Calibration Mathematical Background

Zach Williams

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1 Introduction

This is the mathematical background used for the calibration phase in the Autoguiding Program. The actual calibration involves moving one motor while keeping the other still and repeating with the other motor. While each motor is moving out, the program is taking several sample points.

Our ultimate goal is to translate an (x, y) coordinate pair into motor instructions. However, orthogonal pixel error doesn't directly correlate to motor instructions, i.e. individual x and y error doesn't correspond to any individual motor, rather it requires some combination of both motors. While this isn't an issue for the declination motor, which always moves the camera frame in the positive or negative x-direction, it is an issue for the rotator angle motor at two different declinations. For instance, moving the rotator angle motor at 90° declination could mean moving the camera frame in the positive y-direction while doing the same at 0° declination would mean moving the camera in the positive x-direction.

Moving forward, we can assume that the declination shouldn't make any significant changes after the telescope is moved to its initial position on the guide star. This is true for all equatorial mounts because the rotator angle is the axis that does the vast majority of adjustments with proper polar alignment. We can reason that changing the rotator angle will result in a consistent change in the camera frame over a guiding session.

2 Variables

$\dot{x}$: x pixel rate
$\dot{y}$: y pixel rate
$\dot{\theta}$: motor 1 rate
$\dot{\phi}$: motor 2 rate

3 Procedure

Relating pixel error rates to motor rates...

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix}$$

In order to get matrix abcd, generate sample points for θ motor while keeping ϕ still:

$$(\Delta x_{\theta 1},\ \Delta y_{\theta 1,\ \Delta\theta_1})$$
$$(\Delta x_{\theta 2},\ \Delta y_{\theta 2,\ \Delta\theta_2})$$
$$...$$
$$(\Delta x_{\theta 10},\ \Delta y_{\theta 10,\ \Delta\theta_1 0})$$

Repeat for ϕ motor while keeping θ motor still:

$$(\Delta x_{\phi 1},\ \Delta y_{\phi 1,\ \Delta\phi_1})$$
$$(\Delta x_{\phi 2},\ \Delta y_{\phi 2,\ \Delta\phi_2})$$
$$...$$
$$(\Delta x_{\phi 10},\ \Delta y_{\phi 10,\ \Delta\phi_1 0})$$

Use least squares to get a, b, c, and d.

$$a = \frac{\sum_{i=1}^{10} \Delta x_{\theta i} \Delta\theta_i}{\sum_{i=1}^{10} \Delta\theta_i^2}$$

$$b = \frac{\sum_{i=1}^{10} \Delta x_{\phi i} \Delta\phi_i}{\sum_{i=1}^{10} \Delta\phi_i^2}$$

$$c = \frac{\sum_{i=1}^{10} \Delta y_{\theta i} \Delta\theta_i}{\sum_{i=1}^{10} \Delta\theta_i^2}$$

$$d = \frac{\sum_{i=1}^{10} \Delta y_{\phi i} \Delta\phi_i}{\sum_{i=1}^{10} \Delta\phi_i^2}$$

Compute SVD of abcd to get conversion matrix fghi.

$$\begin{pmatrix} \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} f & g \\ h & i \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

For a random point (x, y), we take the dot product of the conversion matrix and the point to get the calibrated rates.