GW150914, 220 mode ringdown, GR deviation parameter from geometric deviations of final metric

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27/11/23

- We are interested in the ringdown of black holes
- We like to fit damped sinusoids (QNMs)
- Five parameters for each mode: Starting time, amplitude, phase, frequency, damping time
- Perturb around a background BH (Kerr, in GR), to find frequency and damping time (2 parameters -> 2 parameters in standard GR)
- Starting time, amplitude, phase determined by inspiral

General Stationary, Axisymmetric Metrics have more than 2 parameters[1]

$$ds^2 = -[1+h(r,\theta)]\left(1-\frac{2Mr}{\Sigma}\right)dt^2 - [1+h(r,\theta)]\frac{4aMr\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma[1+h(r,\theta)]}{\Delta + a^2\sin^2\theta h(r,\theta)}dr^2 + \Sigma d\theta^2 + \left[\sin^2\theta\left(r^2+a^2+\frac{2a^2Mr\sin^2\theta}{\Sigma}\right) + h(r,\theta)\frac{a^2(\Sigma+2Mr)\sin^4\theta}{\Sigma}\right]d\phi^2$$

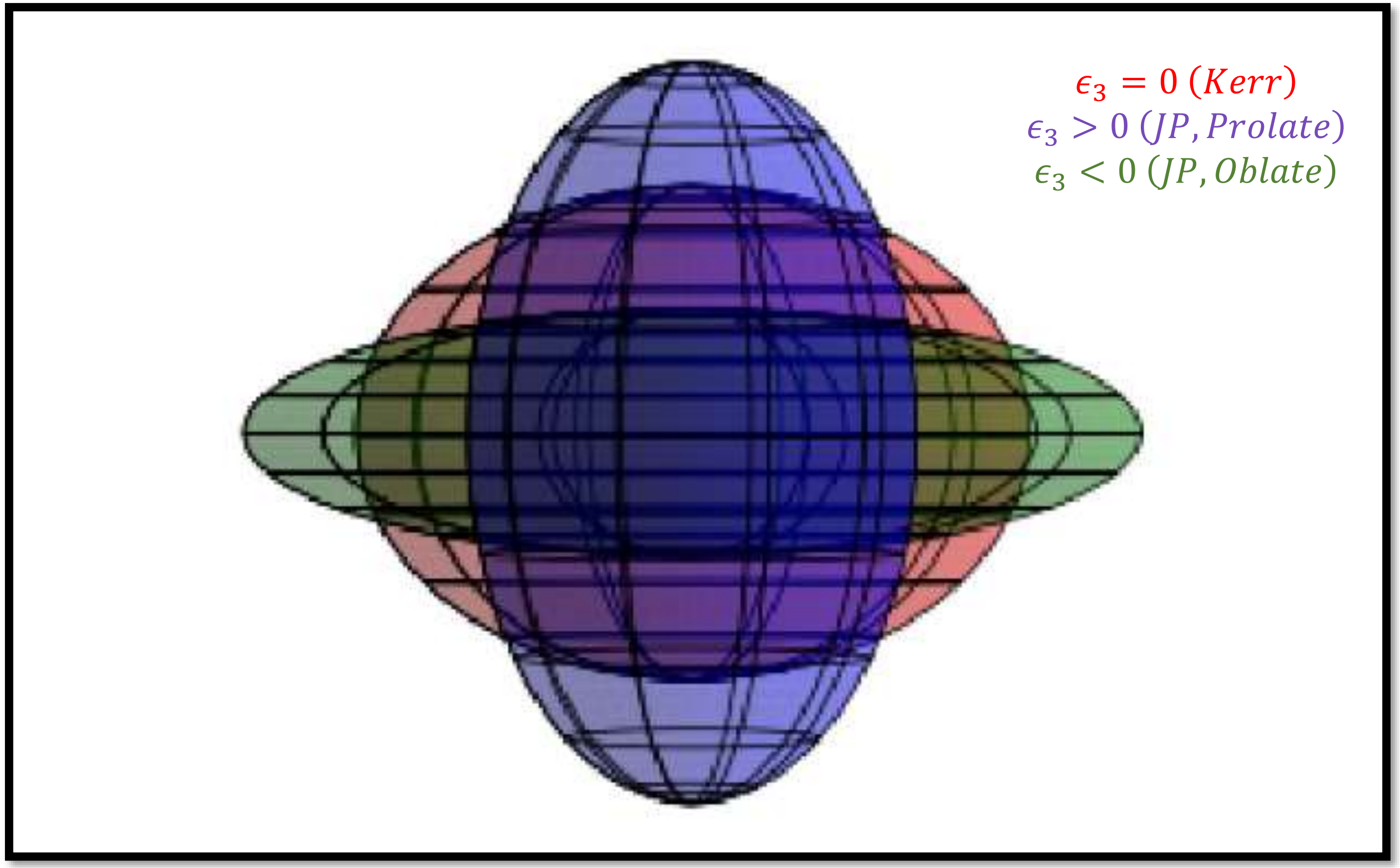
where; $\boxed{h(r,\theta) = \sum_{k=0}^{\infty}\left(\epsilon_{2k} + \epsilon_{2k+1}\frac{Mr}{\Sigma}\right)\left(\frac{M^2}{\Sigma}\right)^k}$

$$\begin{aligned}
g_{tt}^{JP} &= -\left(1-\frac{2Mr}{\Sigma}\right) - \epsilon_3\frac{M^3(r-2M)}{r^4}\\
g_{rr}^{JP} &= \frac{\Sigma}{\Delta} + \epsilon_3\frac{M^3(r-2M)}{\Delta^2}\\
g_{\theta\theta}^{JP} &= \Sigma\\
g_{\phi\phi}^{JP} &= \left(r^2+a^2+\frac{2Ma^2r\sin^2\theta}{\Sigma}\right)\sin^2\theta + \epsilon_3\frac{a^2M^3(r+2M)}{r^3}\\
g_{t\phi}^{JP} &= -\frac{2Mar\sin^2\theta}{\Sigma} - \epsilon_3\frac{2aM^4}{r^4}
\end{aligned}$$

- Non-GR (not a vacuum solution), parametric deviation, reduces to Kerr
- $\epsilon_0 = \epsilon_1 = 0$, for the metric to be asymptotically flat
- $\epsilon_2 = 4.6\text{x}10^{-4}$, Lunar Laser Ranging experiment
- ϵ_3 , first unconstrained parameter $\rightarrow h(r,\theta) = \epsilon_3\frac{M^3 r}{\Sigma^2}$
- Real and imaginary parts of the QNM spectrum for equatorial orbits in Johanssen-Psaltis geometry up to linear order in ϵ_3:[2]

