Problem I: Locality-Sensitive Hashing

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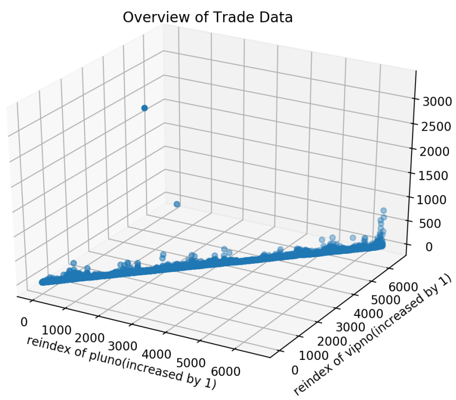
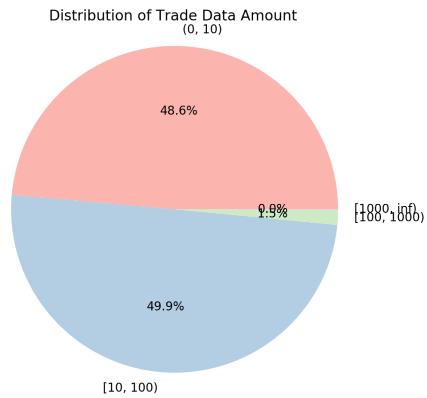
Data Overview & Propressing

Read data as DataFrame, which indexed by pluno and columned by vipno, basic information shows as follows:

```
<class 'pandas.core.frame.DataFrame'>
Index: 2635 entries, 10000004 to 40000700
Columns: 298 entries, 13205496418 to 6222021615015662822
dtypes: float64(298)
memory usage: 6.0+ MB
```

To have a basic impression by Data Visualization:

Overview of Trade Data	Distribution of Trade Data Amount
	

As we can see, it have 2 points above 1k, which lists as follows:

vipno	pluno	amt
2900001356947	14842010	3303.0
2900000350175	15114031	1334.0

Thus, the distribution of the data can be represented by the Pie Chart as above (excluding zero).

Near Neighbor using LSH

Since I use Euclidean Space, following is a brief introduction of Locality-Sensitive Hashing using s-stable distributions.

Let $f_s(t)$ denote the probability density funtion of the absolute value of the s-stable distribution. For the two vectors p, q, let $u = ||p - q||_s$ and let $p(u)$ denote the probility (as a function of u) that p, q collide for a hash function uniformly chosen from family. For a radom vector a whose entries are drawn from from s-stable distribution, $a.p - a.q$ is distributed as cX where X is a random variable drawn from a s-stable distribution. Since b is drawn uniformly from $[0, w]$ it is easy to see that

$$p(u) = Pr_{a,b}[h_{a,b}(p) = h_{a,b}(q)] = \int_0^w \frac{1}{u} f_s(\frac{t}{u})(1 - \frac{t}{w})dt$$

In practice, all L hash tables use the same primary hash function t_1 (used to dermine the index in the hash table) and the same secondary hash function t_2. These two hash functions have the form

$$t_1(a_1, a_2, \ldots, a_k) = ((\Sigma_{i=1}^k r_i' a_i) \; mod \; P) \; mod \; \text{tablesize}$$

$$t_2(a_1, a_2, \ldots, a_k) = (\Sigma_{i=1}^k r_i'' a_i) \; mod \; P)$$

where r' and r'' are random integers, tablesize is the size of the hash tables, and P is a prime.

In the current usage of the LSHash module, I use `sample` to generate random vipno. Since it is a Approximate Near Neighbor, I remove the generated random vipno carefully (rather than remove the first), and the whole code shows as follows:

```
def knn(df, k, coefficient):
    hash_size = int(coefficient * df.shape[1])
    lsh = LSHash(hash_size, input_dim=df.shape[0])
    for vipno in df:
        lsh.index(df[vipno], extra_data=vipno)
    random_column = df[df.columns.to_series().sample(1)]
    random_vip = random_column.columns.values[0]
    logging.info('random vipno: {}'.format(random_vip))
    res = lsh.query(random_column.values.flatten())[0: k + 1]
    print('vipno in ranked order using kNN(k = {}):'.format(k))
    knns = []
    for item in res:
        if item[0][1] != random_vip:
            print(item[0][1])
            knns.append(item[0][1])
    return random_vip, knns[:5]
```

Performance

Here will analyze the performance both in theory and practice.

