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# Tunable Atomic Magnetometer for Detection of Radio-Frequency Magnetic Fields

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We describe an alkali-metal magnetometer for detection of weak magnetic fields in the radio-frequency (rf) range. High sensitivity is achieved by tuning the Zeeman resonance of alkali atoms to the rf frequency and partially suppressing spin-exchange collisions in the alkali-metal vapor. We demonstrate magnetic field sensitivity of  $2 \text{ fT/Hz}^{1/2}$  at a frequency of 99 kHz with a resonance width of 400 Hz. We also derive a simple analytic expression for the fundamental limit on the sensitivity of the rf magnetometer and show that a sensitivity of about  $0.01 \text{ fT/Hz}^{1/2}$  can be achieved in a practical system with a measurement volume of  $200 \text{ cm}^3$ .

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Detection of radio-frequency (rf) fields in the kilohertz to gigahertz frequency range finds numerous applications, from radio communication to detection of NMR and nuclear quadrupole resonance (NQR) signals [1] to fundamental physics measurements, such as axion searches [2]. Inductive pickup coils, commonly used for rf detection, become less efficient below about 1 MHz. Alternative methods of detection with SQUID magnetometers [3] and Rydberg atoms [4] have been explored. Using a resonantly tuned superconducting coil coupled to a SQUID [5], magnetic field sensitivity of  $0.08 \text{ fT/Hz}^{1/2}$  has been obtained at 425 kHz [6]. Alkali-metal atomic magnetometers, which measure the spin precession of optically pumped atoms in a magnetic field, have comparable sensitivity at frequencies below 100 Hz [7]. However, all existing atomic magnetometers [7–11] have been designed for detection of quasi-static magnetic fields, and their sensitivity drops at high frequency as  $1/f$ .

In this Letter, we show that by tuning the Zeeman spin resonance frequency of the atoms to the rf it is possible to obtain similarly high sensitivity to the rf fields. Tunable rf atomic magnetometers open the possibility of detecting fixed high frequency signals, such as those from nuclear quadrupole resonance. They also allow extension of NMR signal detection [12,13] to higher frequencies, which simplifies magnetic resonance imaging (MRI) techniques [14] and reduces environmental magnetic noise and thermal Johnson noise [15]. We demonstrate a resonantly tuned alkali-metal magnetometer operating at 99 kHz with a sensitivity of  $2 \text{ fT/Hz}^{1/2}$  to an oscillating field and  $1 \text{ fT/Hz}^{1/2}$  to a rotating magnetic field. We also derive a simple analytic expression for the fundamental sensitivity of the rf magnetometer, including the effects of spin-projection noise, probe laser photon shot noise, and ac-Stark shift noise due to fluctuations of the probe laser polarization. We find that the experimental sensitivity is close to the fundamental noise limitations. With additional improvements, such as using two counterpropagating op-

tical pumping beams and a larger cell, it is possible to obtain a sensitivity of  $0.01 \text{ fT/Hz}^{1/2}$ .

The magnetometer (Fig. 1) consists of a cell with potassium atoms placed in a static magnetic field  $B_0 \hat{z}$ . A circularly polarized laser optically pumps the atoms along  $\hat{z}$ . A weak rf field perpendicular to the static field and tuned to the Zeeman resonance of the atoms will induce a transverse rotating component of the spin polarization, which is detected with a probe laser directed along  $\hat{x}$ . For alkali-metal atoms the Zeeman transition frequency is given by  $\omega_0 = \gamma B_0 = g_s \mu_B B_0 / (2I + 1) \hbar = 2\pi \times (2.8 \text{ MHz/G}) B_0 / (2I + 1)$ , where  $I$  is the nuclear spin of the atoms. If the atoms are nearly completely polarized along  $\hat{z}$ , their transverse polarization created by a resonant oscillating magnetic field  $B_1 \hat{y} \cos \omega_0 t$  is equal to [16]