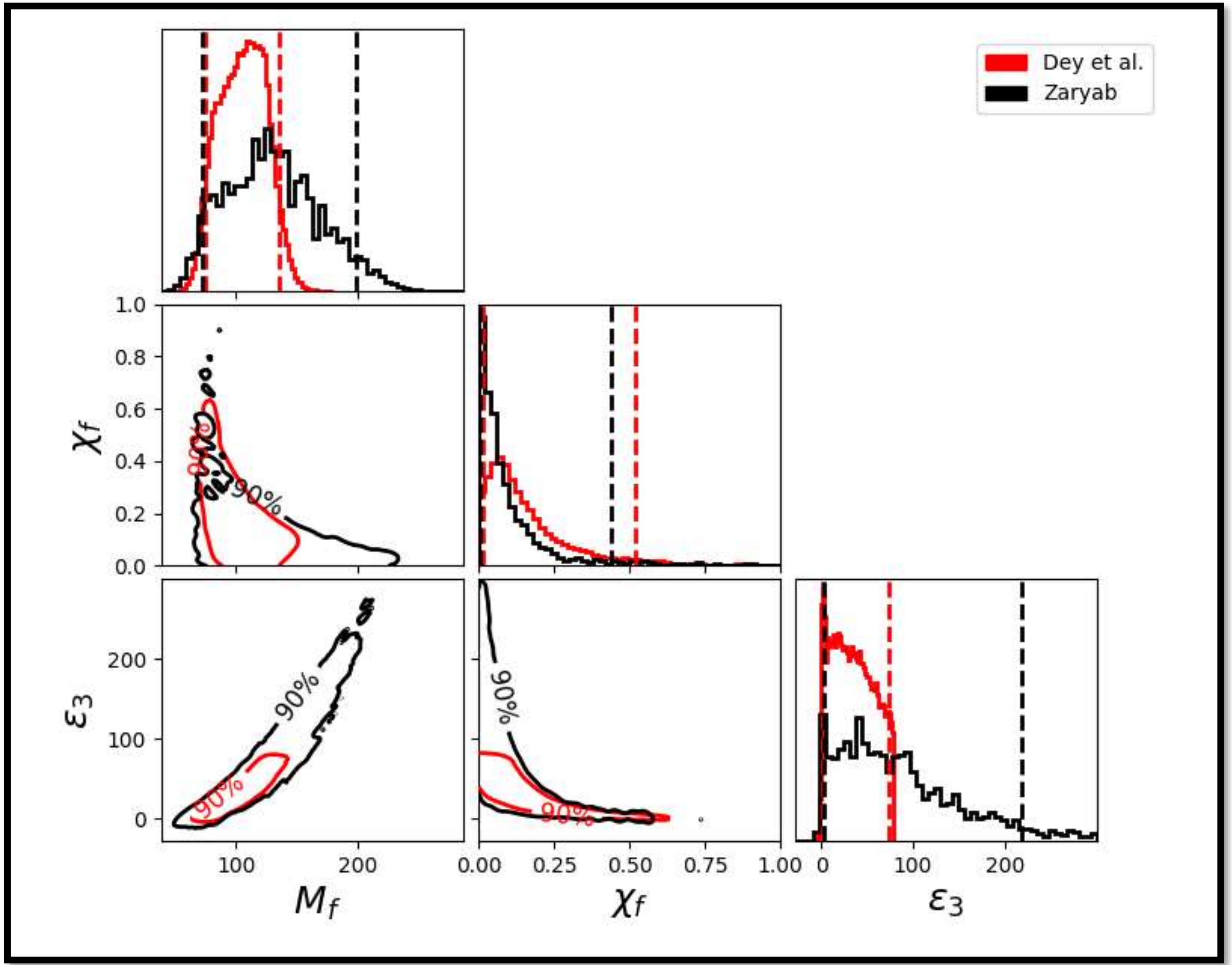
$$\begin{aligned}
\omega_R^{JP} &= \omega_R^K + \epsilon_3\left(\frac{1}{81\sqrt{3}M} + \frac{10}{729M}\chi + \frac{47}{1458\sqrt{3}M}\chi^2\right)\\
\omega_I^{JP} &= \omega_I^K - \epsilon_3\left(\frac{1}{486M}\chi + \frac{16}{2187\sqrt{3}M}\chi^2\right)
\end{aligned}$$

{1} A metric for rapidly spinning black holes suitable for strong-field test of the no-hair theorem https://link.aps.org/doi/10.1103/PhysRevD.83.124015
{2} Probing beyond-Kerr spacetimes with inspiral-ringdown corrections to gravitational waves https://link.aps.org/doi/10.1103/PhysRevD.101.084050



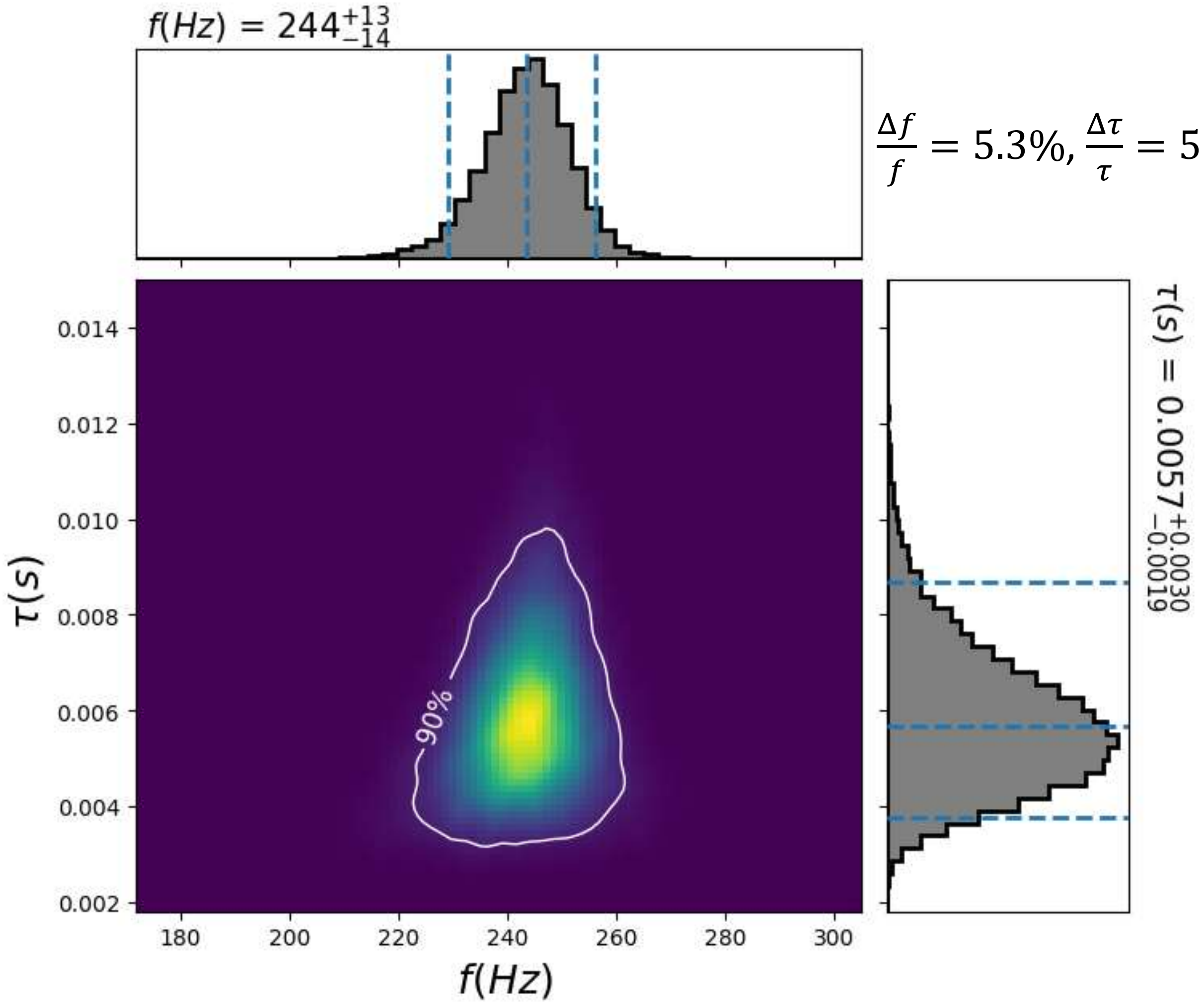
A visual representation of how ϵ_3 affects the ergospheres of Kerr and JP spacetimes.

Posterior Distribution GW150914 220 mode,
(ϵ_3 = [-30,300],M=[20,300])



