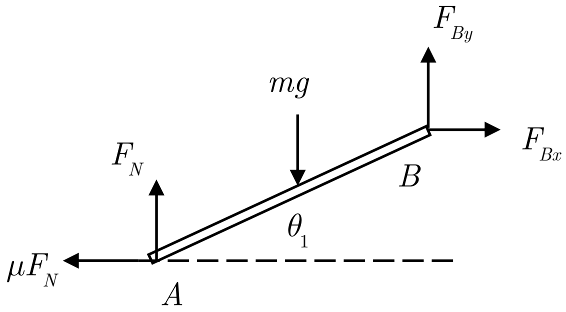
注：本文件是参考解答，不是标准答案

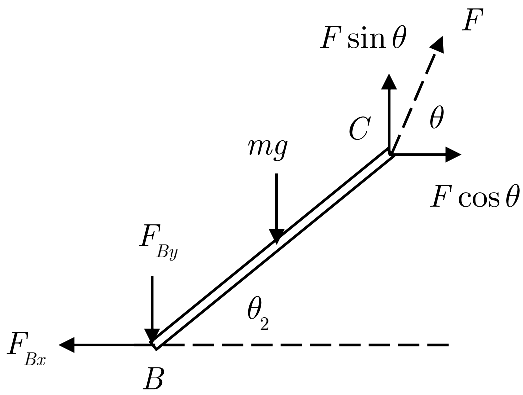
题 26、

分析：有哪几个未知量？能列几个独立方程？



考虑 AB 杆，杆长 L

$$\sum M_B = 0, \quad mg\frac{L}{2}\cos\theta_1 - F_N L\cos\theta_1 - \mu F_N L\sin\theta_1 = 0$$



考虑 BC 杆

$$\sum M_B = 0, \quad F\sin\theta L\cos\theta_2 - F\cos\theta L\sin\theta_2 - mg\frac{L}{2}\cos\theta_2 = 0$$

考虑整体

$$\sum F_x = 0, \quad F\cos\theta - \mu F_N = 0,$$
$$\sum F_y = 0, \quad F\sin\theta + F_N - 2mg = 0.$$

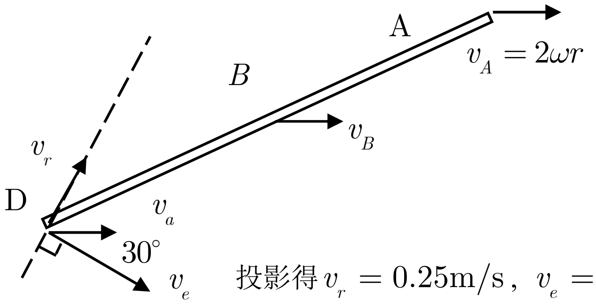
解得：

$$F = \frac{2\mu mg}{\cos\theta + \mu\sin\theta}$$

令 $\frac{dF}{d\theta} = 0$，解得 $\theta_0 = \arctan\mu$

θ_0 代入得最小值 $F_{\min} = \frac{2\mu mg}{\sqrt{\mu^2+1}}$

题 25： （1）v_A, v_B 平行，故有 AD 杆瞬时平移 $v_B = v_A = 2\omega r = 0.5\mathrm{m/s}$



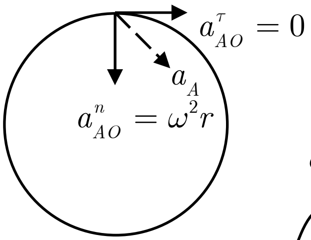
以 D 为动点，动系建在 OE 杆上，

$$v_A = v_a = v_e + v_r$$

大小	√	?	?
方向	√	√	√

投影得 $v_r = 0.25\mathrm{m/s}$，$v_e = \dfrac{\sqrt{3}}{4}\mathrm{m/s}$，$\omega_{O_1E} = \dfrac{v_e}{OD} = \dfrac{15}{4}\mathrm{rad/s}$

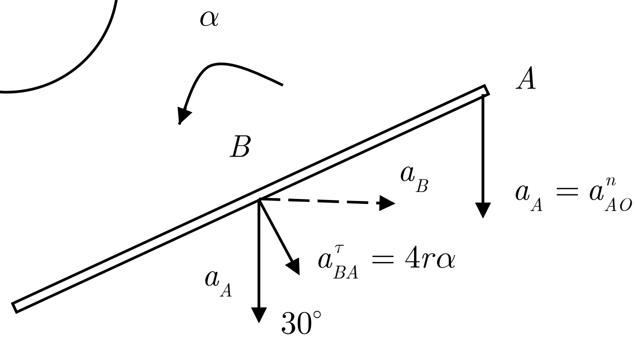
（2）以 O 为基点，列 A 点加速度



$$a_A = a_O + a^n_{AO} + a^\tau_{AO}$$

大小	?	0	√	0
方向	?	√	√	√

（3）以 A 为基点，列 B 点加速度

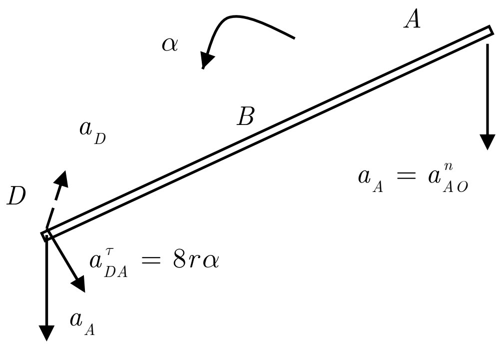


$$a_B = a_A + a^n_{BA} + a^\tau_{BA}$$

大小	?	√	0	?
方向	√	√	√	√

向 y 方向投影，得 $a^\tau_{BA} = \dfrac{-2.5}{\sqrt{3}}$，得 $\alpha_{AD} = \alpha = \dfrac{-2.5}{0.2\sqrt{3}}\mathrm{rad/s^2}$

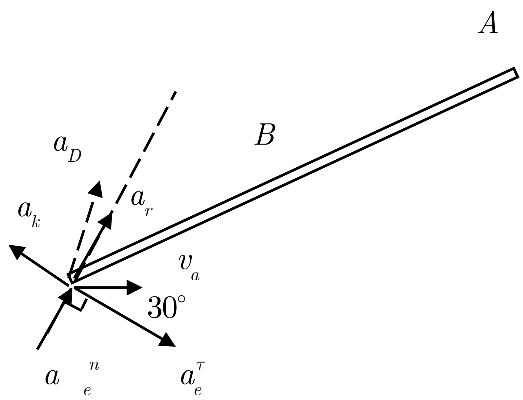
（4）以 A 为基点，列 D 点加速度



$$a_D = a_A + a^n_{DA} + a^\tau_{DA}$$

大小	?	√	0	?
方向	?	√	√	√

（5）以 D 为动点，动系建在 O1E 杆上

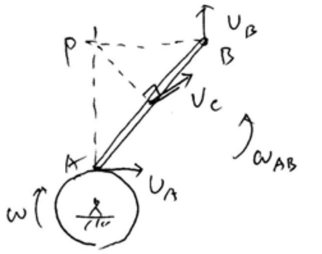


$$a_A + a^\tau_{DA} = a_D = a^n_e + a^\tau_e + a_r + a_k$$

大小	√	√		√	?	?	√
方向	√	√		√	√	√	√

投影得 $\alpha_{O_1E} = 0$

题 24、



解三. ① 动能定理

由瞬心法得 $v_A = \omega_{AO}\overline{AP} = \omega R$

$\Rightarrow \omega_{AB} = \frac{\omega}{2\sqrt3} \Rightarrow v_C = \frac{\omega R}{\sqrt3}$