$$P_x = F_x / F_z = (1/2) \gamma B_1 T_2 \sin \omega_0 t. \quad (1)$$

Hence, the sensitivity is proportional to the transverse spin relaxation time  $T_2$ . While magnetic resonance broadening due to spin-exchange (SE) collisions can be completely eliminated at very low magnetic fields [17], at higher fields it can be only partially suppressed by pumping nearly all atoms into a fully polarized state [18]. The equilibrium electron polarization is equal to  $P_z = s_z R_{\text{OP}} / (R_{\text{OP}} + R_{\text{SD}})$ , where  $R_{\text{OP}}$  is the optical pumping (OP) rate,  $R_{\text{SD}}$  is the electron spin-destruction (SD) rate, and  $s_z$  is the degree of circular polarization of the pumping laser. One can take  $s_z = 1$  because in an optically thick vapor the “wrong” component of the light polarization is quickly absorbed for a well collimated pump beam directed parallel to the magnetic field [19]. The transverse spin relaxation time in the presence of SE and SD collisions can be calculated using the formalism developed in [16] and taking into account partial lifting of the degeneracy between the Zeeman transition frequencies by the Breit-Rabi equation. One can show that, for  $R_{\text{OP}} \gg R_{\text{SD}}$ ,  $s_z = 1$ , and  $I = 3/2$ ,

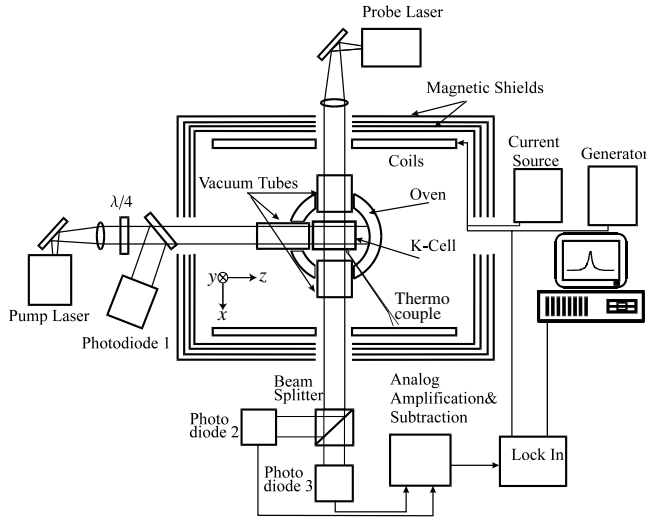


FIG. 1. Experimental setup of the rf magnetometer. A 3.8 cm square aluminosilicate glass cell containing 2.5 atm of  $^4\text{He}$ , 60 Torr of  $\text{N}_2$ , and potassium in natural abundance is heated inside a double wall oven to  $190^\circ\text{C}$ . The K atoms are optically pumped with a broad area diode laser. The transverse spin polarization is measured using optical rotation of off-resonant linearly polarized light with a polarizing beam splitter cube and two balanced photodetectors. A set of coils inside the shields creates the bias magnetic field.

$$T_2^{-1} = \Gamma = \frac{R_{\text{OP}}}{4} + \frac{R_{\text{SE}}R_{\text{SD}}}{R_{\text{OP}}} G(\omega_0, R_{\text{SE}}), \quad (2)$$

$$G(\omega_0, R_{\text{SE}}) = \text{Re} \left[ \frac{R_{\text{SE}} + 4i\omega_0^2/\pi\nu_{\text{HF}}}{5R_{\text{SE}} + 8i\omega_0^2/\pi\nu_{\text{HF}}} \right], \quad (3)$$

where  $R_{\text{SE}}$  is the SE rate and  $\nu_{\text{HF}}$  is the ground state hyperfine splitting. For  $^{39}\text{K}$  with  $\nu_{\text{HF}} = 462$  MHz and density of  $\sim 10^{14} \text{ cm}^{-3}$ ,  $G(\omega_0, R_{\text{SE}}) \approx 1/5$  for  $\omega_0 \leq 2\pi \times 1$  MHz. As a function of the OP rate  $T_2$  has a maximum equal to

