习题5.1

1、证明：1° $\because x_1-x_1=0\in Z$ 且 $y_1-y_1=0\in Z$ 即 $P_1(x_1,y_1)\sim P_1(x_1,y_1)$ 同理 $P_2(x_2,y_2)\sim P_2(x_2,y_2)$

2° $x_2-x_1=-(x_1-x_2)\in Z$，$y_2-y_1=-(y_1-y_2)\in Z$ 即 $P_2(x_2,y_2)\sim P_1(x_1,y_1)$

3° 假设有 $P_2(x_2,y_2)\sim P_3(x_3,y_3)$

$\because \begin{cases} x_1-x_3=(x_1-x_2)+(x_2-x_3)\in Z \\ y_1-y_3=(y_1-y_2)+(y_2-y_3)\in Z \end{cases}$ $\therefore P_1(x_1,y_1)\sim P_3(x_3,y_3)$

证毕

2、解 显然有 $\begin{cases} P\sim P,\ Q\sim Q \\ Q\sim P \end{cases}$ 若有 $Q\sim Z$ 显然 $P\sim Z$ 证毕

$S/\sim$ 的元素为 S 中所有的水平线

3. ~~显然~~ S 共有4种划分，即有4个不同商集

习题5.4

2、证 $A^{-1}(AB)A=BA$ $\therefore AB\sim BA$ 证毕

4. 证 $\begin{cases} (P^{-1}AP)(P^{-1}BP)=P^{-1}ABP \\ (P^{-1}BP)(P^{-1}AP)=P^{-1}BAP \end{cases}$ $\because AB=BA$ $\therefore P^{-1}AP=P^{-1}BP$ 证毕

9. 证：假设 A 可逆 则有 $B-A^{-1}BA=I$

$\because B\sim B$ $\therefore B=A^{-1}BA$，矛盾 $\therefore A$ 不可逆 证毕

习题5.5 $A\alpha=\lambda\alpha$ $A=kLU$ $U\beta=\mu\beta$

1、解 (3) $|A-\lambda I|=\begin{vmatrix} 6-\lambda & 2 & 4 \\ 2 & 3-\lambda & 2 \\ 4 & 2 & 6-\lambda \end{vmatrix}=(2-\lambda)(11-\lambda)$

$\lambda=2$ 时 $A-\lambda I=\begin{pmatrix} 4 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 4 \end{pmatrix}\sim\begin{pmatrix} 2 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 全部特征值 $\left\{k_1\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}+k_2\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}\ \middle|\ k_1,k_2\in K \text{且不同时为}0\right\}$

$\lambda=11$ 时 $A-\lambda I=\begin{pmatrix} -5 & 2 & 4 \\ 2 & -8 & 2 \\ 4 & 2 & -5 \end{pmatrix}\sim\begin{pmatrix} 1 & -2 & 1 \\ -4 & 0 & 5 \\ 0 & 0 & 0 \end{pmatrix}$ 全部特征值为 $\left\{k\begin{bmatrix} 10 \\ 8 \\ 9 \end{bmatrix}\ \middle|\ k\in K,k\neq 0\right\}$

(4) $|A-\lambda I| = \begin{vmatrix} 2-\lambda & -1 & 2 \\ 5 & -3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix} = (\lambda+1)^3$

当$\lambda=-1$时 $A-\lambda I = \begin{bmatrix} 3 & -1 & 2 \\ 5 & -2 & 3 \\ -1 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

∴特征向量为 $\left\{ k\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \middle| k\in K,\ k\neq 0 \right\}$

(5) $|A-\lambda I| = \begin{vmatrix} -\lambda & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2}-\lambda & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2}-\lambda \end{vmatrix} = \lambda(1-\lambda^2)$

$\lambda=0$时 $A-\lambda I = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 即 $\left\{ k_1\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} + k_2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \middle| k_1,k_2\in K \text{ 不同时为0} \right\}$

$\lambda=1$时 $A-\lambda I = \begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{3}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 2 & -1 & -1 \\ 2 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 即 $\left\{ k_1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \middle| k_1,k_2\in K \text{, 不同时为0} \right\}$

$\lambda=-1$时 $A-\lambda I = \begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ 即 $\left\{ k_1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \middle| k_1,k_2\in K \text{ 不同时为0} \right\}$