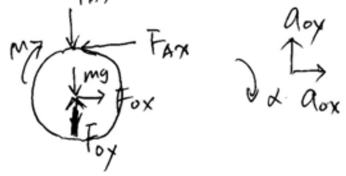
$T_1 = 0$

$T_2 = \frac12\left(\frac12 mR^2\right)\omega^2 + \frac12 m v_C^2 + \frac12\left(\frac{16}{12}mR^2\right)\omega_{AB}^2 = \frac{121}{12\times24} mR^2\omega^2$

$\Sigma W = M\pi - 2Rmg = 2mgR$

由 $T_2 - T_1 = \Sigma W \Rightarrow \omega = \sqrt{\frac{72R}{17g}}$ 或 $2.06\sqrt{\frac{g}{R}}$

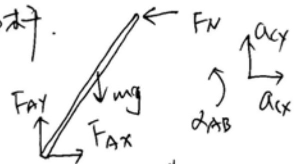
② 考虑轮子, 受力图如右



由定点动量矩定理

$-\frac12 mR^2\alpha = F_{Ax}\cdot R - M \Rightarrow F_{Ax} = \frac{4}{\pi}mg - \frac12 mR\alpha$ ①

③ 考虑AB杆.

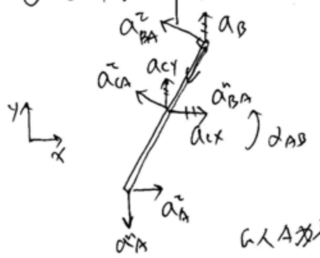


$F_{Ay} - mg = ma_{cy}$ ②

$F_{Ax} - F_N = ma_{cx}$ ③

$F_{Ax}\sqrt3R - F_{Ay}R + F_N\sqrt3R = \frac{16}{12}mR^2\alpha_{AB}$ ④

④ 运动分析. 以A为基点. 列B点加速度 a_B



$a_B = a_A^n + a_A^\tau + a_{BA}^n + a_{BA}^\tau$

大小 ? ω^2R αR $\omega_{AB}^2R\cdot4$? $BA\,\alpha_{AB}$

方向 ✓ ✓ ✓ ✓ ✓

向x方向投影. 得 $\alpha_{AB} = \frac{1}{2\sqrt3}\alpha - \frac{\omega^2}{12\sqrt3}$

以A为基点 列C点加速度 $a_{cx} + a_{cy} = a_A^n + a_A^\tau + a_{CA}^n + a_{CA}^\tau$

? ? ω^2R αR $\omega_{AB}^2 2R$ $\alpha_{AB}\cdot2R$

✓ ✓ ✓ ✓ ✓ ✓

分别向x, y投影得 $\begin{cases} a_{cx} = \frac12 R\alpha \\ a_{cy} = \frac{1}{2\sqrt3}R\alpha - \left(1+\frac{\sqrt3}{12}+\frac{1}{12\sqrt3}\right)\omega^2R \end{cases}$

⑤. 把 a_{cx}, a_{cy} 代入②③式得

$\begin{cases} F_{Ay} = mg + \frac{1}{2\sqrt3}mR\alpha - \left(1+\frac{\sqrt3}{12}+\frac{1}{12\sqrt3}\right)\omega^2Rm \\ F_N = \frac{4}{\pi}mg - mR\alpha \end{cases}$

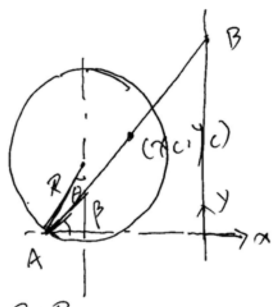
F_{Ay}, F_N 代入④式得

$\left(\frac{8}{\pi}mg - \frac32 mR\alpha\right)\sqrt3R - F_{Ay}R = \frac{1}{12}mR^2\left(\frac{1}{2\sqrt3}\alpha - \frac{\omega^2}{12\sqrt3}\right)$

解得 $\alpha = \frac{3(408+(32+55\sqrt3)\pi)}{289\pi}\frac{g}{R}$ 或 $2.67\frac{g}{R}$

代入 $F_N = \frac{4}{\pi}mg - mR\alpha = -1.39mg$

题三 解析法



$$\begin{cases} \cos\beta = \dfrac{2R+R\sin\theta}{4R} \\ x_c = -2R\cos\beta = -R-\dfrac{R}{2}\sin\theta \\ y_c = R-R\cos\theta+2R\sin\beta \end{cases}$$

β. θ. 是时间函数.

Mathematica 程序

$eq1 = Cos[\beta[t]] - \dfrac{2R+RSin[\theta[t]]}{4R}$;

$x_c = -R - \frac{R}{2} Sin[\theta[t]]$;

$y_c = R - R\,Cos[\theta[t]] + 2R\,Sin[\beta[t]]$;

⋮

解析法通过动能定理

得到 $\omega = \dot{\theta}|_{\theta=180°}$ 后.

再对 $T_2 - T_1 = \sum W$ 两边对时间求导

得到 $\alpha = \ddot{\theta}|_{\theta=180°}$

ω. α 代入两定理得到各力.

分析：目标是建立：

$$\begin{cases} x_c = x_c(\theta) \\ y_c = y_c(\theta) \end{cases}$$

⇓ 再

$$\begin{cases} \dot{x}_c = \dot{x}_c(\theta, \dot{\theta}) \\ \dot{y}_c = \dot{y}_c(\theta, \dot{\theta}) \end{cases}$$

⇓ 再

$$\begin{cases} \ddot{x}_c = \ddot{x}_c(\theta, \dot{\theta}, \ddot{\theta}) \\ \ddot{y}_c = \ddot{y}_c(\theta, \dot{\theta}, \ddot{\theta}) \end{cases}$$

⇓ 得速度关系

$$\begin{cases} v_{cx} = \dot{x}_c|_{\theta=180°} = v_{cx}(\omega) \\ v_{cy} = \dot{y}_c|_{\theta=180°} = v_{cy}(\omega) \end{cases}$$

给动能定理用

其中 $\omega = \dot{\theta}|_{\theta=180°}$

⇓ 得加速度关系

$$\begin{cases} a_{cx} = \ddot{x}_c|_{\theta=180°} = a_{cx}(\omega, \alpha) \\ a_{cy} = \ddot{y}_c|_{\theta=180°} = a_{cy}(\omega, \alpha) \end{cases}$$

给两定理用

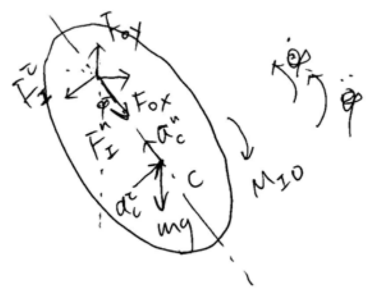
其中 $\alpha = \ddot{\theta}|_{\theta=180°}$

注：可以有中间变量，比如

$$\begin{cases} x_c = x_c(\theta, \beta) \\ y_c = y_c(\theta, \beta) \\ \beta = \beta(\theta) \end{cases}$$

题 23、

题四、



$F_I^\tau = ma_c^\tau = m\ddot{\varphi}a$

$F_I^n = ma_c^n = m\dot{\varphi}^2 a$

$M_{IO} = J_O\ddot{\varphi} = (m\rho_c^2 + ma^2)\ddot{\varphi}$

$$\begin{cases} \Sigma F_x = 0, & F_{Ox} - F_I^\tau\cos\varphi + F_I^n\sin\varphi = 0 \\ \Sigma F_y = 0, & F_{Oy} - F_I^\tau\sin\varphi - F_I^n\cos\varphi - mg = 0 \\ \Sigma M_O = 0, & -mga\sin\varphi - M_{IO} = 0 \end{cases}$$

解得 $\ddot{\varphi} = \frac{-mga\sin\varphi}{J_O}$. $F_{Ox} = -ma\dot{\varphi}^2\sin\varphi - \frac{m^2a^2}{J_O}g\cos\varphi\sin\varphi$.

