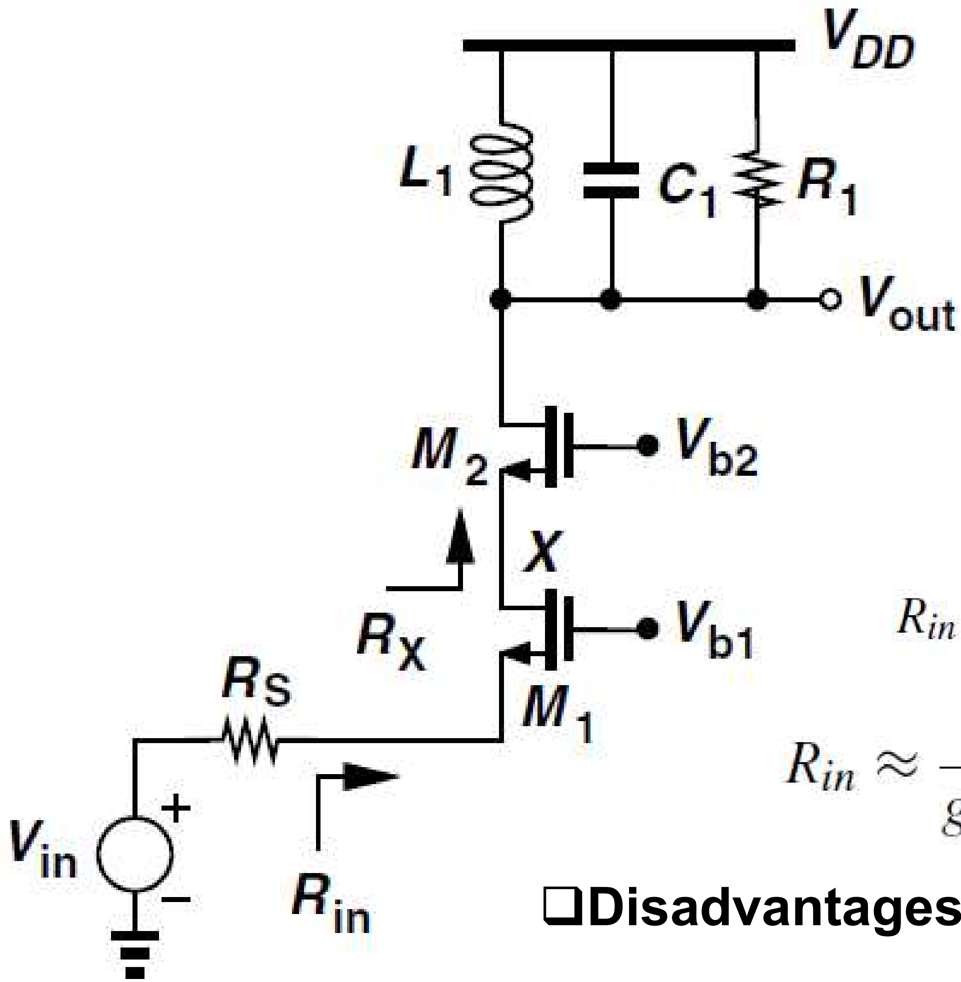
$$R_S = \frac{R_1 + r_O}{1 + g_m r_O}$$

- The voltage gain of the CG stage:

$$\frac{V_{out}}{V_{in}} = \frac{g_m r_O + 1}{2\left(1 + \frac{r_O}{R_1}\right)}$$

[B. Razavi, RF Microelectronics]

Cascode CG Stage



❑The impact of R_1 to the input impedance (gain) can be reduced by incorporating a cascode device

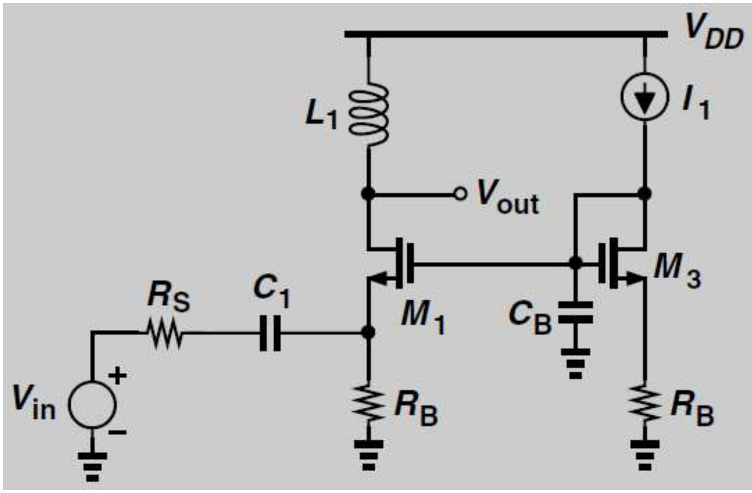
$$R_X = \frac{R_1 + r_{O2}}{1 + g_{m2}r_{O2}}$$