Time & Space Complexity in Theory

Given the parameters k and L, the algorithm has the following performance guarantees:

- preprocessing time: $O(nLkt)$, where t is the time to evaluate a function $h \in \mathcal{F}$ on an input point p.
- space: $O(nL)$, plus the space for storing data points.

- query time: $O(L(kt + dnP_2^k)^L)$

Benchmark in Practice

As the statistics shows, most time cost in file I/O. In the algorithms part, `index` and `query` cost almost all time, since number of functions L is constant, it takes linear time, which is acceptedable.

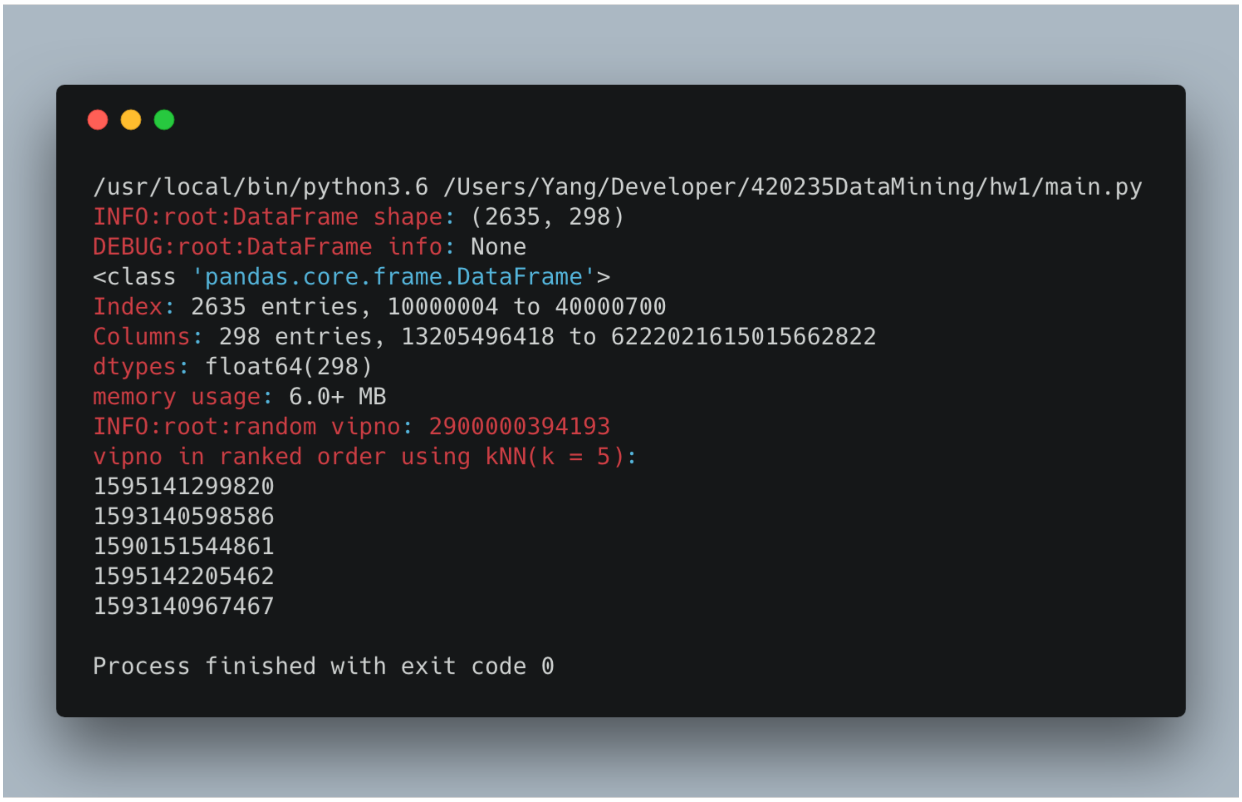
```
Timer unit: 1e-06 s

Total time: 0.073344 s
File: /Users/Yang/Developer/420235DataMining/hw1/q1/knn.py
Function: knn at line 5

Line #      Hits         Time  Per Hit   % Time  Line Contents
==============================================================
     5                                           @profile
     6                                           def knn(df, k, coefficient):
     7         1         13.0     13.0      0.0      hash_size = int(coefficient * df.shape[1])
     8         1        223.0    223.0      0.3      lsh = LSHash(hash_size, input_dim=df.shape[0])
     9       299        443.0      1.5      0.6      for vipno in df:
    10       298      63103.0    211.8     86.0          lsh.index(df[vipno], extra_data=vipno)
    11         1       2874.0   2874.0      3.9      random_column = df[df.columns.to_series().sample(1)]
    12         1          5.0      5.0      0.0      random_vip = random_column.columns.values[0]
    13         1        203.0    203.0      0.3      logging.info('random vipno: {}'.format(random_vip))
    14         1       6445.0   6445.0      8.8      res = lsh.query(random_column.values.flatten())[0: k + 1]
    15         1         12.0     12.0      0.0      print('vipno in ranked order using kNN(k = {}):'.format(k))
    16         1          1.0      1.0      0.0      knns = []
    17         7          4.0      0.6      0.0      for item in res:
    18         6          4.0      0.7      0.0          if item[0][1] != random_vip:
    19         5          7.0      1.4      0.0              print(item[0][1])
    20         5          6.0      1.2      0.0              knns.append(item[0][1])
    21         1          1.0      1.0      0.0      return random_vip, knns[:5]
```

```
Line #    Mem usage    Increment   Line Contents
================================================
     5  112.758 MiB  112.758 MiB   @profile
     6                             def knn(df, k, coefficient):
     7  112.758 MiB    0.000 MiB       hash_size = int(coefficient * df.shape[1])
     8  112.777 MiB    0.020 MiB       lsh = LSHash(hash_size, input_dim=df.shape[0])
     9  130.707 MiB    0.000 MiB       for vipno in df:
    10  130.707 MiB   17.930 MiB           lsh.index(df[vipno], extra_data=vipno)
    11  130.969 MiB    0.262 MiB       random_column = df[df.columns.to_series().sample(1)]
    12  130.969 MiB    0.000 MiB       random_vip = random_column.columns.values[0]
    13  130.969 MiB    0.000 MiB       logging.info('random vipno: {}'.format(random_vip))
    14  131.016 MiB    0.047 MiB       res = lsh.query(random_column.values.flatten())[0: k + 1]
    15  131.016 MiB    0.000 MiB       print('vipno in ranked order using kNN(k = {}):'.format(k))
    16  131.016 MiB    0.000 MiB       knns = []
    17  131.016 MiB    0.000 MiB       for item in res:
    18  131.016 MiB    0.000 MiB           if item[0][1] != random_vip:
    19  131.016 MiB    0.000 MiB               print(item[0][1])
    20  131.016 MiB    0.000 MiB               knns.append(item[0][1])
    21  131.016 MiB    0.000 MiB       return random_vip, knns[:5]
```

Screenshot



```
/usr/local/bin/python3.6 /Users/Yang/Developer/420235DataMining/hw1/main.py
INFO:root:DataFrame shape: (2635, 298)
DEBUG:root:DataFrame info: None
<class 'pandas.core.frame.DataFrame'>
Index: 2635 entries, 10000004 to 40000700
Columns: 298 entries, 13205496418 to 6222021615015662822
dtypes: float64(298)
memory usage: 6.0+ MB
INFO:root:random vipno: 2900000394193
vipno in ranked order using kNN(k = 5):
1595141299820
1593140598586
1590151544861
1595142205462
1593140967467

Process finished with exit code 0
```