$F_{Oy} = mg + ma\dot{\varphi}^2\cos\varphi - \frac{m^2a^2g}{J_O}\sin^2\varphi$.

$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi}\cdot\frac{d\varphi}{dt} = \dot{\varphi}\frac{d\dot{\varphi}}{d\varphi}$

$\dot{\varphi}\,d\dot{\varphi} = \frac{-mga\sin\varphi}{J_O}d\varphi \Rightarrow \frac{1}{2}\dot{\varphi}^2\Big|_0^{\dot{\varphi}} = \frac{mga\cos\varphi}{J_O}\Big|_{\varphi_0}^{\varphi}$

$\Rightarrow \dot{\varphi}^2 = 2\frac{mga}{J_O}(\cos\varphi - \cos\varphi_0)$

代入 F_{Ox}, F_{Oy} 得解.

$F_{Ox} = \cdots$

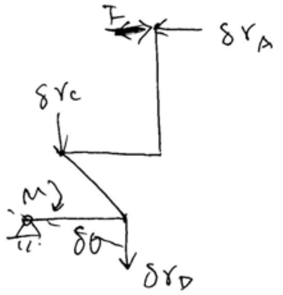
$F_{Oy} = \cdots\cdots$

题 22、

题五.

① 运动条件.

$$\left\{\begin{aligned} &\frac{\delta r_D}{dt} = \frac{\delta r_C}{dt} = a\cdot\frac{\delta\theta}{dt} \\ &\frac{\delta r_C}{b\cdot dt} = \frac{\delta r_A}{3b\,dt} \end{aligned}\right.$$



② 虚位移原理 $\sum F_i \delta r_i = 0$

$$-F\delta r_A + M\delta\theta = 0$$

$$\Rightarrow -3aF\delta\theta + M\delta\theta = 0 \Rightarrow F = \frac{M}{3a}$$

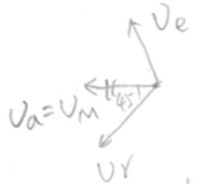
题 21、

期末复习题 (14)

解：(1) 以套筒M点为动点，动系建在AB杆上

$$\vec{v}_M = \vec{v}_a = \vec{v}_e + \vec{v}_r \quad ①$$

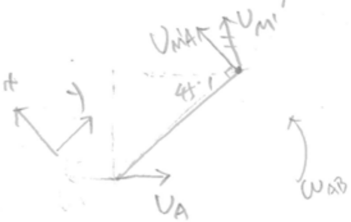
	$\vec{v}_a$	$\vec{v}_e$	$\vec{v}_r$
大小	$-u_0$	?	?
方向	✓	?	✓



(2) 以A为基点，到M'点（即AB杆上与M瞬时重合之点）速度

$$\vec{v}_{M'} = \vec{v}_A + \vec{v}_{M'A} \quad ②$$

	$\vec{v}_{M'}$	$\vec{v}_A$	$\vec{v}_{M'A}$
大小	?	$\omega_0 L$	?
方向	?	✓	✓



注意到 $\vec{v}_e = \vec{v}_{M'}$，联立①、②式，向x方向投影得

$$v_{M'A} = (\omega_0 l + u_0)\frac{\sqrt{2}}{2} = \omega_{AB}\cdot\overline{MA} \Rightarrow \omega_{AB} = \left(\omega_0 + \frac{u_0}{l}\right)\frac{1}{2}$$

向y方向投影，得 $v_r = (\omega_0 l + u_0)\frac{\sqrt{2}}{2}$.

(3) M为动点，AB杆为动系

$$\vec{a}_M = \vec{a}_a = \vec{a}_e + \vec{a}_r + \vec{a}_K \quad ③$$

	$\vec{a}_a$	$\vec{a}_e$	$\vec{a}_r$	$\vec{a}_K$
大小	0	?	?	$2\omega_{AB} v_r$
方向	✓	?	✓	✓

(4) A为基点，到M'加速度

$$\vec{a}_{M'} = \vec{a}_A^\tau + \vec{a}_A^n + \vec{a}_{M'A}^n + \vec{a}_{M'A}^\tau \quad ④$$

	$\vec{a}_{M'}$	$\vec{a}_A^\tau$	$\vec{a}_A^n$	$\vec{a}_{M'A}^n$	$\vec{a}_{M'A}^\tau$
大小	?	0	$\omega_0^2 L$	$\omega_{AB}^2\overline{AM}$	?
方向	?	✓	✓	✓	✓

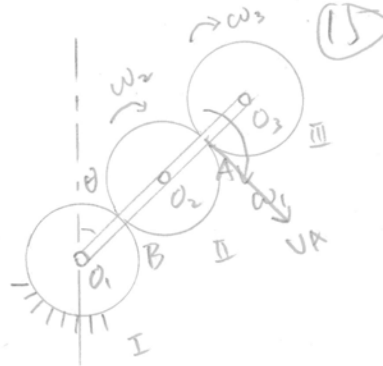
注意到 $\vec{a}_{M'} = \vec{a}_e$，联立③、④式向x方向投影，

得：

$$\alpha_{AB} = \frac{a_{M'A}^\tau}{\overline{MA}} = \frac{1}{2}\left[\left(\omega_0 + \frac{u_0}{l}\right)^2 + \omega_0^2\right]$$

题 20、

解：(1) 应用动能定理



15

$T_1=0$

$T_2=\frac{1}{2}mv_{O_2}^2+\frac{1}{2}(\frac{1}{2}mR^2)\omega_2^2+\frac{1}{2}mv_{O_3}^2+\frac{1}{2}(\frac{1}{2}mR^2)\omega_3^2$

