习题 6.2

1、(1) 原式 $= \frac{5}{2}$

(2) $\lim_{(x,y)\to(0,0)} \left| \frac{\sin(x^3+y^4)}{x^2+y^2} \right| = 0$

$\lim_{(x,y)\to(0,0)} \left| \frac{\sin(x^3+y^4)}{x^2+y^2} \right| \le \lim_{(x,y)\to(0,0)} \left| \frac{x^3+y^4}{x^2+y^2} \right|$

$= \lim_{(x,y)\to(0,0)} \left(\frac{x^2}{x^2+y^2} |x| + \frac{y^2}{x^2+y^2} y^2 \right)$

$\le \lim_{(x,y)\to(0,0)} (|x| + y^2)$

(5) 原式 $= \lim_{(x,y)\to(1,2)} (y-2)$

$= -1$

(6) 原式 $= \lim_{(x,y,z)\to(1,2,0)} \frac{1}{2} \ln(x^2+y^2+z^2)$

$= \frac{1}{2} \ln 5$

2、证明：① 当 $y = x^2$ 时 $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} f(x, x^2) = 0$

② 当 $y = \sqrt{2}x$ 时 $\lim_{(x,y)\to(0,0)} f(x,y) = \frac{-x^4}{3x^4} = -\frac{1}{3}$

此时两种情况极限不同 故 $f(x,y)$ 在 $(0,0)$ 处极限不存在

3、(1)

当 $|x|\le 1, |y|\le 1$ 时 $\lim_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) \ge \lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0$

$\lim_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) \le \lim_{(x,y)\to(0,0)} 2(x+y)\ln(x^2+y^2)$ $\le \lim_{(x,y)\to(0,0)} 4\sqrt{2} \frac{x+y}{\sqrt{2}} \ln \frac{x+y}{\sqrt{2}}$

$= \lim_{t\to 0} 4\sqrt{2} \frac{\ln t}{\frac{1}{t}} = 洛\lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = 0$

$\therefore \lim_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) = 0$

(2) 当 $|x|\le 1, |y|\le 1$ 时 $\lim_{(x,y)\to(0,0)} f(x,y) \le \lim_{(x,y)\to(0,0)} \frac{\frac{1}{2}(x^2+y^2)^2}{(x^2+y^2)x^2y^2} \le \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)^2}{4x^3y^3}$

$= \frac{1}{4} \lim_{(x,y)\to(0,0)} \left(\frac{x}{y^3} + \frac{y}{x^3} + \frac{2}{xy} \right) = +\infty$

$\lim_{(x,y)\to(0,0)} f(x,y) \ge \lim_{(x,y)\to(0,0)} \frac{\frac{1}{3}(x^2+y^2)^2}{\frac{1}{4}(x^2+y^2)^3} = \frac{4}{3} \lim_{(x,y)\to(0,0)} \frac{1}{x^2+y^2} = +\infty$

综上 $\lim_{(x,y)\to(0,0)} f(x,y) \to +\infty$

4、(2) $\lim_{x\to 0}\lim_{y\to 0} f(x,y) = \lim_{x\to 0} \frac{\sin x^2}{x^2} = 1$

$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0} y = 0$