第五章

① (1) $E(X_n)$

$= \cos n\omega\, E(X) + \sin n\omega\, E(Y) = 0$. (常量)

$E(X_n X_{n+m})$

$= E\big((X\cos n\omega + Y\sin n\omega)(X\cos[(n+m)\omega] + Y\sin[(n+m)\omega])\big)$

∵ X 与 Y 不相关

∴ $E(XY)=0 \Rightarrow E(X_n X_{n+m}) = [\cos n\omega \cos(n+m)\omega + \sin n\omega \sin(n+m)\omega]\sigma^2$

$= \sigma^2 \cos(m\omega)$ 只与 m 有关

∴ $\{X_n\}$ 是平稳过程.

(2) $E(Y_n) = \sum_{k=0}^{m} \cos n\omega_k\, E(\xi_k)$

$+ \sum_{k=0}^{m} \sin n\omega_k\, E(\eta_k) = 0$ (常量)

全是0

$E(Y_n Y_{n+j}) = E\left(\sum_{k=0}^{m} \xi_k \cos n\omega_k + \eta_k \sin n\omega_k \cdot \sum_{l=0}^{m} \xi_l \cos(n+j)\omega_l + \eta_l \sin(n+m)\omega_l\right)$

全是0.

由于 $E(\xi_k \xi_l) = E(\eta_l \eta_k) = 0$

$E(\xi_k \eta_l) = 0$, 且 $E(\xi_k \xi_k) = E(\eta_k \eta_k) = \sigma_k^2$

从而 $E(Y_n Y_{n+j}) = \sum_{k=0}^{m}\left[\cos n\omega_k \cos(n+j)\omega_k \cdot D(\xi_k) + \sin n\omega_k \sin(n+j)\omega_k \cdot D(\eta_k)\right]$

$= \sum_{k=0}^{m} \sigma_k^2 \cdot \cos(j\omega_k)$ 只与 m 有关 从而 $\{Y_n\}$ 是平稳过程.

② $X(t) = A\sin(t+\Theta)$, A 与 Θ 独立,

$P(\Theta=\frac{\pi}{4}) = P(\Theta=-\frac{\pi}{4}) = \frac{1}{2}$. $A \sim U[-1,1]$

$E(X(t)) = E(A)E(\sin(t+\Theta)) = 0$ (常量)

$R_X(t,t+\tau) = E(X(t)X(t+\tau))$

$= E(A^2)\cdot E(\sin(t+\Theta)\sin(t+\Theta+\tau))$

而 $E(A^2) = \int_{-1}^{1} \frac{x^2}{2}dx = \int_0^1 x^2 dx = \frac{1}{3}$

$E(\sin(t+\Theta)\sin(\Theta+t+\tau))$

$= \frac{1}{2}\left[\sin(t+\frac{\pi}{4})\sin(t+\frac{\pi}{4}+\tau) + \sin(t-\frac{\pi}{4})\sin(t-\frac{\pi}{4}+\tau)\right]$

$= \frac{1}{2}\cos\tau$.

从而 $R_X(t,t+\tau) = \frac{1}{6}\cos\tau$

∴ $\{X(t)\}$ 是平稳过程.

3. $\mu_X = 0$, $R_X(\tau) = e^{-|\tau|}$. $A \sim U[1,2]$.

A 与 $\{X(t)\}$ 独立. 令 $Y(t) = X(t) - X(0)$,

$Z(t) = \frac{X(t)}{A}$.

$\mu_Y(t) = E(Y(t)) = \mu_X(t) - \mu_X(0) = 0$. (常量)

$\mu_Z(t) = E(Z(t)) = E(A)\, E(X(t)) = 0$. (常量)

$R_Y(t,t+\tau) = E((X(t)-X(0))(X(t+\tau)-X(0)))$

$= R_X(t,t+\tau) - R_X(t) - R_X(t+\tau) + R_X(0)$

$= e^{-|\tau|} - e^{-|t|} - e^{-|t+\tau|} + 1 \neq g(\tau)$

从而 $\{Y(t)\}$ 不是平稳过程.

