

# 习题 5.4

1.  $2x^2 + 3y^2 + 4z^2 - 4x - 6y + 16z + 16 = 0$  即  $\frac{(x-1)^2}{\frac{1}{2}} + \frac{(y-1)^2}{\frac{1}{3}} + \frac{(z+2)^2}{\frac{1}{4}} = 5$

∴ 中心坐标为  $(1, 1, -2)$ ,  $x, y, z$  轴的半轴长为  $\frac{\sqrt{10}}{2}, \frac{\sqrt{15}}{3}, \frac{\sqrt{5}}{2}$

2. 4) 椭圆抛物面



6) 双曲抛物面

3. 4)  $\begin{cases} x = R\sqrt{\frac{5}{2}} \cos \theta + 1 \\ y = R\sqrt{\frac{5}{2}} \cos \theta - 1 \\ z = R(\frac{1}{2} + \sin \theta) \end{cases}$

5)  $\begin{cases} x = \frac{\sqrt{2}}{2} \sin \theta \\ y = \frac{3}{2} \cos \theta \\ z = \cos \theta \end{cases}$

# 习题 5.5

1. 1) 切线方程为  $x-1 = y-2 = z-3$

法平面  $(x-1) + 2(y-1) + 3(z-1) = 0$

2) 联立  $\begin{cases} x^2 + y^2 = R^2 \\ z = x + y \end{cases}$

参数方程为  $\begin{cases} x = R \cos \theta \\ y = R \sin \theta \\ z = R \cos \theta + R \sin \theta \end{cases}$

$P_0 (R \cos \theta, R \sin \theta, R \cos \theta + R \sin \theta)$

切线方程为  $x = y = z = R$

法平面  $2y + 3(z-R) = 0$

3. 证: 不妨设  $a(t) = x_1(t)\vec{i} + y_1(t)\vec{j} + z_1(t)\vec{k}$   
 $b(t) = x_2(t)\vec{i} + y_2(t)\vec{j} + z_2(t)\vec{k}$   
 左式 =  $[a(t) \cdot b(t)]'$

=  $\lim_{\Delta t \rightarrow 0} \frac{a(t+\Delta t) \cdot b(t+\Delta t) - a(t) \cdot b(t)}{\Delta t}$

=  $\lim_{\Delta t \rightarrow 0} \frac{x_1(t+\Delta t)x_2(t+\Delta t) + y_1(t+\Delta t)y_2(t+\Delta t) + z_1(t+\Delta t)z_2(t+\Delta t) - x_1(t)x_2(t) - y_1(t)y_2(t) - z_1(t)z_2(t)}{\Delta t}$

右式 =  $b(t) \lim_{\Delta t \rightarrow 0} \frac{a(t+\Delta t) - a(t)}{\Delta t} + a(t) \lim_{\Delta t \rightarrow 0} \frac{b(t+\Delta t) - b(t)}{\Delta t}$

=  $\lim_{\Delta t \rightarrow 0} \frac{x_1(t+\Delta t)x_2(t+\Delta t) + y_1(t+\Delta t)y_2(t+\Delta t) + z_1(t+\Delta t)z_2(t+\Delta t) - x_1(t)x_2(t) - y_1(t)y_2(t) - z_1(t)z_2(t)}{\Delta t}$

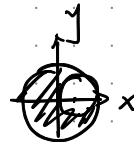
证毕

# 习题 6.1

1. (1)

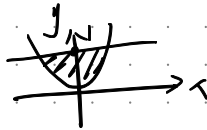
$$\begin{cases} x^2 + y^2 - 2x \geq 0 \\ 4 - x^2 - y^2 > 0 \end{cases}$$

即  $\begin{cases} x^2 + y^2 \leq 4 \\ (x-1)^2 + y^2 \geq 1 \end{cases}$



(2)

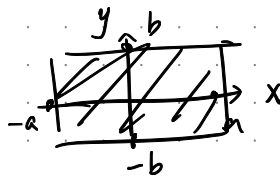
$$x^2 + y^2 < 1$$



(3)

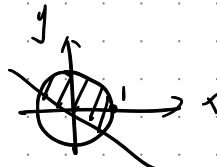
$$x \in [-a, a]$$

$$y \in [-b, b]$$



(4)

$$\begin{cases} x^2 + y^2 \leq 1 \\ x + y > 0 \end{cases}$$



2. (1) 开集, 区域, 有界区域, 不是有界闭区域

(2) 不是开集, 为区域, 有界区域, 不是有界闭区域

(3) 开集 不是区域

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