$$T_{2\text{max}} = (R_{\text{SE}}R_{\text{SD}}/5)^{-1/2}. \quad (4)$$

For comparison, in the regime of low polarization,  $T_2$  is dominated by SE collisions and is equal to  $T_{2\text{SE}} = (R_{\text{SE}}/8)^{-1}$ . If spin relaxation is dominated by alkali-metal collisions, then the maximum linewidth narrowing is given by the ratio  $(5\sigma_{\text{SE}}/\sigma_{\text{SD}})^{1/2}/8$  and is equal to 37 for K atoms with SE cross section  $\sigma_{\text{SE}} = 1.8 \times 10^{-14} \text{ cm}^2$  [20] and SD cross section  $\sigma_{\text{SD}} = 1 \times 10^{-18} \text{ cm}^2$  [21].

We measured experimentally the dependence of the resonance width and the magnetometer response to an rf field as a function of the OP rate, as shown in Fig. 2. A fit based on an exact numerical solution using the density matrix formalism developed in [16] and including the effects of polarized light propagation through the cell [22] closely matches the measured linewidth. An analytic solution given by Eq. (2) is accurate for large  $R_{\text{OP}}$ , including the point of minimum linewidth. From the linewidth fit, we determine  $R_{\text{SE}} = (10.5 \pm 0.8) \times 10^4 \text{ s}^{-1}$  and  $R_{\text{SD}} =$

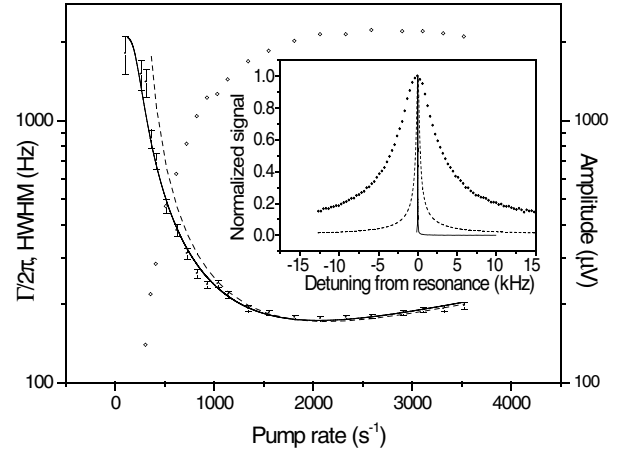


FIG. 2. Comparison of theory and experiment for the dependence of the Zeeman resonance width (points with error bars) and rf signal response (open circles) on optical pumping rate. The solid line shows a fit to a numerical solution of density matrix equations including effects of light propagation, and the dashed line shows the width predicted by Eq. (2). The inset shows the comparison of Zeeman resonances under different modes of operation: (i) (points) at low pump power and high magnetic field; (ii) (dashed line) at the same field with an optimal OP rate; (iii) (solid line) in very low magnetic fields, when SE broadening is turned off [17].

$55 \pm 5 \text{ s}^{-1}$ . We also measured the width of the Zeeman resonance as a function of the magnetic field in very low magnetic fields [17,23], from which we obtained  $R_{\text{SE}} = (9.1 \pm 0.2) \times 10^4 \text{ s}^{-1}$  and  $R_{\text{SD}} = 60 \pm 13 \text{ s}^{-1}$ , in good agreement with values determined at high field. We obtain a linewidth narrowing by a factor of 10, smaller than the maximum possible factor of 37 for K because of contributions from buffer gas and wall collisions. In the inset of Fig. 2 we compare the magnetic resonances with full spin-exchange broadening, partial narrowing due to high spin polarization, and complete elimination of spin-exchange broadening in low magnetic field [17].