$R_Z(t,t+\tau) = E\left(\frac{X(t)}{A}\,\frac{X(t+\tau)}{A}\right)$

$= E(\frac{1}{A^2})\, R_X(\tau) = e^{-|\tau|}\cdot\int_1^2 \frac{1}{x^2}dx$

$= \frac{1}{2}e^{-|\tau|}$ (只与 τ 有关)

从而 $\{Z(t)\}$ 是平稳过程.

4. $X(t)=A\sin t-B\cos t$,

A, B 独立同分布.

$E(A)=\mu$, $E(A^2)=\sigma^2$.

(1) $\mu_X(t)=(\sin t-\cos t)E(A)$

$=(\sin t-\cos t)\mu$.

$R_X(t,t+\tau)=E((A\sin t-B\cos t)(A\sin(t+\tau)-B\cos(t+\tau))$

A与B独立

$=\sin t\sin(t+\tau)E(A^2)+E(B^2)\cos t\cos(t+\tau)$

同分布

$-\cos t\sin(t+\tau)E(A)E(B)-\sin t\cos(t+\tau)E(A)E(B)$

$=(\sigma^2)\cdot\cos\tau-\mu^2\sin(2t+\tau)$

(2) ∵ $\{X(t)\}$ 是宽平稳过程

∴ 必有 $\mu=0$.

(3) $P(A=1)=P(A=-1)=0.5$

$\Rightarrow \mu=0$, $\sigma^2=1$.

∵ $X(0)=-B$.

$P\{X(0)=1\}=\frac{1}{2}=P\{X(0)=-1\}$

而 $X(\frac{\pi}{4})=\frac{\sqrt{2}}{2}(A-B)$

∴ $P\{X(\frac{\pi}{4})=0\}=P(A=B)=\frac{1}{2}$.

$P\{X(\frac{\pi}{4})=\sqrt{2}\}=P\{X(\frac{\pi}{4})=-\sqrt{2}\}=\frac{1}{4}$.

∴ 显然 $\{X(t)\}$ 不是严平稳过程

∴ 其非严平稳过程.

5. 根据 Markov 不等式,

$P\{|X|\geq\varepsilon\}\leq\frac{E(X^k)}{\varepsilon^k}$

(若 X 的 k 阶矩存在)

于是 $P\{|X(t+\tau)-X(t)|\geq\varepsilon\}$

$\leq\frac{1}{\varepsilon^2}E(X^2(t+\tau)+X^2(t)-2X(t)X(t+\tau))$

$=\frac{1}{\varepsilon^2}[2R_X(0)-2R_X(\tau)]$

$=\frac{2}{\varepsilon^2}\cdot(C_X(0)-C_X(\tau))$

证毕.

6. $X(t)=X\cos t$, $X\sim N(1,3)$.

令 $Y(t)=\int_0^t X(u)du=\int_0^t X\cos u\,du$

$=X\sin t$

则 $\mu_Y(t)=E(X\sin t)$

$=\sin t\,E(X)=\sin t$.

$R_{XY}(s,t)=E(X\cos s\cdot X\sin t)$

$=\cos s\sin t\cdot E(X^2)$

$=(1+3)\sin t\cos s=4\sin t\cos s$.

7. 平稳 $\{X(t)\}$

$R_X(\tau)=e^{-a|\tau|}(1+a|\tau|)+1$. $(a>0)$

~~$\lim_{\tau\to\infty}R_X(\tau)=$~~ 由各态历经性定理,

$\lim_{T\to\infty}\frac{1}{T}\int_0^T[e^{-a|\tau|}(1+a|\tau|)+1]-\mu_X^2\,d\tau=0$.

