Calculation of Robotic Arm

Abstract

This document demonstrates the method for calculation of the three rotation angles of the robotic arm. The notation here is not consistent with that in the code.

1 Overview

The method used here includes two main steps: First, we use an ideal model of robotic arm, where all rotation axes are on the same plane, to compute an (approximated) analytic solution. Then, starting from the analytic solution, we use Jacobian Transpose method to get the more accurate numerical solution.

2 Computation of Analytic Solution

For the calculation of analytic solution, we assume that all the three rotation axes are on the same plane. A sample figure is shown below.

For the rotation angle θ_1 of the steer axis, positive x-direction means $0°$. For the rotation angle θ_2 of the major arm and the rotation angle θ_3 of the minor arm, positive s-direction on x-y plane means $0°$.

Let the target position be (x, y, z), and the length of major arm be a_2, the length of minor arm be a_3. Also, we should consider the horizontal offset h_0 and the vertical offset v_0 from the end of the minor arm to the center of the gripper. In

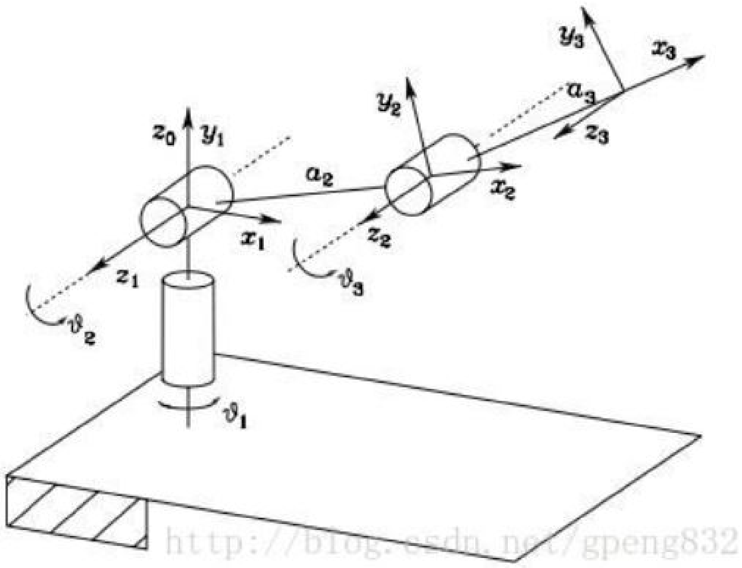

Figure 1: Simplified model of robotic arm[1]

addition, we can account for the height of the legs of the robotic arms, assuming the bottom of the legs is $z = 0$. The height of the leg is h_{leg} Then we have

$$x - \cos(\theta_1)h_0 = \cos(\theta_1)(a_2\cos(\theta_2) + a_3\cos(\theta_2 + \theta_3)) \tag{1}$$

$$y - \sin(\theta_1)h_0 = \sin(\theta_1)(a_2\cos(\theta_2) + a_3\cos(\theta_2 + \theta_3)) \tag{2}$$

$$z + v_0 - h_{leg} = a_2\sin(\theta_2) + a_3\sin(\theta_2 + \theta_3) \tag{3}$$

First, we can get θ_1 through x and y:

$$\theta_1 = \operatorname{atan2}(y, x) \tag{4}$$

Next, with $(1)^2 + (2)^2 + (3)^2$, we get

$$\cos(\theta_3) = \frac{(x - \cos(\theta_1)h_0)^2 + (y - \sin(\theta_1)h_0)^2 + (z + v_0 - h_{leg})^2 - a_2^2 - a_3^2}{2a_2a_3} \tag{5}$$

The structure of our robotic arm suggest the only possibility of $\sin(\theta_3)$, which is $\sin(\theta_3) = -\sqrt{1 - \cos^2(\theta_3)}$.

