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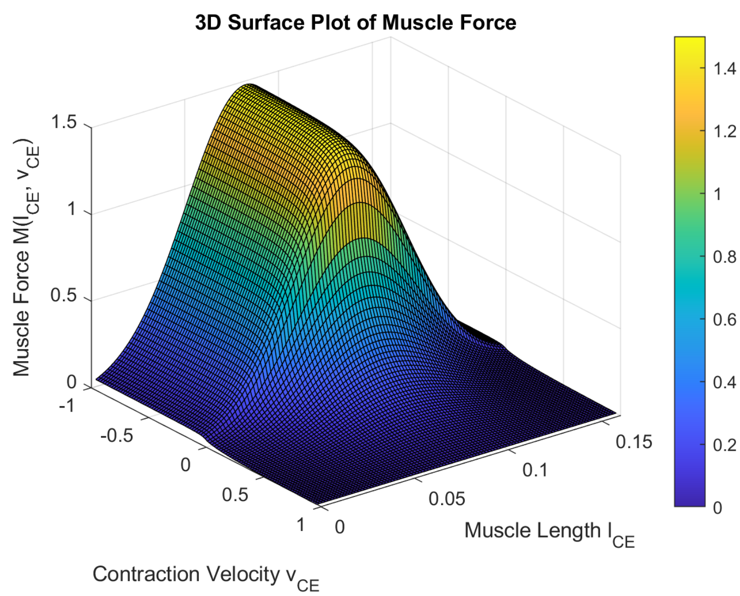
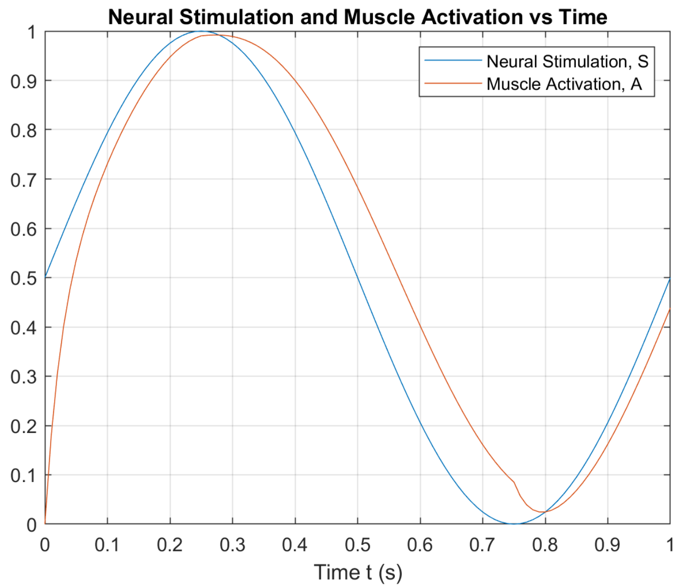
R16-868 F2024
Problem Set 3

1. (a) Start with a contractile element CE. Use Matlab to implement the active force-length relationship $f_l(l_{CE})$ (no passive elasticity) and the force-velocity relationship $f_V(v_{CE})$ of a Hill-type CE. Plot $M(l_{CE}, v_{CE}) = f_l f_v$ for $l_{CE} \epsilon [0, 2l_{opt}]$ and $v_{CE} \epsilon [-v_{max}, +v_{max}]$.
Use this plot to discuss why we feel warm hiking up a hill but cool off on the way down. **Hint:** In your discussion, focus on the large antigravity muscles of our legs.



The graph shows the muscle force-length relationship. This explains why we feel warm hiking up a hill but cool off on the way down. When hiking up a hill, our muscles need to generate more force, which occurs when the muscle length, l_{CE}, increases and causes our muscles to feel warm. We cool off on the way down a hill because our muscles are no longer working against gravity as much and l_{CE} decreases to produce a lower muscle force.

(b) In the second step, consider the muscle activation A. Assume the muscle receives a neural stimulation $S = 0.5sin(2\pi t) + 0.5$. Model the excitation-contraction coupling dynamics $A = f(S)$ as a first order, low-pass filter system with characteristic time $\tau_R = 30ms$ when S is rising and $\tau_F = 80ms$ when S is falling. Simulate 1 second, plot the time traces of S and A, and discuss the behavior of A.



There is a positive correlation between the neural stimulation, S, and muscle activation, A. After time = 0 s, reaches a maximum value of 0.992 at 0.27 seconds and a minimum value of 0.0246 at 0.79 seconds. The values for muscle activation are similar to that of neural stimulation, with a small time delay. For example, the neural stimulation reaches a maximum value of 1 at 0.25 seconds, just before muscle activation reaches its maximum value.