所以 $\mu_X^2=\lim_{T\to\infty}\frac{1}{T}\int_0^T e^{-a|\tau|}(1+a|\tau|)+1\,d\tau$

$=\lim_{T\to\infty}\frac{1}{T}\left[T-\frac{2}{a}e^{-aT}+\frac{2}{a}-Te^{-aT}\right]$

$=1$. $\Rightarrow \mu_X=\pm1$

8. 第二题中的 $\{X(t)\}$,

$R_X(\tau)=\frac{1}{2}\cos\tau$, $E(X(t))=0$

∴ $\lim_{T\to\infty}\frac{1}{T}\int_0^T C_X(\tau)d\tau=0\Rightarrow\{X(t)\}$ 具有均值各态历经性.

第三题中的 $\{Z(t)\}$ (因为只有平稳过程才能各态历经性)

$\lim_{T\to\infty}\frac{1}{T}\int_0^T C_Z(\tau)d\tau=\lim_{T\to\infty}\frac{1}{T}\int_0^T\frac{1}{2}e^{-|\tau|}d\tau$

$=0$

$\Rightarrow\{Z(t)\}$ 具有均值各态历经性.

9. $X(t)=\sqrt{2}X\cos t+Y\sin t$

X与Y独立，

$$f(x)=\begin{cases}1-|x|, & -1<x<1\\ 0, & else.\end{cases}$$

$Y\sim U[-1,1]$

(1) $\mu_X(t)=\sqrt{2}\cos t\,E(X)+\sin t\,E(Y)$

$E(X)=\int_{-1}^{1}x(1-|x|)dx=0$

$E(Y)=0 \Rightarrow \mu_X(t)=0$. 常量

$R_X(t,t+\tau)=E((\sqrt{2}X\cos t+Y\sin t)(\sqrt{2}X\cos(t+\tau)+Y\sin(t+\tau)))$

$=2\cos t\cos(t+\tau)E(X^2)+\sin t\sin(t+\tau)E(Y^2)$

$E(X^2)=\int_{-1}^{1}x^2(1-|x|)dx=\frac{1}{6}$

$E(Y^2)=\int_{-1}^{1}x^2\cdot\frac{1}{2}\cdot dx=\int_0^1x^2dx=\frac{1}{3}$

$\Rightarrow R_X(t,t+\tau)=\frac{1}{3}\cos\tau$ 从而$\{X(t)\}$是平稳过程

(2). $\langle X(t)\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sqrt{2}X\cos t+Y\sin t\,dt=0=\mu_X$

$\Rightarrow$ 具有均值各态历经性

(3). $\langle X(t)X(t+\tau)\rangle$

$=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X(t)X(t+\tau)dt=0\neq R_X(t,t+\tau)$

从而不是各态历经过程.

10. 设$S(t)$是周期为T的函数，$\theta\sim U[0,T]$.
$X(t)=S(t+\theta)$. 证：$\{X(t)\}$为平稳过程.

证明：$\mu_X(t)=E(X(t))$

$=\frac{1}{T}\int_0^T S(t+\theta)d\theta$

$\overset{t+\theta=\alpha}{=}\frac{1}{T}\int_t^{t+T}S(\alpha)d\alpha$

周期性 $\frac{1}{T}\int_0^T S(\alpha)d\alpha$ 为定值.

$R_X(t,t+\tau)$

$=E(X(t)X(t+\tau))$

$=\frac{1}{T}\int_0^T S(t+\theta)S(t+\tau+\theta)d\theta$

从而$\{X(t)\}$是平稳过程

$\overset{t+\theta=\alpha}{=}\frac{1}{T}\int_t^{t+T}S(\alpha)S(\alpha+\tau)d\alpha=\frac{1}{T}\int_0^T S(\alpha)S(\alpha+\tau)d\alpha$ 只与τ有关

(接上)

$\mu_X = \frac{1}{T}\int_0^T s(\alpha)d\alpha$

$= \frac{1}{T}\int_0^T x(t)dt$

$= \frac{1}{T}\cdot\frac{T}{4}\cdot A/2 = \frac{A}{8}$.