With $(1)^2 + (2)^2$ and the solution of θ_3, we can have the unique solution for θ_2:

$$\cos(\theta_2) = \frac{-(z + v_0 - h_{leg})a_3\sin(\theta_3) + (a_2 + a_3\cos(\theta_3))\sqrt{(x - \cos(\theta_1)h_0)^2 + (y - \sin(\theta_1)h_0)^2}}{a_2^2 + a_3^2 + 2a_2a_3} \tag{6}$$

$$\sin(\theta_2) = \frac{(z + v_0 - h_{leg})(a_2 + a_3\cos(\theta_3)) + a_3\sin(\theta_3)\sqrt{(x - \cos(\theta_1)h_0)^2 + (y - \sin(\theta_1)h_0)^2}}{a_2^2 + a_3^2 + 2a_2a_3} \tag{7}$$

3 Calculation of Numerical Solution

Since in reality, the rotation axes are not on the same plane, we need to add two offsets: The x-offset and y-offset between the steer axis and the major arm axis, which are represented by x_{steer} and y_{steer}.
Starting from the analytic solution, we can calculate the more accurate numerical solution by the Jacobian Transpose method. The Jacobian is

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \\ \frac{\partial z}{\partial \theta_1} & \frac{\partial z}{\partial \theta_2} & \frac{\partial z}{\partial \theta_3} \end{bmatrix} \tag{8}$$

Here are the steps in each iteration:

1) Calculate the (x_t, y_t, z_t) values from current value of $\boldsymbol{\theta}$ using the following formulae:

$$x = \cos(\theta_1)\sqrt{((a_2\cos(\theta_2) + a_3\cos(\theta_2 + \theta_3) + h_0 + x_{steer})^2 + y_{steer}^2)} \tag{9}$$
$$y = \sin(\theta_1)\sqrt{((a_2\cos(\theta_2) + a_3\cos(\theta_2 + \theta_3) + h_0 + x_{steer})^2 + y_{steer}^2)} \tag{10}$$
$$z = a_2\sin(\theta_2) + a_3\sin(\theta_2 + \theta_3) - v_0 + h_{leg} \tag{11}$$

2) Take the difference $\Delta\mathbf{e} = (x, y, z) - (x_t, y_t, z_t)$.

3) Compute the transposed Jacobian from the current value of $\boldsymbol{\theta}$.

4) $\Delta\boldsymbol{\theta} = \alpha\mathbf{J}^T(\boldsymbol{\theta})\Delta\mathbf{e}$, where α is an user-defined learning rate.

5) $\boldsymbol{\theta} = \boldsymbol{\theta} + \Delta\boldsymbol{\theta}$

The iteration will end until the norm of $\Delta\mathbf{e}$ goes below a preset value.

4 Calculation of the Rotation Angle of the Bent Arm

In the calculation above, we have calculated the rotation angle of the minor arm (θ_3). However, what we need is the rotation angle of the bent arm. We can use the following model to calculate the rotation angle of the bent arm: In the triangle

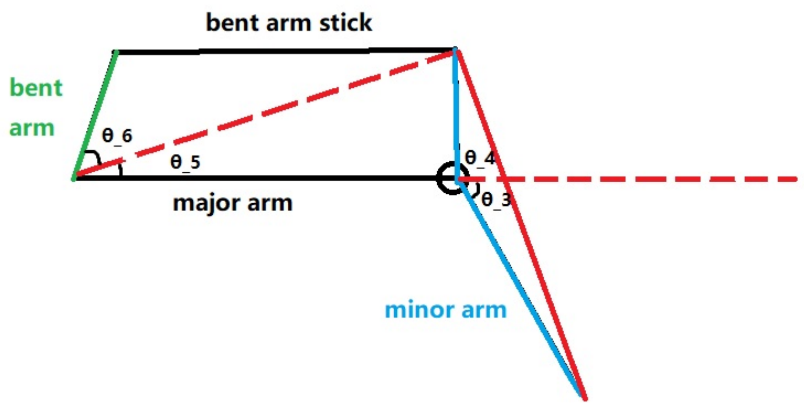

Figure 2: Calculation of Rotation Angle of Bent Arm

formed by the three endpoints of the minor arm, we can calculate the obtuse angle ($\theta_3 + \theta_4$), which means that we can get the value of θ_4. In this way, the triangle including θ_5 is solvable: We can calculate θ_5 and the red edge. Then, we can solve the triangle including θ_6. The angle $\theta_5 + \theta_6$ is the rotation angle of the bent arm.

5 Reference

Both figure [1] and the basic idea of the analytic solution comes from
https://blog.csdn.net/gpeng832/article/details/78966193
The idea of the numerical solution comes from
https://blog.csdn.net/liujiandu101/article/details/81383499