$\sum W=mg2R(1-\cos\theta)+mg4R(1-\cos\theta)$

运动学条件：$v_{O_2}=2R\dot{\theta}$　$v_{O_3}=4R\dot{\theta}$

I，II 两轮重合点 B 是瞬心 $\Rightarrow \omega_2=\frac{v_{O_2}}{R}=2\dot{\theta}$

II，III 轮重合点 A 和 A' 速度相同 $\Rightarrow v_A=v_{A'}$

其中 $v_A=2R\omega_2=4R\dot{\theta}$，得 $\omega_3=\frac{v_{O_3}-v_{A'}}{R}=0$

由 $T_2-T_1=\sum W$ 得 $\dot{\theta}^2=\frac{6}{11}\frac{g}{R}(1-\cos\theta)$　①

杆到水平时 $\omega=\dot{\theta}|_{\theta=90^\circ}=\sqrt{\frac{6g}{11R}}$

(2) 求杆加速度 α

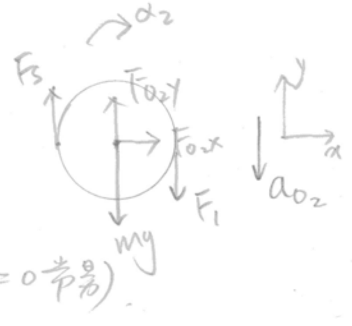
对①式两边对时间求导得

$2\dot{\theta}\ddot{\theta}=\frac{6g}{11R}\sin\theta\cdot\dot{\theta} \Rightarrow$ 杆到水平时 $\alpha=\ddot{\theta}|_{\theta=90^\circ}=\frac{3g}{11R}$

(3) 求 F_s、

考虑轮 II，由动量矩定理得

$F_1R+F_sR=\frac{1}{2}mR^2\alpha_2$　②



运动学条件 $\alpha_2=\frac{d\omega_2}{dt}|_{\theta=90^\circ}=2\alpha=\frac{6g}{11R}$

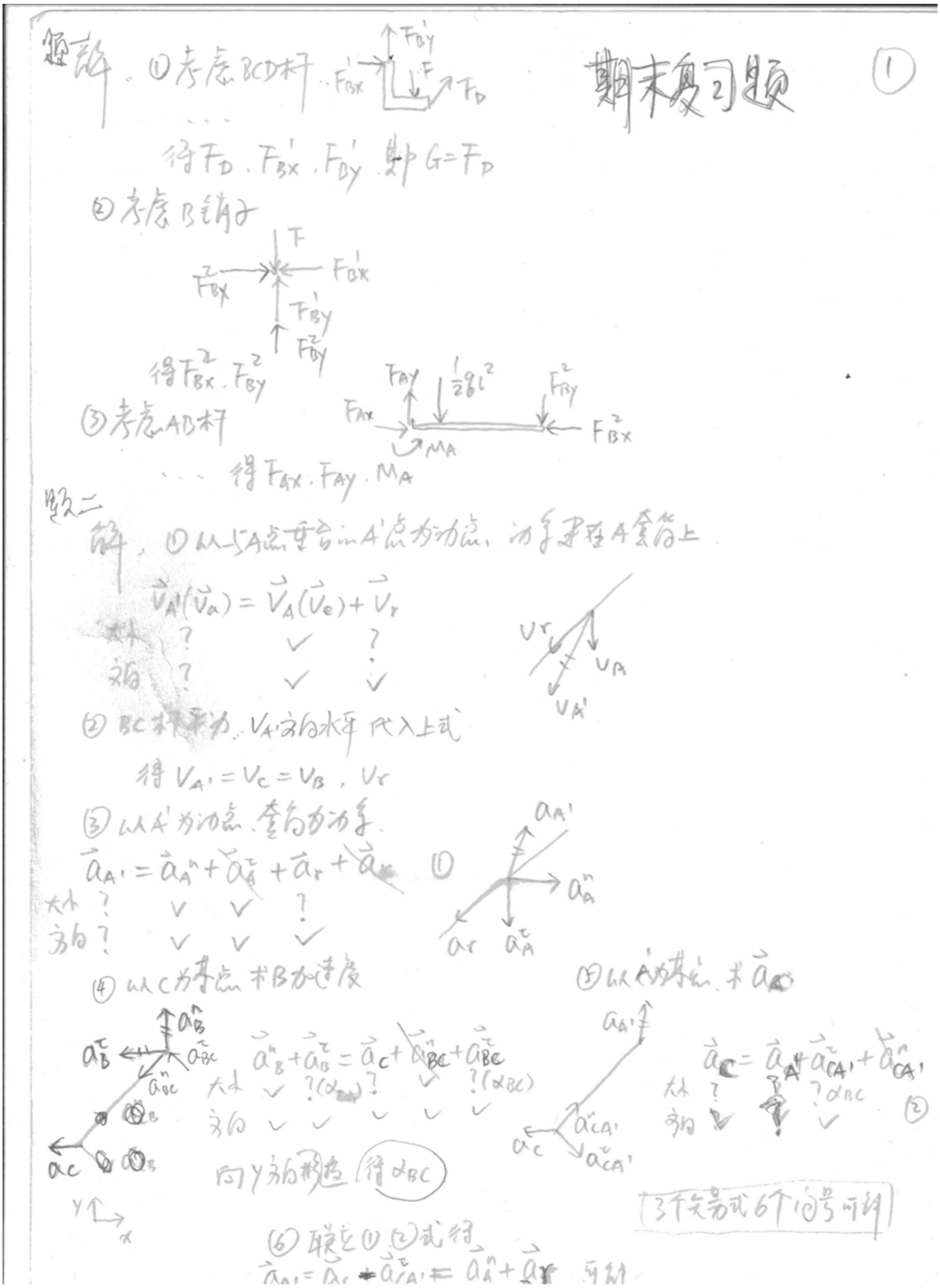
对轮 III 用动量矩定理可得 $F_1=0$（因为 $\omega_3=0$ 常量）

代入②式得 $F_s=\frac{3}{11}mg$

对轮 II y 方向动量定理得

$F_{O_2y}+F_s-mg=-ma_{O_2}=-m\cdot 2R\alpha \Rightarrow F_{O_2y}=\frac{2}{11}mg$

题 18、题 19、



题解、① 考虑BCD杆 …

得 F_D、F'_{Bx}、F'_{By} 即 $G=F_D$

期末复习题 ①

② 考虑B销子

得 F^2_{Bx}、F^2_{By}

③ 考虑AB杆

…得 F_{Ax}、F_{Ay}、M_A

题二

解，① 以与A点重合的A'点为动点，动系建在A套筒上

$\vec{V}_{A'}(\vec{V}_a)=\vec{V}_A(\vec{V}_e)+\vec{V}_r$

大小 ? ✓ ?

方向 ? ✓ ✓

② BC杆平动，$V_{A'}$方向水平 代入上式

得 $V_{A'}=V_C=V_B$，V_r

③ 以A'为动点，套筒为动系

$\vec{a}_{A'}=\vec{a}^n_A+\vec{a}^\tau_A+\vec{a}_r+\vec{a}_k$ ①

大小 ? ✓ ✓ ?

