GradientDescent

November 26, 2024

1 Data Generation

[1]:
```
%matplotlib inline
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as plt

dim_theta = 10
data_num = 1000
scale = .1

theta_true = np.ones((dim_theta,1))
print('True theta:', theta_true.reshape(-1))

A = np.random.uniform(low=-1.0, high=1.0, size=(data_num,dim_theta))
y_data = A @ theta_true + np.random.normal(loc=0.0, scale=scale,␣
↪size=(data_num, 1))

A_test = np.random.uniform(low=-1.0, high=1.0, size=(50, dim_theta))
y_test = A_test @ theta_true + np.random.normal(loc=0.0, scale=scale, size=(50,␣
↪1))
```

```
True theta: [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

2 Solving for the exact mean squared loss (solving Ax = b)

[85]:
```
'''
Using numpy's lstsq() to get the closed-form least squares solution
'''
theta_pred = la.lstsq(A, y_data)[0]

print('Empirical theta: ', theta_pred.reshape(-1))
```

```
Empirical theta:  [1.01117113 0.99879624 0.99707909 1.01475117 1.00365197
0.98530922
 1.00109998 0.99715709 0.99246624 0.99988042]
```

3 SGD Variants Noisy Function

[11]:
```
batch_size = 1
max_iter = 1000
lr = 0.001
theta_init = np.random.random((10,1)) * 0.1
```

[15]:
```
def noisy_val_grad(theta_hat, data_, label_, deg_=2.):
    gradient = np.zeros_like(theta_hat)
    loss = 0

    for i in range(data_.shape[0]):
        x_ = data_[i, :].reshape(-1,1)
        y_ = label_[i, 0]
        err = np.sum(x_ * theta_hat) - y_

        grad = deg_ * np.abs(err) ** (deg_ - 1) * np.sign(err) * x_
        l = np.abs(err) ** deg_

        loss += l / data_.shape[0]
        gradient += grad / data_.shape[0]

    return loss, gradient
```

4 Running SGD Variants

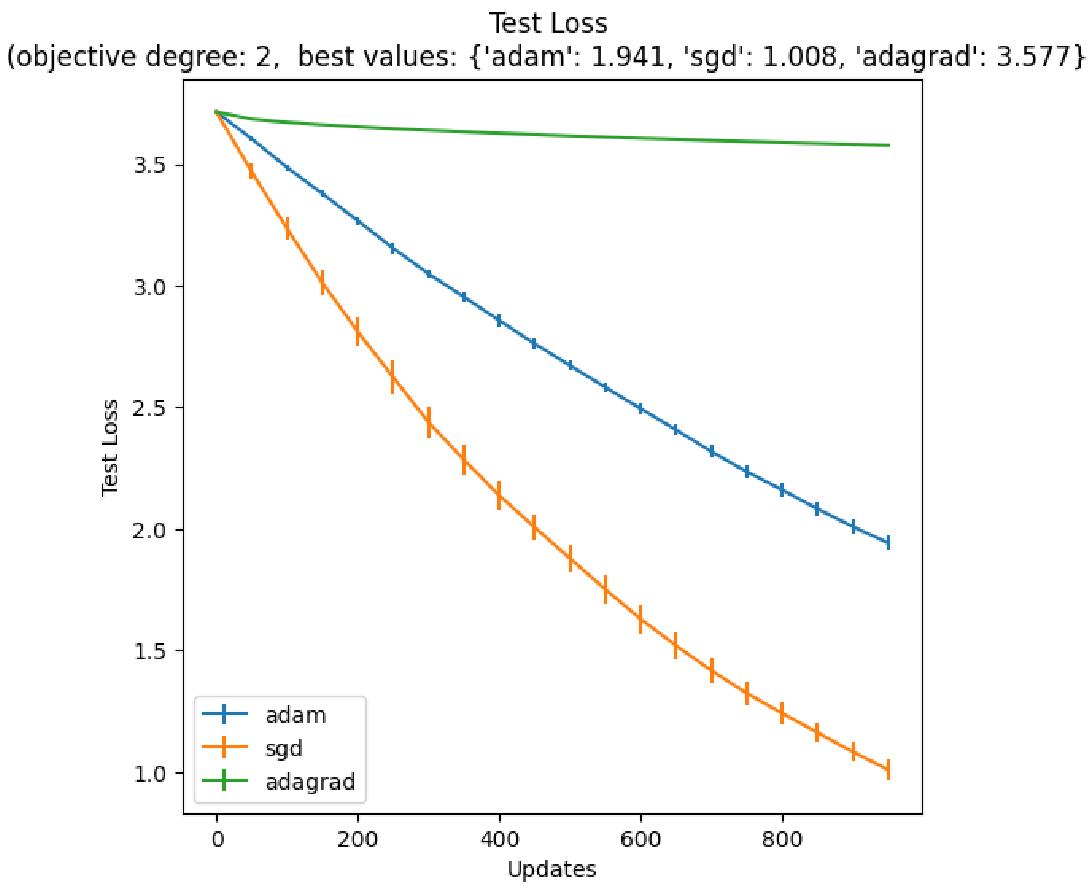
[95]:
```
#@title Parameters
deg_ = 2 #@param {type: "number"}
num_rep = 15 #@param {type: "integer"}
max_iter = 1000 #@param {type: "integer"}
fig, ax = plt.subplots(figsize=(6, 6))
best_vals = {}
test_exp_interval = 50 #@param {type: "integer"}
grad_artificial_normal_noise_scale = 0. #@param {type: "number"}

