

FA7 Exponential Distribution

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1. Records show that job submissions to a computer center have a Poisson distribution with an average of four per minute.

Let T be the time in minutes between submissions.

$P(T < 0.25) = P(\text{the time between submissions is less than 0.25 minute (15 seconds)})$

$P(T > 0.5) = P(\text{the time between submissions is greater than 0.5 minute (30 seconds)})$

$P(0.25 \leq T \leq 1) = P(\text{the time between submissions is between 0.25 and 1 minute})$

With $\lambda = 4$ per minute, use R to obtain:

```
lambda <- 4
time_int <- 0.25

prob_lt_025 <- pexp(time_int, lambda)
prob_lt_025
```

(a) $P(T \leq 0.25) = P(\text{time between submissions is at most 15 seconds});$

```
## [1] 0.6321206
```

```
time_int <- 0.5

prob_lt_05 <- 1 - pexp(time_int, lambda)
prob_lt_05
```

(b) $P(T > 0.5) = P(\text{time between submissions is greater than 30 seconds});$

```
## [1] 0.1353353
```

```
time_int <- 0.25
up_lim <- 1

prob_bw_025_1 <- pexp(up_lim, lambda) - pexp(time_int, lambda)
prob_bw_025_1
```

(c) $P(0.25 < T < 1) = P(\text{time between submissions is between 15 seconds and 1 minute})$.

```
## [1] 0.3495638
```

3. The average rate of job submissions in a computer center is 2 per minute.

If it can be assumed that the number of submissions per minute has a Poisson distribution, calculate the probability that:

```
lambda <- 2
1-ppois(2, lambda)
```

(a) more than two jobs will arrive in a minute; $P(X > 2) = 1 - P(X \leq 2)$

```
## [1] 0.3233236
```

```
1- pexp(0.5, lambda)
```

(b) at least 30 seconds will elapse between any two jobs; $P(X \geq 0.5)$

```
## [1] 0.3678794
```

```
pexp(0.5, lambda)
```

(c) less than 30 seconds will elapse between jobs; $P(X < 0.5)$

```
## [1] 0.6321206
```

(d) a job will arrive in the next 30 seconds, if no jobs have arrived in the last 30 seconds
 $P(X \leq 0.5 \mid X > 0.5) = P(X < 0.5)$

```
pexp(0.5, lambda)
```

```
## [1] 0.6321206
```

7. A website receives an average of 15 visits per hour, which arrive following a Poisson distribution.

```
lambda <- 15
1- pexp(0.167, lambda)
```

(a) Calculate the probability that at least 10 minutes will elapse without a visit. $P(X > 0.167)$

```
## [1] 0.0816756
```

```
ppois(7, lambda)
```

(b) What is the probability that in any hour, there will be less than eight visits?

```
## [1] 0.01800219
```

(c) Suppose that 15 minutes have elapsed since the last visit, what is the probability that a visit will occur in the next 15 minutes. $P(X < 0.25)$

```
pexp(0.25, lambda)
```

```
## [1] 0.9764823
```

```
qpois(0.75, lambda)
```

(d) Calculate the top quartile, and explain what it means.

```
## [1] 18
```

This means that in 75% of the hours, the website shall have 18 or less visits each hour and only 25% of any hour will the website have more than 18 visits.