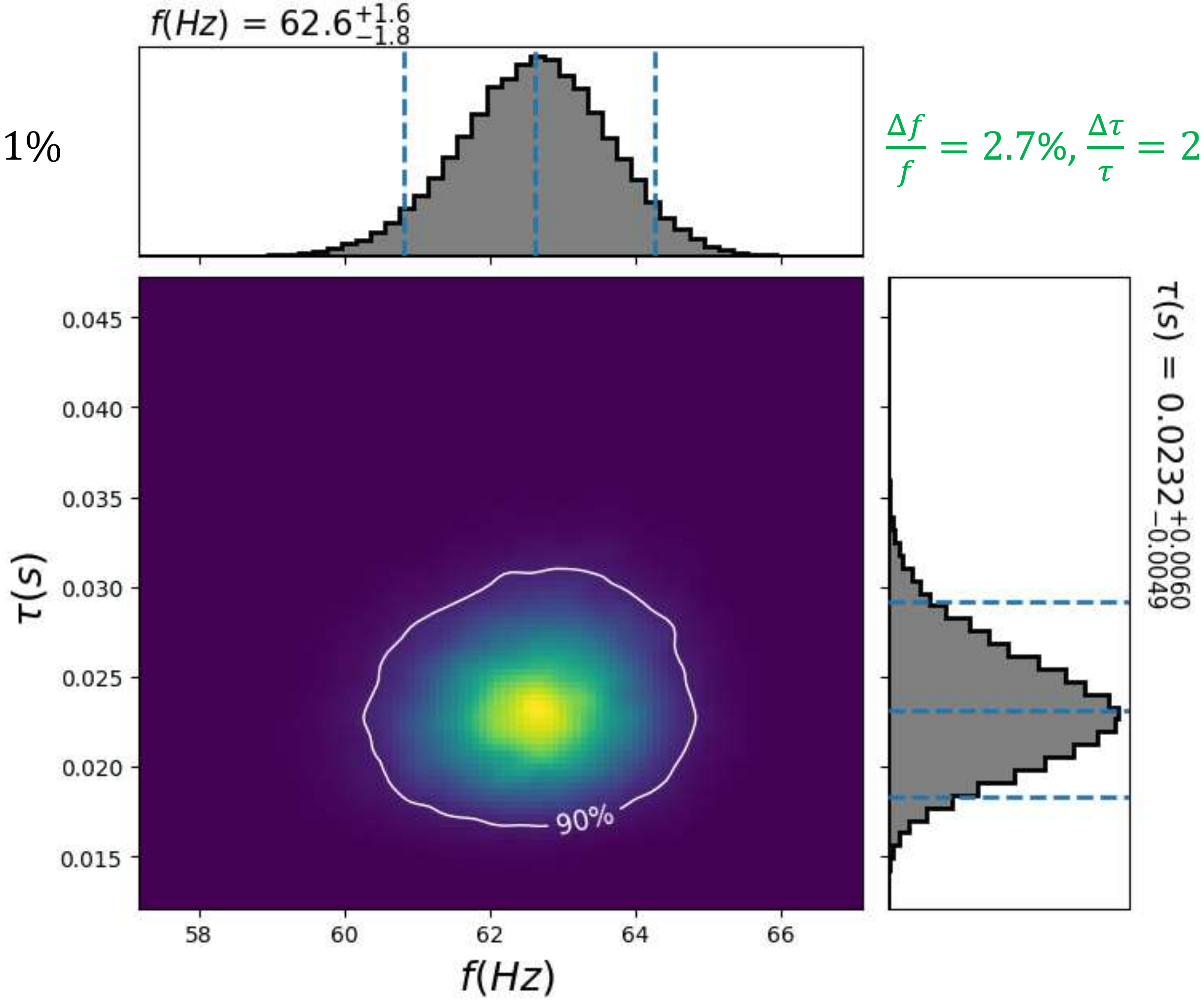
Frequency-Tau Distribution

- GW150914

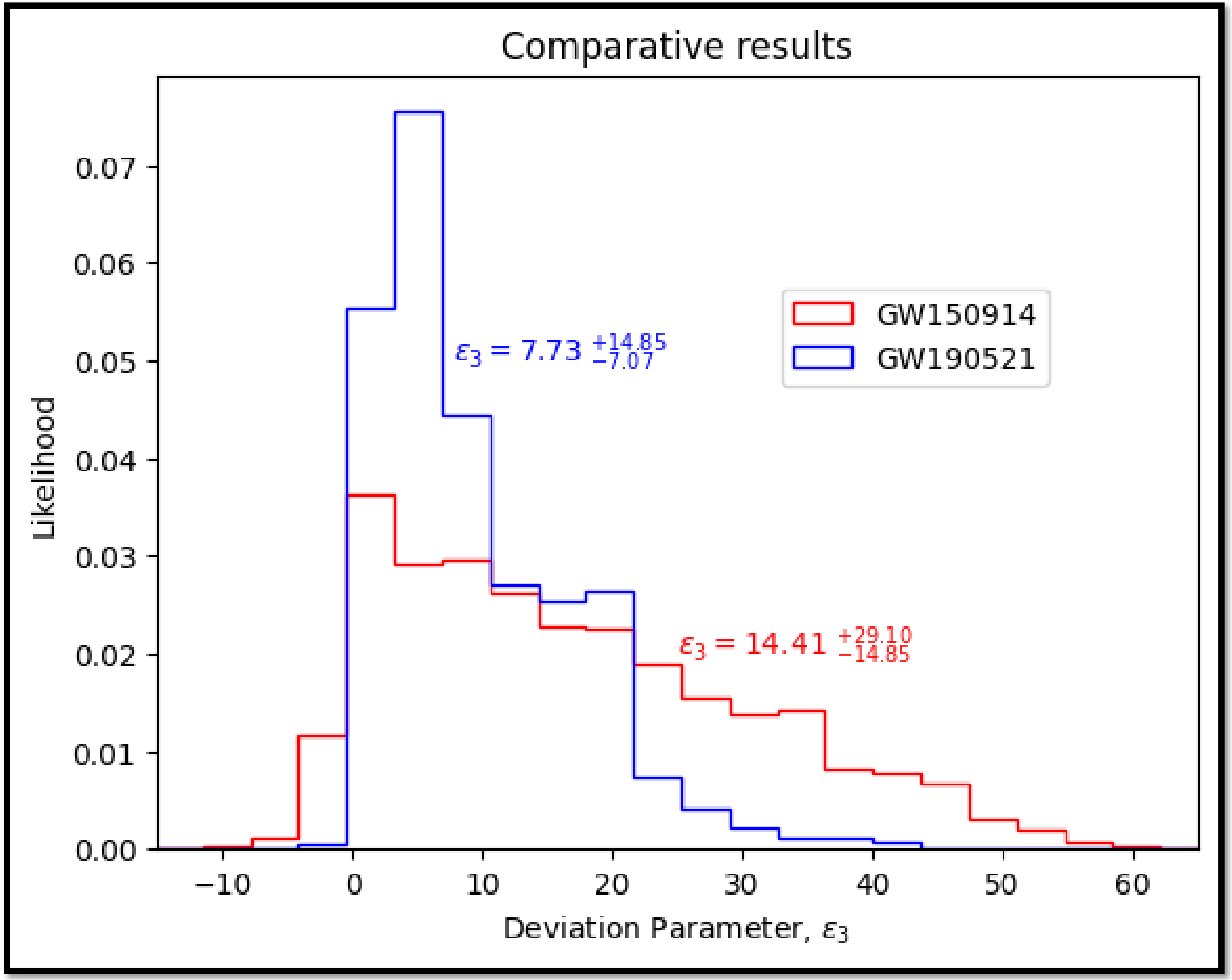


$\frac{\Delta f}{f} = 5.3\%, \frac{\Delta \tau}{\tau} = 51\%$

- GW190521



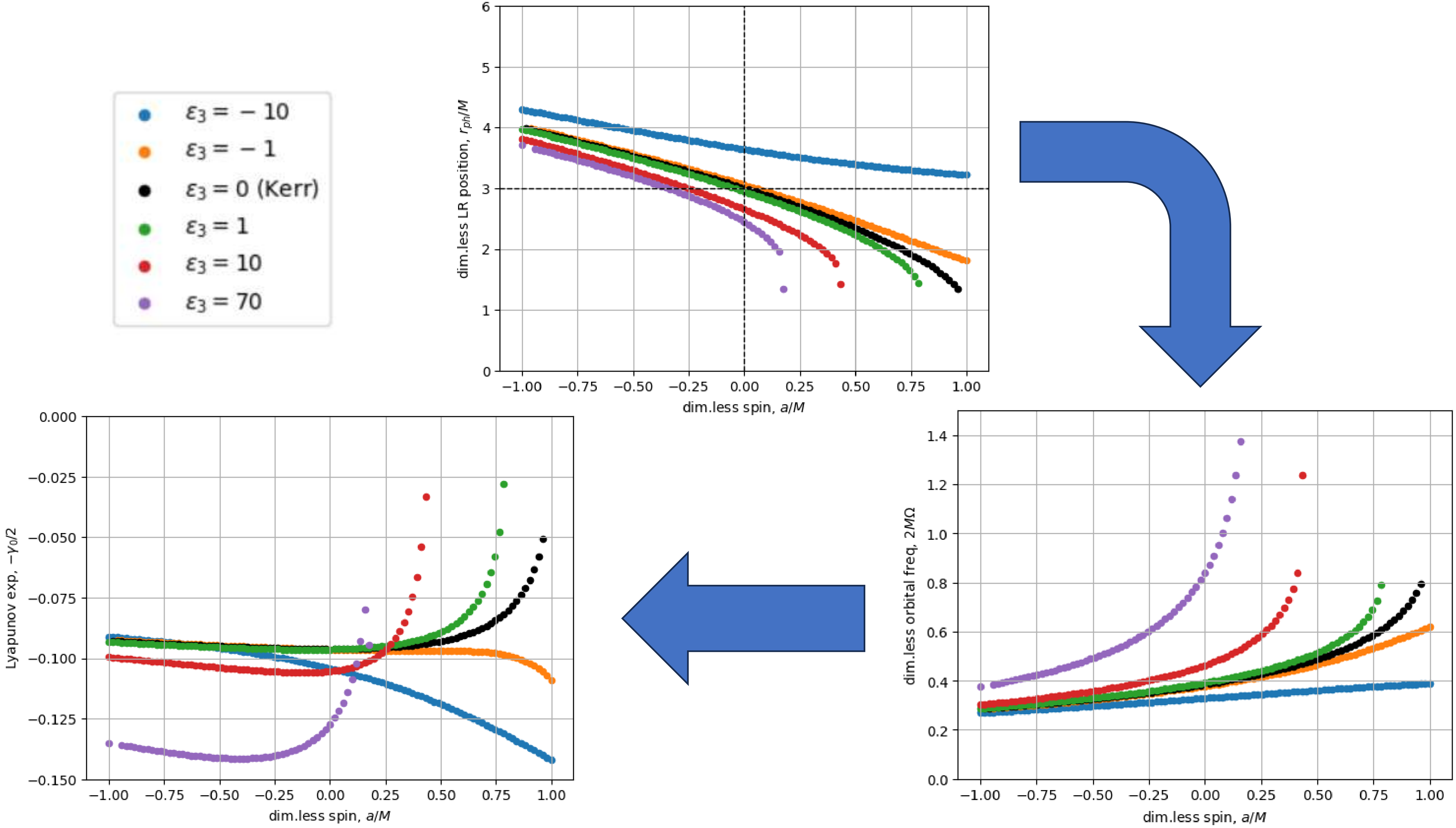
$\frac{\Delta f}{f} = 2.7\%, \frac{\Delta \tau}{\tau} = 23\%$