# dictionary for keeping track of parameters
params = {}

for method_idx, method in enumerate(['adam', 'sgd', 'adagrad']):
    test_loss_mat = []
    train_loss_mat = []

    for replicate in range(num_rep):
        if replicate % 20 == 0:
            print(method, replicate)
```

```
        if method == 'adam':
            beta_1 = 0.9
            beta_2 = 0.999
            m = np.zeros((dim_theta, 1))
            v = np.zeros((dim_theta, 1))
            epsilon = 1e-8

        if method == 'adagrad':
            epsilon = 1e-8
            squared_sum = np.zeros((dim_theta, 1))

        theta_hat = theta_init.copy()
        test_loss_list = []
        train_loss_list = []

        for t in range(max_iter):
            idx = np.random.choice(data_num, batch_size) # Split data
            train_loss, gradient = noisy_val_grad(theta_hat, A[idx,:], y_data[idx,:], deg_=deg_)
            artificial_grad_noise = np.random.randn(10, 1) * grad_artificial_normal_noise_scale + np.sign(np.random.random((10, 1)) - 0.5) * 0.
            gradient = gradient + artificial_grad_noise
            train_loss_list.append(train_loss)

            if t % test_exp_interval == 0:
                test_loss, _ = noisy_val_grad(theta_hat, A_test[:,:], y_test[:,:], deg_=deg_)
                test_loss_list.append(test_loss)

            if method == 'adam':
                m = beta_1 * m + (1-beta_1) * gradient
                v = beta_2 * v + (1-beta_2) * (gradient * gradient)
                m_hat = m / (1 - beta_1 ** (t+1))
                v_hat = v / (1 - beta_2 ** (t+1))
                theta_hat = theta_hat - lr * (m_hat/(np.sqrt(v_hat)+epsilon))

            elif method == 'adagrad':
                squared_sum = squared_sum + (gradient * gradient)
                theta_hat = theta_hat - lr * gradient * (1/np.sqrt(squared_sum+epsilon))

            elif method == 'sgd':
                theta_hat = theta_hat - lr * gradient

        test_loss_mat.append(test_loss_list)
        train_loss_mat.append(train_loss_list)
```

```
    print(method, 'done')
    x_axis = np.arange(max_iter)[::test_exp_interval]

    test_loss_np = np.array(test_loss_mat)

    '''
    Hints:
    1. Use test_loss_np in np.mean() with axis = 0
    '''
    test_loss_mean = np.mean(test_loss_np, axis=0)

    '''
    Hints:
    1. Use test_loss_np in np.std() with axis = 0
    2. Divide by np.sqrt() using num_rep as a parameter
    '''
    test_loss_se = np.std(test_loss_np, axis=0) / np.sqrt(num_rep)

    plt.errorbar(x_axis, test_loss_mean, yerr=2.5*test_loss_se, label=method)
    best_vals[method] = min(test_loss_mean)