In Fig. 3 we show the magnetic field sensitivity of the rf magnetometer. The narrow peak at 99 kHz is due to an oscillating rf field of known amplitude applied to calibrate the sensitivity. The broader peak is due to the ambient magnetic field noise as well as fluctuations of the pump and probe lasers that excite transverse spin polarization. The sensitivity of the magnetometer is equal to  $2 \text{ fT/Hz}^{1/2}$  for an oscillating field and  $1 \text{ fT/Hz}^{1/2}$  for a rotating magnetic field. The width of the noise peak ( $\text{FWHM} = 400 \text{ Hz}$ ) gives the bandwidth of the magnetometer. The optical detection noise in the absence of the pump laser corresponds to  $0.7 \text{ fT/Hz}^{1/2}$ . Note that thermal Johnson noise, which limits the performance for dc fields [17], is negligible at higher frequencies. Our preliminary results indicate that the rf magnetometer can operate in a completely unshielded environment with comparable sensitivity.

The fundamental sensitivity of the magnetometer is limited by three effects: spin-projection noise, photon

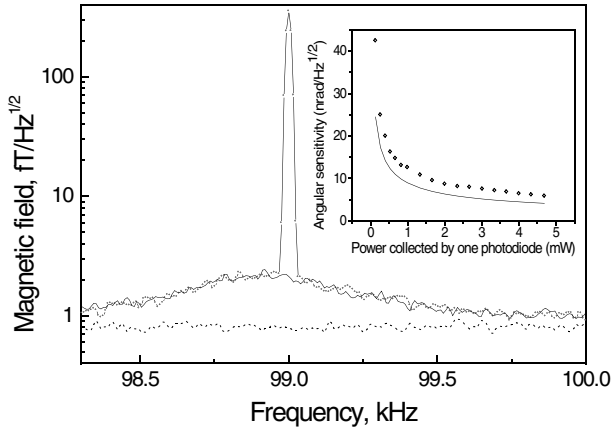


FIG. 3. The spectrum of the rf magnetometer near 99 kHz. The sharp peak is due to a calibration rf field and the broader peak is due to magnetic field and pump laser fluctuations. The magnetic field sensitivity to an oscillating magnetic field is  $2 \text{ fT/Hz}^{1/2}$ . Optical noise measured in the absence of the pump beam (dashed line) corresponds to  $0.7 \text{ fT/Hz}^{1/2}$ . Inset: Angular sensitivity  $\delta\phi$  (circles) in the absence of the pump beam as a function of the probe laser power compared with the photon shot-noise limit (solid line).

shot noise, and ac-Stark shifts [24]. The spin-projection noise arises from the uncertainty relation  $\delta F_x \delta F_y \geq |F_z|/2$  for noncommuting operators of total angular momentum  $[F_x, F_y] = iF_z$ . Without spin squeezing  $\delta F_x = \delta F_y$ , and for  $N$  atoms the uncertainty is given by  $\delta F_x = \sqrt{F_z/2N}$ . After a continuous measurement for a time  $t$  the uncertainty  $\langle \delta F_x \rangle_t$  is given by  $\langle \delta F_x \rangle_t = \delta F_x [(2/t) \int_0^t (1 - \tau/t) K(\tau) d\tau]^{1/2}$ , where  $K(\tau) = \exp(-\tau/T_2)$  is the spin ensemble time-correlation function [25]. For  $t \gg T_2$ ,  $\langle \delta F_x \rangle_t = \delta F_x (2T_2/t)^{1/2}$ , while for  $t = T_2$ ,  $\langle \delta F_x \rangle_t \approx 0.86 \delta F_x$ . Setting  $\delta P_x^{\text{rms}} = \langle \delta F_x \rangle_t / F_z$  in Eq. (1) the spin-projection noise in  $T_{\text{rms}}/\text{Hz}^{1/2}$  for  $t = 1/(2BW) \gg T_2$  is equal to

$$\delta B_{\text{spn}} = \frac{1}{\gamma} \sqrt{\frac{8}{F_z n V T_2}}, \quad (5)$$

where  $V$  is the active measurement volume given by the intersection of the pump and probe laser beams and  $n$  is the density of the alkali-metal atoms.