Comparative results
Likelihood
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
−10
0
10
20
30
40
50
60
Deviation Parameter, ε_3
GW150914
GW190521
$\varepsilon_3 = 7.73\,^{+14.85}_{-7.07}$
$\varepsilon_3 = 14.41\,^{+29.10}_{-14.85}$

Conclusions

- Ringdown is sensitive to post-merger metric
- Spin is anti-correlated with ϵ_3 in the ringdown
- GW190521 is a better candidate for ringdown analysis
- A single ringdown mode is unlikely to be competitive with the inspiral
- $\omega_R^{330} = \frac{3}{2}\omega_R^{220}$, in the eikonal-limit
- Use multimode observation GW190521

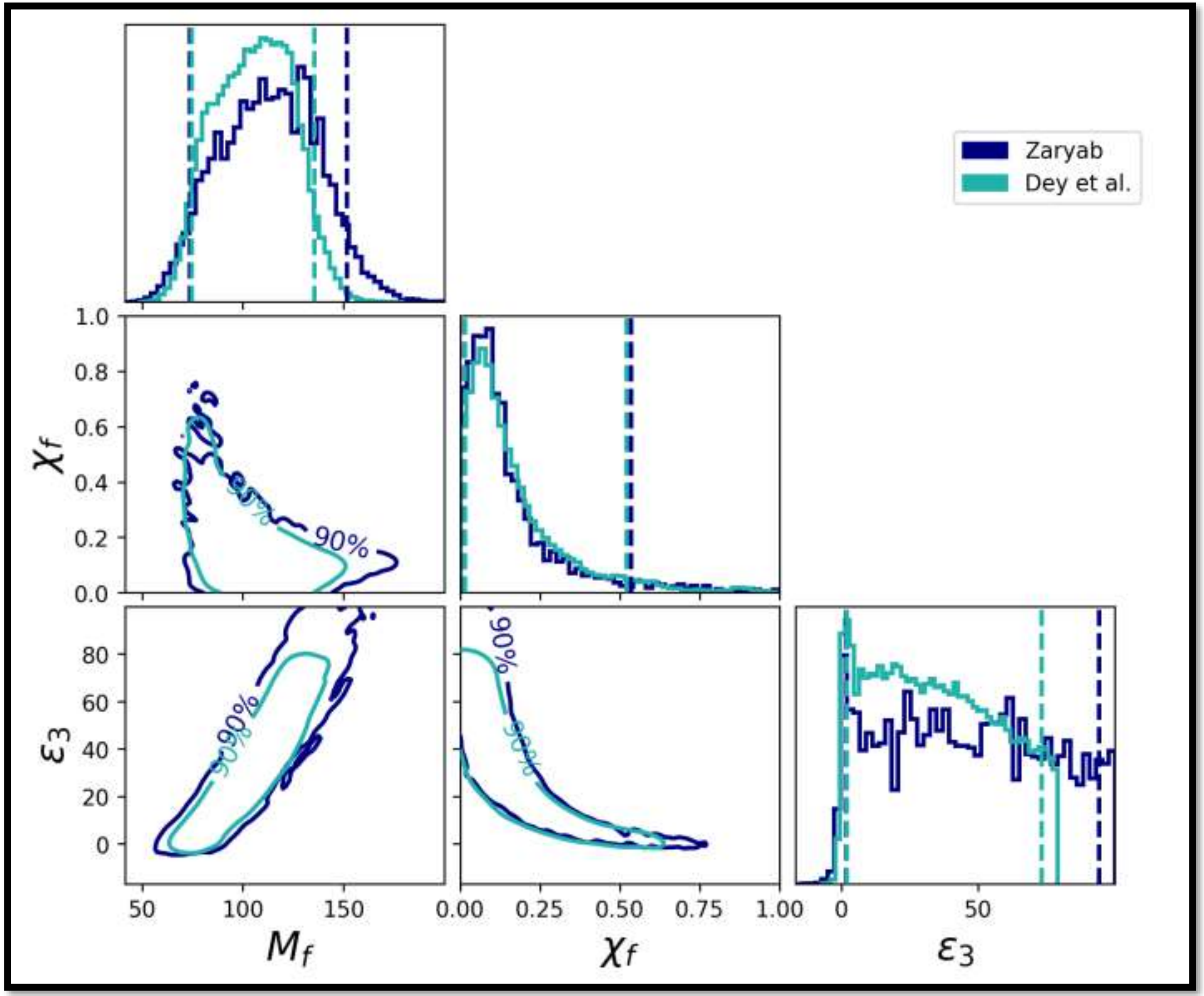
THANK YOU!



$\varepsilon_3 = -10$
$\varepsilon_3 = -1$
$\varepsilon_3 = 0$ (Kerr)
$\varepsilon_3 = 1$
$\varepsilon_3 = 10$
$\varepsilon_3 = 70$
dim.less LR position, r_{ph}/M
dim.less spin, a/M
dim.less orbital freq, $2M\Omega$
dim.less spin, a/M
Lyapunov exp, $-\gamma_0/2$
dim.less spin, a/M

Posterior Distribution GW150914, (ϵ_3 = [-30,100],M=[20,200])

Dey et al. priors
- ϵ_3 = [-30,100]
- χ = [0,0.99]
- M=[20,200]



Post-Kerr QNM Validity [5]

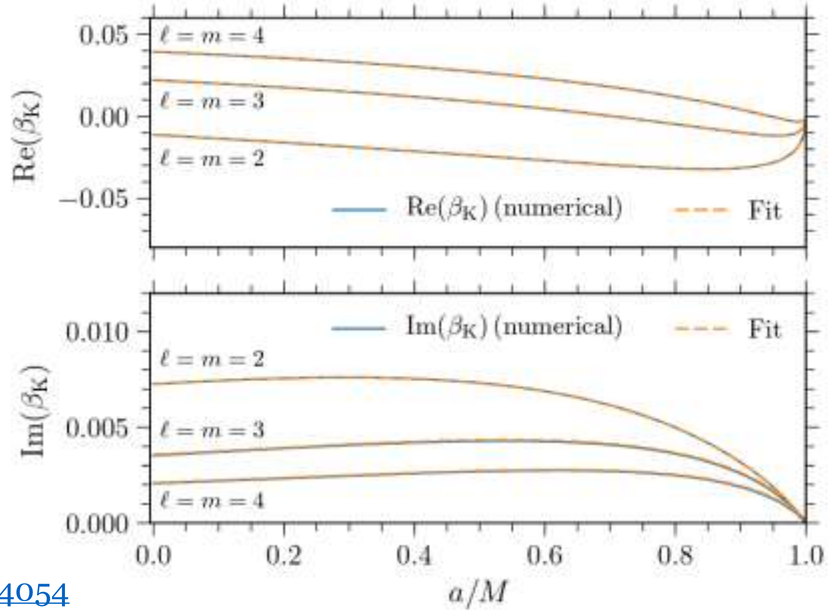
- QNM spectrum used for GW analysis based on Kerr metric
- Ensure that it is valid for a non-Kerr geometry
- Introduce an offset function, β_K