    # this only gets the parameters for one replicate (the last one) but it's␣
 ↪okay because we only need one to answer q3.
    params[method] = theta_hat

best_vals = { k: int(v * 1000) / 1000. for k,v in best_vals.items() } # A weird␣
 ↪way to round numbers
plt.title(f'Test Loss \n(objective degree: {deg_},  best values: {best_vals})')
plt.ylabel('Test Loss')
plt.legend()
plt.xlabel('Updates')
```

```
adam 0
adam done
sgd 0
sgd done
adagrad 0
adagrad done
```

[95]: Text(0.5, 0, 'Updates')



[94]:
```
print(params["adam"].reshape(-1))
```

```
[0.35388391 0.27269951 0.31367533 0.36023661 0.28976016 0.36528544
 0.33467022 0.36753296 0.29897261 0.30564255]
```

[ ]:

NeuralNetworks

November 26, 2024

1 Imports for Python libraries

[1]:
```
%matplotlib inline
%config InlineBackend.figure_format = 'retina'

import numpy as np
import torch
import torchvision
import matplotlib.pyplot as plt
from time import time
from torchvision import datasets, transforms
from torch import nn
from torch import optim
```

2 Set up the mini-batch size

[14]:
```
#@title Batch Size
mini_batch_size = 64 #@param {type: "integer"}
```

3 Download the dataset, pre-process, and divide into mini-batches

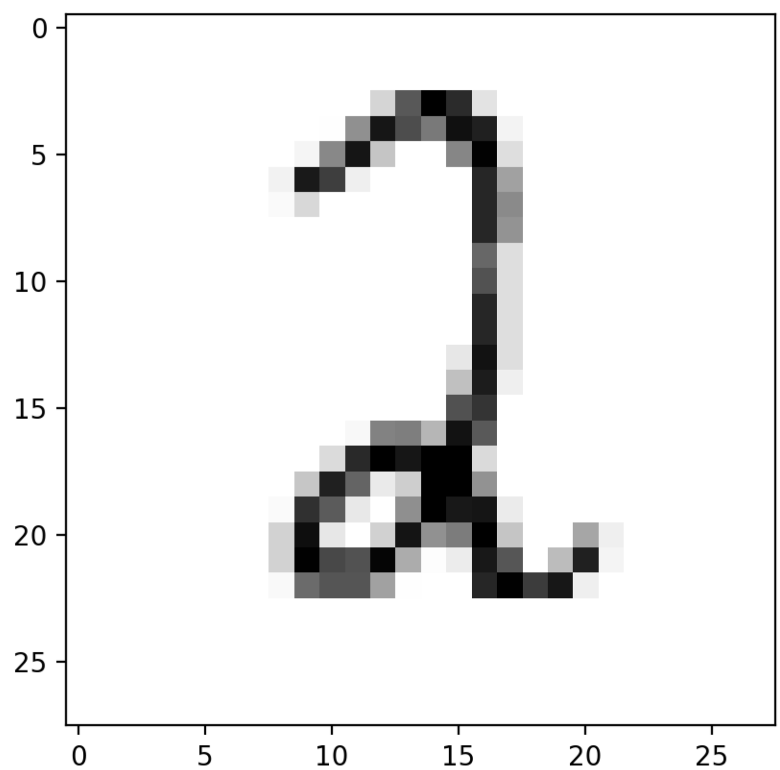
[15]:
```
### Define a transform to normalize the data
transform = transforms.Compose([transforms.ToTensor(), transforms.Normalize((0.
 ↪5,), (0.5,)),])

### Download and load the training data
trainset = datasets.MNIST('MNIST_data/', download=True, train=True,␣
 ↪transform=transform)
valset = datasets.MNIST('MNIST_data/', download=True, train=False,␣
 ↪transform=transform)

trainloader = torch.utils.data.DataLoader(trainset, batch_size=mini_batch_size,␣
 ↪shuffle=True)
valloader = torch.utils.data.DataLoader(valset, batch_size=mini_batch_size,␣
 ↪shuffle=True)
```

