Problem1

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Problem 1

In the solution, I use the functions from the user's guide of SAS so that the survival function will be different from the theoretical function in PPT. For other functions, I will further explain in detail.

Part 1

Interval	Time Period	Events	Censor	At risk at the beginning of the interval	Average number at risk in the interval
1	[0,4)	2	1	20	19.5
2	[4,8)	1	1	17	16.5
3	[8,12)	0	3	15	13.5
4	[12,16)	1	2	12	11

The life-table estimates are computed by counting the numbers of censored and uncensored observations that fall into each of the time intervals $[t_{i-1}, t_i), i = 1, 2, ..., k+1$, where $t_0 = 0$ and $t_{k+1} = \infty$. Let n_i be the number of units entering the interval $[t_{i-1}, t_i)$, and let d_i be the number of events occurring in the interval. Let $b_i = t_i - t_{i-1}$, and let $n_i^{'} = n_i - w_i/2$, where w_i is the number of units censored in the interval. The **effective sample size** of the interval $[t_{i-1}, t_i)$ is denoted by $n_i^{'}$. Let t_{mi} denote the midpoint of $[t_{i-1}, t_i)$.

Part 2 The conditional probability is an event in $[t_{i-1}, t_i)$ is estimated by

$$\hat{q}_i = \frac{d_i}{n_i^{'}}$$

and its estimated standard error is

Interval	Time Period	Survival probability	PDF
1	[0,4)	1	0.025641
2	[4,8)	0.897436	0.013598
3	[8,12)	0.843046	0
4	[12,16)	0.843046	0.01916

Interval	Time Period	Hazard	se(S(t))
1	[0,4)	0.027027	0
2	[4,8)	0.015625	0.0687
3	[8,12)	0	0.0833
4	[12,16)	0.02381	0.0833

The conditional probability of an event in is estimated by

$$\hat{q}_i = \frac{d_i}{n_i^{'}}$$

and its estimated standard error is

$$\hat{\sigma}(\hat{q}_i) = \sqrt{\frac{\hat{q}_i \hat{p}_i}{n_i^{`}}}$$

where $\hat{p}_i = 1 - \hat{q}_i$.

The estimate of the survival function at t_i is

$$\hat{S}(t_i) = \begin{cases} 1 & i = 0 \\ \hat{S}(t_{i-1}) p_{i-1} & i > 0 \end{cases}$$

and its estimated standard error is

$$\hat{\sigma}(\hat{S}(t_i)) = \hat{S}(t_i) \sqrt{\Sigma_{j=1}^{i-1} \frac{\hat{q}_j}{n_j^{`} \hat{p}_j}}$$

The density function at t_{mi} is estimated by

$$\hat{f}(t_{mi}) = \frac{\hat{S}(t_{i-1}) q_{\hat{i-1}}}{b_i}$$

The estimated hazard function at t_{mi} is

$$\hat{h}(t_{mi}) = \frac{2\hat{q}_i}{b_i(1 + \hat{p}_i)}$$