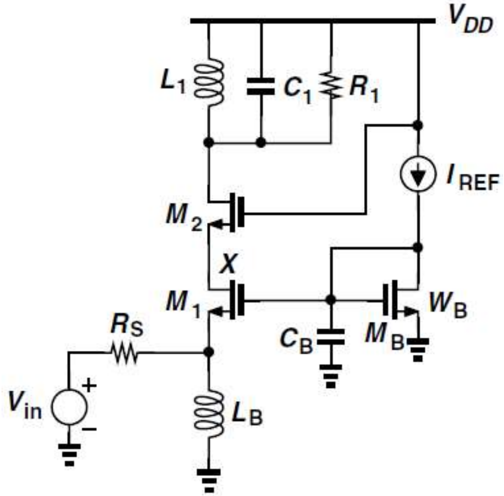
$$R_{in} = \left(\frac{R_1 + r_{O1}}{1 + g_{m2}r_{O2}} + r_{O1}\right) \div (1 + g_{m1}r_{O1})$$

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{R_1}{g_{m1}r_{O1}g_{m2}r_{O2}} + \frac{1}{g_{m1}r_{O1}g_{m2}}$$

❑Disadvantages: Noise contribution of M_2 and voltage headroom limitation *[B. Razavi, RF Microelectronics]*

Noise-Headroom Tradeoff





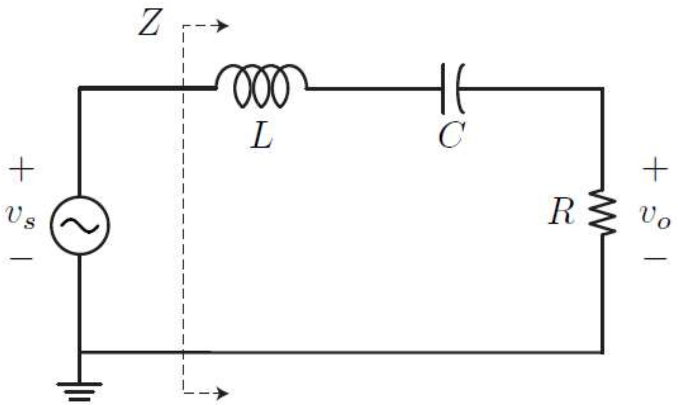
❑There is an up limit for the source voltage of M_1

❑This value of R_B cannot be very large, which degrades the gain and noise behavior of the circuit considerably

❑CG stages often employ an inductor for the bias path, which minimizes the additional noise and significantly improves the input matching.

[B. Razavi, RF Microelectronics]

Series RLC Circuits



❑The RLC circuit shown is deceptively simple. The impedance seen by the source is simply given by

$$Z = j\omega L + \frac{1}{j\omega C} + R = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right)$$

❑The impedance is purely real at the resonant frequency when Im(Z) = 0, or $w = \pm\frac{1}{\sqrt{LC}}$. At resonance the impedance takes on a minimal value.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Quality Factor

- So what's the magic about this circuit? The first observation is that at resonance, the voltage across the reactance can be larger, in fact much larger, than the voltage across the resistors R. In other words, this circuit has voltage gain. Of course it does not have power gain, for it is a passive circuit. The voltage across the inductor is given by

$$v_L = j\omega_0 Li = j\omega_0 L\frac{v_s}{Z(j\omega_0)} = j\omega_0 L\frac{v_s}{R} = jQ \times v_s$$

- where we have defined a circuit Q factor at resonance as

$$Q = \frac{\omega_0 L}{R}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Voltage Multiplication

- It's easy to show that the same voltage multiplication occurs across the capacitor