```
dataiter = iter(trainloader)
images, labels = next(dataiter)
print(type(images))
print(images.shape)
print(labels.shape)
```

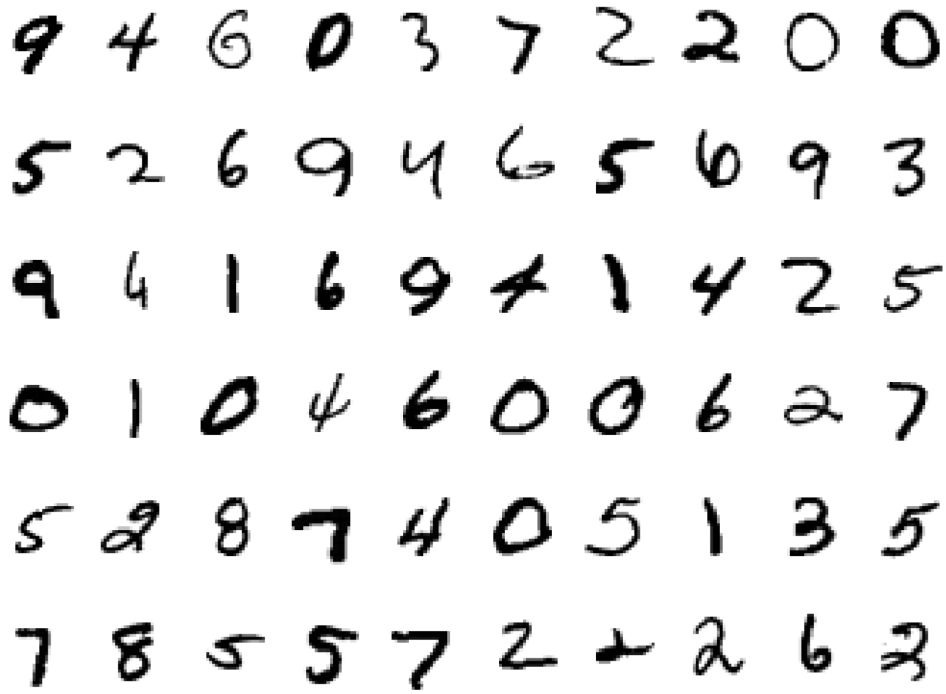
```
<class 'torch.Tensor'>
torch.Size([64, 1, 28, 28])
torch.Size([64])
```

4 Explore the processed data

[4]:
```
plt.imshow(images[0].numpy().squeeze(), cmap='gray_r'); # Change the index of␣
↪images[] to get different numbers
```



[7]:
```
figure = plt.figure()
num_of_images = 60
for index in range(1, num_of_images + 1):
    plt.subplot(6, 10, index)
    plt.axis('off')
    plt.imshow(images[index].numpy().squeeze(), cmap='gray_r')
```



5 Set up the neural network

[19]:
```python
# Please change the runtime to GPU if you'd like to have some speed-up on Colab
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

### Layer details for the neural network
input_size = 784
hidden_sizes = [128, 64]
output_size = 10

### Build a feed-forward network
model = nn.Sequential(
    nn.Linear(input_size, hidden_sizes[0]), # Fully Connected Layer
    nn.ReLU(), # Activation
    nn.Linear(hidden_sizes[0], hidden_sizes[1]), # Fully Connected Layer
    nn.ReLU(), # Activation
    nn.Linear(hidden_sizes[1], output_size), # Fully Connected Layer
    nn.LogSoftmax(dim=1) # (Log) Softmax Layer: Output a probability␣
 ↪distribution and apply log
)
print(model)
model.to(device)
```

```
Sequential(
  (0): Linear(in_features=784, out_features=128, bias=True)
  (1): ReLU()
  (2): Linear(in_features=128, out_features=64, bias=True)
  (3): ReLU()
  (4): Linear(in_features=64, out_features=10, bias=True)
  (5): LogSoftmax(dim=1)
)
```

[19]:
```
Sequential(
  (0): Linear(in_features=784, out_features=128, bias=True)
  (1): ReLU()
  (2): Linear(in_features=128, out_features=64, bias=True)
  (3): ReLU()
  (4): Linear(in_features=64, out_features=10, bias=True)
  (5): LogSoftmax(dim=1)
)
```

6 Set up the optimization model

[20]:
```
#@title Optimizer
lr = 3e-3 #@param {type: "number"}
optimizer = optim.Adam(model.parameters(), lr=lr) # Feel free to try out other␣
↪optimizers as you see fit!
```

7 Set up the loss function to optimize over

[21]:
```
time0 = time()
epochs = 15
criterion = nn.NLLLoss() # Negative log likelihood loss function is used
images, labels = next(iter(trainloader))
images = images.view(images.shape[0], -1).to(device)