方向 ? ✓ ✓ ✓

④ 以C为基点 求B加速度

$\vec{a}^n_B+\vec{a}^\tau_B=\vec{a}_C+\vec{a}^n_{BC}+\vec{a}^\tau_{BC}$

大小 ✓ ?(α_{AB}) ? ✓ ?(α_{BC})

方向 ✓ ✓ ✓ ✓ ✓

向y方向投影 得 α_{BC}

⑤ 以A'为基点，求 a_C

$\vec{a}_C=\vec{a}_{A'}+\vec{a}^\tau_{CA'}+\vec{a}^n_{CA'}$

大小 ? ? ?α_{BC}

方向 ✓ ? ✓ ②

3个矢量式 6个未知量可解

⑥ 联立① ②式得

$\vec{a}_{A'}=\vec{a}_C+\vec{a}^\tau_{CA'}=\vec{a}^n_A+\vec{a}_r$ 可解

题 16、题 17、

解：① 由动能定理

$T_1=0$

$T_2=\frac{1}{2}mv_c^2+\frac{1}{2}J_c\omega^2$

$v_c=(R+\frac{R}{4})\omega$，$J_c=J_O-m(\frac{R}{4})^2$

②

$\Sigma W=\cdots\cdots$

由 $T_2-T_1=\Sigma W$ 解得 $\omega=\sqrt{\frac{\pi g}{2R}}$

② y方向动量定理

$F_N-mg\cos 30°=ma_{cy}$ ①

$\vec{a}_{cx}+\vec{a}_{cy}=\vec{a}_O+\vec{a}_{cO}^n+\vec{a}_{cO}^\tau$

向y方向投影得

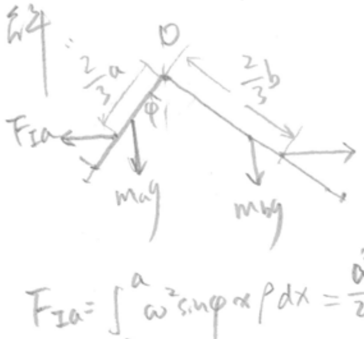
$a_{cy}=-a_{cO}^n=-\frac{R}{4}\omega^2$ 代入①式，得 $F_N=(\frac{\sqrt{3}}{2}-\frac{\pi}{8})mg$

③ x方向动量定理和动量矩定理

$$\begin{cases}-mg\sin 30°+2\pi RK+F_s=ma_{cx} & ② \\ F_s(R+\frac{R}{4})+2\pi RK\cdot\frac{R}{4}=J_c\alpha\end{cases}$$

其中 $\vec{a}_{cx}=\vec{a}_{cO}^\tau+\vec{a}_O=\alpha\frac{R}{4}+\alpha R$ 代入②式 得 $\alpha=\frac{g}{16R}$

解：



$\Sigma M_O=0$，……

得 $\omega^2=\cdots$

$F_{Ia}=\int_0^a\omega^2\sin\varphi\, x\rho\, dx=\frac{a^2}{2}\rho\omega^2\sin\varphi$

$m_a=\rho a$

题 14、题 15、

解：①

由虚功原理：

$\sum \vec{F}_i \cdot \delta \vec{r}_i = 0$ ③

$F\delta r_B \cos 60^\circ - F\delta r_D \cos 45^\circ + Q\delta r_D \cos 45^\circ = 0$ ①

由速度投影定理

$\delta r_D \cos 0^\circ = \delta r_C \cos 45^\circ$

$\delta r_C \cos 60^\circ = \delta r_B \cos 30^\circ$，得 δr_B 与 δr_D 关系代入①式

得 $F = \cdots$

解：① 以 M 为动点、ABC 杆为动系

$\vec{v}_M = \vec{v}_e + \vec{v}_r$

大小 ? √ ?

方向 √ √ √

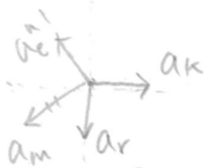
求得 v_a, v_r

② 以 M 为动点、ABC 杆为动系 求 a_M

$\vec{a}_M = \vec{a}_e^n + \vec{a}_e^\tau + \vec{a}_r + \vec{a}_K$ ①

大小 ? √ ? √

方向 ? √ √ √

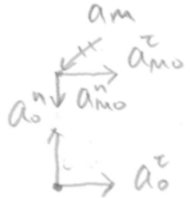


③ 以 O 为基点，求 a_M

$\vec{a}_M = \vec{a}_O^n + \vec{a}_O^\tau + \vec{a}_{MO}^n + \vec{a}_{MO}^\tau$ ②

大小 ? √ ?(α_O) √ ?(α_O)

方向 ? √ √ √ √



$\frac{v_O^2}{R+r}$, $v_O = \omega_O r$, $\omega_O = \frac{v_M}{2r}$

$a_O^\tau = \alpha_O r$

$a_{MO}^n = \omega_O^2 r$

$\omega_O = \frac{\sqrt{3}}{2}\omega$，$\alpha_O = \frac{\omega^2}{2}$

④ 联立①②式

$\vec{a}_O^\tau + \vec{a}_{MO}^n + \vec{a}_{MO}^\tau = \vec{a}_e^n + \vec{a}_r + \vec{a}_K$

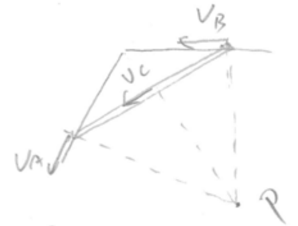
?(α_O) ?(α_O) ?

题 13、

解：① 由动能定理

$T_1=0$

$T_2=\frac{1}{2}\frac{1}{12}mL^2\omega^2+\frac{1}{12}mv_C^2$

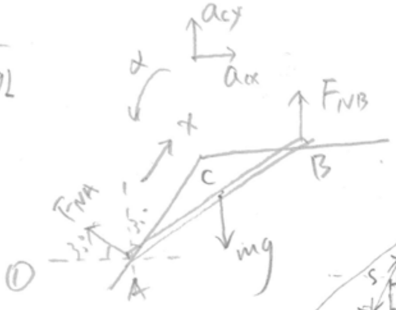


运动学得 $v_C=\frac{\sqrt{3}}{2}L\omega^2$

$\Sigma W=mg\frac{L}{4}$

由 $T_2-T_1=\Sigma W$ 得 ω

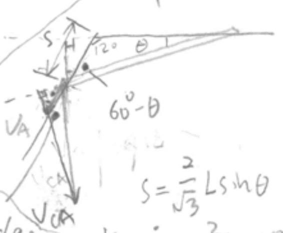
由 $v_A=\omega L$ 得 $v_A=\sqrt{\frac{3}{5}gL}$



② 由动量定理和动量矩定理

$$\begin{cases} F_{NB}\frac{\sqrt{3}}{2}\frac{L}{2}-F_{NA}\frac{\sqrt{3}}{2}\frac{L}{2}=\frac{1}{12}mL^2\alpha \\ a_{cx}m=-\frac{\sqrt{3}}{2}F_{NA} \\ a_{cy}m=F_{NB}+\frac{1}{2}F_{NA}-mg \end{cases} \quad ①$$

解析法



$180°-(90°-60°+\theta)$

$S=\frac{2}{\sqrt{3}}L\sin\theta$

$V_A=\dot{S}=\frac{2}{\sqrt{3}}L\cos\theta\cdot\dot{\theta}$

$\vec{V}_C=\vec{V}_A+\vec{V}_{CA}$

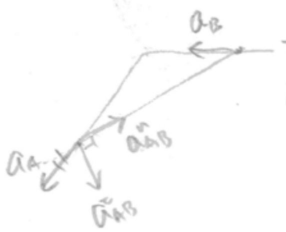
$|\vec{V}_C|=[\dot{S}^2+\frac{1}{4}L^2\dot{\theta}^2-L\dot{S}\dot{\theta}\cos(\frac{\pi}{6}+\theta)]$