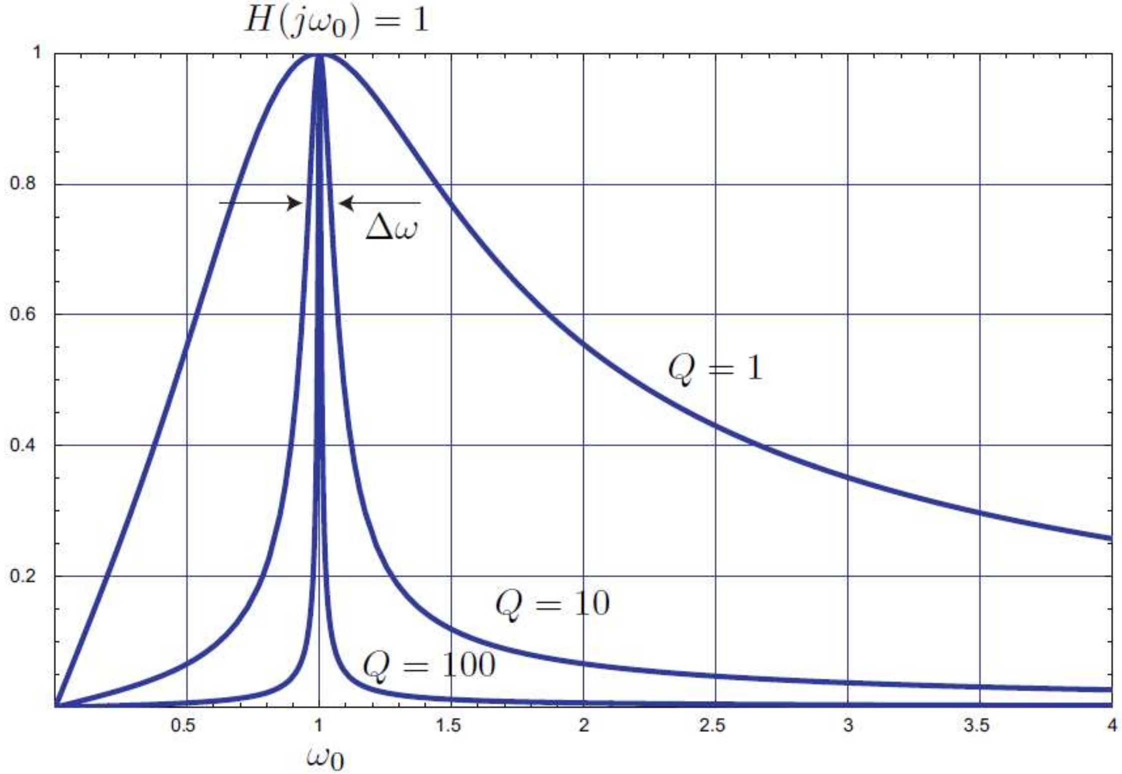
$$V_C = \frac{1}{j\omega_0 C} i = \frac{1}{j\omega_0 C} \frac{V_S}{\mathrm{Z}(j\omega_0)} = \frac{1}{j\omega_0 RC} V_S = -jQ * V_S$$

- This voltage multiplication property is the key feature of the circuit that allows it to be used as an impedance transformer.
- It's important to distinguish this Q factor from the intrinsic Q of the inductor and capacitor. For now, we assume the inductor and capacitor are ideal.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Circuit Transfer Function

❑Let's now examine the transfer function of the circuit



❑As we plot the magnitude of the transfer function, we see that the selectivity of the circuit is also related inversely to the Q factor.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Selectivity Bandwidth (I)

- In the limit that $Q \to \infty$, the circuit is infinitely selective and only allows signals at resonance ω_0 to travel to the load.
- Note that the peak gain in the circuit is always unity, regardless of Q, since at resonance the L and C together disappear and effectively all the source voltage appears across the load.
- The selectivity of the circuit lends itself well to filter applications. To characterize the 3dB bandwidth, let's compute the frequency when the magnitude squared of the transfer function drops by half

$$|H(j\omega)|^2 = \frac{\left(\omega \frac{\omega_0}{Q}\right)^2}{\left(\omega_0^2 - \omega^2\right)^2 + \left(\omega \frac{\omega_0}{Q}\right)^2} = \frac{1}{2}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Selectivity Bandwidth (II)