The transverse spin polarization induced by the rf field causes optical rotation of the probe beam's linear polarization. The polarization is rotated by an angle  $\phi$  [26]:

$$\phi = \frac{1}{2} l r_e c f n P_x D(\nu), \quad (6)$$

where  $l$  is the length of the cell along the probe direction,  $r_e = 2.8 \times 10^{-13} \text{ cm}$  is the classical electron radius,  $f \approx 1/3$  is the typical oscillator strength of the  $D1$  transition, and the dispersion profile  $D(\nu)$  is given by  $D(\nu) = (\nu - \nu_0)/[(\nu - \nu_0)^2 + (\Delta\nu/2)^2]$ , where  $\Delta\nu$  is the optical FWHM and  $\nu_0$  is the frequency of the  $D1$  transition. The

polarization rotation angle  $\phi$  is determined from the photocurrents of the two photodiodes,  $I_1$  and  $I_2$ ,  $\phi = (I_2 - I_1)/4I_1$ . The shot noise in the rotation angle  $\phi$  in  $\text{rad/Hz}^{1/2}$  is given by

$$\delta\phi = 1/\sqrt{2\Phi_{\text{pr}}\eta}, \quad (7)$$

where  $\Phi_{\text{pr}}$  is the flux of the probe beam photons arriving at the polarimeter and  $\eta$  is the photodiode quantum efficiency. In the inset of Fig. 3 we show that our detection system approaches the photon shot-noise angle sensitivity at 99 kHz. The excess noise is due to optical interference effects which convert laser frequency noise to amplitude noise. Combining Eqs. (1), (6), and (7) we obtain the photon shot-noise magnetic field sensitivity

$$\delta B_{\text{psn}} = \frac{4}{\gamma l r_e c f n T_2 D(\nu) \sqrt{2\Phi_{\text{pr}}\eta}}. \quad (8)$$

Quantum fluctuations in the polarization of the linearly polarized probe laser also contribute to the magnetic noise due to the ac-Stark shift (light shift) [27]. The light shift is equivalent to a fluctuating magnetic field  $B^{\text{LS}}$  and is given by the complex OP rate of the probe laser [18]

$$R_{\text{pr}} s_x + i g_s \mu_B B_x^{\text{LS}}/\hbar = \frac{r_e c f \Phi_{\text{pr}} s_x / A}{\Delta\nu/2 - i(\nu - \nu_0)}, \quad (9)$$

where  $A$  is the probe beam cross section and  $s_x$  is its degree of circular polarization. The component of  $B_x^{\text{LS}}$  corotating with the spins causes fluctuations that are proportional to the fluctuating probe polarization per unit bandwidth, which is given for unsqueezed light by  $\delta s_x = \sqrt{2/\Phi_{\text{pr}}}$  [28]. Hence the magnetic noise due to the light shift is

$$\delta B_{\text{lsn}} = \frac{r_e c f D(\nu) \sqrt{2\Phi_{\text{pr}}}}{8\gamma A}. \quad (10)$$

If the probe beam is detuned far from resonance so that  $D(\nu) \approx 1/(\nu - \nu_0)$ , the total magnetic noise can be written in terms of the optical depth on resonance  $\text{OD} = \sigma_0 n l$ , where  $\sigma_0$  is the resonance absorption cross section, and the probe beam OP rate  $R_{\text{pr}}$ ,

$$\delta B = \frac{1}{\gamma \sqrt{nV}} \sqrt{\frac{4}{T_2} + \frac{R_{\text{pr}} \text{OD}}{32} + \frac{8}{R_{\text{pr}} \text{OD} T_2^2 \eta}}. \quad (11)$$