$$\boxed{\omega_{\rm obs} = \sigma + \beta_{\rm K}}$$

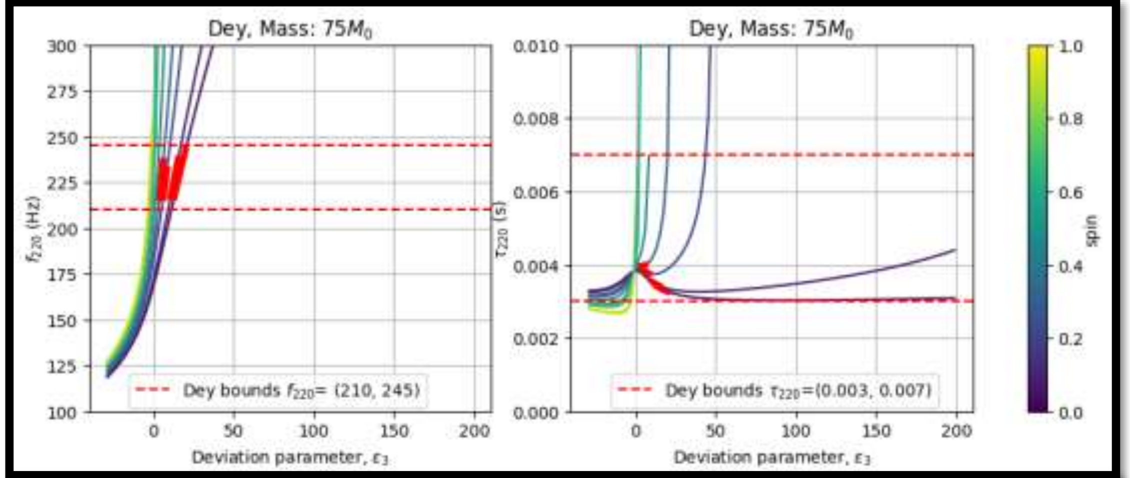
$$\boxed{\omega_{\rm K} = \sigma_{\rm K} + \beta_{\rm K}}$$

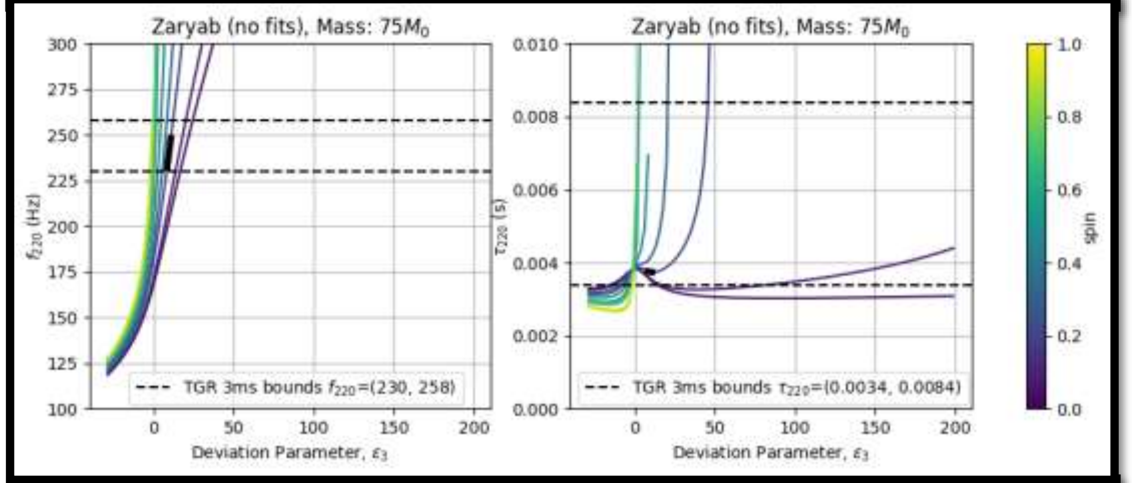
$$\boxed{\omega_{\rm obs} - \omega_{\rm K} = \sigma - \sigma_{\rm K} \neq 0}$$

$$\boxed{\sigma = \sigma_{\rm K} + \delta\sigma}$$



{5} Post-Kerr black hole spectroscopy https://link.aps.org/doi/10.1103/PhysRevD.96.064054


Dey, Mass: 75M_0
Dey, Mass: 75M_0
f_{220} (Hz)
τ_{220} (s)
Dey bounds f_{220}= (210, 245)
Dey bounds τ_{220}=(0.003, 0.007)
Deviation parameter, ϵ_3
Deviation parameter, ϵ_3
spin


Zaryab (no fits), Mass: 75M_0
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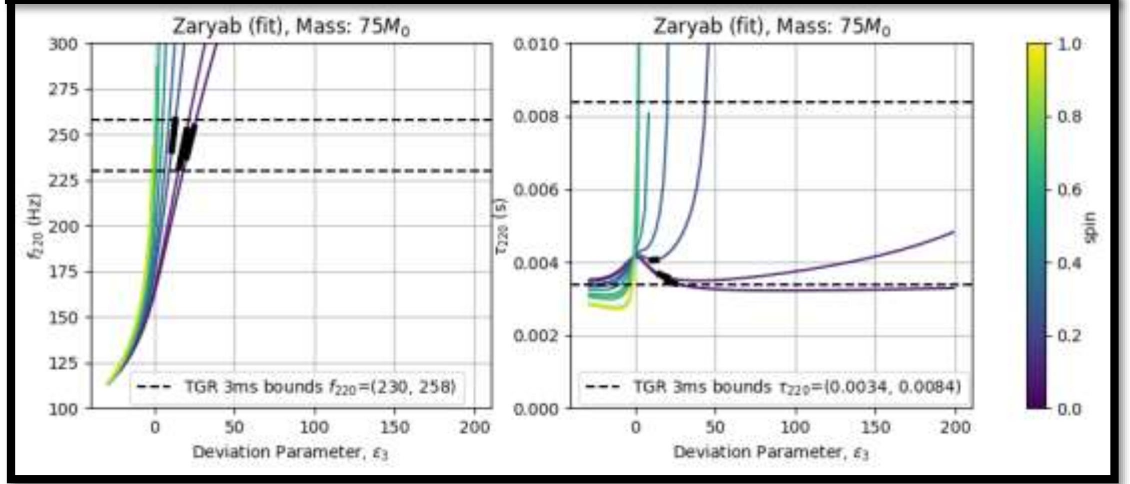
f_{220} (Hz)
τ_{220} (s)
TGR 3ms bounds f_{220}=(230, 258)
TGR 3ms bounds τ_{220}=(0.0034, 0.0084)
Deviation Parameter, ϵ_3
Deviation Parameter, ϵ_3
spin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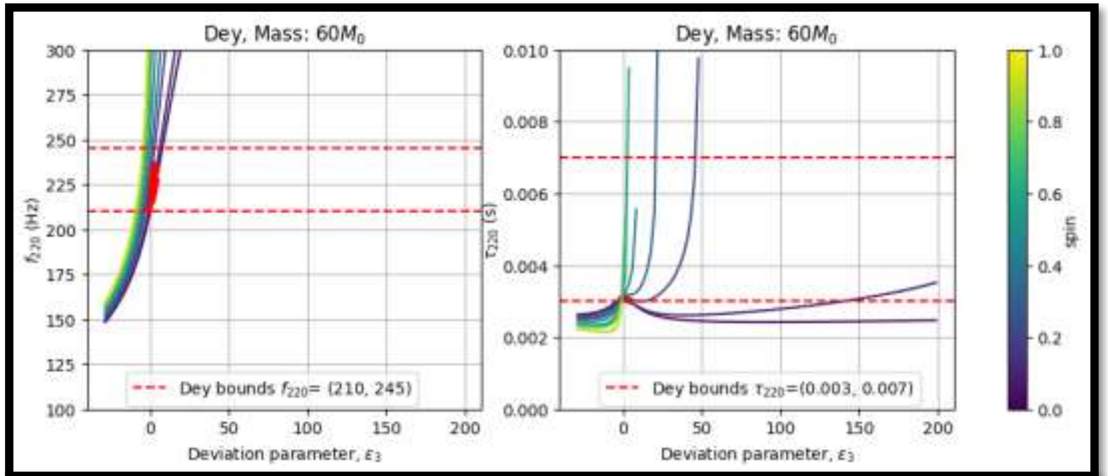