$T_1=0$

$T_2=\frac{1}{24}mL^2\dot{\theta}^2+\frac{1}{2}m|\vec{V}_C|^2$

要找到 a_{cx}, a_{cy} 与 α 之间的关系

以B为基点求 a_A



	$\vec{a}_A=$	$\vec{a}_B+$	$\vec{a}^n_{AB}+$	$\vec{a}^\tau_{AB}$
大小	?	?	√	?(α)
方向	√	√	√	√

向x方向投影 得 $a_B=a_B(\alpha)$

以B为基点求 a_C

	$\vec{a}_{cx}+$	$\vec{a}_{cy}=$	$\vec{a}_B+$	$\vec{a}^n_{CB}+$	$\vec{a}^\tau_{CB}$
大小	?	?	?(α)	√	?(α)
方向	√	√	√	√	√

分别向x、y方向投影得 $a_{cx}=a_{cx}(\alpha)$，$a_{cy}=a_{cy}(\alpha)$

代入①式解得

$\alpha=\frac{3\sqrt{3}g}{10L}$，$F_{NA}=\frac{11}{20}mg$，$F_{NB}=\frac{13}{20}mg$

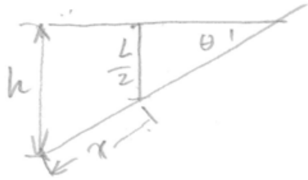
题 11、题 12、

解：弹簧原长 $= L$

5

弹簧伸长量 $x = 2L - \frac{L}{2\sin\theta} - L$,

物块高度 $h = 2L\sin\theta$



变分得 $\delta h = 2L\cos\theta\,\delta\theta$

$\delta x = \frac{L\cos\theta}{2\sin^2\theta}\cdot\delta\theta$

由虚功原理：

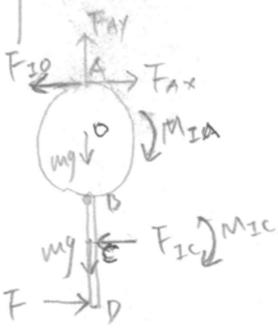
$\sum F_i \delta r_i = 0$

$\Rightarrow P\delta h - F\delta x = 0$

$F = x\cdot k$

解得 $8P\sin^3\theta - 2kL\sin\theta + kL = 0$

解：① 整体受力图如下

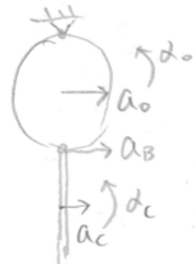


其中 $F_{IO} = ma_O$ $F_{IC} = ma_C$ $M_{IA} = J_A\alpha_O$

$M_{IC} = J_C\alpha_C$, $J_A = \frac{1}{2}mr^2 + mr^2$, $J_C = \frac{1}{12}m(2r$

由运动学条件：$a_O = \alpha_O r$

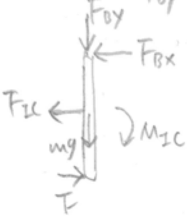
$a_C = a_B + \alpha_C r = 2r\alpha_O + r\alpha_C$



② 考虑AB轮受力图

$\sum M_A = 0$ …… 得 $F_{Bx} = \frac{3}{4}r^2\alpha_0$

③ 考虑BD杆



$\sum F_x = 0$. $F - F_{Bx} - ma_C = 0$

$\sum M_C = 0$,

$Fr + F_{Bx}r - J_C\alpha_C = 0$

将运动学条件代入后得：

$\alpha_0 = -\frac{2F}{5mr}$, $\alpha_C = -\frac{21F}{10mr}$

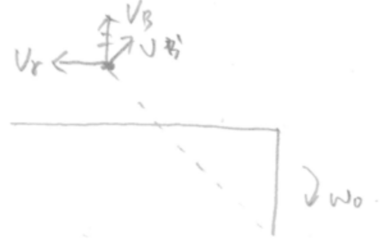
题 9、题 10、

解、①以B为动点，AO为动系 ⑥

$\vec{v}_B = \vec{v}_{B'} + \vec{v}_r$

大小 ? ✓ ?

方向 ✓ ✓ ✓



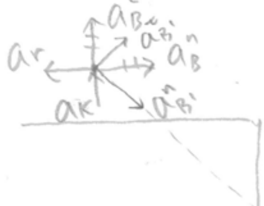
$\omega_{O_1} = \dfrac{v_B}{R} = 2\omega_0$

②以B为动点 AO为动系

$\vec{a}_B^n + \vec{a}_B^\tau = \vec{a}_{B'}^n + \cancel{\vec{a}_{B'}^\tau} + \vec{a}_r + \vec{a}_K$

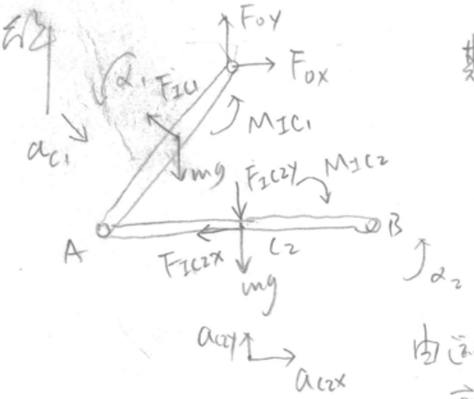
大小 ✓ ? ✓ ? ✓

方向 ✓ ✓ ✓ ✓ ✓



$\alpha_{O_1} = \dfrac{a_B^\tau}{R} = 2\omega_0^2$

解



其中 $M_{IC_1} = J_C\alpha_1 \quad J_C = \frac{1}{12}mL^2$

$F_{IC_1} = m\frac{L}{2}\alpha_1$

$M_{IC_2} = J_C\alpha_2$

$F_{IC2x} = m a_{C2x}$

$F_{IC2y} = m a_{C2y}$

由运动学条件、A为基点，求 a_{C_2}

$\vec{a}_{C2x} + \vec{a}_{C2y} = \cancel{\vec{a}_A^n} + \vec{a}_A^\tau + \cancel{\vec{a}_{CA}^n} + \vec{a}_{CA}^\tau$

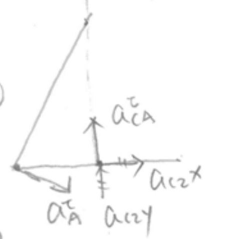
向x方向投影得 $a_{C2x} = a_A^\tau \cdot \cos 30^\circ = \frac{\sqrt{3}}{2}L\alpha_1$

向y方向投影得 $a_{C2y} = \frac{L}{2}\alpha_2 - \frac{1}{2}L\alpha_1$

考虑整体

$\Sigma M_O = 0, \ldots\ldots$

$f(\alpha_1, \alpha_2) = 0$ ①



考虑AB杆

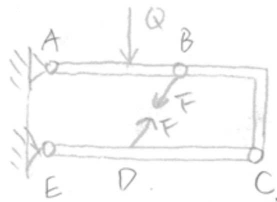
$\Sigma M_A = 0$

$g(\alpha_1, \alpha_2) = 0$ ②

联立①②式 解得 α_1，α_2

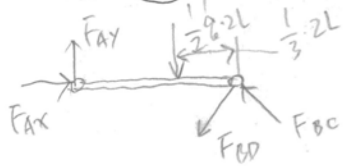
题 8、

解：



⑦

① 考虑AB杆（不含铆钉，取成分离体）、受力图如下



$\Sigma M_A = 0$，…… 得 $F_{BC} = f(F_{BD})$ ①

② 考虑EC杆

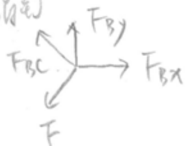
$\Sigma M_E = 0$ …… 得 $F_{BC} = g(F_{BD})$ ②