(c) Implement an activation-dependent contractile element in Simulink whose force output is $F_{CE} = F_{max}^{iso} f_l(l_{CE}) f_v(v_{CE}) A$. Simulate the muscle behavior for the following experiment: At $t = 0$, the muscle stimulation S is set to its maximum while the muscle is locked at the length $l_m = l_{opt}$. At time $t_q = 1$s, the lock is released instantaneously and the muscle pulls against the mass $m = 400$kg. Determine the slope of the muscle length change right after t = t_q (use a v_{CE} plot as guide).

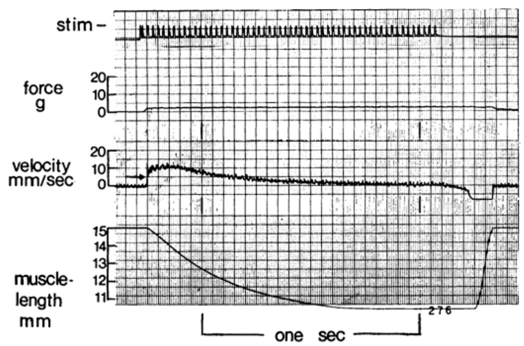
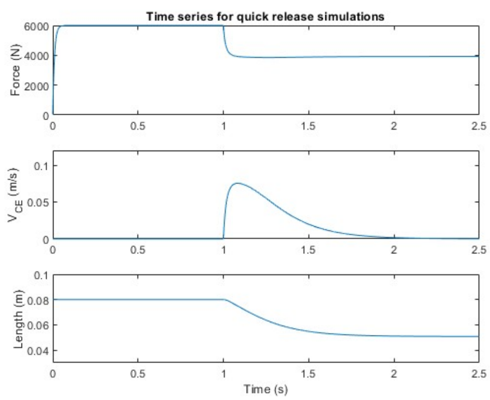


FIG. 4. Representative afterloaded isotonic contraction of a rat soleus muscle. From above downward are tetanic stimulus artifact, force, velocity of shortening, and muscle length.



Comparing our simulation at 400kg to experimental data from,[1] We see some of the same shapes created, in particular the velocity and length graphs produce similar outputs. Because the data is from rat muscles, the magnitudes are extremely different.

[1] W W Parmley, L A Yeatman, and E H Sonnenblick. "Differences between isotonic and isometric force-velocity relations in cardiac and skeletal muscle". en. In: *Am. J. Physiol.* 219.2 (Aug. 1970), pp. 546–550.

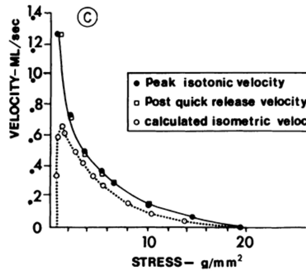
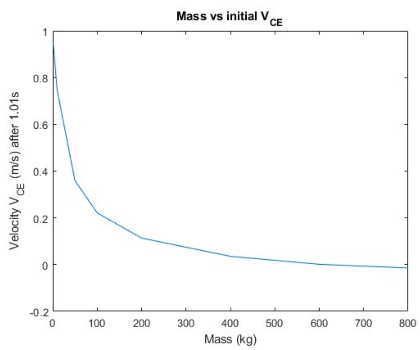


FIG. 5 *A:* Series elastic extension curve of a representative sonles muscle obtained by quick release methods. *B:* Inverse slope (dp/dl) of curve in *panel A* is plotted as a function of load (P) to produce linear relation: dp/dl = *K*P + *C*. *C:* Comparative force-velocity relations obtained from isotonic (●), quick release (□), and isometric (○) contractions.



Again comparing our simulation to the real data in[2] we see a very similar curve produced

Alcazar et al.[3] describes the force-velocity (F-V) relationship in skeletal muscle, which shows that a muscle generates less force as it shortens faster. This relationship, discovered by Hill in 1938, is often shown in quick-release experiments where muscles are allowed to shorten quickly under a load. The resulting F-V graph usually has a downward curve, indicating high force at low velocities and decreasing force as velocity increases. While traditionally modeled as a simple hyperbolic curve, newer studies suggest a more complex double-hyperbolic shape, with force leveling off at both high and low extremes due to muscle structure and control mechanisms.

For applications in robotics and prosthetics, this relationship is crucial to mimic natural muscle performance. Comparing simulated time

[2]Parmley, Yeatman, and Sonnenblick, "Differences between isotonic and isometric force-velocity relations in cardiac and skeletal muscle".

[3]Julian Alcazar et al. "On the shape of the force-velocity relationship in skeletal muscles: The linear, the hyperbolic, and the double-hyperbolic". en. In: *Front. Physiol.* 10 (June 2019), p. 769.

histories of force and velocity to this F-V curve can help validate the model's accuracy. In practical terms, if the simulation captures this curve shape—strong force at low speeds and a significant drop at higher speeds—it suggests the model behaves similarly to real muscle. This type of graph is important in designing exoskeletons and actuators because it guides how much force and control are needed for natural and efficient movement.