Dey, Mass: $60M_0$
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Dey bounds τ_{220}=(0.003, 0.007)
f_{220} (Hz)
τ_{220} (s)
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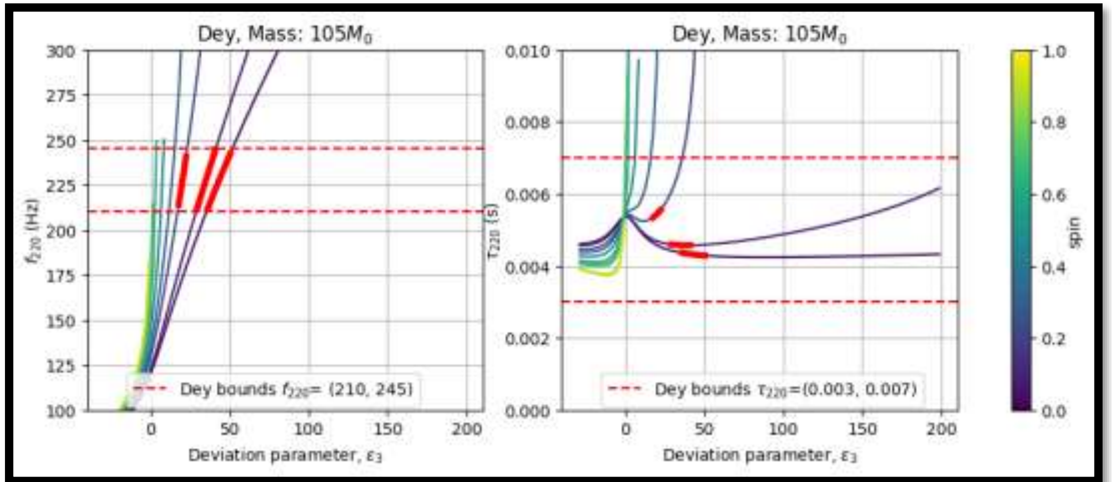

Dey, Mass: $105M_0$
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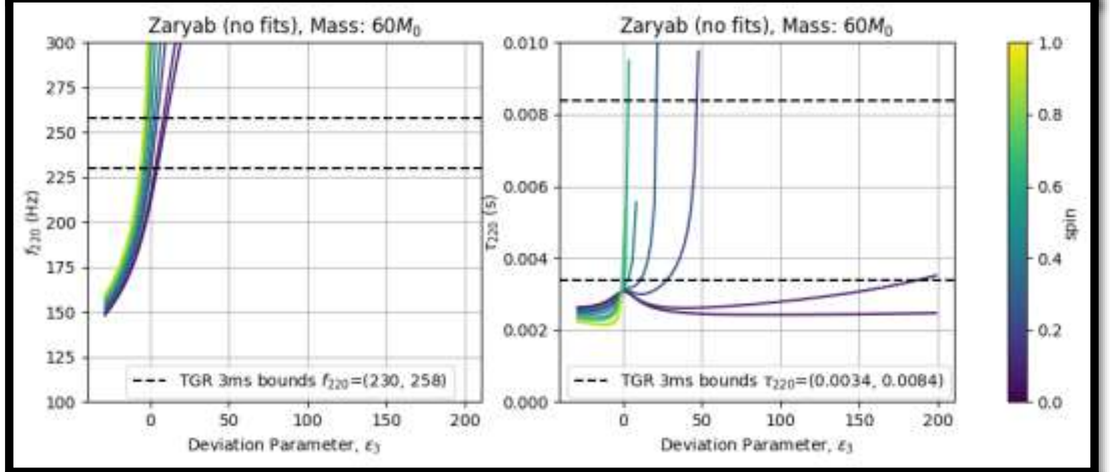
Dey bounds f_{220}= (210, 245)
Dey bounds τ_{220}=(0.003, 0.007)
f_{220} (Hz)
τ_{220} (s)
Deviation parameter, ϵ_3
Deviation parameter, ϵ_3
spin



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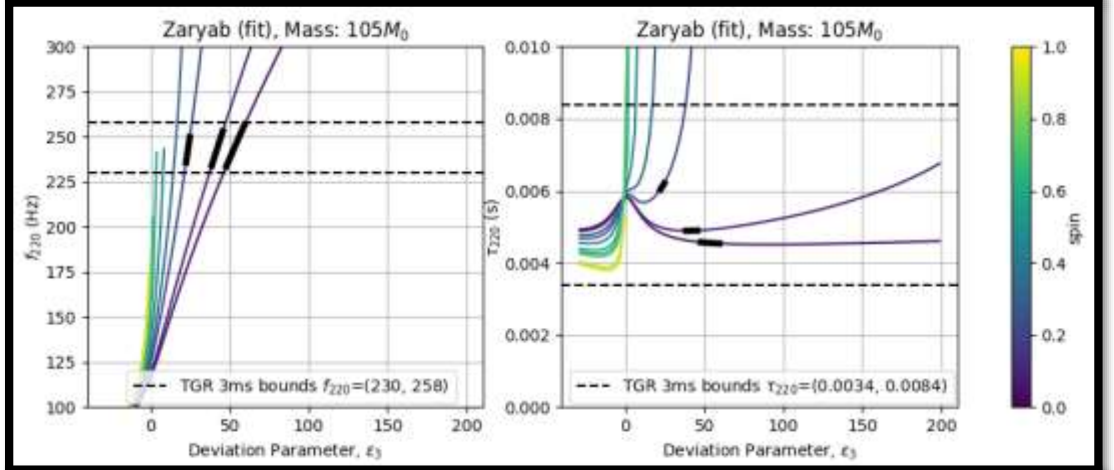

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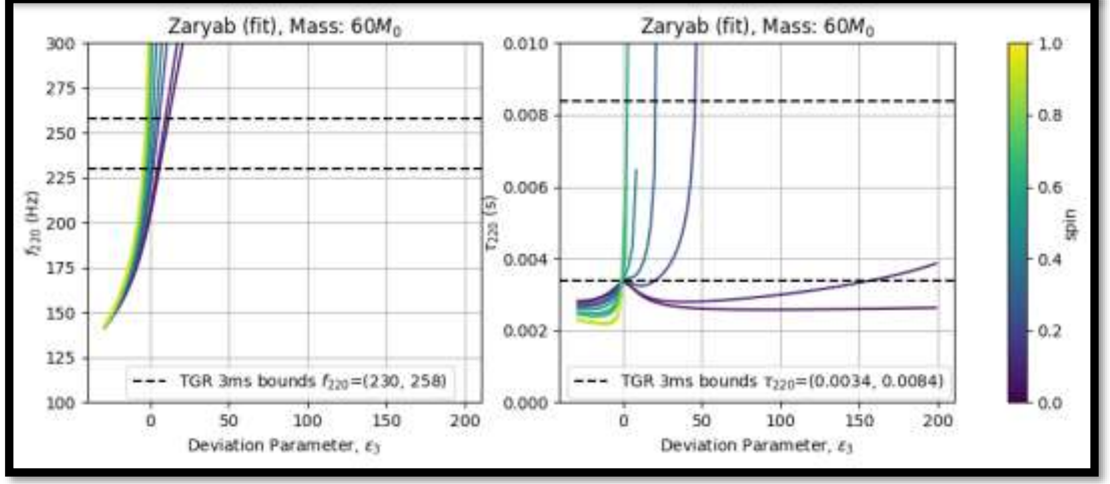

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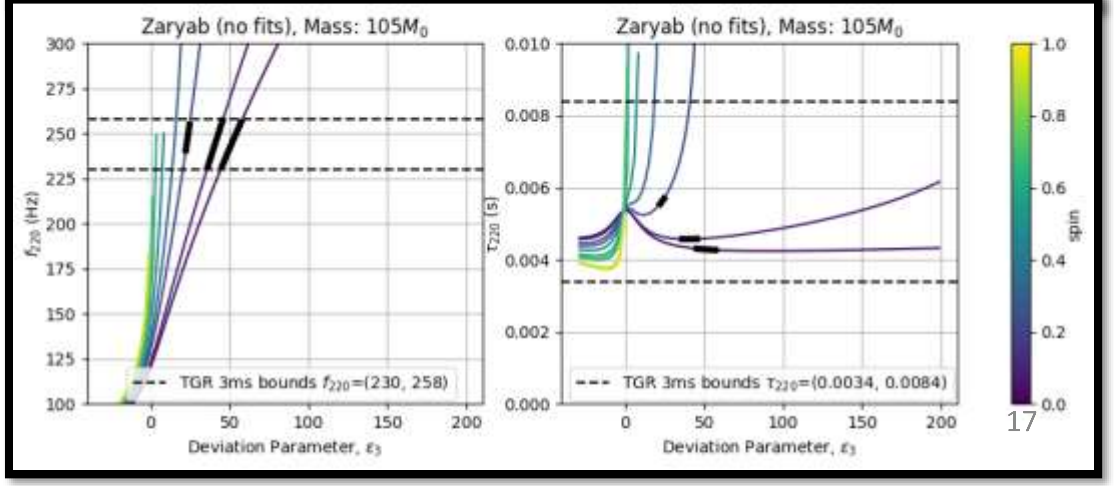
Deviation Parameter, ϵ_3
spin



