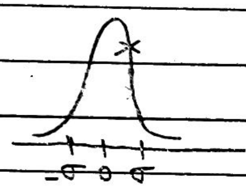
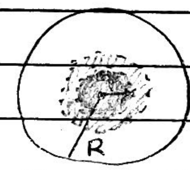
Problem 2

If the incoming beam pattern is gaussian then the intensity at any given point on the dish depends on how far you are from the center and the shape of the gaussian (σ).

2D representation:



looking down on ~~the dish:~~



The area of the entire dish is:

$$A = \pi R^2$$

But the effective area depends on the integrated gaussian:

$$A_{eff} = \int_0^R e^{(-r^2/2\sigma^2)} 2\pi r dr$$

To properly normalize we need to calculate $\frac{A_{eff}}{A}$, however this will eventually > 1 if σ is too large causing spillover (the signal will go beyond the dish and hit the ground). To understand the fraction of the beam that stays on the dish:

$$f = \frac{\int_0^R e^{-r^2/2\sigma^2} 2\pi r dr}{\int_0^\infty e^{-r^2/2\sigma^2} 2\pi r dr}$$

See code for remainder of problem.