$R_X(\frac{T}{8}) = \frac{1}{T}\int_0^T x(t)x(t+\frac{T}{8})dt$

$= \frac{1}{T}\int_0^{\frac{T}{8}} \frac{8A}{T}t\cdot(A-\frac{8A}{T}t)dt$

$= \frac{A^2}{48}$

(2) $\langle x(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)dt$

$= \lim_{T\to\infty}\frac{1}{2T}\int_0^{0+2T} x(t)dt$ (化为面积计算)

$= \lim_{T\to\infty}\frac{1}{2T}\cdot\frac{T}{4}\cdot A = \frac{A}{8}$.

11. 平稳过程 $\{X(t)\}$ $P\{X(t+T_0)=X(t)\}=1$ 称为 T_0 的平稳过程. 证: $\{X(t)\}$ 以 T_0 为周期 $\Leftrightarrow R_X(\tau)=R_X(\tau+T_0)$

证明: "$\Rightarrow$":

$R_X(\tau+T_0) = E(X(t)X(t+\tau+T_0))$

$\because P\{X(t+T_0)=X(t)\}=1$

$\therefore R_X(\tau+T_0) = E(X(t)X(t+\tau)) = R_X(\tau)$

即 $R_X(\tau)$ 以 T_0 为周期

"$\Leftarrow$": 若 $R_X(\tau)=R_X(\tau+T_0)$

有: $\underline{E\{(X(t+T_0)-X(t))^2\}}$ 证明 $P\{X=Y\}=1$ 考虑 $E\{(X-Y)^2\}=0$ 则 $P\{X=Y\}=1$

$= E\{X^2(t+T_0)+X^2(t)-2X(t)X(t+T_0)\}$

$\underset{相关函数}{\underline{利用自}}\ 2R_X(0)-2R_X(T_0) \underset{周期性}{\overset{R_X(\tau)}{=}} 0$.

从而 $P\{X(t+T_0)=X(t)\}=1$

从而 $\{X(t)\}$ 以 T_0 为周期.

至此 充要性证毕

12. $E(X_i)=\mu$, $D(X_i)=\sigma^2$ 且独立

$Y_n = X_n X_{n+1} X_{n+2}$.

(1) $\mu_Y(n) = E(X_n X_{n+1} X_{n+2}) \overset{独立}{=} \mu^3$ (常数)

$R_Y(n,n+m) = E(X_n X_{n+1} X_{n+2} X_{n+m} X_{n+m+1} X_{n+m+2})$

$= \begin{cases} \mu^6, & m\geq 3 \\ (\mu^2+\sigma^2)^{3-m}\mu^{2m}, & 0\leq m\leq 2 \end{cases}$ 与 m 有关

$n\ \ n+1\ \ n+2$

$m=0$ $(\mu^2+\sigma^2)^3$

$m=1$, $(\sigma^2+\mu^2)^2\mu^2$

$m=2$ $(\mu^2+\sigma^2)(\mu^2)^2$

$m\geq 3$, μ^6

从而 $\{Y_n\}$ 是平稳过程

(2) $\langle Y_n\rangle = \lim_{N\to\infty}\frac{1}{2N}\sum_{n=-N}^{N} Y_n$ 不好算 换方法

考虑 $C_Y(m) = R_Y(m) - \mu_Y^2$

$= \begin{cases} 0, & m\geq 3 \\ (\mu^2+\sigma^2)^{3-m}\mu^{2m}-\mu^6, & 0\leq m\leq 2 \end{cases}$

$\therefore \lim_{m\to\infty} C_Y(m)=0$

$\therefore \lim_{N\to\infty}\frac{1}{N}\sum_{m=0}^{N} C_Y(m) = 0$

从而 Y_n 均值具有各态历经性

$\therefore \langle Y_n\rangle = \mu_Y = \mu^3$

13. X_0, ε_1, ε_2 ··· 独立. $\{X_n\}$ 平稳.

$E(X_0)=\mu$, $D(X_0)=\sigma^2$.