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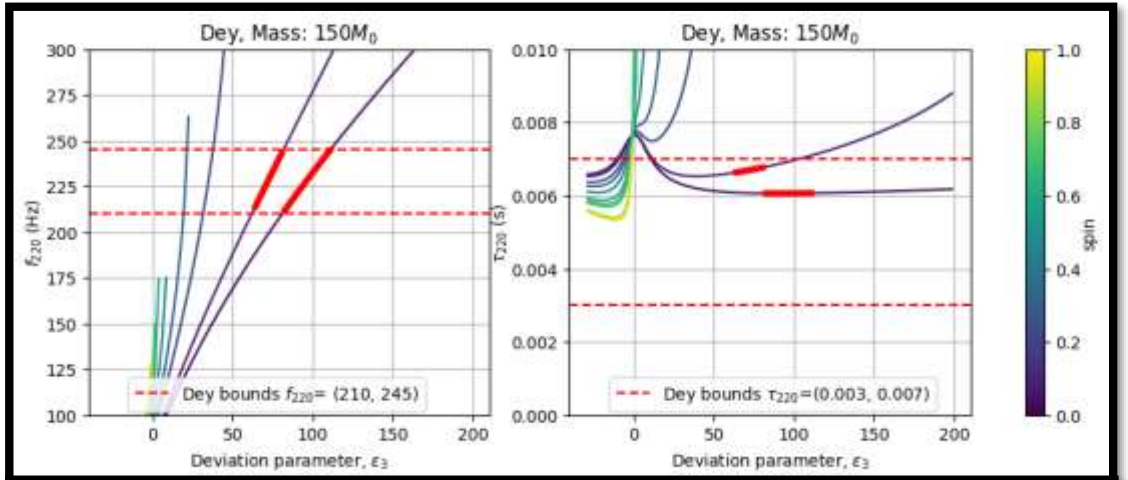
Dey, Mass: 150M_0
Dey, Mass: 150M_0
f_{220} (Hz)
τ_{220} (s)
Deviation parameter, ϵ_3
Dey bounds f_{220}= (210, 245)
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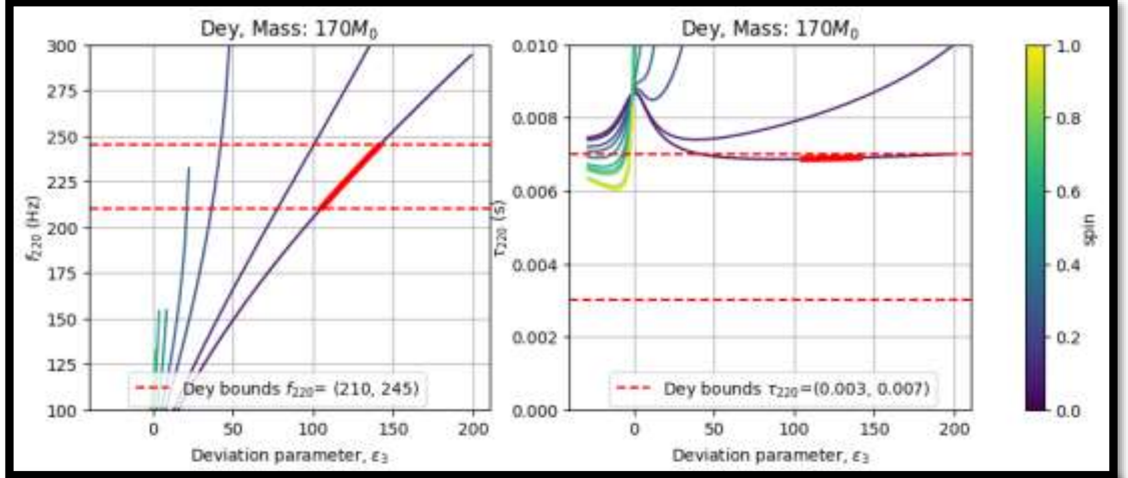
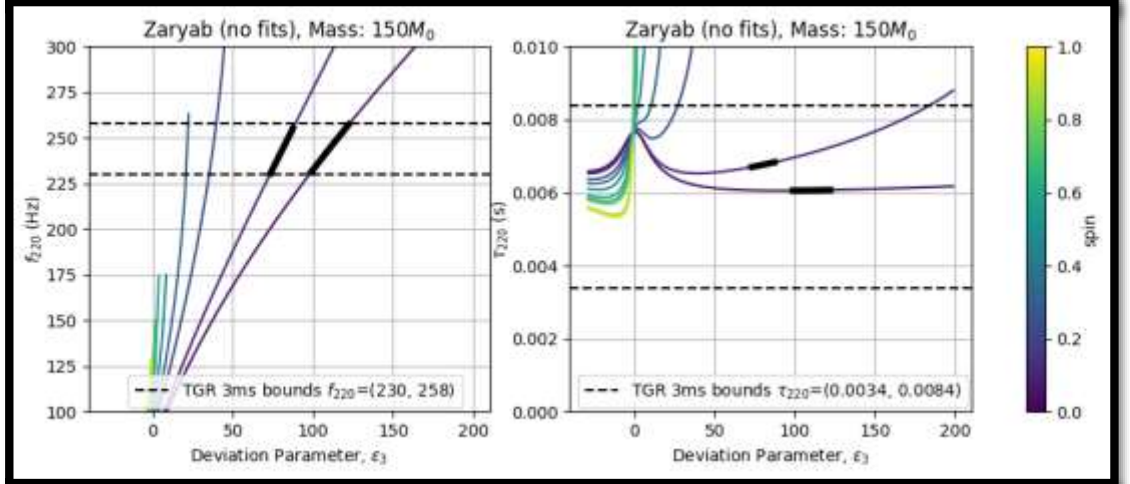
spin


