习题 6.2

1、(1) 原式 $= \frac{5}{2}$

(2) $\lim\limits_{(x,y)\to(0,0)} \left|\frac{\sin(x^3+y^4)}{x^2+y^2}\right| \geq 0$

$$\lim_{(x,y)\to(0,0)} \left|\frac{\sin(x^3+y^4)}{x^2+y^2}\right| \leq \lim_{(x,y)\to(0,0)} \left|\frac{x^3+y^4}{x^2+y^2}\right|$$

$$= \lim_{(x,y)\to(0,0)} \left(\frac{x^2}{x^2+y^2}|x| + \frac{y^2}{x^2+y^2}y^2\right)$$

$$\leq \lim_{(x,y)\to(0,0)} (|x| + y^2)$$

(5) 原式 $= \lim\limits_{(x,y)\to(1,2)} (y-2)$

$= -1$

(6) 原式 $= \lim\limits_{(x,y,z)\to(1,-2,0)} \frac{1}{2}\ln(x^2+y^2+z^2)$

$= \frac{1}{2}\ln 5$

2、证明：① 当 $y = x^2$ 时 $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} f(x,x^2) = 0$

② 当 $y = \sqrt{2}x^2$ 时 $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \frac{-x^4}{3x^4} = -\frac{1}{5}$

此时两种情况极限不同 故 $f(x,y)$ 在 $(0,0)$ 处极限不存在

3、(1)

当 $|x| \leq 1, |y| \leq 1$ 时 $\lim\limits_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) \geq \lim\limits_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0$

$$\lim_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) \leq \lim_{(x,y)\to(0,0)} 2(x+y)\ln(x^2+y^2) \leq \lim_{(x,y)\to(0,0)} 4\sqrt{2}\frac{x+y}{\sqrt{2}}\ln\frac{x+y}{\sqrt{2}}$$

$$= \lim_{t\to0} 4\sqrt{2}\frac{\ln t}{\frac{1}{t}} = 4\sqrt{2}\lim_{t\to0}\frac{\frac{1}{t}}{-\frac{1}{t^2}} = 0$$

$\therefore \lim\limits_{(x,y)\to(0,0)} (x+2y)\ln(x^2+y^2) = 0$

(2) 当 $|x| \leq 1, |y| \leq 1$ 时 $\lim\limits_{(x,y)\to(0,0)} f(x,y) \leq \lim\limits_{(x,y)\to(0,0)} \frac{\frac{1}{2}(x^2+y^2)^2}{(x^2+y^2)x^2y^2} \leq \lim\limits_{(x,y)\to(0,0)} \frac{(x^2+y^2)^2}{4x^3y^3}$

$$= \frac{1}{4}\lim_{(x,y)\to(0,0)}\left(\frac{x}{y^3} + \frac{y}{x^3} + \frac{2}{xy}\right) = +\infty$$

$$\lim_{(x,y)\to(0,0)} f(x,y) \geq \lim_{(x,y)\to(0,0)} \frac{\frac{1}{3}(x^2+y^2)^2}{\frac{1}{4}(x^2+y^2)^3} = \frac{4}{3}\lim_{(x,y)\to(0,0)}\frac{1}{x^2+y^2} = +\infty$$

综上 $\lim\limits_{(x,y)\to(0,0)} f(x,y) \to +\infty$

4、(2) $\lim\limits_{x\to 0}\lim\limits_{y\to 0} f(x,y) = \lim\limits_{x\to 0} \frac{\sin x^2}{x^2} = 1$

$\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y) = \lim\limits_{y\to 0} y = 0$

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