logps = model(images) # Model spits out the log probability of image belonging␣
↪to different classes
loss = criterion(logps, labels.to(device))
```

8 Train the neural network

[22]:
```
for e in range(epochs):
    running_loss = 0
    for images, labels in trainloader:
        # Flatten MNIST images into a 784 long vector
        images = images.view(images.shape[0], -1).to(device)
        labels = labels.to(device)
```

```
        # Training pass
        optimizer.zero_grad()

        output = model(images).to(device)
        loss = criterion(output, labels)

        # backpropagation: calculate the gradient of the loss function w.r.t␣
↪model parameters
        loss.backward()

        # And optimizes its weights here
        optimizer.step()

        running_loss += loss.item()
    else:
        print("Epoch {} - Training loss: {}".format(e, running_loss/
↪len(trainloader)))
print("\nTraining Time (in minutes) =", (time()-time0)/60)
```

```
Epoch 0 - Training loss: 0.34144784285744495
Epoch 1 - Training loss: 0.1740617364394798
Epoch 2 - Training loss: 0.14824035517803863
Epoch 3 - Training loss: 0.12345252463868114
Epoch 4 - Training loss: 0.11443911887891987
Epoch 5 - Training loss: 0.10384408547542989
Epoch 6 - Training loss: 0.0993284839158282
Epoch 7 - Training loss: 0.09590731687776284
Epoch 8 - Training loss: 0.09157071839238622
Epoch 9 - Training loss: 0.08829374302188302
Epoch 10 - Training loss: 0.07945390294806393
Epoch 11 - Training loss: 0.07720497616464699
Epoch 12 - Training loss: 0.07428096306788524
Epoch 13 - Training loss: 0.07470227380091798
Epoch 14 - Training loss: 0.06939642548617939

Training Time (in minutes) = 3.182766552766164
```

9 Evaluate the trained neural network

[23]:
```
correct_count, all_count = 0, 0
for images, labels in valloader:
    for i in range(len(labels)):
        img = images[i].view(1, 784).to(device)
        labels = labels.to(device)
        # Forward pass only during evaluation
        with torch.no_grad():
```

```
            logps = model(img)

        # Output of the network are log-probabilities, need to take exponential␣
 ↪for probabilities
        ps = torch.exp(logps)
        probab = list(ps.cpu().numpy()[0])
        pred_label = probab.index(max(probab))
        true_label = labels.cpu().numpy()[i]
        if true_label == pred_label:
            correct_count += 1
        all_count += 1

print("Number Of Images Tested =", all_count)
print("\nModel Accuracy =", (correct_count/all_count))
```

```
Number Of Images Tested = 10000

Model Accuracy = 0.97
```

10 Predict using the trained neural network

[ ]:
```
def view_classify(img, ps):
    """ Function for viewing an image and it's predicted classes."""
    ps = ps.data.numpy().squeeze()

    fig, (ax1, ax2) = plt.subplots(figsize=(6,9), ncols=2)
    ax1.imshow(img.resize_(1, 28, 28).numpy().squeeze())
    ax1.axis('off')
    ax2.barh(np.arange(10), ps)
    ax2.set_aspect(0.1)
    ax2.set_yticks(np.arange(10))
    ax2.set_yticklabels(np.arange(10))
    ax2.set_title('Class Probability')
    ax2.set_xlim(0, 1.1)
    plt.tight_layout()
```

[ ]:
```
images, labels = next(iter(valloader))

img = images[0].view(1, 784).to(device)
# Turn off gradients
with torch.no_grad():
    logps = model(img)

# Output of the network are log-probabilities, need to take exponential for␣
 ↪probabilities
ps = torch.exp(logps)
probab = list(ps.cpu().numpy()[0])
```

```python
print("Predicted Digit =", probab.index(max(probab)))
view_classify(img.cpu().view(1, 28, 28), ps.cpu())
```