- This happens when

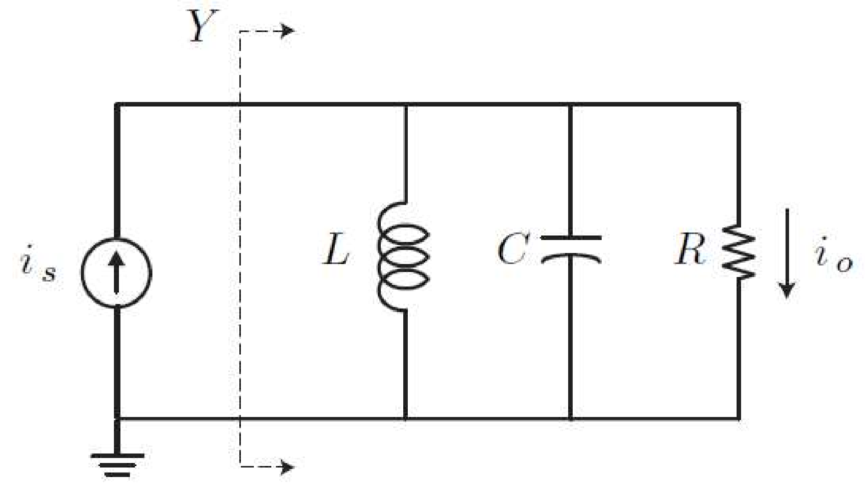
$$\left(\frac{\omega_0^2 - \omega^2}{\omega_0\omega/Q}\right)^2 = 1$$

- Solving the above equation yields four solutions, corresponding to two positive and two negative frequencies. The 3-dB bandwidth is characterized by the difference between these frequencies, or the bandwidth, given by

$$\Delta\omega = \omega_+ - \omega_- = \frac{\omega_0}{Q}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Parallel RLC



❑The admittance of the circuit is given by

$$Y = j\omega C + \frac{1}{j\omega L} + G = G + j\omega C\left(1 - \frac{1}{\omega^2 LC}\right)$$

which has the same form as before. The resonant frequency also occurs when Im(Y) = 0, or when $\boldsymbol{\omega = \omega_0 = \pm\frac{1}{\sqrt{LC}}}$.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Current Multiplication

- Likewise, at resonance the admittance takes on a minimal value. Equivalently, the impedance at resonance is maximum.
- This property makes the parallel RLC circuit an important element in tuned amplifier loads. It's also easy to show that at resonance the circuit has a current gain of Q

$$i_C = j\omega_0 C v_o = j\omega_0 C \frac{i_s}{Y(j\omega_0)} = j\omega_0 C \frac{i_s}{G} = jQ \times i_s \quad i_L = -jQ \times i_s$$

where $Q = \frac{\omega_0 C}{G}$

- The equivalent expressions for the circuit Q factor are given by the inverse of the previous relations

$$Q = \frac{\omega_0 C}{G} = \frac{R}{\omega_0 L} = \frac{R}{\frac{1}{\sqrt{LC}} L} = \frac{R}{\sqrt{\frac{L}{C}}} = \frac{R}{Z_0}$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Circuit Transfer Function

- The transfer function of parallel RLC

$$H(j\omega) = \frac{i_o}{i_s} = \frac{v_o G}{v_o Y(j\omega)} = \frac{G}{j\omega C + \frac{1}{j\omega L} + G}$$

- which can be put into the same canonical form as before

$$H(j\omega) = \frac{j\omega \frac{\omega_0}{Q}}{\omega_0^2 + (j\omega)^2 + j\frac{\omega\omega_0}{Q}}$$

- We have appropriately re-defined the circuit Q to correspond the parallel RLC circuit.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Parallel Resonance

❑At resonance, the real terms in the denominator cancel

$$Z(j\omega_0) = \frac{j\frac{R}{Q}}{\underbrace{1 + \left(\frac{j\omega_0}{\omega_0}\right)^2}_{=0} + j\frac{1}{Q}} = R$$