$E(\varepsilon_n)=0$, $D(\varepsilon_n)=1$

令 $X_n = \lambda X_{n-1}+\varepsilon_n$, $0<\lambda<1$.

(1) $\mu_X(n) = E(X_n) = \lambda E(X_{n-1}) + E(\varepsilon_n)$

$\Rightarrow E(X_n) = \lambda^n E(X_0) = \lambda^n\mu$ (数列递推)

又 $X_n = \lambda^n X_0 + \sum_{i=1}^{n}\lambda^{n-i}\varepsilon_i$

~~$D(X_n)$~~

~~$R_X(m)=$~~ $R_X(n,n+m)$

$= E(X_n\cdot X_{n+m})$

$= E(\lambda^n X_0 + \sum_{i=1}^{n}\lambda^{n-i}\varepsilon_i,\ \lambda^{n+m}X_0 + \sum_{k=1}^{n+m}\lambda^{n+m-k}\varepsilon_k)$

$= \lambda^{2n+m}E(X_0^2) + \sum_{i=1}^{n}\sum_{k=1}^{n+m}\lambda^{2n+m-k-i}E(\varepsilon_i\varepsilon_k)$

只有 $i=k$, $E(\varepsilon_i\varepsilon_k)=1$ 否则为零

$-\frac{\lambda^m}{\lambda^2-1}$

$= \lambda^{2n+m}(\sigma^2+\mu^2) + \sum_{i=1}^{n}\lambda^{2n+m-2i} = \lambda^m(\sigma^2+\mu^2+\frac{1}{\lambda^2-1})\lambda^{2n}$

(接上) 表达式

$R_Y(m) = \lambda^m(\sigma^2+\mu^2+\frac{1}{\lambda^2-1})\lambda^{2n} - \frac{\lambda^m}{\lambda^2-1}$

应只与 m 有关，且 $\mu_Y = \mu \cdot \lambda^n$ 应为常数

$\Rightarrow \begin{cases} \mu = 0 \\ \sigma^2 = \frac{1}{1-\lambda^2} \end{cases}$

(2) 由上讨论，$\mu_X = 0$

$R_X(m) = \frac{\lambda^m}{1-\lambda^2}$ $(m>0)$

则若 $m<0$，$R_X(n, n+m)$

$\underset{n+m=n'}{=}$ $R_X(n', n'+(-m)) = \frac{\lambda^{-m}}{1-\lambda^2} = \frac{\lambda^{|m|}}{1-\lambda^2}$

当 $m=0$，$R_X(0) = \frac{1}{1-\lambda^2}$

从而 $R_X(m) = \frac{\lambda^{|m|}}{1-\lambda^2}$

(3) $C_X(m) = \frac{\lambda^{|m|}}{1-\lambda^2}$

$\therefore \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N} C_X(n) = 0$

以有均值各态历经性。

14. $Y(t) = aX(t-\tau_1) + N(t)$，

$a \le 1$

(1) 平稳、相关 $\Rightarrow$ $\{X(t)\}$ 与 $\{N(t)\}$ 均是平稳过程

$\Rightarrow R_{XY}(\tau) = E(X(t)aX(t+\tau-\tau_1) + X(t)N(t+\tau))$

$= aR_X(\tau-\tau_1) + R_{XN}(\tau)$

(2) 于是 $R_{XN}(\tau) = E(X(t)N(t+\tau))$

$= E(X(t))E(N(t+\tau)) = 0$

$\therefore R_{XY}(\tau) = aR_X(\tau-\tau_1)$

15. $S_X(\omega) = \frac{1}{\omega^4+5\omega^2+6}$

$= \frac{1}{(\omega^2+2)(\omega^2+3)} = \frac{1}{\omega^2+2} - \frac{1}{\omega^2+3}$

又 $e^{-a|\tau|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2a}{a^2+\omega^2}$

于是 $R_X(\tau) = \frac{1}{2\sqrt{2}} e^{-\sqrt{2}|\tau|} - \frac{1}{2\sqrt{3}} e^{-\sqrt{3}|\tau|}$

16. $R_X(\tau) = e^{-|\tau|} + e^{-|\tau|}\cos\pi\tau$.