联立①、②式 得 $F_{BD} = \cdots$

③ 考虑AB杆

$\Sigma F_x = 0$
$\Sigma F_y = 0$ 得 $F_{Ax} = \cdots$ $F_{Ay} = \cdots$

④ 考虑铆钉



$\begin{cases}\Sigma F_x = 0 \\ \Sigma F_y = 0\end{cases}$ 得 $F_{Bx} = \cdots$，$F_{By} = \cdots$

题 7、

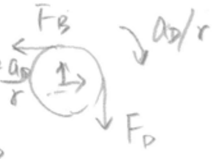
11

解：① 考虑物块D

由 $F_D - mg = -ma_D$

得 $F_D = mg - ma_D$

考虑轮O

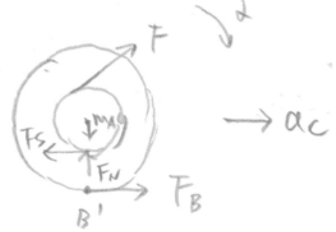


由 $(F_D - F_B)r = \frac{1}{2}mr^2\frac{a_D}{r}$

得 $F_B = mg - \frac{3}{2}ma_D$

② 考虑塔轮

动量定理和动量矩定理

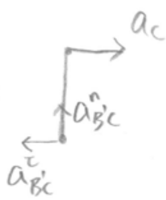


$F\cos\beta - F_s + F_B = m_A a_C$ ③

$Fr + F_s r - F_B R = J_C\alpha$ ④

运动条件 $a_C = \alpha \cdot r$ ①

以C为基点求 $a_{B'}$



$\vec{a}_{B'x} + \vec{a}_{B'y} = \vec{a}_C + \vec{a}^n_{B'C} + \vec{a}_{B'C}$

得 $\vec{a}_{B'x} = a_D = a_C - a^\tau_{B'C}$

$= \alpha r - \alpha R$ ②

①②代入③④ 解得 $F_s = (1 + \frac{9\sqrt{3}}{23})mg$

Y方向动量定理

$F_N + F\sin\beta - m_A g = 0$ 得 $F_N = \frac{7}{2}mg$

纯滚动条件 $F_s \le f_s F_N$ 得 $f_s \ge 0.48$

题 6、

12

解：① B端未离开墙时，杆作定轴转动

由动能定理

$T_1 = 0$

$T_2 = \frac{1}{2}\cdot\frac{1}{3}ml^2\dot{\theta}^2$，

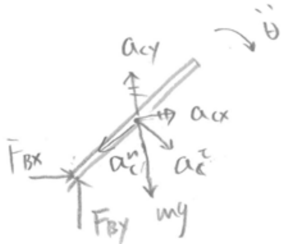
$\Sigma W = mg\frac{L}{2}(1-\cos\theta)$

由 $T_2 - T_1 = \Sigma W$，得 $\dot{\theta} = \sqrt{\frac{3g}{L}(1-\cos\theta)}$

两边求导得

$\ddot{\theta} = \frac{3g}{2L}\sin\theta$.

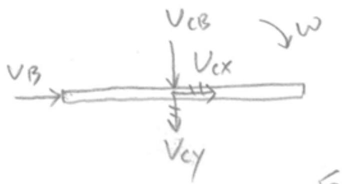
$\vec{a}_{cx} + \vec{a}_{cy} = \vec{a}_c^{\tau} + \vec{a}_c^{n}$



$$\begin{cases} F_{Bx} = ma_{cx} \\ F_{By} - mg = ma_{cy} \\ a_{cx} = a_c^{\tau}\cos\theta - a_c^{n}\sin\theta \\ a_{cy} = -a_c^{\tau}\sin\theta - a_c^{n}\cos\theta \end{cases} \Rightarrow \begin{cases} F_{Bx} = \frac{3}{4}mg\sin\theta(3\cos\theta - 2) \\ F_{By} = \cdots \end{cases}$$

当 $F_{Bx}=0$，即 $\theta_1 = \arccos\frac{2}{3}$ 时，B端离开墙，初角速度 $\dot{\theta}_1 = \cdots\cdots$

② x方向动量守恒得，$V_{cx} = \frac{1}{2}L\dot{\theta}_1\cos\theta_1 = \cdots$



	$\vec{V}_{cx}$ +	$\vec{V}_{cy}$ =	$\vec{V}_B$ +	$\vec{V}_{CB}$
大小	✓	?	?	?(ω)
方向	✓	✓	✓	✓

向y轴投影，得 $V_{cy} = \frac{L}{2}\omega$.

由动能定理（全程） $mg\frac{L}{2} = \frac{1}{2}m(V_{cx}^2 + V_{cy}^2) + \frac{1}{2}\cdot\frac{1}{12}ml^2\omega^2$ 得 $\omega = \sqrt{\frac{8g}{3L}}$

题 5、

解. ① ABC杆瞬时平动. 10

$V_C = V_A = 2R\cdot\frac{U}{R}$

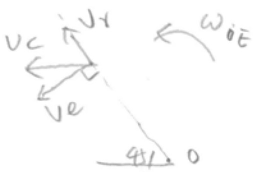
以C为动点, EO杆为动系

$\vec{V}_C = \vec{V}_e + \vec{V}_r$

大小 √ ? ?

方向 √ √ √

解得 $V_e = \cdots = \omega_{OE}\cdot\overline{OC} \Rightarrow \omega_{OE} = \cdots$



② 以O_1为基点, 求a_A

$\vec{a}_A = \vec{a}_{O_1} + \vec{a}^n_{AO_1} + \cancel{\vec{a}^\tau_{AO_1}}$

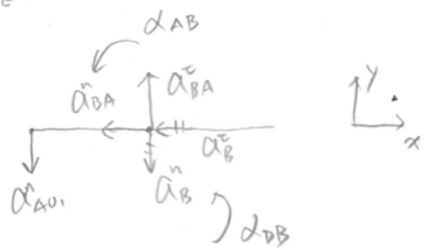
以A为基点, 求a_B

$\vec{a}^\tau_B + \vec{a}^n_B = \vec{a}^n_{AO_1} + \cancel{\vec{a}^n_{BA}} + \vec{a}^\tau_{BA}$

大小 ?(α_{DB}) √ √ ?(α_{AB})