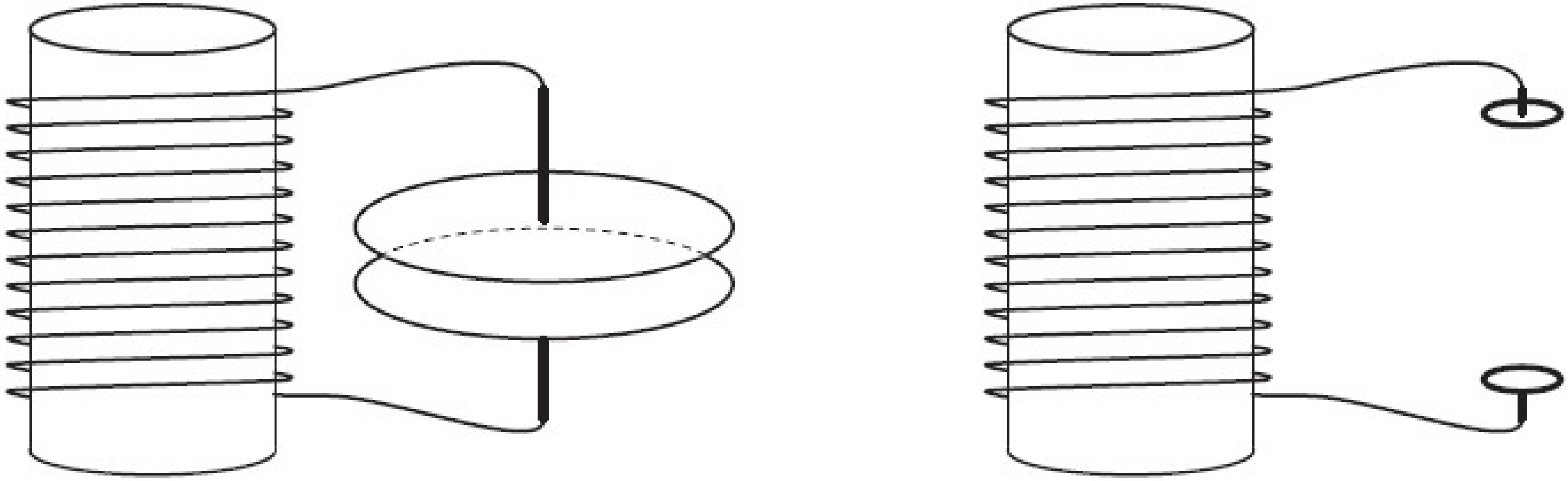
❑It's not hard to see that this circuit has the same half power bandwidth as the series RLC circuit, since the denominator has the same functional form

$$\frac{\Delta\omega}{\omega_0} = \frac{1}{Q}$$

❑plot of this impedance versus frequency has the same form as before multiplied by the resistance R.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

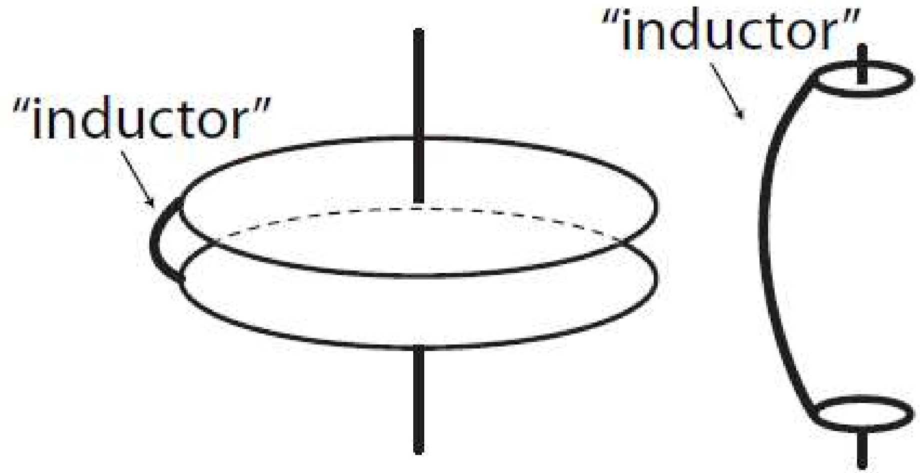
LC-Tank Frequency Limit



- ❑What's the highest resonance frequency we can achieve with a "lumped" component LC tank?
- ❑Can we make C and L arbitrarily small?
- ❑Clearly, to make C small, we just move the plates apart and use smaller plates. But what about L?

[Ali Niknejad, EE142&EE242 of UC Berkeley]