于是 $e^{-|\tau|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{\omega^2+1}$

$e^{-|\tau|}\cos\pi\tau \overset{\mathcal{F}}{\longrightarrow} \frac{1}{2\pi}\left(\frac{2}{\omega^2+1}\right) * \pi[\delta(\omega-\pi) + \delta(\omega+\pi)]$

$= \frac{1}{(\omega-\pi)^2+1} + \frac{1}{(\omega+\pi)^2+1}$

$\Rightarrow S_X(\omega) = \frac{2}{\omega^2+1} + \frac{1}{(\omega-\pi)^2+1} + \frac{1}{(\omega+\pi)^2+1}$

17.

(1) $E(X(t)) = 0$ (常)

$R_X(t, t+\tau) = E((A\cos t + B\sin t + C)(A\cos(t+\tau) + B\sin(t+\tau) + C))$

由于 A, B, C 独立同分布

$\therefore E(AB) = E(AC) = E(BC) = 0$

从而 $R_X(t, t+\tau) = \frac{1}{3}(1+\cos\tau)$

$\therefore \{X(t)\}$ 是平稳过程

(2) $\langle X(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}(A\cos t + B\sin t + C)\,dt$

$= C$

$\therefore P\{\langle X(t)\rangle = \mu_X\} = P\{C=0\} = 0$

$\therefore$ 不具有均值各态历经性

(3) ~~$S_X(\omega)$~~ $R_X(\tau) = \frac{1}{3}(1+\cos\tau)$

$\Rightarrow S_X(\omega) = \frac{2\pi}{3}\delta(\omega) + \frac{\pi}{3}[\delta(\omega-1) + \delta(\omega+1)]$

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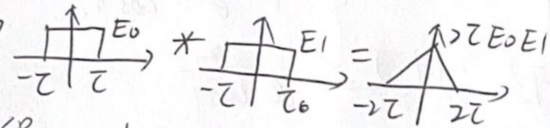
$$S_X(\omega)=\begin{cases}2\delta(\omega)+1-|\omega|, & |\omega|<1\\ 0, & \text{else.}\end{cases}$$

对于 $1 \xrightarrow{F} 2\pi\delta(\omega)$

$\Rightarrow \frac{1}{\pi} \xrightarrow{F} 2\delta(\omega)$

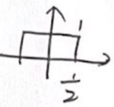
对 $1-|\omega|$，是三角波

由



得 $\tau=\frac{1}{2}$，$E_0=E_1=1$

由 傅立叶变换调制性质，且 $\frac{\sin(\frac{1}{2}t)}{\pi t} \xrightarrow{F}$

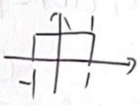


$\cancel{R_X(\tau)=\frac{1}{2\pi}\left[\frac{\sin(\frac{1}{2}t)}{\pi t}\right]^2}$

$$R_X(\tau)=2\pi\left[\frac{\sin(\frac{1}{2}\tau)}{\pi\tau}\right]^2$$

$$\Rightarrow \text{原 } R_X(\tau)=2\pi\left[\frac{\sin(\frac{1}{2}\tau)}{\pi\tau}\right]^2+\frac{1}{\pi}$$