In the experiment the intensity and detuning of the probe laser were optimized to maximize the sensitivity, giving  $T_2 = 0.8 \text{ ms}$  and  $R_{\text{pr}} \approx 0.5 \text{ ms}^{-1}$ . With  $\eta = 0.8$ ,  $V = 4 \text{ cm}^3$ ,  $l = 2 \text{ cm}$ ,  $n = 7.4 \times 10^{13} \text{ cm}^{-3}$ , determined from the SE rate, and  $\sigma_0 = 1.8 \times 10^{-13} \text{ cm}^2$ , determined from pressure broadening of the absorption line, we have  $\text{OD} = 26$ , and Eq. (11) gives magnetic field sensitivity of  $0.11 \text{ fT/Hz}^{1/2}$ . The ideal shot noise of the optical rotation measurements corresponds to a magnetic field of  $0.5 \text{ fT/Hz}^{1/2}$ , not very far from the theoretical limit.

In a larger optical cell it is easier to reduce the effects of buffer gas and wall relaxation. The effect of the probe laser on the maximum relaxation time  $T_{2\max} = [R_{\text{SE}}(R_{\text{SD}} + R_{\text{pr}})/5]^{-1/2}$  can be reduced by sufficient detuning of the probe laser, and the sensitivity can be optimized by adjusting the OD so that the last two terms in Eq. (11) are equal. Then the fundamental limit on the sensitivity is set by the SE and SD cross sections,

$$\delta B_{\min} = \frac{2}{\gamma} \sqrt{\frac{\bar{v}[\sigma_{\text{SE}}\sigma_{\text{SD}}/5]^{1/2}}{V} \left(1 + \frac{1}{4\sqrt{\eta}}\right)}, \quad (12)$$

where  $\bar{v}$  is the mean relative thermal velocity of alkali-metal collisions. For photodiode efficiency  $\eta = 80\%$ , the total noise of the magnetometer is close to the spin-projection noise. For a  $V = 100 \text{ cm}^3$  cell, the magnetic field sensitivity for K vapor is equal to  $0.01 \text{ fT/Hz}^{1/2}$ .

We also performed a detailed numerical simulation of the magnetometer to estimate other limiting effects, such as light attenuation, relaxation by the buffer gas, and the effects of the  $D2$  line in K. In particular, absorption of the pump laser causes the OP rate to deviate from the optimal value that maximizes  $T_2$ . However, using two counterpropagating pump beams it is possible to obtain a nearly uniform pumping rate since optical pumping light is attenuated approximately linearly in a dense alkali-metal vapor [22]. From numerical estimates we find that the sensitivity of  $10^{-17} \text{ T/Hz}^{1/2}$  can be achieved in a practical system with a cell volume of  $200 \text{ cm}^3$ .

It is interesting to consider whether quantum-nondemolition (QND) techniques can improve the sensitivity of an rf magnetometer. While previous QND measurements have been demonstrated near zero field [11,29], it is possible to perform a QND measurement in a rotating frame by sending short pulses of probe light at frequency  $2\omega_0$ , so the light shift back-reaction noise contributes only to the out-of-phase component of spin precession. The probe beam parameters can then be optimized to achieve exactly spin-projection noise sensitivity given by Eq. (5). However, this is not a significant improvement compared to Eq. (12). It is also possible to use QND measurements to obtain an increase in sensitivity on a time scale much shorter than  $T_2$  [11,24,29] and improve broadband field measurements. However, an isolated magnetic field transient would excite a spin precession signal that lasts on the order of  $T_2$ , and its detectability would not be significantly improved by fast sub-shot-noise measurements.

In conclusion, we have demonstrated a tunable rf atomic magnetometer with a fT-level sensitivity. Atomic rf magnetometers appear particularly promising for detection of NQR signals from explosives. An NQR signal from a small TNT mine is about 4 fT with a bandwidth of 1 kHz and cannot be detected with present pickup coil technology without signal averaging [1]. In contrast, an atomic magnetometer such as described here could detect the signal with signal-to-noise ratio  $>10$ . Another promising appli-

cation is the detection of NMR signals and MRI in moderate magnetic fields of 10–100 G without the need for superconducting magnets or detectors.