Dey, Mass: 170M_0
Dey, Mass: 170M_0
f_{220} (Hz)
τ_{220} (s)
Deviation parameter, ϵ_3
Dey bounds f_{220}= (210, 245)
Dey bounds τ_{220}=(0.003, 0.007)
spin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Zaryab (no fits), Mass: 150M_0
f_{220} (Hz)
τ_{220} (s)
Deviation Parameter, ϵ_3
TGR 3ms bounds f_{220}=(230, 258)
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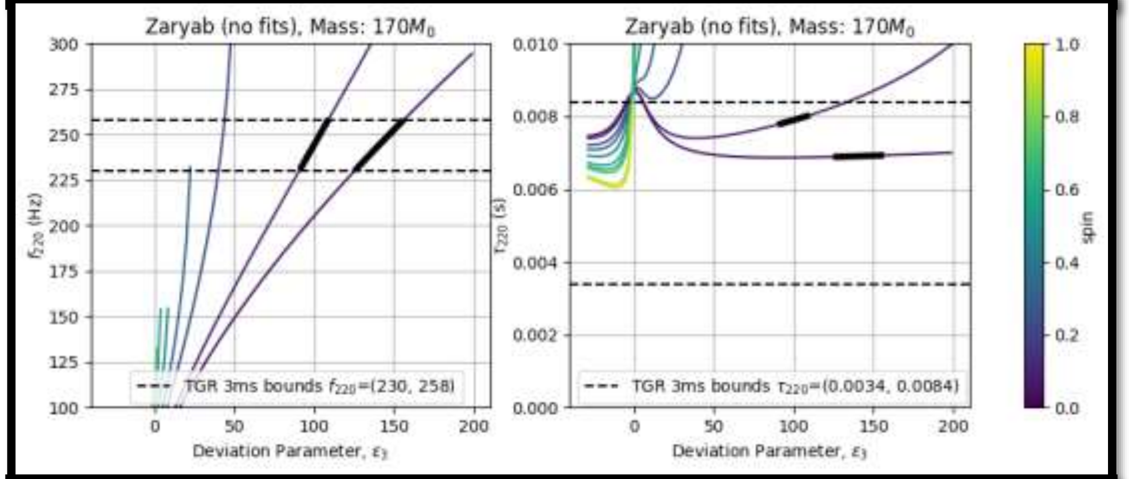
spin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Zaryab (no fits), Mass: 170M_0
f_{220} (Hz)
τ_{220} (s)
Deviation Parameter, ϵ_3
TGR 3ms bounds f_{220}=(230, 258)
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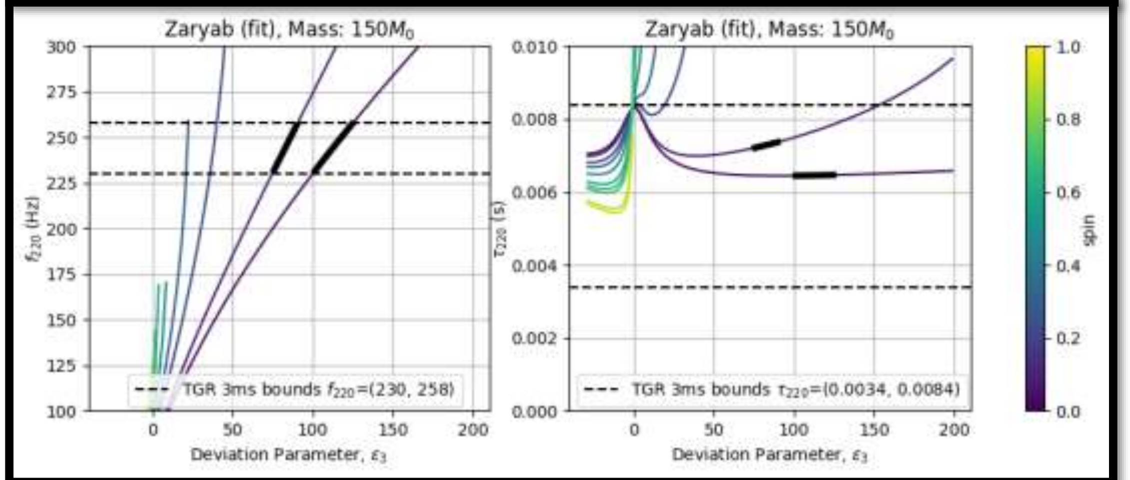
spin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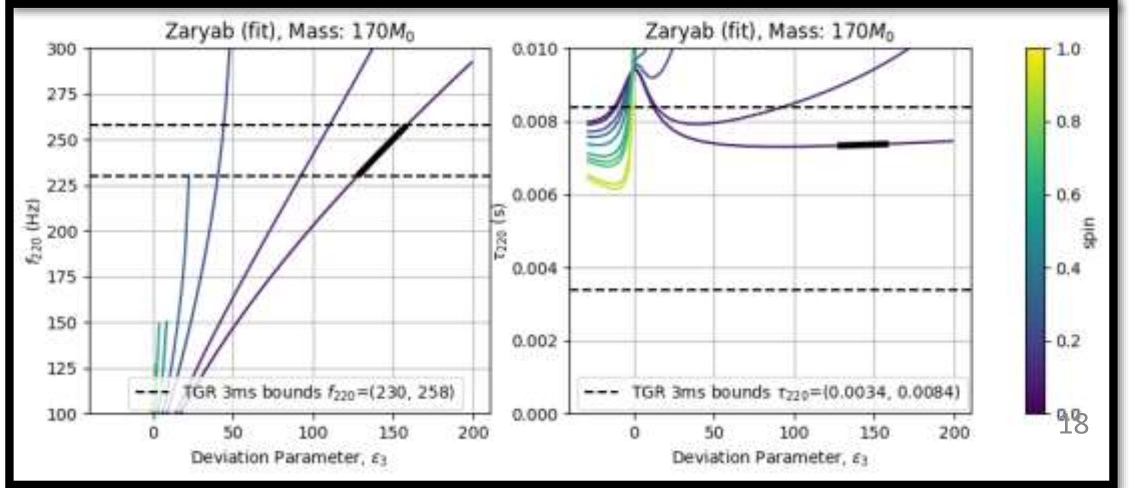
Zaryab (fit), Mass: 170M_0
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τ_{220} (s)
Deviation Parameter, ϵ_3
TGR 3ms bounds f_{220}=(230, 258)
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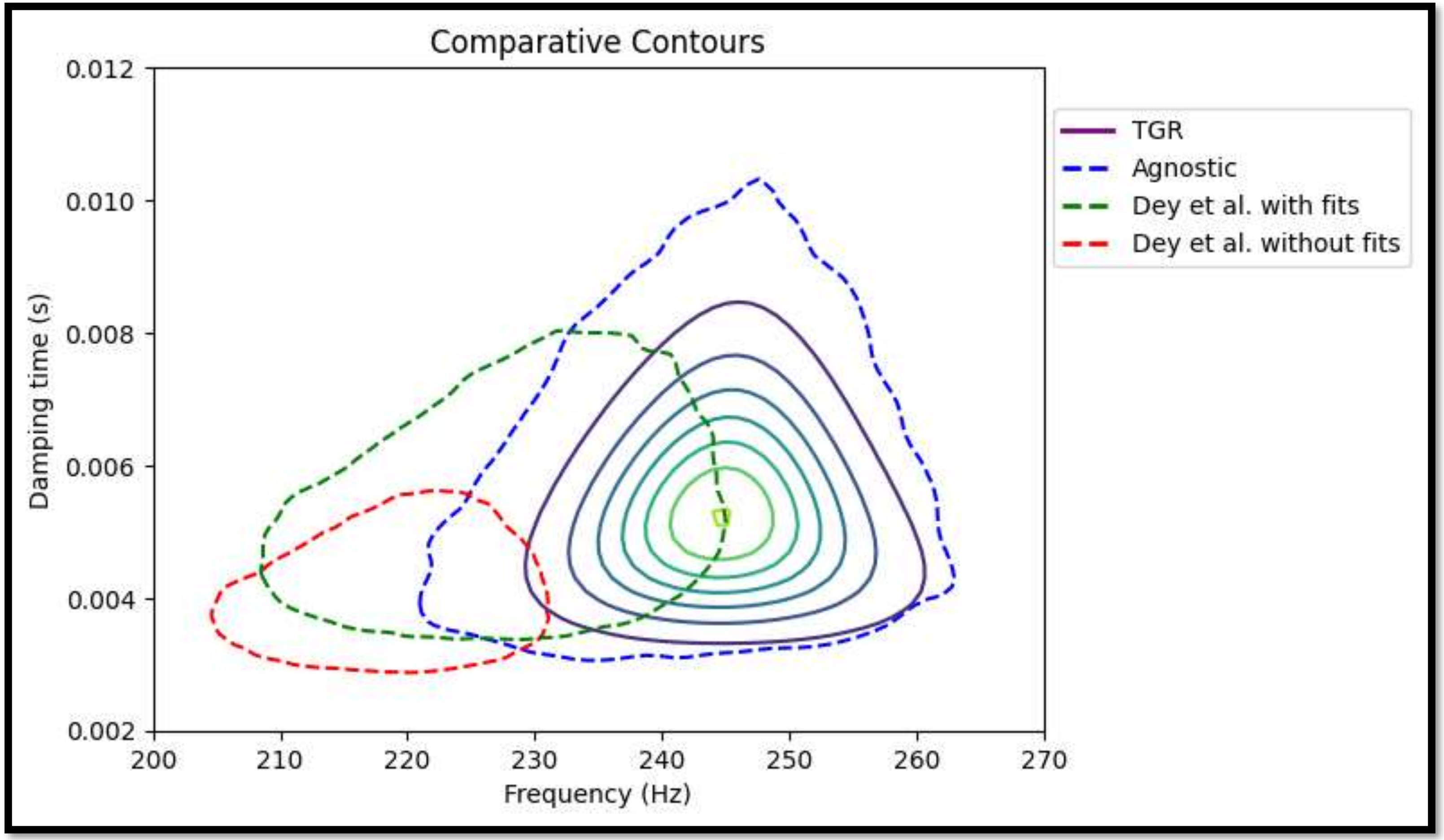
spin

Extras


Comparative Contours
0.012
0.010
0.008
0.006
0.004
0.002
Damping time (s)
200
210
220
230
240
250
260
270
Frequency (Hz)
TGR
Agnostic
Dey et al. with fits
Dey et al. without fits