19. $S_X(\omega)=\begin{cases}1, & |\omega|\le 1\\ 0, & |\omega|>1\end{cases}$



$$R_X(\tau)=\frac{\sin\tau}{\pi\tau}$$

$\because \lim_{\tau\to\infty} R_X(\tau)=0$ 存在

$\therefore \{X(t)\}$ 具有均值各态历经性

$\Leftrightarrow \mu_X=\lim_{\tau\to\infty} R_X(\tau)=0$

20. $Y(t)=X(t)\cos(t+\theta)$

$E(X(t))=0$

已证明：$E(Y(t))=E(X(t))E(\cos(t+\theta))=0$ (常)

$R_Y(t,t+\tau)=E(X(t)X(t+\tau)\cos(t+\theta)\cos(t+\tau+\theta))$

$=R_X(\tau)\cdot\frac{1}{2}\left[\cos(t+\frac{\pi}{4})\cos(t+\tau+\frac{\pi}{4})+\cos(t-\frac{\pi}{4})\cos(t+\tau-\frac{\pi}{4})\right]$

$=\frac{1}{2}R_X(\tau)\cos\tau \Rightarrow \{Y(t)\}$ 是平稳过程

$$S_Y(\omega)=\left(\frac{1}{2}S_X(\omega)\cdot\frac{1}{2\pi}\right)*\pi[\delta(\omega-1)+\delta(\omega+1)]$$

$$=\frac{1}{4}[S_X(\omega+1)+S_X(\omega-1)]$$

21. $Y(t)=X(t+L)-X(t)$

证明 $\cancel{S_Y(\omega)=e^{-j\omega L}S_X(\omega)-S_X(\omega)}$

先算：$R_Y(t,t+\tau)=E((X(t+L)-X(t))(X(t+\tau+L)-X(t+\tau)))$

$$=R_X(\tau)-R_X(\tau+L)+R_X(\tau)-\begin{cases}R_X(\tau-L), & \tau>L\\ R_X(L-\tau), & L>\tau.\end{cases}$$

而 $R_X(L-\tau)$ 是偶函数 $=R_X(\tau-L)$

$\therefore R_Y(\tau)=2R_X(\tau)-R_X(\tau+L)-R_X(\tau-L)$

$\xrightarrow{F} 2S_X(\omega)-S_X(\omega)\cdot\left[e^{-j\omega L}+e^{j\omega L}\right]$

$=2S_X(\omega)-2S_X(\omega)\cos(\omega L)$

$=2S_X(\omega)(1-\cos\omega L)$ 证毕

22. $X(t)=\alpha\cos(t+\theta)$

$Y(t)=\beta\cos(t+\theta)$

$R_{XY}(t,t+\tau)$

$=E(\alpha\cos(t+\theta)\beta\cos(t+\tau+\theta))$

$=\alpha\beta\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{1}{2}\cos(2t+\tau+2\theta)+\frac{1}{2}\cos(\tau)\right]d\theta$

$=\frac{\alpha\beta}{4\pi}\cdot 2\pi\cos\tau=\frac{\alpha\beta}{2}\cos\tau$

$\Rightarrow S_{XY}(\omega)=\frac{\alpha\beta\pi}{2}[\delta(\omega-1)+\delta(\omega+1)]$

23. $R_{XY}(\tau)$ 不相关 $\Leftrightarrow Cov=0$

$=E(X(t)Y(t+\tau))$

$=Cov(X(t)Y(t+\tau))+\mu_X\mu_Y=\mu_X\mu_Y$

$\Rightarrow S_{XY}(\omega)=2\pi\mu_X\mu_Y\delta(\omega)$

$R_{XZ}(\tau)=R_X(\tau)+R_{XY}(\tau)$

$\cancel{E(X(t)\cdot(X(t)}$

$\xrightarrow{F} S_X(\omega)+2\pi\mu_X\mu_Y\delta(\omega)$

[注]. 傅立叶变换调制性质：

$x(t)\cdot y(t)\xrightarrow{F}\frac{1}{2\pi}X(j\omega)*Y(j\omega)$

$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta.$