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- [1] A. N. Garroway *et al.*, IEEE Trans. Geosci. Remote Sens. **39**, 1108 (2001).
  - [2] R. Bradley *et al.*, Rev. Mod. Phys. **75**, 777 (2003).
  - [3] R. C. Black *et al.*, Appl. Phys. Lett. **66**, 1267 (1995).
  - [4] I. Ogawa, S. Matsuki, and K. Yamamoto, Phys. Rev. D **53**, R1740 (1996).
  - [5] M. Simmonds, W. Fertig, and R. Giffard, IEEE Trans. Magn. **15**, 478 (1979).
  - [6] H. C. Seton, J. M. S. Hutchison, and D. M. Bussell, IEEE Trans. Appl. Supercond. **7**, 3213 (1997).
  - [7] I. K. Kominis, T. W. Kornack, J. C. Allred, and M. V. Romalis, Nature (London) **422**, 596 (2003).
  - [8] D. Budker, V. Yashchuk, and M. Zolotarev, Phys. Rev. Lett. **81**, 5788 (1998).
  - [9] C. Affolderbach, M. Stahler, S. Knappe, and R. Wynands, Appl. Phys. B **75**, 605 (2002).
  - [10] E. B. Aleksandrov *et al.*, Opt. Spectrosc. (USSR) **78**, 292 (1995).
  - [11] J. M. Geremia, J. K. Stockton, A. C. Doherty, and H. Mabuchi, Phys. Rev. Lett. **91**, 250801 (2003).
  - [12] I. M. Savukov and M. V. Romalis, Phys. Rev. Lett. **94**, 123001 (2005).
  - [13] V. V. Yashchuk *et al.*, Phys. Rev. Lett. **93**, 160801 (2004).
  - [14] C. Meriles, D. Sakellariou, A. Trabesinger, V. Demas, and A. Pines, Proc. Natl. Acad. Sci. U.S.A. **102**, 1840 (2005).
  - [15] T. Varpula and T. Pautanen, J. Appl. Phys. **55**, 4015 (1984).
  - [16] S. Appelt *et al.*, Phys. Rev. A **58**, 1412 (1998).
  - [17] J. C. Allred, R. N. Lyman, T. W. Kornack, and M. V. Romalis, Phys. Rev. Lett. **89**, 130801 (2002).
  - [18] S. Appelt, A. B. Baranga, A. R. Young, and W. Happer, Phys. Rev. A **59**, 2078 (1999).
  - [19] B. Chann, E. Babcock, L. W. Anderson, and T. G. Walker, Phys. Rev. A **66**, 033406 (2002).
  - [20] E. B. Aleksandrov *et al.*, Opt. Spectrosc. (USSR) **87**, 329 (1999); **93**, 488(E) (2002).
  - [21] S. Kadlecik, L. Anderson, and T. Walker, Nucl. Instrum. Methods Phys. Res., Sect. A **402**, 208 (1998).
  - [22] T. G. Walker and W. Happer, Rev. Mod. Phys. **69**, 629 (1997).
  - [23] I. M. Savukov and M. V. Romalis, Phys. Rev. A **71**, 023405 (2005).
  - [24] M. Auzinsh *et al.*, Phys. Rev. Lett. **93**, 173002 (2004).
  - [25] W. A. Gardner, *Introduction to Random Processes* (McGraw-Hill, New York, 1990).
  - [26] W. Happer and B. Mathur, Phys. Rev. **163**, 12 (1967).
  - [27] M. Fleischhauer, A. B. Matsko, and M. O. Scully, Phys. Rev. A **62**, 013808 (2000).
  - [28] P. Grangier, R. E. Slusher, B. Yurke, and A. LaPorta, Phys. Rev. Lett. **59**, 2153 (1987).
  - [29] A. Kuzmich, L. Mandel, and N. P. Bigelow, Phys. Rev. Lett. **85**, 1594 (2000).