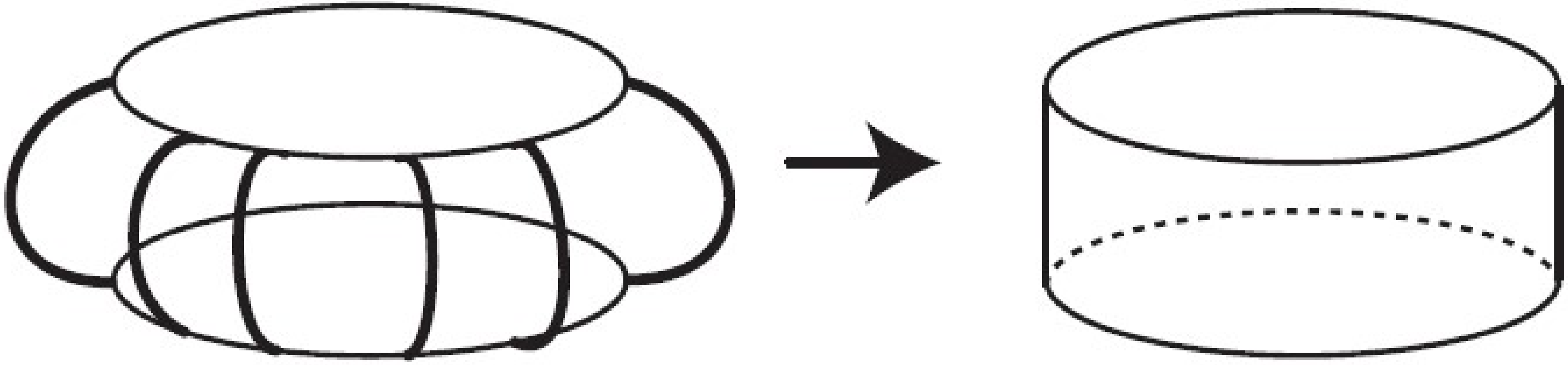
Small Capacitor→Bigger Inductor



- As shown above, the smallest "inductor" has zero turns, or it's just straight wire connected to the capacitor.
- Since inductance is defined for a loop, the capacitor is actually now part of the inductor and defines the inductance of the circuit.
- To make the inductance smaller requires that we increase the capacitor (bring plates closer).

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Feynman’s Can



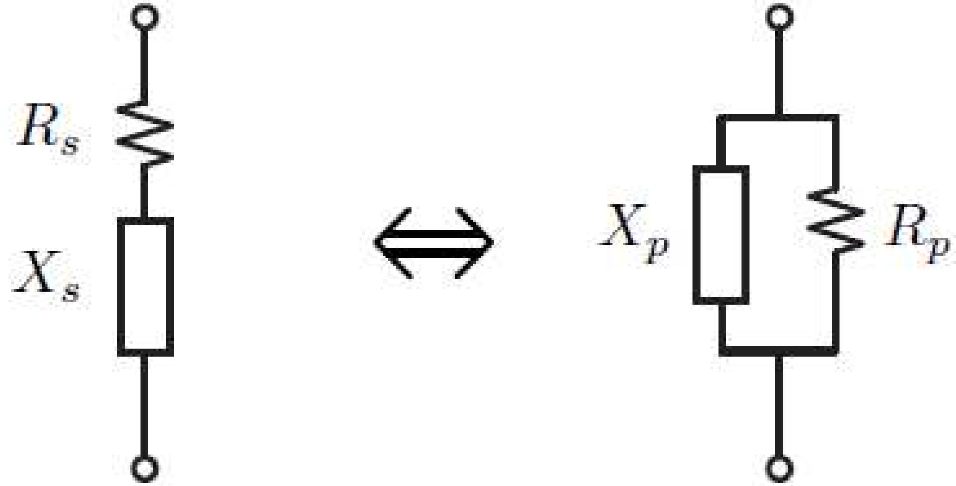
❑Given a fixed plate spacing and size, we can also be clever and keep adding inductors in parallel to reduce the inductance.

❑In the limit, we end up with a “can”!

❑High frequency resonators are built this way from the outset, rather than from lumped components.

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Shunt-Series Transformation (I)



❑Series to parallel transformation is widely used. Consider the impedance shown above, which we wish to represent as a parallel impedance.

❑We can do this at a single frequency as long as the impedance of the series network equals the impedance of the shunt network

[Ali Niknejad, EE142&EE242 of UC Berkeley]

Shunt-Series Transformation (II)

- Equating the real and imaginary parts

$$R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2} \qquad R_p = R_s(1 + Q^2)$$

$$X_s = \frac{R_p^2 X_p}{R_p^2 + X_p^2} \qquad X_p = X_s(1 + Q^{-2}) \approx X_s$$

- which can be simplified by using the definition of Q

$$Q_s = \frac{X_s}{R_s} = \frac{R_p^2 X_p}{R_p X_p^2} = \frac{R_p}{X_p} = Q_p = Q$$

[Ali Niknejad, EE142&EE242 of UC Berkeley]