方向 √ √ √ √

向y方向投影, 得 $\alpha_{AB} = \cdots$

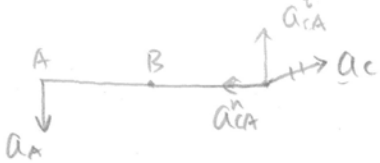


③ 以A为基点, 求a_C

$\vec{a}_C = \vec{a}_A + \vec{a}^n_{CA} + \vec{a}^\tau_{CA}$ ①

大小 ? √ √

方向 ? √ √

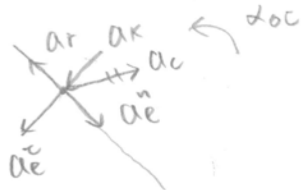


以C为动点, EO杆为动系

$\vec{a}_C = \vec{a}^n_e + \vec{a}^\tau_e + \vec{a}_r + \vec{a}_k$ ②

大小 ? √ ?(α_{OE}) ? √

方向 ? √ √ √ √

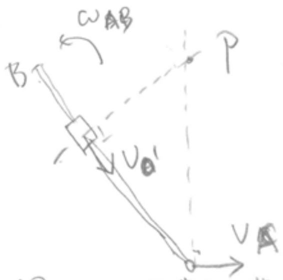


联立①、②式得 $\alpha_{OE} = \cdots$

题 3、题 4、

解：

8



① 瞬心法

$\omega_{AB}=\frac{V_A}{\overline{PA}}=\cdots$

$\vec{V}_{O'}=\omega_{AB}\overline{PO'}=\vec{v}_r$

② 以A为基点，求$a_{O'}$

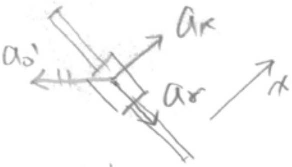
$\vec{a}_{O'}=\cancel{\vec{a}_A}+\vec{a}^n_{O'A}+\vec{a}^\tau_{O'A}$ ①

	$\vec{a}_{O'}$	$\vec{a}_A$	$\vec{a}^n_{O'A}$	$\vec{a}^\tau_{O'A}$
大小	?	✓	✓	?(α_{AB})
方向	?	✓	✓	✓

以O'为动点，套筒O为动系

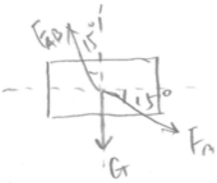
$\vec{a}_{O'}=\cancel{\vec{a}_e}+\vec{a}_r+\vec{a}_k$ ②

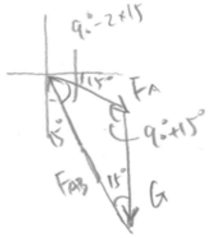
	$\vec{a}_{O'}$	$\vec{a}_r$	$\vec{a}_k$
大小	?	?	✓
方向	?	✓	✓



联立①②式向x方向投影，得$\alpha_{AB}=\cdots$

解：① 考虑A块

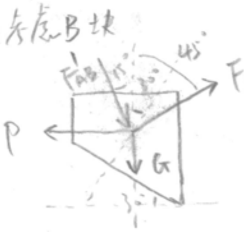




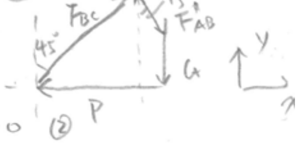
$\vec{F}_A+\vec{F}_{AB}+\vec{F}_G=\vec{0}$

$\frac{G}{\sin(90^\circ-2\times15^\circ)}=\frac{F_{AB}}{\sin(90^\circ+15^\circ)}=\frac{F_A}{\sin15^\circ}$ ①

② 考虑B块



$\vec{P}+\vec{F}'_{AB}+\vec{F}_{BC}+\vec{G}=\vec{0}$



$\sum F_x=0,\ F_{AB}\sin15^\circ-P+F_{BC}\cos45^\circ=0$ ②

$\sum F_y=0,\ -F_{AB}\cos15^\circ-G+F_{B}\cos45^\circ=0$ ③

联立①②③得 $P=2.366G$

题 2、

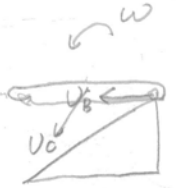
解 ① 动能定理

$T_1 = 0$

$T_2 = \frac{1}{2} m_2 v_B^2 + \frac{1}{2} m_1 v_C^2 + \frac{1}{2} J_C \omega^2$

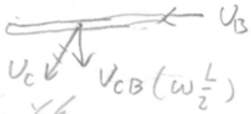
$\Sigma W = mg\frac{L}{2}$

(13)



运动学条件，以B为基点

$\vec{v}_C = \vec{v}_B + \vec{v}_{CB}$



得 $v_C^2 = \omega^2 \frac{L}{4} + v_B^2$

注意：两角度得不到 v_B 和 ω 的关系：

系统在x方向上动量守恒得

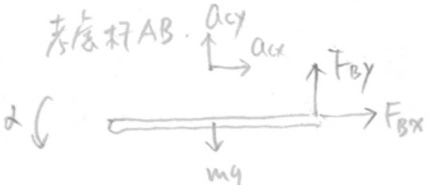
$(m_2\vec{v}_B + m_1\vec{v}_C)|_x = 0$

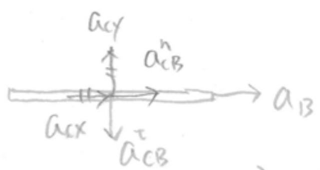
$\Rightarrow -m_2 v_B - m_1 v_B = 0 \Rightarrow v_B = 0$

由 $\Sigma W = T_2 - T_1$ 解得：$\omega^2 = \frac{3g}{L}$

②

考虑杆AB.





$\vec{a}_{Cx} + \vec{a}_{Cy} = \vec{a}_B + \vec{a}_{CB}^n + \vec{a}_{CB}^\tau$

向y方向投影得

$$\begin{cases} m a_{Cy} = -mg - F_{By} \\ F_{By}\frac{L}{2} = J_C \alpha \\ a_{Cy} = -a_{CB}^\tau = -\alpha\frac{L}{2} \end{cases}$$

得 $\alpha = \frac{3g}{2L}$，$F_{By} = \frac{1}{4}mg$

题 1、

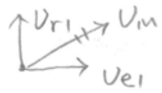
9

解：① M销钉为动点，杆CDE为动系

$\vec{v}_M = \vec{v}_{r1} + \vec{v}_{e1}$ ①

大小 ? ? ✓

方向 ? ✓ ✓

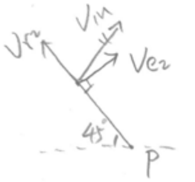


再M为动点，FG为动系

$\vec{v}_M = \vec{v}_{r2} + \vec{v}_{e2}$ ②

大小 ? ? ✓

方向 ? ✓ ✓



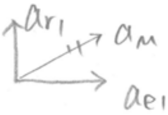
联立两式，得 $v_{r2} = -100\sqrt{2}$，$v_{r1} = 0$，代入①式得 $\vec{v}_M = \cdots$

(2) M动点，CDE动系

$\vec{a}_M = \vec{a}_{e1} + \vec{a}_{r1}$ ①

大小 ? ✓ ?

方向 ? ✓ ✓

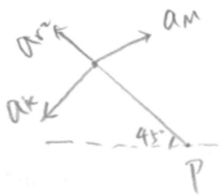


再M动点，FG动系

$\vec{a}_M = \vec{a}_{e2} + \vec{a}_{r2} + \vec{a}_k$ ②

大小 ? ? ? ✓

方向 ? ? ✓ ✓

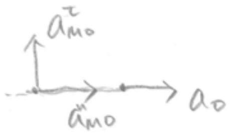


以O为基点，求 $\vec{a}_{e2}$

$\vec{a}_{e2} = \vec{a}_O + \vec{a}^n_{MO} + \vec{a}^\tau_{MO}$ ③

大小 ? ✓

方向 ? ✓



联立①、②、③式

得 $a_{r1} = 300$，$a_{r2} = -100\sqrt{2}$，代入①式得 $\vec{a}_M = \cdots$